A class of asymptotic solution for the time delay wind field model of an ocean*

Zhou Xian-Chun(周先春)[a)b)c)†], Shi Lan-Fang(石兰芳)[d)], and Mo Jia-Qi(莫嘉琪)[e)]

[a)] College of Electronic and Information Engineering, Nanjing University of Information Science and Technology (NUIST), Nanjing 210044, China

[b)] Jiangsu Technology and Engineering Center for Meteorological Sensor Network, Nanjing University of Information Science and Technology (NUIST), Nanjing 210044, China

[c)] Jiangsu Key Laboratory of Meteorological Observation and Information Processing, Nanjing University of Information Science and Technology (NUIST), Nanjing 210044, China

[d)] College of Mathematics and Statistics, Nanjing University of Information Science and Technology (NUIST), Nanjing 210044, China

[e)] Department of Mathematics, Anhui Normal University, Wuhu 241003, China

(Received 21 June 2013; revised manuscript received 20 September 2013; published online 20 February 2014)

A time delay model of a two-layer barotropic ocean with Rayleigh dissipation is built. Using the improved perturbation method, an analytic asymptotic solution of a better approximate degree is obtained in the mid-latitude wind field, and the physical meaning of the corresponding solution is also discussed.

Keywords: two-layers barotropic, time delay model, asymptotic method

PACS: 02.30.Mv **DOI:** 10.1088/1674-1056/23/4/040202

1. Introduction

Variations of coupled air–sea systems have been widely investigated.[1–3] An atmospheric wind anomaly will cause an ocean flow anomaly. Many scholars have discussed this phenomenon. Cabanes *et al.*[4] considered wind stress driven sine Sverdrup balance and first order baroclinic Rossby waves. Qiu and Chen[5] used a wind stress driven reduced-gravity model to discuss the dynamic mechanism of temporal and spatial variations of sea surface height in the south Pacific. Considering the effect of the mid-latitude surface west wind stress anomaly on sea flow, the anomaly is that mid-latitude has a maximal wind stress which decays gradually toward the north and south. So wind stress in the meridional zone shows no uniform distribution. In order to overcome the limit of a horizontal one-dimensional model some scholars[6–13] built a two-dimensional barotropic quasi-equilibrium ocean linear model under wind forcing, considering Rayleigh friction. Recently, many scholars[14–17] have studied nonlinear problems and built many approximate methods. Using the perturbation method and other methods, the authors *et al.*[18–23] also considered a class of nonlinear problem. In this paper, we consider a time delay weakly nonlinear model in the absence of a base flow and mass change between two levels by vertical stationary atmospheric wind stress and seek its analytic asymptotic solution.

2. Wind field of the two-layer barotropic ocean model

Considering a time delay weakly nonlinear model in the absence of a base flow and mass change between two levels, as well as Rayleigh friction induced by vertical stationary atmospheric wind stress:[6–9]

$$\begin{aligned}&(f_0+\beta y)v_k-\frac{\partial(a_k\Phi_1+\Phi_2)}{\partial x}-\gamma\delta_k(u_1-u_2)\\&=\varepsilon F_k(u_k(x,y-\mu),v_k(x,y-\mu)),\end{aligned}\tag{1}$$

$$\begin{aligned}&(f_0+\beta y)u_k+\frac{\partial(a_k\Phi_1+\Phi_2)}{\partial y}+\gamma\delta_k(v_1-v_2))\\&=\varepsilon G_k(u_k(x,y-\mu),v_k(x,y-\mu)),\end{aligned}\tag{2}$$

$$\frac{\partial u_k}{\partial x}+\frac{\partial v_k}{\partial y}=0,\tag{3}$$

where t is the time variable, x is the zonal variable, and y is the meridional variable; $k=1,2$ indicate up and down levels respectively; u_k is the zonal wind speed and v_k is the meridional wind speed; γ is the Rayleigh friction factor, which is a constant value; f_0 is the Coriolis parameter at y_0 where $\beta=(\partial f/\partial y)_{y=y_0}$, y_0 is the center position of β channel; Φ is the geopotential height, where $a_1=1$, $0<a_2=\rho_1/\rho_2<1$; ρ_1 and ρ_2 are up and down level densities respectively. Tracer coefficients δ_k are defined as $\delta_1=1$ and $\delta_2=-1$; εF_k and εG_k are nonlinear force perturbed term stresses and let each of them be a sufficiently smooth bounded function, ε and μ are

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11202106 and 61302188), the Specialized Research Fund for the Doctoral Program of Higher Education, China (Grant No. 20123228120005), the Fund from the Jiangsu Sensor Network and Modern Meteorological Equipment Preponderant Discipline Platform, China, the Natural Science Fundation from the Universities of Jiangsu Province, China (Grant No. 13KJB170016), and the Advance Research Foundation in NUIST of China (Grant Nos. 20110371 and 20110385).

†Corresponding author. E-mail: zhouxc2008@163.com

© 2014 Chinese Physical Society and IOP Publishing Ltd

both small positive parameters.

In this article, we consider an ideal ocean climate in a middle-high latitude under stationary wind forcing and assume that the β channel in the x direction is a semi-infinite space. Let the boundary conditions of this model of west coast ($x=0$) and $y=y_0\pm M$ for β channel be

$$u_k|_{x=0}=(-1)^k a\cos\frac{\pi}{2M}(y-y_0),\quad v_k|_{x=0}=0, \tag{4}$$

$$\left.\frac{\partial u_k}{\partial y}\right|_{y=y_0\pm M}=\left.\frac{\partial v_k}{\partial y}\right|_{y=y_0\pm M}=0, \tag{5}$$

where a and M are both constants.

3. Improving perturbed solving method for sea–air time delay model

In order to obtain approximate solutions for the sea–air time delay model, we need to improve the perturbation method. That is, we need to deal with the delay function, which contains small parameter μ. Therefore, we first develop the time delay functions of μ as follows:

$$\begin{aligned}u_k(x,y-\mu)&=u_k(x,y)-\mu\frac{\partial u_k(x,y)}{\partial y}+\frac{\mu^2}{2!}\frac{\partial^2 u_k(x,y)}{\partial y^2}\\&\quad-\frac{\mu^3}{3!}\frac{\partial^3 u_k(x,y)}{\partial y^3}+\cdots,\end{aligned} \tag{6}$$

$$\begin{aligned}v_k(x,y-\mu)&=v_k(x,y)-\mu\frac{\partial v_k(x,y)}{\partial y}+\frac{\mu^2}{2!}\frac{\partial^2 v_k(x,y)}{\partial y^2}\\&\quad-\frac{\mu^3}{3!}\frac{\partial^3 v_k(x,y)}{\partial y^3}+\cdots.\end{aligned} \tag{7}$$

And let

$$u_k=\sum_{i,j=0}^{\infty}u_{kij}(x,y)\varepsilon^i\mu^j,\quad k=1,2, \tag{8}$$

$$v_k=\sum_{i,j=0}^{\infty}v_{kij}(x,y)\varepsilon^i\mu^j,\quad k=1,2. \tag{9}$$

Substitute Eqs. (6)–(9) into Eqs. (1)–(5), expand the nonlinear terms in ε and μ, equate the coefficients of the same powers of ε and μ, and let the coefficient of $\varepsilon^0\mu^0$ equal zero, then we will obtain

$$(f_0+\beta y)v_{k00}-\frac{\partial(a_k\Phi_1+\Phi_2)}{\partial x}-\gamma\delta_k(u_{100}-u_{200})=0, \tag{10}$$

$$(f_0+\beta y)u_{k00}+\frac{\partial(a_k\Phi_1+\Phi_2)}{\partial y}+\gamma\delta_k(v_{100}-v_{200})=0, \tag{11}$$

$$\frac{\partial u_{k00}}{\partial x}+\frac{\partial v_{k00}}{\partial y}=0, \tag{12}$$

$$u_{k00}|_{x=0}=(-1)^k a\cos\frac{\pi}{2M}(y-y_0),\quad v_{k00}|_{x=0}=0, \tag{13}$$

$$\begin{aligned}u_{k00}&=(-1)^k a\exp\left[-\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right)x\right]\\&\quad\times\cos\frac{\pi}{2M}(y-y_0),\quad k=1,2.\end{aligned} \tag{14}$$

Let

$$u_{k00}=-\frac{\partial W_{k00}}{\partial y},\quad v_{k00}=\frac{\partial W_{k00}}{\partial x},\quad k=1,2. \tag{15}$$

From Eqs. (12) and (15), we can obtain[8,9]

$$\frac{\partial(W_{100}+W_{200})}{\partial x}=0, \tag{16}$$

$$\begin{aligned}&W_{100}-W_{200}\\&=-\frac{4aM}{\pi}\exp\left[-\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right)x\right]\\&\quad\times\sin\frac{\pi}{2M}(y-y_0).\end{aligned} \tag{17}$$

From Eqs. (10)–(17), it is easy to see that

$$\begin{aligned}v_{k00}&=(-1)^{k+1}\frac{2aM}{\pi}\left[\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right]\\&\quad\times\exp\left[-\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}x\right)\right]\\&\quad\times\sin\frac{\pi}{2M}(y-y_0).\quad k=1,2,\end{aligned} \tag{18}$$

$$\begin{aligned}u_{k00}&=(-1)^k a\exp\left[-\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right)x\right]\\&\quad\times\cos\frac{\pi}{2M}(y-y_0),\quad k=1,2.\end{aligned} \tag{19}$$

Note Substituting Eqs. (18) and (19) into Eqs. (10)–(14), we can also obtain the geopotential heights Φ_1 and Φ_2. But their constitutions are omitted.

Substituting Eqs. (6)–(9) into Eqs. (1)–(5), expanding the nonlinear terms in ε and μ, equating the coefficients of the same powers of ε, μ, letting the coefficient of $\varepsilon^i\mu^j$ $(i,j=0,1,2,\ldots,i\neq 0)$ equal zero, then we have

$$(f_0+\beta y)v_{kij}-\frac{\partial(a_k\Phi_1+\Phi_2)}{\partial x}-\gamma\delta_k(u_{1ij}-u_{2ij})=\bar{F}_{kij}, \tag{20}$$

$$(f_0+\beta y)u_{kij}-\frac{\partial(a_k\Phi_1+\Phi_2)}{\partial y}+\gamma\delta_k(v_{1ij}-v_{2ij})=\bar{G}_{kij}, \tag{21}$$

$$\frac{\partial u_{kij}}{\partial x}+\frac{\partial v_{kij}}{\partial y}=0, \tag{22}$$

$$\begin{aligned}u_{kij}|_{x=0}&=v_{kij}|_{x=0}=\left.\frac{\partial u_{kij}}{\partial y}\right|_{y=y_0\pm M}\\&=\left.\frac{\partial v_{kij}}{\partial y}\right|_{y=y_0\pm M}=0,\end{aligned} \tag{23}$$

where

$$\begin{aligned}\bar{F}_{kij}&=\frac{(-1)^j}{i!\,j!}\left[\frac{\partial^i}{\partial\varepsilon^i}\frac{\partial^j}{\partial\mu^j}F_k\left(\sum_{j=0}^{\infty}u_k\varepsilon^i\mu^j,\sum_{j=0}^{\infty}v_k\varepsilon^i\mu^j\right)\right]_{\varepsilon=\mu=0},\\&\quad k=1,2,\quad i,j=0,1,2,\cdots,i\neq 0,\\\bar{G}_{kij}&=\frac{(-1)^j}{i!\,j!}\left[\frac{\partial^i}{\partial\varepsilon^i}\frac{\partial^j}{\partial\mu^j}G_k\left(\sum_{j=0}^{\infty}u_k\varepsilon^i\mu^j,\sum_{j=0}^{\infty}v_k\varepsilon^i\mu^j\right)\right]_{\varepsilon=\mu=0},\\&\quad k=1,2,\quad i,j=0,1,2,\ldots,i\neq 0,\end{aligned}$$

which are known functions successively. Let

$$u_{kij}=-\frac{\partial W_{kij}}{\partial y},\quad v_{kij}=\frac{\partial W_{kij}}{\partial x},\quad k=1,2, \tag{24}$$

from Eqs. (20)–(24), we can have

$$W_{1ij}-W_{2ij} = -\left[C_{ij}\exp\left[-\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right)x\right] + H_{ij}(x,y)\right]\sin\frac{\pi}{2M}(y-y_0). \tag{25}$$

where C_{ij} are arbitrary constants, and

$$\begin{aligned} &H_{ij}(x,y)\\ &= \frac{1}{2\gamma}\int_0^x\left[\frac{\partial}{\partial y}(\bar{F}_{1ij}(\xi,y)-\bar{F}_{2ij}(\xi,y)) + \frac{\partial}{\partial\xi}(\bar{G}_{1ij}(\xi,y)-\bar{G}_{2ij}(\xi,y))\right]\\ &\times\exp\left[-\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right)(x-\xi)\right]\mathrm{d}\xi. \end{aligned}$$

From Eqs. (24) and (25), we obtain

$$\begin{aligned} &-u_{1ij}+u_{2ij}\\ &= \frac{\pi}{2M}\left[C_{ij}\exp\left[-\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right)x\right] + H_{ij}(x,y)\right]\cos\frac{\pi}{2M}(y-y_0)\\ &+\frac{\partial H_{ij}}{\partial y}\sin\frac{\pi}{2M}(y-y_0). \end{aligned}$$

Substituting $x=0$, and considering condition (23), we have $C_{ij}=0$. Thus

$$-u_{1ij}+u_{2ij} = \frac{\pi}{2M}[H_{ij}(x,y)]\cos\frac{\pi}{2M}(y-y_0) + \frac{\partial H_{ij}}{\partial y}\sin\frac{\pi}{2M}(y-y_0), \tag{26}$$

and

$$\begin{aligned} &\beta\left(\frac{\partial u_{1ij}}{\partial x}+\frac{\partial u_{2ij}}{\partial x}\right)\\ &= -\frac{\partial}{\partial y}\left[\frac{\partial}{\partial y}(\bar{F}_{1ij}+\bar{F}_{2ij})+\frac{\partial}{\partial x}(\bar{G}_{1ij}+\bar{G}_{2ij})\right],\\ &u_{1ij}+u_{2ij}\\ &= -\frac{1}{\beta}\frac{\partial}{\partial y}\int_0^x\left[\frac{\partial}{\partial y}(\bar{F}_{1ij}(\xi,y)+\bar{F}_{2ij}(\xi,y)) + \frac{\partial}{\partial x}(\bar{G}_{1ij}(\xi,y)+\bar{G}_{2ij}(\xi,y))\right]\mathrm{d}\xi. \end{aligned} \tag{27}$$

Then from Eqs. (26) and (27), we have

$$\begin{aligned} u_{kij} = &-\frac{1}{2\beta}\frac{\partial}{\partial y}\int_0^x\left[\frac{\partial}{\partial y}(\bar{F}_{1ij}(\xi,y)+\bar{F}_{2ij}(\xi,y)) + \frac{\partial}{\partial x}(\bar{G}_{1ij}(\xi,y)+\bar{G}_{2ij}(\xi,y))\right]\mathrm{d}\xi\\ &+(-1)^k\frac{\pi}{4M}[H_{ij}(x,y)]\cos\frac{\pi}{2M}(y-y_0)\\ &+\frac{(-1)^k}{2}\frac{\partial H_{ij}}{\partial y}\sin\frac{\pi}{2M}(y-y_0). \end{aligned}$$

$$k=1,2,\ \ i,j=0.1,2,\ldots,\ i\neq 0. \tag{28}$$

Considering conditions (22) and (23) and $C_{ij}=0$, we have

$$\begin{aligned} v_{kij} = &\frac{1}{2\beta}\left[\frac{\partial}{\partial y}(\bar{F}_{1ij}+\bar{F}_{2ij})+\frac{\partial}{\partial x}(\bar{G}_{1ij}+\bar{G}_{2ij})\right]\\ &-\frac{(-1)^k}{2}\frac{\partial H_{ij}}{\partial x}\sin\frac{\pi}{2M}(y-y_0),\\ &k=1,2,\ \ i,j=0.1,2,\ldots,\ i\neq 0. \end{aligned} \tag{29}$$

From Eqs. (18), (19), (28), and (29), substituting u_{kij}, v_{kij}, $k=1,2$ into Eq. (6), we then obtain perturbation solutions for the original time delay problem (Eqs. (1)–(5)) u_{kmper} and v_{kmper} $(k=1,2)$:

$$\begin{aligned} u_{kmper} = &(-1)^k a\exp\left[-\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right)x\right]\\ &\times\cos\frac{\pi}{2M}(y-y_0)\\ &+\sum_{i=1,j=0}^{m}\left[-\frac{1}{2\beta}\frac{\partial}{\partial y}\int_0^x\left[\frac{\partial}{\partial y}(\bar{F}_{1ij}(\xi,y)+\bar{F}_{2ij}(\xi,y))\right.\right.\\ &\left.+\frac{\partial}{\partial x}(\bar{G}_{1ij}(\xi,y)+\bar{G}_{2ij}(\xi,y))\right]\mathrm{d}\xi\\ &+(-1)^k\frac{\pi}{4M}[H_{ij}(x,y)]\cos\frac{\pi}{2M}(y-y_0)\\ &\left.+\frac{(-1)^k}{2}\frac{\partial H_{ij}}{\partial y}\sin\frac{\pi}{2M}(y-y_0)\right]\varepsilon^i\mu^j+O(\lambda),\\ &k=1,2,\ \ \lambda=\min(\varepsilon^{m+1}\mu^m,\varepsilon^m\mu^{m+1}),\\ &0<\varepsilon,\ \ \mu\ll 1, \end{aligned} \tag{30}$$

$$\begin{aligned} v_{kmper} = &(-1)^k\frac{2aM}{\pi}\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right)\\ &\times\exp\left[-\left(\frac{\beta}{4\gamma}+\frac{1}{2}\sqrt{\left(\frac{\beta}{2\gamma}\right)^2+\left(\frac{\pi}{M}\right)^2}\right)x\right]\\ &\times\sin\frac{\pi}{2M}(y-y_0)\\ &+\sum_{i=0,j=0}^{m}\left[\frac{1}{2\beta}\left[\frac{\partial}{\partial y}(\bar{F}_{1ij}+\bar{F}_{2ij})+\frac{\partial}{\partial x}(\bar{G}_{1ij}+\bar{G}_{2ij})\right]\right.\\ &\left.-\frac{(-1)^k}{2}\frac{\partial H_{ij}}{\partial x}\sin\frac{\pi}{2M}(y-y_0)\right]\varepsilon^i\mu^j+O(\lambda)\\ &k=1,2,\ \ \lambda=\max(\varepsilon^{m+1}\mu^m,\varepsilon^m\mu^{m+1}),\\ &0<\varepsilon,\ \mu\ll 1. \end{aligned} \tag{31}$$

4. Example

Observations show that there exists a maximal surface westerly intensity near 40° N over the west Pacific Ocean. Thus, let the center of β channel be at 40° N, and half width $M=10^3$ km, $\mu=3\times10^{-7}$ s^{-1}, $\beta=1.7536\times10^{-8}$ km$^{-1}\cdot$s^{-1},[8,9] for convenience, then we set nonlinear perturbation wind stress $F_k=\varepsilon u_k(x,y-\mu)$, $G_k=0$, parameter $a=1$. So the time delay weak nonlinear maritime climate

model under atmospheric wind stress forcing Eqs. (1)–(5) can be transformed into

$$\begin{aligned}&(f_0+1.7536\times10^{-11}y)v_k-\frac{\partial(a_k\Phi_1+\Phi_2)}{\partial x}\\&-3\times10^{-7}\delta_k(u_1-u_2)\\=\;&\varepsilon u_k(x,y-\mu),\end{aligned}\tag{32}$$

$$\begin{aligned}&(f_0+1.7536\times10^{-11}y)u_k+\frac{\partial(a_k\Phi_1+\Phi_2)}{\partial x}\\&+3\times10^{-7}\delta_k(v_1-v_2)=0,\end{aligned}\tag{33}$$

$$\frac{\partial u_k}{\partial x}+\frac{\partial v_k}{\partial y}=0,\tag{34}$$

$$u_k|_{x=0}=(-1)^k\cos(y-y_0),\quad v_k|_{x=0}=0,\tag{35}$$

$$\frac{\partial u_k}{\partial y}\Big|_{y=y_0\pm10^6}==\frac{\partial v_k}{\partial y}\Big|_{y=y_0\pm10^6}=0.\tag{36}$$

Now, we come to seek the approximate perturbation solutions for the model of Eqs. (32)–(36). Set

$$u_{k\mathrm{per}}=\sum_{i,j=0}^{\infty}u_{kij}\varepsilon^i\mu^j,\quad v_{k\mathrm{per}}=\sum_{i,j=0}^{\infty}v_{kij}\varepsilon^i\mu^j,\quad k=1,2.\tag{37}$$

Substituting Eq. (37) into Eqs. (32)–(36), expanding nonlinear terms in ε and μ, equating the coefficients of the same powers of ε, μ, letting the coefficient of $\varepsilon^0\mu^0$ equal zero, then we will have u_{k00} and v_{k00} as follows:

$$\begin{aligned}u_{k00}=\;&(-1)^k\exp\left[-\left(\frac{1.7536\times10^{-4}}{12}\right.\right.\\&\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]\\&\times\cos\frac{\pi}{2\times10^6}(y-y_0),\quad k=1,2,\end{aligned}\tag{38}$$

$$\begin{aligned}v_{k00}=\;&(-1)^k\frac{2\times10^6}{\pi}\left(\frac{1.7536\times10^{-4}}{12}\right.\\&\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)\\&\times\exp\left[-\left(\frac{1.7536\times10^{-4}}{12}\right.\right.\\&\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]\\&\times\sin\frac{\pi}{2\times10^6}(y-y_0),\quad k=1,2.\end{aligned}\tag{39}$$

From Eqs. (38) and (39), the simulation results of approximate solutions u_{k00} and v_{k00} $(k=1,2)$ for the weak nonlinear maritime climate model of Eqs. (32)–(36), which is under stationary atmospheric wind stress forcing, are obtained and shown in Figs. 1–4 ($\varepsilon=0$ and $\varepsilon=0.5$). From the figures, it follows that they accord with physical facts.

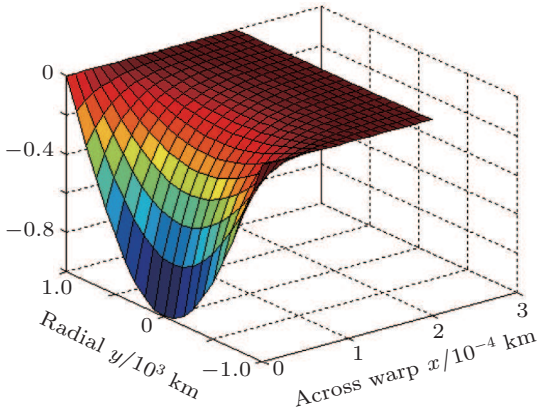


Fig. 1. (color online) Simulated contour u_{100}.

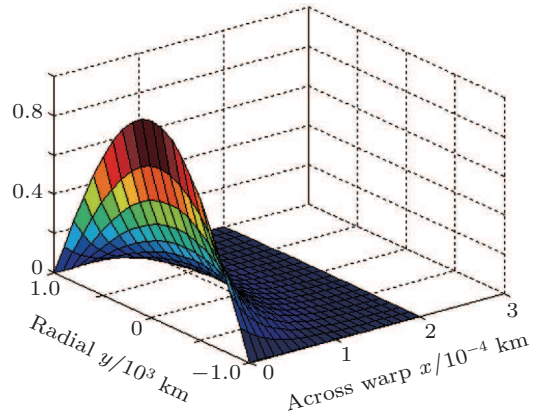


Fig. 2. (color online) Simulated contour u_{200}.

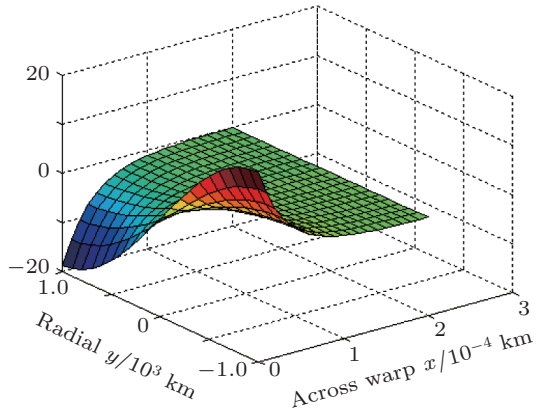


Fig. 3. (color online) Simulated contour v_{100}.

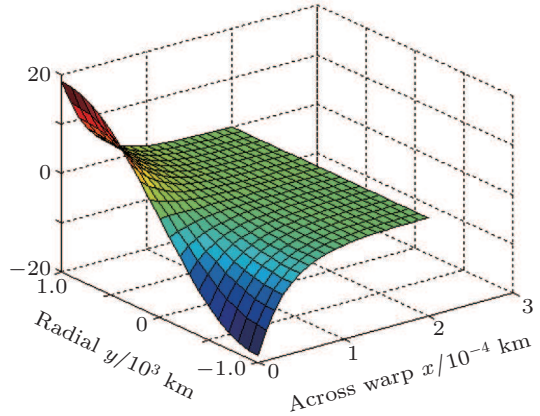


Fig. 4. (color online) Simulated contour v_{200}.

Note that from Eqs. (38) and (39), we can also obtain the geopotential heights Φ_i $(i=1,2)$. But their constitutions are omitted too.

Substituting Eq. (37) into Eqs. (32)–(36), expanding nonlinear terms in ε and μ, equating the coefficients of the same powers of ε, μ, and letting the coefficient of $\varepsilon^1\mu^0$, $\varepsilon^1\mu^1$ equal zero, we have u_{k10}, v_{k10} and u_{k11}, v_{k11}

$$\begin{aligned}
u_{k10} &= (-1)^k \frac{\pi}{24\times 10^5}\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\\
&\quad \left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]x\\
&\quad \times \sin\frac{\pi}{10^6}(y-y_0), \qquad (40)\\
v_{k10} &= -\frac{1}{1.7536\times 10^{-8}}\left[\frac{\pi}{2\times 10^6}\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\right.\\
&\quad \left.\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]\right.\\
&\quad \left.\times \sin\frac{\pi}{2\times 10^6}(y-y_0)\right]\\
&\quad -(-1)^k\frac{10\pi}{12}\left[1-x\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\\
&\quad \left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)\right]\\
&\quad \times\left[\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\right.\\
&\quad \left.\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]\right.\\
&\quad \left.\times \sin^2\frac{\pi}{2\times 10^6}(y-y_0)\right], \qquad (41)\\
u_{k11} &= -(-1)^k\left(\frac{1}{12\times 10^{-7}}\right)\left(\frac{\pi}{2\times 10^6}\right)^3\\
&\quad \times\left[\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\right.\\
&\quad \left.\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]x\right.\\
&\quad \left.\times\cos\frac{\pi}{10^6}(y-y_0)\right], \qquad (42)\\
v_{k11} &= (-1)^{k+1}\frac{1}{24\times 10^{-7}}\left(\frac{\pi}{2\times 10^6}\right)^2\\
&\quad \times(1+x)\left(\frac{1.7536\times 10^{-4}}{12}\right.\\
&\quad \left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)\\
&\quad \times\left[\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\right.\\
&\quad \left.\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]\right.\\
&\quad \left.\times\sin\frac{\pi}{10^6}(y-y_0)\right]. \qquad (43)
\end{aligned}$$

Thus, we obtain the asymptotic solutions $u_{k11\mathrm{per}}$ and $v_{k11\mathrm{per}}$ for the time delay weak nonlinear maritime climate model under atmospheric wind stress forcing Eqs. (32)–(36) as follows:

$$\begin{aligned}
u_{k11\mathrm{per}} &= (-1)^k\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\\
&\quad \left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]\\
&\quad \times\cos\frac{\pi}{2\times 10^6}(y-y_0),\\
&\quad +\varepsilon(-1)^k\frac{\pi}{24\times 10^5}\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\\
&\quad \left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]x\\
&\quad \times\sin\frac{\pi}{10^6}(y-y_0)\\
&\quad +\varepsilon\mu\,(-1)^k\frac{10^7}{3}\left(\frac{\pi}{2\times 10^6}\right)^3\\
&\quad \times\left[\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\right.\\
&\quad \left.\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]\right.\\
&\quad \left.\times\left[\frac{3}{2}\sin^2\frac{\pi}{2\times 10^6}(y-y_0)-1\right]x+O(\lambda)\right],\\
&\quad k=1,2,\ \ \lambda=\max(\varepsilon^2\mu,\ \varepsilon\mu^2),\\
&\quad 0<\varepsilon,\quad \mu\ll 1, \qquad (44)\\
v_{k11\mathrm{per}} &= (-1)^k\frac{2\times 10^6}{\pi}\left(\frac{1.7536\times 10^{-4}}{12}\right.\\
&\quad \left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)\\
&\quad \times\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\\
&\quad \left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]\\
&\quad \times\sin\frac{\pi}{2\times 10^6}(y-y_0)\\
&\quad -\varepsilon\left[\frac{1}{1.7536\times 10^{-8}}\left[\frac{\pi}{2\times 10^6}\right.\right.\\
&\quad \times\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\\
&\quad \left.\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]\right.
\end{aligned}$$

$$\times \sin\frac{\pi}{2\times 10^6}(y-y_0)\Bigg]\Bigg]$$
$$+(-1)^k\frac{10\pi}{12}\left[1-x\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.$$
$$\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)\Bigg]$$
$$\times\left[\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\right.$$
$$\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]$$
$$\times\sin^2\frac{\pi}{2\times 10^6}(y-y_0)\Bigg]$$
$$+\varepsilon\mu(-1)^{k+1}\frac{10^7}{12}\left(\frac{\pi}{2\times 10^6}\right)^2$$
$$\times\left[-1+\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.$$
$$\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\Bigg]$$
$$\times\left[\exp\left[-\left(\frac{1.7536\times 10^{-4}}{12}\right.\right.\right.$$
$$\left.\left.+\frac{1}{2}\sqrt{\left(\frac{1.7536\times 10^{-4}}{6}\right)^2+\left(\frac{\pi}{10^6}\right)^2}\right)x\right]$$
$$\times\sin\frac{\pi}{10^6}(y-y_0)\Bigg],$$
$$k=1,2,\ \lambda=\max(\varepsilon^2\mu,\ \varepsilon\mu^2),$$
$$0<\varepsilon,\ \mu\ll 1. \tag{45}$$

5. Simulated contour for asymptotic solution

From Eqs. (44) and (45), the simulated contours $u_{k10\text{per}}$ and $v_{k10\text{per}}$ $(k=1,2)$ with $\varepsilon=0.01$ and $\mu=0$ of perturbation solutions for the time delay weak nonlinear maritime climate model under atmospheric wind stress forcing Eqs. (32)–(36) are obtained and shown in Figs. 5–8.

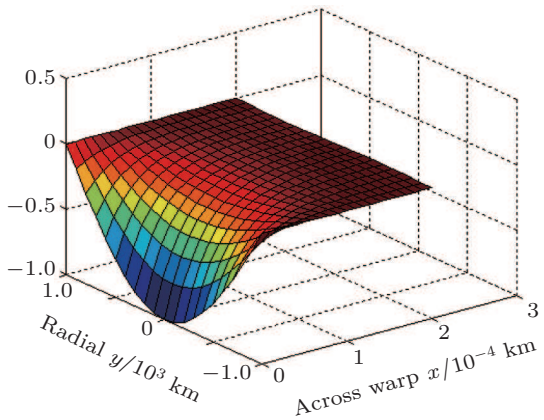


Fig. 5. (color online) Perturbation solution of $u_{110\text{per}}$.

From Figs. 5–8, we can to see that, as $\varepsilon=0.01$, $\mu=0$, there is a small correction as $\varepsilon=\mu=0$ (Figs. 1–4).

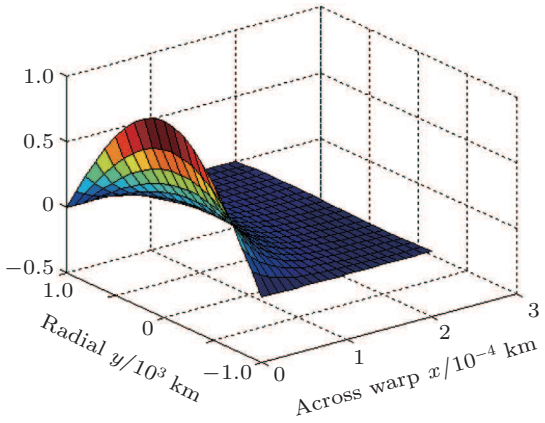


Fig. 6. (color online) Perturbation solution of $u_{210\text{per}}$.

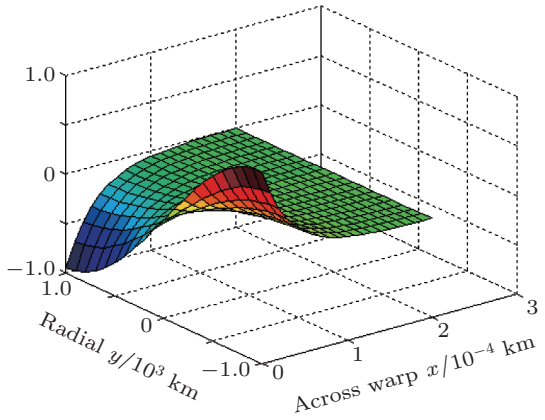


Fig. 7. (color online) Perturbation solution of $v_{110\text{per}}$.

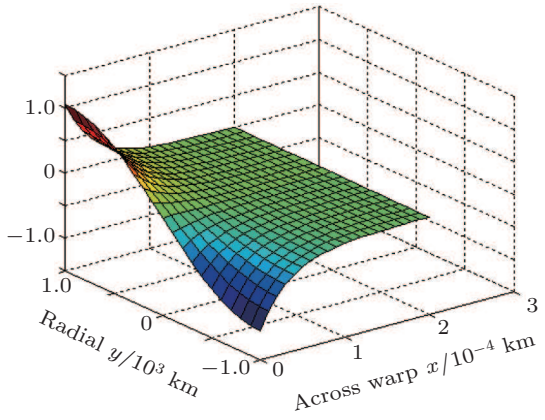


Fig. 8. (color online) Perturbation solution of $v_{210\text{per}}$.

From Eqs. (44) and (45), the simulated contours $u_{k11\text{per}}$ and $v_{k11\text{per}}$ $(k=1,2)$ with $\varepsilon=0.01$, and $\mu=0.02$ of perturbation solutions for the time delay weak nonlinear maritime climate model under atmospheric wind stress forcing Eqs. (32)–(36) are obtained and shown in Figs. 9–12.

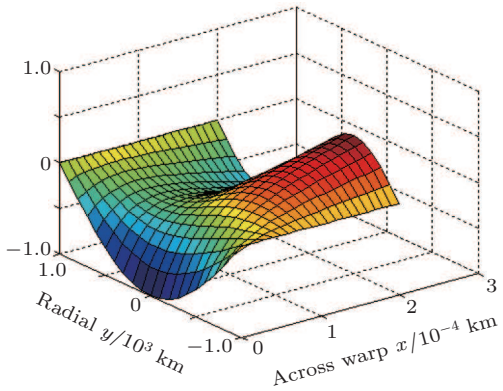


Fig. 9. (color online) Perturbation solution of $u_{111\text{per}}$.

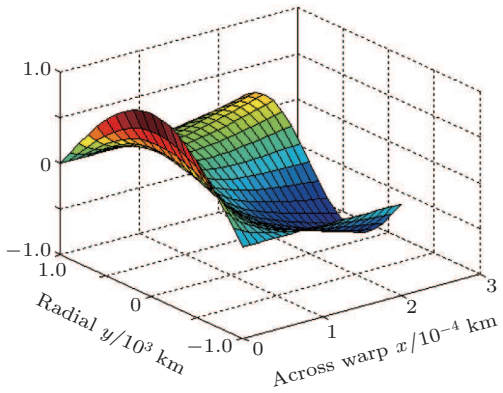


Fig. 10. (color online) Perturbation solution of $u_{211\text{per}}$.

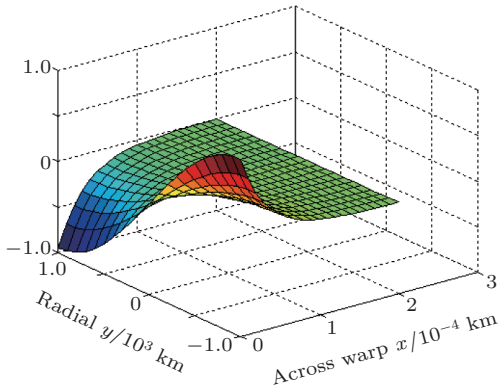


Fig. 11. (color online) Perturbation solution of $v_{111\text{per}}$.

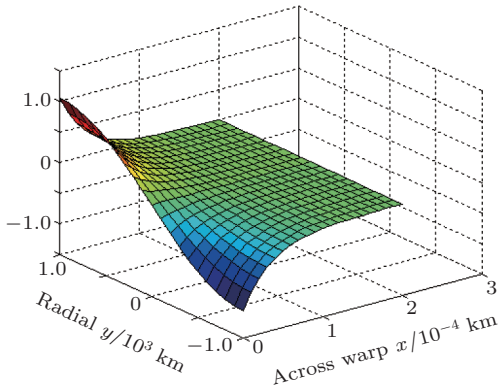


Fig. 12. (color online) The perturbation solution of $v_{211\text{per}}$.

From Figs. 9–12, we can also see that with $\varepsilon = 0.01$ and $\mu = 0.02$, there is a smaller correction as in the case $\varepsilon = \mu = 0$ (Figs. 1–4).

Note that we can also obtain higher order terms and more accurate asymptotic solutions using the same method repeatedly.

6. Physical meaning of the asymptotic solution

The results of the asymptotic solution can obtain the strong flow near the west boundary, which is more accordant with the actual situation of the flow. The flow in the upper layer is convergent to center dimensions (40° N) and then moves to the east. The asymptotic solution of the low layer flow reverses its direction but with the same magnitude as that of an upper layer flow. The asymptotic solutions of the flow are qualitatively consistent with the actual situation, just because the asymptotic solutions near the west boundary give a partially strong flow, in contrast, interior gyres appear to be much weaker.

7. Summary

The time delay atmospheric mid-latitude western wind field anomaly will cause a mid-latitude upper layer ocean flow anomaly, which is a complicated natural phenomenon. In order to study the time delay nonlinear model for the mid-latitude upper layer ocean flow, we sometimes need to solve them using an approximate method. In this paper, the perturbation method is a simple and valid method. This method differs from a general numerical simulation method. The analytic solutions obtained using the asymptotic perturbation method can be analytically operated repeatedly. Thus, from the asymptotic perturbation solutions we can perform the differential and integral operations sequentially and obtain the corresponding physical quantities. Then we can make predictions and take the necessary measures in due time to reduce unnecessary damage.

References

[1] Zhou T J, Wu B and Wang B 2009 *J. Climate* **22** 1159
[2] Zhou T J and Zou L 2010 *J. Climate* **23** 6009
[3] Zhou T J and Zhang J 2011 *J. Climate* **24** 1053
[4] Cabanes C, Huck T and De Verderce A G 2006 *J. Phys. Oceanogyr.* **36** 1739
[5] Qiu B and Chen S M 2006 *J. Phys. Oceanogyr.* **36** 1751
[6] Zhang Y C, Lu K C and Zhang M 2011 *Sci. Meteorodog. Sin.* **31** 11 (in Chinese)
[7] Lu K C, Zhang Y C and Zhang M 2011 *Sci. Meteorodog. Sin.* **31** 17 (in Chinese)

[8] Zhang Y C, Lu K C and Zhang M 2012 *Clinmatic and Environmental Research* **17** 217
[9] Zhang H Q and Ding Q 2009 *Chin. Phys. Lett.* **26** 104701
[10] Lin W T, Zhang Yu and Mo J Q 2013 *Chin. Phys. B* **22** 030205
[11] Mo J Q and Lin W T 2008 *Acta Phys. Sin.* **57** 1291 (in Chinese)
[12] Lin Y H, Lin W T and Mo J Q 2009 *Acta Phys. Sin.* **58** 6692 (in Chinese)
[13] Mo J Q, Lin Y W and Lin W T 2010 *Acta Phys. Sin.* **59** 6701 (in Chinese)
[14] Barbu L and Morosanu G 2007 *Singularly Perturbed Boundary-Value Problems* (Basel: Birkhauserm Verlag AG)
[15] Ramos M 2009 *J. Math. Anal. Appl.* **352** 246
[16] D'Aprile T and Pistoia A 2010 *J. Diff. Eqs.* **248** 556
[17] Faye L, Frenod E and Seck D 2011 *Discrete Contin. Dyn. Sys.* **29** 1001
[18] Shi L F, Lin W T, Lin Y H and Mo J Q 2013 *Acta Phys. Sin.* **62** 010201 (in Chinese)
[19] Shi L F and Mo J Q 2013 *Acta Phys. Sin.* **62** 040203 (in Chinese)
[20] Mo J Q 2009 *Sci. China G* **39** 568
[21] Mo J Q 2011 *Acta Phys. Sin.* **60** 090203 (in Chinese)
[22] Mo J Q 2010 *Commun. Theor. Phys.* **53** 440
[23] Mo J Q, Lin W T and Lin Y H 2011 *Chin. Phys. B* **20** 070205