Revivals of Zitterbewegung in the Bernevig–Hughes–Zhang model in the quantum spin Hall regime with a laser pulse*

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We theoretically investigate the Zitterbewegung (ZB) behavior of electrons in the Bernevig–Hughes–Zhang model with a short laser pulse. To obtain a steady picture, we fix the electron on the Landau levels with a magnetic field. The ZB motion and the electromagnetic radiations in the quantum spin Hall regime are given. We find that over a shorter time, the electromagnetic radiations show a quasi-classical cyclotron oscillation, while over a longer time, they exhibit a clear revival picture. The resulting revival time and excited electric field are large enough to make experimental detection accessible.

Keywords: quantum well, Zitterbewegung, electromagnetic radiation

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1. Introduction

A trembling motion called Zitterbewegung (ZB) is associated with relativistic particles like electrons and gives a surprising phenomenon of the Dirac equation's unification of quantum mechanics and special relativity. It is now understood as originating from the interference between the positive and the negative energy states. Earlier studies pointed out that the ZB characteristic frequency of the electron in free space can be as high as $2m_0c^2 \sim 10^{20}$ Hz,[1] which is experimentally inaccessible in the original electron system.

Due to the development of material and technology in recent years, ZB has rearoused people's attention. It was argued that ZB may be not only restricted to the relativistic particles.[2,3] As long as the interferences between the positive and the negative states exist, the oscillation of the particle will happen. Therefore, a few possible candidate systems have been proposed to observe the ZB, such as graphene,[4–8] photon crystals,[9,10] ultracold atom systems,[11,12] and the bounded Dirac oscillator.[13,14] In particular, Gerritsma *et al* have reported an important progress in experiment with trapped ions and laser excitations, which simulates the ZB motion by adjusting the parameters of the Dirac equation in the relativistic region.[15] To observe the ZB successfully, the related duration time and observables should lie within the detectable region.

Here we investigate the ZB motion in a HgTe/CdTe quantum well (QW) system described by the Bernevig–Hughes–Zhang (BHZ) model with the help of a laser pulse. The QW is a two-dimensional (2D) electron system. An important benefit of the system is that its topological order can be controlled by the thickness of the HgTe layer; below the critical thickness $d_c \approx 6.3$ nm, the system is a trivial insulator, while above d_c, it behaves as a topological insulator (TI).[16,17] In Ref. [18], Shi *et al* studied the electron orbital motion and the accompanying ZB in the TI with an in-plane electric field. In this work, we try to study the ZB from the electromagnetic radiation accompanying the trembling motion. To stablise the electron motion, a perpendicular magnetic field is added to the system to fix the electron states on the Landau levels (LLs). Our study will focus on the attractive quantum spin Hall (QSH) phase. The results show that the electromagnetic radiations exhibit the cyclotron motion in a shorter time while giving a clear revival behavior after a long time. In addition, when choosing proper experimental conditions, the ZB revival time can be of several tens of nanoseconds (ns) and the excited electric field can be as high as millivolts (mV), which may provide a new route to observe the electron ZB motion.

2. Model and method

It is found that in the HgTe/CdTe QW system, the electronic properties close to the Γ point can be well described by the BHZ model,[16] which is a 4×4 Dirac matrix near the mass inversion thickness d_c. In the basis of $(|E+\rangle, |H+\rangle, |E-\rangle, |H-\rangle)^{\mathrm{T}}$, where E (H) refers to the s (p) orbital of the electron and $\pm$ is the degenerate Kramer partner representing the up and down spin components,[19] the Hamiltonian reads ($\hbar = 1$)

$$\boldsymbol{H} = \begin{pmatrix} h_+(\boldsymbol{k}) & 0 \\ 0 & h_-(\boldsymbol{k}) \end{pmatrix},$$
$$h_+(\boldsymbol{k}) = \epsilon(\boldsymbol{k}) + \boldsymbol{d}(\boldsymbol{k})\cdot\boldsymbol{\sigma}, \quad h_-(\boldsymbol{k}) = h_+^*(-\boldsymbol{k}), \qquad (1)$$

where $\boldsymbol{\sigma}$ denotes the Pauli matrix, and $\boldsymbol{k}$ is the 2D crystal momentum. To the lowest order in k, $\epsilon(\boldsymbol{k}) = -Dk^2$, $\boldsymbol{d}(\boldsymbol{k}) =$

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$(Ak_x, -Ak_y, M - Gk^2)$, where A, D, G are the parameters determined by the material structure, and M gives the energy offset between the s and p orbitals and changes with the width of the QW. When M turns from positive to negative, the phase transition from the trivial insulator to the QSH state occurs.

When a perpendicular magnetic field B is applied on the QW, it will destroy the time-reversal symmetry of the system.[20] We use the Landau gauge $\boldsymbol{A} = [0, Bx]$ to describe the magnetic field, which can be included into the system via the Peierls substitution $\boldsymbol{k} \to \boldsymbol{k} + e\boldsymbol{A}$. To diagonalize the Hamiltonian, we introduce the dimensionless variable $\xi = x/l + k_y l$, where $l = \sqrt{1/(eB)}$ is the magnetic length and the raising and lowering operators are $a^+ = \sqrt{1/2}(\xi - \partial/\partial_\xi)$, $a = \sqrt{1/2}(\xi + \partial/\partial_\xi)$. Then the diagonal blocks become $h_\pm = h_{\rm HO} + h^\pm_{\rm AJC}$,[21] where $h_{\rm HO} = -2DB(a^+a + 1/2) + [M - 2BG(a^+a + 1/2)]\sigma_3$ represents the harmonic-oscillator-like part and is diagonal in the chosen basis, while $h^+_{\rm AJC} = {\rm i}\sqrt{2B}A(a^+\sigma^+ - a\sigma^-)$ and $h^-_{\rm AJC} = {\rm i}\sqrt{2B}A(a\sigma^+ - a^+\sigma^-)$ are the anti-Jaynes–Cummings terms. As the two blocks in the Hamiltonian are decoupled, they can be diagonalized in respective bases.[17] In Fig. 1, we show the dependence of LLs on the magnetic field in the trivial insulator regime and the QSH regime. It is shown that in the QSH regime, the LLs ± 0 cross each other under a strong magnetic field.[17] Consequently, the system can give distinct values of the Hall conductivity depending on the position of the Fermi surface.[17] We will concentrate on the attractive QSH regime and consider the minor magnetic field.

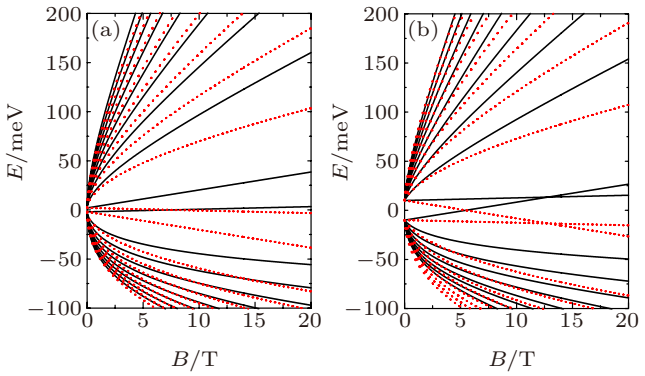


Fig. 1. (color online) Dependence of the LLs on the magnetic field B for spin up (black solid) and down (red dotted) components with the material parameters $A = 364.5$ meV·nm, $G = -686$ meV·nm^2, and $D = -512$ meV·nm^2: (a) $M = 2$ meV in the trivial insulator regime, (b) $M = -10$ meV in the QSH region.

The plane-wave state is taken as the initial state[4,5,22,23] and the Fermi energy is supposed to be situated at the neutral point where all negative LLs are occupied. To induce the interference between the positive and the negative LLs, a Gaussian-shaped laser pulse is chosen to act on the system along the x direction.[5,6] The Hamiltonian describing the laser pulse is

$$W(t) = -exE_0\,{\rm e}^{-\alpha t^2/\tau^2}\cos(\omega_{\rm L} t), \tag{2}$$

where E_0 is the amplitude of the laser field, α is the modulated factor controlling the pulse width, τ is the pulse duration, and $\omega_{\rm L}$ represents the frequency of the laser.

3. Main results and discussion

We consider the up-spin block of the BHZ model and the down-spin block will be discussed later. When the laser pulse acts on the QW, the electrons will be excited and hop between different LLs. Because the interaction energy between the laser pulse and the QW in Eq. (2) is much smaller than the characteristic energy of the QW, we can use the the time-dependent perturbation theory to deal with the external laser field. The initial wave function ψ_i will be modified as a linear combination of the eigenstates in the system

$$\Psi_i(t) \simeq {\rm e}^{-{\rm i}E_i t}\psi_i + \sum_f c_{if}(t)\,{\rm e}^{-{\rm i}E_f t}\psi_f, \tag{3}$$

here the transition amplitude $c_{if}(t)$ between the initial state i and the final state f is given by $c_{if}(t) = -{\rm i}\int_{-\infty}^{t} {\rm d}t' W_{if}(t')\,{\rm e}^{{\rm i}\omega_{if}t'}$, where the matrix element is $W_{if}(t) = \int_{-\infty}^{\infty} {\rm d}r\psi_i^+(r)W(t)\psi_f(r)$ and the transition frequency is $\omega_{if} = E_i - E_f$. Although the electron hopping is excited by the external field, as the laser is a short pulse, the subsequent electron motion still reflects the intrinsic behavior of the system. The time-dependent transition amplitude is approximated as $c_{if}(t) = c_{if}(\infty)$. With the help of two dimensionless variables $m_{if} = \langle i|x|f\rangle/l$, and

$$n_{if} = \frac{\sqrt{\alpha}}{\sqrt{\pi}\tau}\int_{-\infty}^{\infty} {\rm e}^{-\alpha t^2/\tau^2}\,{\rm e}^{{\rm i}\omega_{if}t}\cos(\omega_{\rm L} t)\,{\rm d}t,$$

the c_{if} can be expressed as

$$c_{if} = \frac{{\rm i}\sqrt{\pi}eE_0\tau l}{\sqrt{\alpha}} m_{if} n_{if}.$$

For the nonzero dipole transition process, a selection rule $\Delta = |i| - |f| = \pm 1$ is obtained for intraband and interband transitions, and the rule is the same as that in the previous studies related with the LL transition.[5,6,21,24,25] In addition, the hopping processes within the negative LLs are neglected as they are of higher order.

The ZB phenomenon can be investigated from its indirect response to the external fields,[1] here we study the electromagnetic radiations.[5] The idea is that the electron hopping processes between different LLs lead to the variation of the position, accompanying the acceleration of its motion. In classical electrodynamics, the charged particle motion is closely related with the dipole moment, which in mode i is defined as $D_{ir}(t) = e\langle\Psi_i(t)|r|\Psi_i(t)\rangle$. The electromagnetic radiation will be caused by the acceleration of the oscillating dipole moment. The related excited electric field is shown as[26]

$$E_r(t) = \frac{\sin\theta}{4\pi\epsilon_0 c^2 R}\frac{\partial^2 D_r(t)}{\partial t^2},$$

where R and θ are the distance and the angle between the observer and the sample, respectively, ϵ_0 is the vacuum dielectric constant, and c is the light speed. The electromagnetic radiations appear along the direction that the laser acts while they vanish in the transverse direction. Therefore the dipole moment and the excited electric field for mode i are given by

$$D_{ix}(t) = -g_0 \sum_f m_{if}^2 n_{if} \sin(\omega_{if} t),$$
$$E_{ix}(t) = \frac{g_0 \sin\theta}{4\pi\epsilon_0 c^2 R} \sum_f m_{if}^2 n_{if} \omega_{if}^2 \sin(\omega_{if} t), \quad (4)$$

where $g_0 = 2\sqrt{\pi} e^2 l^2 E_0 \tau/(\hbar\sqrt{\alpha})$. The total results are obtained by summing over all states under the Fermi surface [6] $D_x(t) = \sum_i D_{ix}(t)$ and $E_x(t) = \sum_i E_{ix}(t)$, which suggests that the actual process is a collective effect including many modes. In our simulation, the chosen laser pulse has a duration of $\tau = 1.6$ fs, which is the shortest laser pulse created experimentally so far within the visible range.[27] The corresponding wavelength is $\lambda_L = 650$ nm. We take the laser intensity to be 1.0×10^9 W/cm^2 and the modulated factor $\alpha = 2\ln 2$. In addition, we choose other parameters as $\theta = \pi/4$ and $R = 1$ cm.

In Fig. 2, we give the evolution of the electromagnetic radiation with time for several modes and the total behavior when $B = 1$ T. In Fig. 2(a), it shows that the dipole moment for each state exhibits a quasi-classical cyclotron oscillation, the period is given by $T_{\rm cl}^0 = 2\pi/\omega_{0,1}$ or

$$T_{\rm cl}^i \simeq \frac{4\pi}{\omega_{i,i-1} + \omega_{i,i+1}}, \quad (i > 0).$$

According to our model, there is only one hopping process for 0h LL, while for other LLs, two hopping frequencies are involved. The total behavior of the dipole moment demonstrates the ZB motion of electrons in the system. More importantly, the cyclotron period and magnitude of mode $i = 0$ are much larger than those of the other modes, of which the oscillations tend to be vanishing. This is because the dipole moment is largely constrained by n_{if}, where

$$n_{if} = \left[e^{-(\omega_{if}+\omega_L)^2\tau^2/(4\alpha)} + e^{-(\omega_{if}-\omega_L)^2\tau^2/(4\alpha)} \right]/2.$$

As ω_{if} increases with increasing i, n_{if} decays quickly. From Fig. 2(b), we find that there will be certain contributions of other states to the total excited electric field because ω_{if}^2 plays an indispensable role in Eq. (4). However, the contribution of the other modes is still smaller compared to that of the $i = 0$ mode.

From the above analysis, the electron begins with a cyclotron motion and then evolves and spreads. In Fig. 3, the evolution picture of electromagnetic radiations over a longer time is given. It is shown that the amplitude of the dipole moment exhibits a well revival behavior with period $T_{\rm r}^{\rm d}$ in Fig. 3(a). Such a revival picture reminds us of the previous ZB studies which show a similar behavior, where the initial states are Gaussian-type wavepackets.[7,8,14] For the excited electric field in Fig. 3(b), initially the strength is small, then it increases with time and reaches a strong resonate peak at $T_{\rm r}^{\rm e}$. After the peak, the electric field decays instantly and goes into the next revival period. We can see that in the first several periods, the peak shape can be well kept, after that it will be gradually lost. After about eight periods, the electric field gives a chaos-like picture, where the peak and the minimum value become obscure. It is worth pointing out that the cyclotron movement and the electric field are not synchronous and $T_{\rm r}^{\rm d} > T_{\rm r}^{\rm e}$. However, they all enter the region of ns, which is longer than the previous studies [6,8,14] and favors the actual observation.

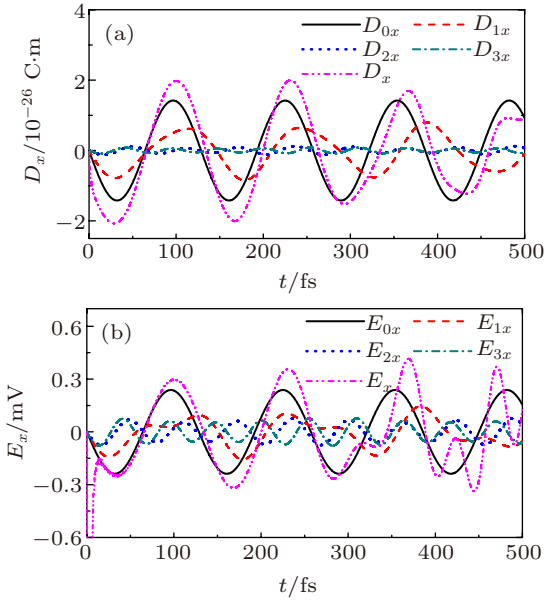


Fig. 2. (color online) Time dependences of (a) dipole moments D_x and (b) excited electric fields E_x for several states i ($i = 0, 1, 2, 3$) and the total results. The magnetic field is $B = 1$ T.

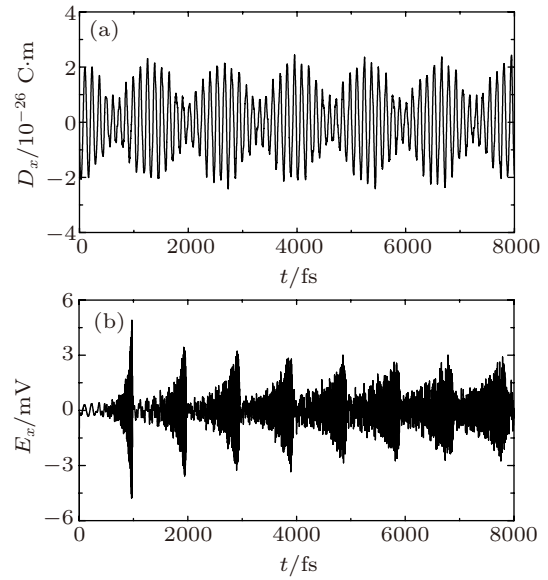


Fig. 3. Time dependences of (a) total dipole moment D_x and (b) excited electric field E_x with longer time. The magnetic field is $B = 1$ T.

To further investigate the ZB, in Fig. 4, we plot the normalized frequency spectra of the electromagnetic radiations corresponding to Fig. 3. In Fig. 4(a), there is only one strong main peak owing to the hopping process from 0h LL to 1e LL (see the arrow). While in Fig. 4(b), besides the main peak, other frequencies have certain contributions to the electric field, which also show certain oscillations. Because for some initial LL i, the hopping to level $i+1$ gives a relative larger value, while that to $i-1$ contributes a smaller value. These results suggest that E_x is determined by the superposition of many transitions and the strong peak value is caused by the constructive interference of these transitions. Because of the stability of the LLs, we expect that the frequency spectra from the electromagnetic radiation due to the ZB motion between LLs may provide a feasible route to analyze the internal structure information of the HgTe QW, similar to the experimental studies of graphene which analyzed the electron band velocity from the infrared spectroscopy of LLs.[24,25]

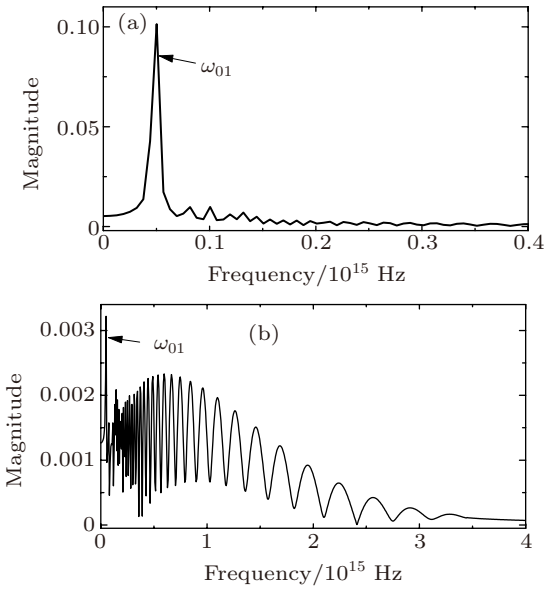


Fig. 4. Normalized Fourier frequency spectra of (a) D_x and (b) E_x, corresponding to Figs. 3(a) and 3(b), respectively. The main peaks correspond to the frequency ω_{01}.

Now we discuss the role of the magnetic field. The magnetic field stabilizes the electron on the LLs, reducing the influence of the disorder and impurity scattering. Moreover, the magnetic field can affect the ZB evolution through the revival period T_r^e and the peak value of the excited electric field E_x^p, which are numerically shown in Fig. 5. We find that T_r^e decreases exponentially with increasing B and reduces to only several tens of fs around 8 T, while E_x^p exhibits an increasing trend. When the magnetic field is higher than 4 T, the peak value oscillates due to the superposition of many modes. As the longer revival time and larger excited electric field are the key factors in experimental observation, an appropriate magnetic field should be chosen. In addition, as the HgTe QW has a translation symmetry along the transverse direction, we speculate that the degeneracy degree owned by the LLs should also be taken into account when evaluating the observables, which may push the electric field to a much larger value.[6]

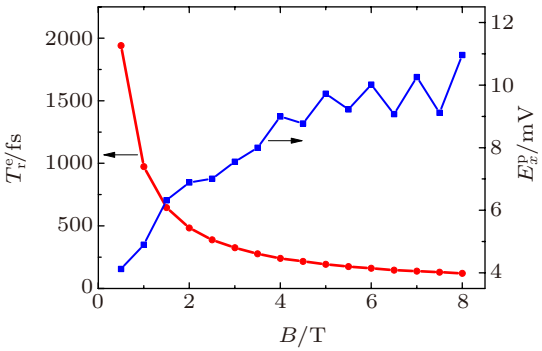


Fig. 5. (color online) Dependences of the revival time T_r^e and the peak value of the excited electric field E_x^p on the magnetic field B.

The above results only concern the up-spin block h_+. As h_+ and h_- are mixed with equal weights in the BHZ model, the electromagnetic radiations should include the contributions from both blocks. So we investigate the ZB evolution determined by h_-. The results show a similar revival picture, only the location of the main peak deviates a little from the up-spin part due to the difference of the energy bands.

4. Conclusion

In this paper, we have theoretically studied the ZB phenomenon from the electromagnetic radiations of the BHZ model in the QSH regime. Although many interference events contribute to the ZB, only one main frequency dominates, the ZB motion gives a well revival behavior. More importantly, when choosing proper conditions such as the magnetic field and the laser pulse, the excited electric field and the revival period can be much larger compared to those in the previous studies,[5,6] which favors the observation in experiment. Our work broadens the ZB study and we expect further theoretical and experimental investigations.

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