# Optical properties of the electromagnetic waves propagating in an elliptical cylinder multilayer structure

A. Abdoli-Arani†

*Department of Laser and Photonics, Faculty of Physics, University of Kashan, Kashan, Iran*

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Theoretical description of the wave propagation in an elliptical cylinder multilayer structure under the conditions of H polarization and E polarization is presented. A transfer matrix method has been developed for elliptical cylinder waves. The formulas of reflection and transmission coefficients for an elliptical cylinder multilayer structure are driven. Reflection and transmission coefficients of elliptical cylinder waves by a single elliptical cylinder interface is presented. The obtained formulas can be generalized to the cold plasma filled structures and thus the obtained results in the limit of circular cylinder structures are investigated.

**Keywords:** transfer matrix method, elliptical cylinder multilayer structure, wave propagation

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## 1. Introduction

It is well known that when the index of refraction of the medium is periodically modulated, the propagation of photons is forbidden in a certain range of frequency (for the Bragg reflector this region corresponds to the reflection band).[1] In these constructions, Bragg reflectors localize light in one direction, i.e., they act like a material with a one-dimensional photon band gap. Bragg reflectors,[2] i.e., periodic sequences of pairs of quarter-wave layers, are part of various optoelectronic devices, e.g., vertical-emission lasers.[3] A further example of the use of Bragg reflectors is in distributed-feedback lasers, where a diffraction grating serves as the Bragg reflector for a waveguide mode.[4] In addition, the optical fibres have wide applications in the fields of radar feed line, optical communication, fibre lasers, microwave heating applicator, electron accelerator, and so on.[5,6] When a circular cylinder fibre is compared with an elliptical cylinder fibre, the attenuation of the dominant mode in an elliptic fibre with the same cross-section area will be reduced and the transmission modes in the elliptic fibre will be stable.[7,8] Therefore, an elliptic fibre is superior to a circular one in some aspects, and has wider applications. Attenuation effects and power flow expressions were achieved for wave propagation in a surface wave transmission line with an elliptical cross section.[6] It was found that some modes in the guide have lower attenuation than the corresponding modes in a circular guide. In conventional optical fibers, light confinement is achieved through total internal reflection and photons propagate mainly in the high index center core. A completely different confinement mechanism, Bragg reflection, provides an alternative way of guiding photons. This possibility was first pointed out by Yeh *et al.*[9] where the concept of Bragg fiber was proposed. The experimental fabrication of Bragg fibers has been recently reported.[10] The analysis of Bragg fibers, however, is much more complicated than that of conventional fibers. In the matrix formalism, Yeh *et al.*[9] used four independent parameters to describe the solution of Maxwell equations in each layer of the Bragg fiber and the parameters in neighbor dielectric layers were related via a $4 \times 4$ matrix. However, a simple periodic dielectric multilayer structure known as a one-dimensional photonic crystal is easier to fabricate compared to the two- and three-dimensional photonic crystals. In addition, one-dimensional photonic crystals can be used to explore many fundamental and interesting optical properties, such as the existence of photonic band gaps as well as the feature of omnidirectional mirror.[11,12] In one-dimensional photonic crystal, the wave propagation properties can be analytically investigated by the familiar transfer matrix method in Cartesian coordinates.[13] In addition, cylindrical wave transfer matrix method has been developed in Ref. [14]. They developed an elegant transfer matrix method in cylindrical coordinates which, in fact, is an analogous version of Abeles theory in Cartesian coordinates. Moreover, based on the use of such transfer matrix method, studies of photonic band structures in metallic and superconducting cylindrical photonic crystals have also been available.[15,16] Although a transfer-matrix method has been developed for cylindrical waves and analysis of optical properties in cylindrical dielectric photonic crystal[17] and Bragg reflectors for cylindrical waves[18] in E and H polarization cases have been investigated. We develop a transfer-matrix method for elliptical cylinder waves in elliptical Bragg reflectors. Therefore, the purpose of this paper is to give a detailed theoretical formulas on the wave propagation in an elliptical cylinder multilayer structure under the conditions of H polar-

†Corresponding author. E-mail: abdoliabbas@kashanu.ac.ir

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ization and E polarization. We shall derive the formulae of reflection and transmission coefficients for an elliptical cylinder multilayer structure. We study different model structures, including the single elliptical cylinder interface and the multilayer structures. All the results of these structures are given for both E polarization and H polarization.

## 2. Basic equations

We start with introducing the elliptical cylinder coordinates. The orthogonal elliptical cylinder coordinate system $(\xi,\eta,z)$ is convenient to solve Maxwell's equations in the structures with elliptical boundaries. Elliptical cylinder coordinates are related to their rectangular counterpart via[19]

$$x = d\cosh\xi\cos\eta,\quad y = d\sinh\xi\sin\eta,\quad z = z,$$

where $0 \le \xi \le \infty$, $0 \le \eta \le 2\pi$, and $d = \sqrt{a^2 - b^2}$ is the semi-focal length of the elliptic expressed in term of the semi-major and semi-minor axes $a$ and $b$. The elliptic boundary is defined by $\xi = \xi_0$, where $\xi_0 = \tanh^{-1}(b/a)$. Figure 1 shows an elliptical cylinder multilayer structure. Maxwell's equations are written as

$$\nabla \times \boldsymbol{H} = \frac{\partial}{\partial t}(\varepsilon \boldsymbol{E}), \tag{1}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}(\mu \boldsymbol{H}). \tag{2}$$

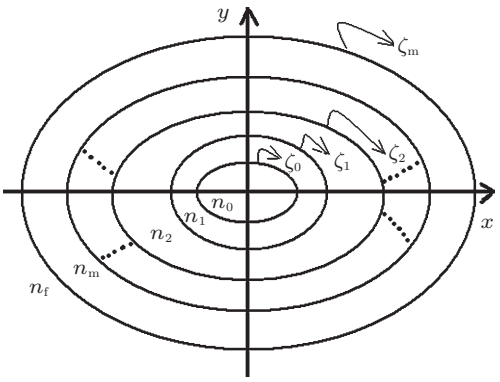



**Fig. 1.** Sketch of the elliptical cylinder multilayer structure, in which the $m$-layer system is bounded by the media of refractive indices, $n_0$ and $n_f$. The subscript 0 is known as the starting medium, whereas the final medium is indexed by the subscript f.

We assume that the temporal part of all the fields is $\exp(\mathrm{i}\omega t)$ and for a given layer permeability and permittivity are $\varepsilon = \varepsilon_r\varepsilon_0$ and $\mu$, respectively. In elliptical cylinder coordinate and a monochromatic wave, equations (1) and (2) can be expanded as

$$\frac{1}{h}\frac{\partial E_z}{\partial \eta} - \frac{\partial E_\eta}{\partial z} = -\mathrm{i}\omega\mu H_\xi, \tag{3}$$

$$\frac{\partial E_\xi}{\partial z} - \frac{1}{h}\frac{\partial E_z}{\partial \xi} = -\mathrm{i}\omega\mu H_\eta, \tag{4}$$

$$\frac{1}{h^2}\left[\frac{\partial(hE_\eta)}{\partial \xi} - \frac{\partial(hE_\xi)}{\partial \eta}\right] = -\mathrm{i}\omega\mu H_z, \tag{5}$$

$$\frac{1}{h}\frac{\partial H_z}{\partial \eta} - \frac{\partial H_\eta}{\partial z} = \mathrm{i}\omega\varepsilon E_\xi, \tag{6}$$

$$\frac{\partial H_\xi}{\partial z} - \frac{1}{h}\frac{\partial H_z}{\partial \xi} = \mathrm{i}\omega\varepsilon E_\eta, \tag{7}$$

$$\frac{1}{h^2}\left[\frac{\partial(hH_\eta)}{\partial \xi} - \frac{\partial(hH_\xi)}{\partial \eta}\right] = \mathrm{i}\omega\varepsilon E_z, \tag{8}$$

where $h = d\sqrt{\cosh^2\xi - \cos^2\eta}$. We will describe the propagation of elliptical cylinder waves in this medium as either diverging from $\xi = 0$ or converging to it with respect to the normal to the symmetry axis of the system $z$. This implies that the derivative $\partial/\partial z = 0$. Equations (3)–(5) in this case takes the form

$$\frac{1}{h}\frac{\partial E_z}{\partial \eta} = -\mathrm{i}\omega\mu H_\xi, \tag{9}$$

$$\frac{1}{h}\frac{\partial E_z}{\partial \xi} = \mathrm{i}\omega\mu H_\eta, \tag{10}$$

$$\frac{1}{h^2}\left[\frac{\partial(hE_\eta)}{\partial \xi} - \frac{\partial(hE_\xi)}{\partial \eta}\right] = -\mathrm{i}\omega\mu H_z. \tag{11}$$

Likewise, equations (6)–(8) can be reduced to

$$\frac{1}{h}\frac{\partial H_z}{\partial \eta} = \mathrm{i}\omega\varepsilon E_\xi, \tag{12}$$

$$\frac{1}{h}\frac{\partial H_z}{\partial \xi} = -\mathrm{i}\omega\varepsilon E_\eta, \tag{13}$$

$$\frac{1}{h^2}\left[\frac{\partial(hH_\eta)}{\partial \xi} - \frac{\partial(hH_\xi)}{\partial \eta}\right] = \mathrm{i}\omega\varepsilon E_z. \tag{14}$$

It is well known that solutions for Eqs. (3)–(5) and (6)–(8) can be classified as two modes. One is called the E polarization which has three non-zero components, $E_z$, $H_\eta$, $H_\xi$. The other is H polarization having nonzero components $H_z$, $E_\eta$, $E_\xi$. On the other hand, it is noted that for an elliptical cylinder multilayer structure where each layer filled with the cold collisionless plasma we can consider $\varepsilon_l = \varepsilon_0[1 - \omega_{pl}^2/\omega^2]$ in the $l$-th layer where $\omega_{pl} = (n_{0l}e^2/m_e\varepsilon_0)^{1/2}$ is the plasma frequency and $n_{0l}$ is the density of electrons in the plasma $l$-th layer and $\varepsilon_0$ is the electric permittivity in free space.

## 3. Transfer matrix method in elliptical cylinder system

In this section we will present transfer matrix method in elliptical cylinder system for the H polarization and the E polarization.

### 3.1. H polarization

In this case, with Eqs. (11)–(13) the governing differential equation for $H_z$ can be obtained in the following form:

$$\frac{1}{h^2}\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right)H_z + k^2 H_z = 0. \tag{15}$$

Equation (15) is a Mathieu equation giving a well-known solution and eigenvalue[19]

$$H_z = \sum_{m=0}^{\infty}[C_m Ce_m(\xi,q_i)+F_m Fey_m(\xi,q_i)]ce_m(\eta,q_i) + \sum_{m=1}^{\infty}[S_m Se_m(\xi,q_i)+G_m Gey_m(\xi,q_i)]se_m(\eta,q_i), \quad (16)$$

where $ce_m(\eta,q_i)$, $se_m(\eta,q_i)$ are the even and odd solutions of the angular Mathieu equation, $Ce_m(\xi,q_i)$, $Se_m(\xi,q_i)$ are the even and odd solutions of the radial Mathieu equation of the first kind and $Fey_m(\xi,q_i)$; $Gey_m(\xi,q_i)$ are the even and odd radial Mathieu functions of the second kind, respectively, and subscript $m$ represents the order of the Mathieu functions. $C_m$, $S_m$, $F_m$, $G_m$ are arbitrary computable constants and $q_i = k_i^2 d^2/4$ and $k_i^2 = \omega^2\varepsilon_i\mu_i$ that $k_i = \omega n_i/c$ is the wave number in $i$-th medium and $c$ is the speed of light in free space and $n_i$ is the refractive index of $i$-th medium. For any $m$, there are even and odd solutions in the following form:

$$H_{zme} = [C_m Ce_m(\xi,q)+F_m Fey_m(\xi,q)]ce_m(\eta,q) = V^{(\mathrm{H})}(\xi)ce_m(\eta,q), \quad (17)$$

and

$$H_{zms} = [S_m Se_m(\xi,q)+G_m Gey_m(\xi,q)]se_m(\eta,q). \quad (18)$$

We consider even mode and obtain the non-zero electric fields in the following form:

$$E_\xi = \frac{-\mathrm{i}}{\omega\varepsilon h}V^{(\mathrm{H})}(\xi)ce'_m(\eta,q), \quad (19)$$

$$E_\eta = \frac{1}{h}U^{(\mathrm{H})}(\xi)ce_m(\eta,q), \quad (20)$$

where

$$V^{(\mathrm{H})}(\xi) = [C_m Ce_m(\xi,q)+F_m Fey_m(\xi,q)],$$
$$U^{(\mathrm{H})}(\xi) = \frac{\mathrm{i}}{\omega\varepsilon}[C_m Ce'_m(\xi,q)+F_m Fey'_m(\xi,q)]. \quad (21)$$

It is seen that $V$ and $U$ can be used to respectively determine the non-zero electric field components $E_\xi$ and $E_\eta$ according to Eqs. (12) and (13). In order to obtain values of the functions $(V, U)$ at an arbitrary point $\xi$ we use the transfer matrix formalism and relate $V^{(\mathrm{H})}(\xi)$, $U^{(\mathrm{H})}(\xi)$ to the corresponding vector at some other point $\xi_0 < \xi$:[14]

$$\begin{pmatrix} V^{(\mathrm{H})}(\xi) \\ U^{(\mathrm{H})}(\xi) \end{pmatrix} = \boldsymbol{R}^{(\mathrm{H})} \begin{pmatrix} V^{(\mathrm{H})}(\xi_0) \\ U^{(\mathrm{H})}(\xi_0) \end{pmatrix} = \begin{pmatrix} R_{11}^{(\mathrm{H})} & R_{12}^{(\mathrm{H})} \\ R_{21}^{(\mathrm{H})} & R_{22}^{(\mathrm{H})} \end{pmatrix} \begin{pmatrix} V^{(\mathrm{H})}(\xi_0) \\ U^{(\mathrm{H})}(\xi_0) \end{pmatrix}. \quad (22)$$

This matrix equation relates the two non-zero electric fields at two distinct positions $\xi_0$ and $\xi$. The elements of transfer matrix $\boldsymbol{R}^{(\mathrm{H})}$ can be found when the vector $[V^{(\mathrm{H})}(\xi), U^{(\mathrm{H})}(\xi)]$ has been set at a special value of [1,0] or [0,1]:[14]

$$\begin{pmatrix} V^{(\mathrm{H})}(\xi_0) \\ U^{(\mathrm{H})}(\xi_0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} V^{(\mathrm{H})}(\xi_0) \\ U^{(\mathrm{H})}(\xi_0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (23)$$

Therefore, matrix elements $\boldsymbol{R}^{(\mathrm{H})}$ can be expressed as

$$R_{11}^{(\mathrm{H})} = \frac{1}{\delta}[Ce_m(\xi,q)Fey'_m(\xi_0,q) - Fey_m(\xi,q)Ce'_m(\xi_0,q)], \quad (24)$$

$$R_{21}^{(\mathrm{H})} = \frac{\mathrm{i}}{\omega\varepsilon\delta}[Ce'_m(\xi,q)Fey'_m(\xi_0,q) - Fey'_m(\xi,q)Ce'_m(\xi_0,q)], \quad (25)$$

$$R_{12}^{(\mathrm{H})} = \frac{\mathrm{i}\omega\varepsilon}{\delta}[Ce_m(\xi,q)Fey_m(\xi_0,q) - Fey_m(\xi,q)Ce_m(\xi_0,q)], \quad (26)$$

$$R_{22}^{(\mathrm{H})} = \frac{-1}{\delta}[Ce'_m(\xi,q)Fey_m(\xi_0,q) - Fey'_m(\xi,q)Ce_m(\xi_0,q)]. \quad (27)$$

The determinant of the transfer matrix is obtained as

$$\det \boldsymbol{R}^{(\mathrm{H})} = \frac{\Delta^{(\mathrm{H})}}{\delta^{(\mathrm{H})}}, \quad (28)$$

where

$$\delta^{(\mathrm{H})} = Ce_m(\xi_0,q)Fey'_m(\xi_0,q) - Fey_m(\xi_0,q)Ce'_m(\xi_0,q), \quad (29)$$

and

$$\Delta^{(\mathrm{H})} = Ce_m(\xi,q)Fey'_m(\xi,q) - Fey_m(\xi,q)Ce'_m(\xi,q). \quad (30)$$

In the limit of a circular cylinder, $\xi \to \infty$, by using the asymptotic forms of Mathieu functions (22) we have $ce_m(\eta,q) \to \cos m\varphi$, $Ce_m(\xi,q) \to p'_m \mathrm{J}_m(k\rho)$, $Fey_m(\xi,q) \to p'_m Y_m(k\rho)$ and $Ce'_m(\xi,q) \to p'_m k\rho \mathrm{J}_m(k\rho)$, $Fey'_m(\xi,q) \to p'_m k\rho Y_m(k\rho)$, where $\mathrm{J}_m$ is a Bessel function, $Y_m$ is a Neumann function and $p'_m$ is a constant. Therefore, in this case matrix elements $\boldsymbol{R}^{(\mathrm{H})}$ are convert to[17]

$$R_{11}^{(\mathrm{H})} \to \frac{\pi}{2}k\rho_0[\mathrm{J}_m(k\rho)Y'_m(k\rho_0) - Y_m(k\rho)\mathrm{J}'_m(k\rho_0)], \quad (31)$$

$$R_{21}^{(\mathrm{H})} \to \frac{\mathrm{i}\pi}{2}k\rho_0 p[\mathrm{J}'_m(k\rho)Y'_m(k\rho_0) - Y'_m(k\rho)\mathrm{J}'_m(k\rho_0)], \quad (32)$$

$$R_{12}^{(\mathrm{H})} \to \frac{\mathrm{i}\pi}{2}k\rho_0/p[\mathrm{J}_m(k\rho)Y_m(k\rho_0) - Y_m(k\rho)\mathrm{J}_m(k\rho_0)], \quad (33)$$

$$R_{22}^{(\mathrm{H})} \to \frac{\pi}{2}k\rho_0[\mathrm{J}_m(k\rho_0)Y'_m(k\rho) - Y_m(k\rho_0)\mathrm{J}'_m(k\rho)], \quad (34)$$

where $p = (\mu/\varepsilon)^{1/2}$ and $\mathrm{J}_m(k\rho_0)Y'_m(k\rho_0) - Y_m(k\rho_0)\mathrm{J}'_m(k\rho_0) = 2/\pi k\rho$.

### 3.2. E polarization

For the case of E polarization, with Eqs. (9), (10), and (14), the governing differential equation for $E_z$ can be obtained as

$$\frac{1}{h^2}\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right)E_z + k^2 E_z = 0. \quad (35)$$

Equation (35) is a Mathieu equation giving a well-known solution and eigenvalue[19]

$$E_z = \sum_{m=0}^{\infty}[C_m Ce_m(\xi,q_i)+F_m Fey_m(\xi,q_i)]ce_m(\eta,q_i) + \sum_{m=1}^{\infty}[S_m Se_m(\xi,q_i)+G_m Gey_m(\xi,q_i)]se_m(\eta,q_i). \quad (36)$$

For any $m$ there are even and odd solutions in the following form:

$$E_{zme} = [C_m Ce_m(\xi,q)+F_m Fey_m(\xi,q)]ce_m(\eta,q) = V(\xi)ce_m(\eta,q), \quad (37)$$

and

$$E_{zms} = [S_m Se_m(\xi,q)+G_m Gey_m(\xi,q)]se_m(\eta,q). \quad (38)$$

In this case, we consider odd mode and obtain the non-zero fields in the form

$$H_\xi = \frac{\mathrm{i}}{\omega\mu h}V^{(\mathrm{E})}(\xi)se'_m(\eta,q), \quad (39)$$

$$H_\eta = \frac{1}{h}U^{(\mathrm{E})}(\xi)se_m(\eta,q), \quad (40)$$

where

$$V^{(\mathrm{E})}(\xi) = [S_m Se_m(\xi,q)+G_m Gey_m(\xi,q)],$$
$$U^{(\mathrm{E})}(\xi) = \frac{-\mathrm{i}}{\omega\mu}[S_m Se'_m(\xi,q)+G_m Gey'_m(\xi,q)]. \quad (41)$$

For modes that $H_\xi \neq 0$, this implies that we must add to the energy flux in the radial direction a rotating energy flux that propagates around the structure. Similar to H polarization case, we consider

$$\begin{pmatrix} V^{(\mathrm{E})}(\xi) \\ U^{(\mathrm{E})}(\xi) \end{pmatrix} = \boldsymbol{R}^{(\mathrm{E})} \begin{pmatrix} V^{(\mathrm{E})}(\xi_0) \\ U^{(\mathrm{E})}(\xi_0) \end{pmatrix}, \quad (42)$$

where elements of the transfer matrix are obtained as

$$R_{11}^{(\mathrm{E})} = \frac{1}{\delta}[Se_m(\xi,q)Gey'_m(\xi_0,q) - Gey_m(\xi,q)Se'_m(\xi_0,q], \quad (43)$$

$$R_{21}^{(\mathrm{E})} = \frac{-\mathrm{i}}{\omega\mu\delta}[Se'_m(\xi,q)Gey'_m(\xi_0,q) - Gey'_m(\xi,q)Se'_m(\xi_0,q], \quad (44)$$

$$R_{12}^{(\mathrm{E})} = \frac{-\mathrm{i}\omega\mu}{\delta}[Se_m(\xi,q)Gey_m(\xi_0,q) - Gey_m(\xi,q)Se_m(\xi_0,q], \quad (45)$$

$$R_{22}^{(\mathrm{E})} = \frac{-1}{\delta}[Se'_m(\xi,q)Gey_m(\xi_0,q) - Gey'_m(\xi,q)Se_m(\xi_0,q]. \quad (46)$$

The determinant of the transfer matrix for this case is determined as

$$\det \boldsymbol{R}^{(\mathrm{E})} = \frac{\Delta^{(\mathrm{E})}}{\delta^{(\mathrm{E})}}, \quad (47)$$

where

$$\delta^{(\mathrm{E})} = Se_m(\xi_0,q)Gey'_m(\xi_0,q) - Gey_m(\xi_0,q)Se'_m(\xi_0,q), \quad (48)$$

$$\Delta^{(\mathrm{E})} = Se_m(\xi,q)Gey'_m(\xi,q) - Gey_m(\xi,q)Se'_m(\xi,q). \quad (49)$$

In the limit of a circular cylinder, similarly we have $se_m(\eta,q) \to \sin(m\varphi)$, $Se_m(\xi,q) \to s'_m \mathrm{J}_m(k\rho)$, $Gey_m(\xi,q) \to s'_m Y_m(k\rho)$ and $Se'_m(\xi,q) \to s'_m k\rho \mathrm{J}_m(k\rho)$, $Gey'_m(\xi,q) \to s'_m k\rho Y_m(k\rho)$, where $s'_m$ is a constant. In this case, the matrix elements convert to the same form as, H polarization, but with a replacement of $p \to (\varepsilon/\mu)^{1/2}$.[18]

## 4. Transfer matrix in a traveling-elliptical wave basis

### 4.1. H polarization

For the problem of wave propagation, it is often convenient to consider the field within a structure as a sum of waves traveling in opposite directions. It is convenient to express the field solution as the sum of two contrary propagating waves, i.e., a superposition of ingoing (converging) and outgoing (diverging) waves. In elliptical cylinder symmetry and for even mode these two waves are generally represented by[19]

$$Me_m^{(1),(2)}(\xi,q) = Ce_m(\xi,q) \pm \mathrm{i}Fey_m(\xi,q). \quad (50)$$

For H polarization, the magnetic and electric fields of outgoing elliptical cylinder wave take the form

$$H_z^+ = AMe_m^{(2)}(\xi,q)Ce_m(\xi,q), \quad (51)$$

$$E_\eta^+ = \frac{\mathrm{i}}{\omega\varepsilon h}AMe_m^{(2)'}(\xi,q)Ce_m(\xi,q), \quad (52)$$

and the magnetic and electric fields of ingoing elliptical cylinder wave are presented as

$$H_z^- = AMe_m^{(1)}(\xi,q)Ce_m(\xi,q), \quad (53)$$

$$E_\eta^- = \frac{\mathrm{i}}{\omega\varepsilon h}AMe_m^{(1)'}(\xi,q)Ce_m(\xi,q). \quad (54)$$

The total field of both $H_z$ and $E_\eta$ can be written as

$$H_z = H_z^+ + H_z^-,$$
$$E_\eta = E_\eta^+ + E_\eta^- = \frac{\mathrm{i}}{\omega\varepsilon h}[L_m^{(\mathrm{H2})}H_z^+ + L_m^{(\mathrm{H1})}H_z^-], \quad (55)$$

where

$$L_m^{(\mathrm{H1,2})} = \frac{Me_m^{(1,2)'}(\xi,q)}{Me_m^{(1,2)}(\xi,q)}, \quad (56)$$

and for $l$-th layer we introduce

$$L_{ml}^{(\mathrm{H1,2})} = \frac{Me_m^{(1,2)'}(\xi,q_l)}{Me_m^{(1,2)}(\xi,q_l)}. \quad (57)$$

Now we can define matrix $\boldsymbol{P}$ that converts the magnetic field at inner boundary $\xi = \xi_0$ to a point of $\xi = \xi$ inside the layer. Therefore, we can write in matrix form

$$\begin{pmatrix} H_z^+(\xi,\eta) \\ H_z^-(\xi,\eta) \end{pmatrix} = \boldsymbol{P}^{(\mathrm{H})} \begin{pmatrix} H_z^+(\xi_0,\eta) \\ H_z^-(\xi_0,\eta) \end{pmatrix}, \quad (58)$$

where

$$P^{(\mathrm{H})} = \begin{pmatrix} \dfrac{Me_m^{(2)}(\xi,q)}{Me_m^{(2)}(\xi_0,q)} & 0 \\ 0 & \dfrac{Me_m^{(1)}(\xi,q)}{Me_m^{(1)}(\xi_0,q)} \end{pmatrix}. \tag{59}$$

It is noted that matrix $\boldsymbol{P}$ plays the similar role as the propagation matrix. Based on the continuity of the tangential components of the electric and magnetic fields $H_z$, $E_\eta$, we can express the interface conditions in terms of $H_z^+$, $H_z^-$. We introduce matrix $\boldsymbol{D}$ that can convert the basis $(H_z^+, H_z^-)$ to $(H_z, E_\eta)$, namely

$$\begin{pmatrix} H_z \\ E_\eta \end{pmatrix} = \boldsymbol{D}^{(\mathrm{H})} \begin{pmatrix} H_z^+ \\ H_z^- \end{pmatrix}, \tag{60}$$

where

$$\boldsymbol{D}^{(\mathrm{H})} = \begin{pmatrix} 1 & 1 \\ \dfrac{\mathrm{i}}{\omega\varepsilon h} L_m^{(2)} & \dfrac{\mathrm{i}}{\omega\varepsilon h} L_m^{(1)} \end{pmatrix}. \tag{61}$$

Let us consider how the field transforms when we pass through a boundary between two uniform concentric layers. The traveling-wave amplitude will change due to reflection from the boundary. The condition for continuity of the tangential components of the field at the boundary is written in the form

$$\begin{aligned} \begin{pmatrix} H_{z2}^+ \\ H_{z2}^- \end{pmatrix} &= \boldsymbol{D}_2^{(H)-1} \boldsymbol{D}_1^{(\mathrm{H})} \begin{pmatrix} H_{z1}^+ \\ H_{z1}^- \end{pmatrix} \\ &= \begin{pmatrix} d_{11}^{(\mathrm{H})} & d_{12}^{(\mathrm{H})} \\ d_{21}^{(\mathrm{H})} & d_{22}^{(\mathrm{H})} \end{pmatrix} \begin{pmatrix} H_{z1}^+ \\ H_{z1}^- \end{pmatrix}. \end{aligned} \tag{62}$$

Thus, the transfer matrix through the boundary separating the first and second layers is a product of the direct and inverse transformation matrices $\boldsymbol{D}_2^{(H)-1}\boldsymbol{D}_1^{(\mathrm{H})}$. This matrix is regarded as transmission matrix that links the amplitudes of the waves on the two sides of the interface and it has components as

$$\begin{aligned} d_{11}^{(\mathrm{H})} &= \frac{-\varepsilon_2}{W_2} Me_m^{(1)}(\xi,q_2) Me_m^{(2)}(\xi,q_2) \\ &\quad \times \left[ \frac{L_m^{(\mathrm{H1})}(\xi,q_2)}{\varepsilon_2} - \frac{L_m^{(\mathrm{H2})}(\xi,q_1)}{\varepsilon_1} \right], \end{aligned} \tag{63}$$

$$\begin{aligned} d_{12}^{(\mathrm{H})} &= \frac{-\varepsilon_2}{W_2^{\mathrm{H}}} Me_m^{(1)}(\xi,q_2) Me_m^{(2)}(\xi,q_2) \\ &\quad \times \left[ \frac{L_m^{(\mathrm{H1})}(\xi,q_2)}{\varepsilon_2} - \frac{L_m^{(\mathrm{H1})}(\xi,q_1)}{\varepsilon_1} \right], \end{aligned} \tag{64}$$

$$\begin{aligned} d_{21} &= \frac{\varepsilon_2}{W_2^{\mathrm{H}}} Me_m^{(1)}(\xi,q_2) Me_m^{(2)}(\xi,q_2) \\ &\quad \times \left[ \frac{L_m^{(\mathrm{H2})}(\xi,q_2)}{\varepsilon_2} - \frac{L_m^{(\mathrm{H2})}(\xi,q_1)}{\varepsilon_1} \right], \end{aligned} \tag{65}$$

$$\begin{aligned} d_{22}^{(\mathrm{H})} &= \frac{\varepsilon_2}{W_2^{\mathrm{H}}} Me_m^{(1)}(\xi,q_2) Me_m^{(2)}(\xi,q_2) \\ &\quad \times \left[ \frac{L_m^{(\mathrm{H2})}(\xi,q_2)}{\varepsilon_2} - \frac{L_m^{(\mathrm{H1})}(\xi,q_1)}{\varepsilon_1} \right], \end{aligned} \tag{66}$$

where

$$\begin{aligned} W_2^{\mathrm{H}} &= Me_m^{(1)}(\xi,q_2) Me_m'^{(2)}(\xi,q_2) \\ &\quad - Me_m'^{(1)}(\xi,q_2) Me_m^{(2)}(\xi,q_2). \end{aligned} \tag{67}$$

In the limit of a circular cylinder, $\xi \to \infty$, by using the asymptotic forms of Mathieu functions[19] we have $Me_m^{(1,2)}(\xi,q) \to p'_m H_m^{(1,2)}(k\rho)$ where $H_m^{(1,2)}$ is Hankel functions of the first, second kind and $p'_m$ is a constant.

### 4.2. E polarization

In elliptical cylinder symmetry and for odd mode these two waves are generally represented by

$$Ne_m^{(1),(2)}(\xi,q) = Se_m(\xi,q) \pm \mathrm{i} Gey_m(\xi,q). \tag{68}$$

A monochromatic elliptical cylinder wave propagating in the direction of increasing $\xi$ has the following form for the case of E polarization:

$$E_z^- = ANe_m^{(2)}(\xi,q) Se_m(\xi,q), \tag{69}$$

$$H_\eta^- = \frac{-\mathrm{i}}{\omega\mu h} ANe_m^{(2)'}(\xi,q) Se_m(\xi,q), \tag{70}$$

and a converging wave is given by

$$E_z^+ = ANe_m^{(1)}(\xi,q) Se_m(\xi,q), \tag{71}$$

$$H_\eta^+ = \frac{-\mathrm{i}}{\omega\mu h} ANe_m^{(1)'}(\xi,q) Se_m(\xi,q). \tag{72}$$

Denoting the amplitudes of the electric field for diverging and converging waves as $E_z^+$ and $E_z^-$, respectively, we can write

$$\begin{aligned} &E_z = E_z^+ + E_z^-, \\ &H_\eta = H_\eta^+ + H_\eta^- = \frac{-\mathrm{i}}{\omega\mu h} [L_m^{(\mathrm{E1})} E_z^+ + L_m^{(\mathrm{E2})} E_z^-], \end{aligned} \tag{73}$$

where

$$L_m^{(\mathrm{E1,2})} = \frac{Ne_m^{(1,2)'}(\xi,q)}{Ne_m^{(1,2)}(\xi,q)}, \tag{74}$$

and for $l$-th layer we rewrite

$$L_{ml}^{(\mathrm{E1,2})} = \frac{Ne_m^{(1,2)'}(\xi,q_l)}{Ne_m^{(1,2)}(\xi,q_l)}. \tag{75}$$

Transforming from the $(E_z^+, E_z^-)$ to the $(E_z, H_\eta)$ basis, we can write in matrix form

$$\begin{pmatrix} H_z \\ E_\eta \end{pmatrix} = \boldsymbol{D}^{(\mathrm{E})} \begin{pmatrix} H_z^+ \\ H_z^- \end{pmatrix}, \tag{76}$$

where the transformation matrix is

$$\boldsymbol{D}^{(\mathrm{E})} = \begin{pmatrix} 1 & 1 \\ \dfrac{-\mathrm{i}}{\omega\mu h} L_m^{(\mathrm{E1})} & \dfrac{-\mathrm{i}}{\omega\mu h} L_m^{(\mathrm{E2})} \end{pmatrix}. \tag{77}$$

From Eq. (77) together with the condition of continuity of the tangential field components at the interface of two layers labeled 1 and 2, we have

$$\begin{pmatrix} E_{z2}^{+} \\ E_{z2}^{-} \end{pmatrix} = \boldsymbol{D}_2^{(\mathrm{E})-1}\boldsymbol{D}_1^{(\mathrm{E})}\begin{pmatrix} E_{z1}^{+} \\ E_{z1}^{-} \end{pmatrix} = \begin{pmatrix} d_{11}^{(\mathrm{E})} & d_{12}^{(\mathrm{E})} \\ d_{21}^{(\mathrm{E})} & d_{22}^{(\mathrm{E})} \end{pmatrix}\begin{pmatrix} E_{z1}^{+} \\ E_{z1}^{-} \end{pmatrix}. \tag{78}$$

Thus, the matrix elements of $\boldsymbol{D}_2^{(\mathrm{E})-1}\boldsymbol{D}_1^{(\mathrm{E})}$ can be obtained, with the results

$$d_{11}^{(\mathrm{E})} = \frac{\mu_2}{W_2^{\mathrm{E}}}Ne_m^{(1)}(\xi,q_2)Ne_m^{(2)}(\xi,q_2) \times \left[\frac{L_m^{(\mathrm{E2})}(\xi,q_2)}{\mu_2} - \frac{L_m^{(\mathrm{E1})}(\xi,q_1)}{\mu_1}\right], \tag{79}$$

$$d_{12}^{(\mathrm{E})} = \frac{\mu_2}{W_2^{\mathrm{E}}}Ne_m^{(1)}(\xi,q_2)Ne_m^{(2)}(\xi,q_2) \times \left[\frac{L_m^{(2)}(\xi,q_2)}{\mu_2} - \frac{L_m^{(\mathrm{E2})}(\xi,q_1)}{\mu_1}\right], \tag{80}$$

$$d_{21}^{(\mathrm{E})} = \frac{-\mu_2}{W_2^{\mathrm{E}}}Ne_m^{(1)}(\xi,q_2)Ne_m^{(2)}(\xi,q_2) \times \left[\frac{L_m^{(\mathrm{E1})}(\xi,q_2)}{\mu_2} - \frac{L_m^{(\mathrm{E1})}(\xi,q_1)}{\mu_1}\right], \tag{81}$$

$$d_{22}^{(\mathrm{E})} = \frac{-\mu_2}{W_2^{\mathrm{E}}}Ne_m^{(1)}(\xi,q_2)Ne_m^{(2)}(\xi,q_2) \times \left[\frac{L_m^{(\mathrm{E1})}(\xi,q_2)}{\mu_2} - \frac{L_m^{(\mathrm{E2})}(\xi,q_1)}{\mu_1}\right], \tag{82}$$

where

$$W_2^{\mathrm{E}} = Ne_m^{(1)}(\xi,q_2)Ne_m^{\prime(2)}(\xi,q_2) - Ne_m^{\prime(1)}(\xi,q_2)Ne_m^{(2)}(\xi,q_2). \tag{83}$$

In the limit of a circular cylinder, $\xi \to \infty$, we have $Ne_m^{(1,2)}(\xi,q) \to s_m' H_m^{(1,2)}(k\rho)$. Therefore, in the limit

$$L_{ml}^{(\mathrm{E1},2)} \to \frac{H_m^{(1,2)'}(k_l\rho)}{H_m^{(1,2)}(k_l\rho)}, \tag{84}$$

$$d_{11}^{(\mathrm{E})} \to \frac{\mathrm{i}\pi}{4}k_0\rho H_m^{(1)}(k_2\rho)H_m^{(1)}(k_2\rho) \times [p_2 L_{m1}^{(\mathrm{E2})} - p_1 L_{m1}^{(\mathrm{E1})}], \tag{85}$$

$$d_{12}^{(\mathrm{E})} \to \frac{\mathrm{i}\pi}{4}k_0\rho H_m^{(1)}(k_2\rho)H_m^{(1)}(k_2\rho) \times [p_2 L_{m2}^{(\mathrm{E2})} - p_1 L_{m1}^{(\mathrm{E2})}], \tag{86}$$

$$d_{21}^{(\mathrm{E})} \to \frac{-\mathrm{i}\pi}{4}k_0\rho H_m^{(1)}(k_2\rho)H_m^{(1)}(k_2\rho) \times [p_2 L_{m2}^{(\mathrm{E1})} - p_1 L_{m1}^{(\mathrm{E1})}], \tag{87}$$

$$d_{22}^{(\mathrm{E})} \to \frac{-\mathrm{i}\pi}{4}k_0\rho H_m^{(1)}(k_2\rho)H_m^{(1)}(k_2\rho) \times [p_2 L_{m2}^{(\mathrm{E1})} - p_1 L_{m1}^{(\mathrm{E2})}]. \tag{88}$$

## 5. Reflection and transmission coefficients for elliptical cylinder surfaces: a single elliptical cylinder interface

### 5.1. H polarization

Let us consider a diverging wave incident from medium 1 on the interface between layers 1 and 2. Based on Eq. (62), it is direct to have the reflection coefficient $r_{\mathrm{d}}$ and the transmission coefficient $t_{\mathrm{d}}$ that should satisfy the following relation:

$$\begin{pmatrix} t_{\mathrm{d}}^{(\mathrm{H})} \\ 0 \end{pmatrix} = \boldsymbol{D}_2^{(H)(-1)}\boldsymbol{D}_1^{(\mathrm{H})}\begin{pmatrix} 1 \\ r_{\mathrm{d}}^{(\mathrm{H})} \end{pmatrix}. \tag{89}$$

The amplitude coefficients for reflection and transmission of light are given by

$$r_{\mathrm{d}}^{(\mathrm{H})} = -\frac{d_{21}^{(\mathrm{H})}}{d_{22}^{(\mathrm{H})}} = \frac{L_m^{(\mathrm{H2})}(\xi,q_2)/\varepsilon_2 - L_m^{(\mathrm{H2})}(\xi,q_1)/\varepsilon_1}{L_m^{(\mathrm{H1})}(\xi,q_1)/\varepsilon_1 - L_m^{(\mathrm{H2})}(\xi,q_2)/\varepsilon_2}, \tag{90}$$

$$t_{\mathrm{d}}^{(\mathrm{H})} = \frac{\det(D_2^{(\mathrm{H})(-1)}D_1^{(\mathrm{H})})}{d_{22}^{(\mathrm{H})}} = \frac{Me_m^{(1)}(\xi,q_2)Me_m^{(2)}(\xi,q_2)}{\varepsilon_1 W_2^{\mathrm{H}}} \times \frac{(L_m^{(\mathrm{H1})}(\xi,q_1) - L_m^{(\mathrm{H2})}(\xi,q_1))(L_m^{(\mathrm{H1})}(\xi,q_2) - L_m^{(\mathrm{H2})}(\xi,q_2))}{(L_m^{(\mathrm{H2})}(\xi,q_2)/\varepsilon_2 - L_m^{(\mathrm{H1})}(\xi,q_1)/\varepsilon_1)}. \tag{91}$$

In above calculation to simplify one can consider $D_2^{(\mathrm{H})(-1)}D_2^{(\mathrm{H})} = 1$, as

$$\frac{Me_m^{(1)}(\xi,q_2)Me_m^{(2)}(\xi,q_2)}{W_2^{\mathrm{H}}} \times [L_m^{(\mathrm{H2})}(\xi,q_2) - L_m^{(\mathrm{H1})}(\xi,q_2)] = 1. \tag{92}$$

It is mentioned that these obtained formulae of wave reflection and transmission between two media are analogous to the usual Fresnel's equations in the planar geometry.

### 5.2. E polarization

Let us consider the arrival of a diverging wave on a cylindrical boundary between medium 1 and medium 2. The amplitudes of the incident, reflected, and transmitted waves, 1, $r_{\mathrm{d}}$, and $t_{\mathrm{d}}$, respectively, are related by

$$\begin{pmatrix} 1 \\ r_{\mathrm{d}}^{(\mathrm{E})} \end{pmatrix} = \boldsymbol{D}_2^{(\mathrm{E})(-1)}\boldsymbol{D}_1^{(\mathrm{E})}\begin{pmatrix} t_{\mathrm{d}} \\ 0 \end{pmatrix}. \tag{93}$$

The amplitude coefficients for reflection and transmission of light are given by

$$r_{\mathrm{d}}^{(\mathrm{E})} = \frac{d_{21}^{(\mathrm{E})}}{d_{11}^{(\mathrm{E})}} = \frac{L_m^{(\mathrm{E1})}(\xi,q_2)/\mu_2 - L_m^{(\mathrm{E1})}(\xi,q_1)/\mu_1}{L_m^{(\mathrm{E1})}(\xi,q_1)/\mu_1 - L_m^{(\mathrm{E2})}(\xi,q_2)/\mu_2}, \tag{94}$$

$$t_{\mathrm{d}}^{(\mathrm{E})} = \frac{1}{d_{11}^{(\mathrm{E})}} = W_2^{\mathrm{E}}\{\mu_2 Me_m^{(1)}(\xi,q_2)Me_m^{(2)}(\xi,q_2) \times [L_m^{(2)}(\xi,q_2)/\mu_2 - L_m^{(\mathrm{E1})}(\xi,q_1)/\mu_1]\}^{-1}. \tag{95}$$

## 6. Reflection and transmission in elliptical cylinder multilayer structure

### 6.1. H polarization

We now derive the analytical expressions for reflection and transmission coefficients in elliptical cylinder multilayer structure. Using these formulae, we can examine the wave properties in an elliptical Bragg reflector. It is assumed that an outgoing wave is incident on the interface, $\xi = \xi_0$, between 0 and 1, and then propagates into the final medium $f$, which is assumed to extend from $\xi = \xi_f$ to $\xi = \infty$. The amplitudes of the magnetic field and electric fields at $\xi = \xi_0$ and $\xi = \xi_f$ can be written in terms of the amplitude reflection and transmission coefficients $r_d^{(H)}$ and $t_d^{(H)}$ together with the transfer matrix $R^{(H)}$, with the results

$$\begin{pmatrix} 1 + r_d^{(H)} \\ \dfrac{i}{\omega\varepsilon_l} L_m^{(H2)}(\xi, q_l) + \dfrac{i}{\omega\varepsilon_l} L_m^{(H1)}(\xi, q_l) r_d^{(H)} \end{pmatrix} = \boldsymbol{R}^{(H)-1} \begin{pmatrix} t_d^{(H)} \\ \dfrac{i}{\omega\varepsilon_f} L_m^{(H2)}(\xi, q_f) r_d^{(H)} \end{pmatrix}. \tag{96}$$

The inverse matrix $\boldsymbol{R}^{(H)}$ that is $\boldsymbol{R}^{(H-1)}$ is denoted by

$$\boldsymbol{R}^{(H)-1} = \frac{\delta^{(H)}}{\Delta^{(H)}} \begin{pmatrix} R_{22}^{(H)} & -R_{12}^{(H)} \\ -R_{21}^{(H)} & R_{11}^{(H)} \end{pmatrix}, \tag{97}$$

where

$$\begin{aligned} \delta^{(H)} &= Ce_m(\xi_0, q) Fey'_m(\xi_0, q) \\ &\quad - Fey_m(\xi_0, q) Ce'_m(\xi_0, q), \end{aligned} \tag{98}$$

$$\begin{aligned} \Delta^{(H)} &= Ce_m(\xi, q) Fey'_m(\xi, q) \\ &\quad - Fey_m(\xi, q) Ce'_m(\xi, q), \end{aligned} \tag{99}$$

and

$$\det \boldsymbol{R}^{(H)} = \frac{\Delta^{(H)}}{\delta^{(H)}}. \tag{100}$$

Therefore, the reflection coefficient and transmission coefficient can be determined, namely

$$r_d^{(H)} = \frac{\left[-R_{21}^{(H)} - \dfrac{i}{\omega\varepsilon_l} L_m^{(H2)}(\xi, q_l) R_{22}^{(H)}\right] + \dfrac{i}{\omega\varepsilon_f} L_m^{(H2)}(\xi, q_f) \left[R_{11}^{(H)} + \dfrac{i}{\omega\varepsilon_l} L_m^{(H2)}(\xi, q_l) R_{12}^{(H)}\right]}{\left[R_{21}^{(H)} + \dfrac{i}{\omega\varepsilon_l} L_m^{(1)}(\xi, q_l) R_{22}^{(H)}\right] + \dfrac{i}{\omega\varepsilon_f} L_m^{(H2)}(\xi, q_f) \left[-R_{11}^{(H)} - \dfrac{i}{\omega\varepsilon_l} L_m^{(H1)}(\xi, q_l) R_{12}^{(H)}\right]}, \tag{101}$$

$$t_d^{(H)} = \frac{\dfrac{i}{\omega\varepsilon_l}[L_m^{(H1)}(\xi, q_l) - L_m^{(H2)}(\xi, q_l)]}{\left[R_{21}^{(H)} + \dfrac{i}{\omega\varepsilon_l} L_m^{(H1)}(\xi, q_l) R_{22}^{(H)}\right] + \dfrac{i}{\omega\varepsilon_f} L_m^{(H2)}(\xi, q_f) \left[-R_{11}^{(H)} - \dfrac{i}{\omega\varepsilon_l} L_m^{(H1)}(\xi, q_l) R_{12}^{(H)}\right]}. \tag{102}$$

In limit of circular cylinder[17]

$$r_d^{(H)} \to \frac{[-R_{21}^{(H)} - i p_0 L_{ml}^{(2)} R_{22}^{(H)}] + i p_f L_{mf}^{(H2)} [R_{11}^{(H)} + i p_f L_{ml}^{(H2)} R_{12}^{(H)}]}{[R_{21}^{(H)} + i p_l L_{ml}^{(1)} R_{22}^{(H)}] + i p_f L_{mf}^{(H2)} [-R_{11}^{(H)} + i p_l L_{ml}^{(H1)} R_{12}^{(H)}]}, \tag{103}$$

$$t_d^{(H)} \to \frac{-4\sqrt{\varepsilon_r} p_l}{\pi k \rho_0 H_m^{(2)}(k_l \rho_0) H_m^{(1)}(k_l \rho_0) \left[[R_{21}^{(H)} + i p_l L_{ml}^{(1)} R_{22}^{(H)}] + i p_f L_{mf}^{(H2)} [-R_{11}^{(H)} - i p_l L_{ml}^{(H1)} R_{12}^{(H)}]\right]}. \tag{104}$$

### 6.2. E polarization

Let us consider a cylindrical layered structure located between media with indices of refraction $n_f$ and $n_l$. Let a diverging cylindrical wave fall on the boundary of this layered structure from the medium with index of refraction $n_f$. The amplitudes of the electric and magnetic fields at the inner and outer boundaries of the structure are related to one another by the transfer matrix through the layered structure $\boldsymbol{R}^{(E)}$ as follows:

$$\begin{pmatrix} 1 + r_d^{(E)} \\ \dfrac{i}{\omega\varepsilon_f} L_m^{(E1)}(\xi, q_l) + \dfrac{i}{\omega\varepsilon_f} L_m^{(E2)}(\xi, q_f) r_d \end{pmatrix} = \boldsymbol{R}^{(E)} \begin{pmatrix} t_d^{(E)} \\ \dfrac{i}{\omega\varepsilon_l} L_m^{(E1)}(\xi, q_l) r_d^{(E)} \end{pmatrix}. \tag{105}$$

From above equation we obtain the amplitude reflection and transmission coefficients as

$$r_d^{(E)} = \frac{\left[R_{21}^{(E)} - \dfrac{i}{\omega\varepsilon_f} L_m^{(E1)}(\xi, q_f) R_{11}\right] + \dfrac{i}{\omega\varepsilon_l} L_m^{(E1)}(\xi, q_l) \left[R_{22}^{(E)} - \dfrac{i}{\omega\varepsilon_f} L_m^{(E1)}(\xi, q_f) R_{12}^{(E)}\right]}{\left[\dfrac{i}{\omega\varepsilon_f} L_m^{(E2)}(\xi, q_f) R_{11}^{(E)} - R_{21}^{(E)}\right] + \dfrac{i}{\omega\varepsilon_l} L_m^{(E1)}(\xi, q_l) \left[\dfrac{i}{\omega\varepsilon_f} L_m^{(E2)}(\xi, q_f) R_{12} - R_{22}^{(E)}\right]}, \tag{106}$$

$$t_{\mathrm{d}}^{(\mathrm{E})}=\frac{\dfrac{\mathrm{i}}{\omega\varepsilon_{\mathrm{f}}}\left[L_m^{(\mathrm{E2})}(\xi,q_{\mathrm{f}})-L_m^{(\mathrm{E1})}(\xi,q_{\mathrm{f}})\right]}{\left[\dfrac{\mathrm{i}}{\omega\varepsilon_{\mathrm{f}}}L_m^{(\mathrm{E2})}(\xi,q_{\mathrm{f}})R_{11}^{(\mathrm{E})}-R_{21}^{(\mathrm{E})}\right]+\dfrac{\mathrm{i}}{\omega\varepsilon_l}L_m^{(\mathrm{E1})}(\xi,q_l)\left[\dfrac{\mathrm{i}}{\omega\varepsilon_{\mathrm{f}}}L_m^{(\mathrm{E2})}(\xi,q_{\mathrm{f}})R_{12}^{(\mathrm{E})}-R_{22}^{(\mathrm{E})}\right]}. \tag{107}$$

Similarly, we can obtain expressions for the reflection and transmission coefficients of a converging wave.

## 7. Conclusion

In this work the wave propagation in the elliptical cylinder multilayer structure has been theoretically treated based on the elliptical cylinder wave under H polarization and E polarization. The transfer matrix method in an elliptical cylinder coordinates has been described. Formulae such as the reflection and transmission coefficients have been given. With the derived formulae, we have investigated the reflection responses for the basic elliptical cylinder structures. The obtained results can be generalized for the elliptical cylinder multilayer structures that each layer filled by cold collisionless plasma. The obtained results in the limit of circular cylinder structures have been investigated.

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