Dynamics of quantum discord in a two-qubit system under classical noise*

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We study the quantum discord dynamics of two noninteracting qubits that are, respectively, subject to classical noise. The results show that the dynamics of quantum discord are dependent on both the coupling between the qubits and classical noise, and the average switching rate of the classical noise. In the weak-coupling Markovian region, quantum discord exhibits exponent decay without revival, and can be well protected by increasing the average classical noise switching rate. While in the strong-coupling non-Markovian region, quantum discord reveals slowly decayed oscillations with quick revival by decreasing the average switching rate of the classical noise. Thus, our results provide a new method of protecting quantum discord in a two-qubit system by controlling the coupling between the qubits and classical noise, and the average switching rate of the classical noise.

Keywords: quantum discord, non-Markovian region, classical noise

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1. Introduction

Any bipartite quantum state contains both classical and quantum correlations. The total correlation in a bipartite quantum state is quantified by quantum mutual information.[1,2] The quantum part of the total correlation is called quantum discord (QD), and was initially introduced by Ollivier and Zurek.[3] Over the past decade, the dynamics of QD have been greatly studied.[4–11] This is because QD has been identified as an important physical resource in quantum information processing.[12–14]

In general, QD is different from entanglement and may even be nonzero for certain separable states.[15,16] The authors found that QD may be smaller or larger than the entanglement, as measured with concurrence or negativity. Bipartite quantum system entanglement has been characterized and quantified completely, whereas QD has been characterized and quantified only for particular cases,[15–19] with some analytical results obtained for a restricted subset of X-states.[20–23] Recently, many efforts have been devoted to research on classical correlation (CC) and QD under different systems.[8–11] The dynamics of CC and QD have been studied in the presence of nondissipative decoherence in Ref. [11], where the authors showed that QD is not destroyed by decoherence, but CC decays for times, t, less than decoherence time, $\bar{t}$ ($\bar{t} = -\ln(|C_3|)/2\gamma$). However, when t is greater than $\bar{t}$, CC does not change in time, only the QD is lost. Besides, the behaviors of CC and QD mediated by the cavity in a two-qubit system have also been reported.[24,25] All of the schemes mentioned above have been proposed to investigate the dynamics of CC and QD, which are influenced by quantum environment systems. Little attention has been paid to the dynamics of QD induced by system and classical noise environment.

In this paper, we consider the QD dynamics of a system composed of two noninteracting qubits, each trapped in a classical noise environment. We mainly focus on the QD behavior where two initially entangled qubits experience evolution in the presence of weak-coupling classical noise in the Markovian region and strong-coupling classical noise in the non-Markovian region. We find that the dynamics of QD exhibit exponent decay or oscillations depending on the coupling between the qubits and the classical noise, and the average switching rate of the classical noise.

The layout of this paper is as follows. In Section 2, we introduce the model, and we give the expression of QD for the bipartite X-state in Section 3. The QD dynamics of the qubit system under classical noise are discussed in Section 4, and finally, the conclusion is given in Section 5.

2. Physical model

Let us consider a system of two independent qubits caused by uncorrelated sources of classical random noise. The Hamiltonian describing a single qubit subjected to classical random noise is given by[26,27]

$$H = -\frac{1}{2}B_0\sigma_z - \frac{1}{2}\sum_{i=1}^{n} S_i(t)\boldsymbol{g}\cdot\boldsymbol{\sigma}, \tag{1}$$

where B_0 is the energy splitting between the ground and excited states in the qubit system, $\boldsymbol{g}$ is the coupling strength between the qubits and the classical random noise, $S_i(t)$ is the

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classical random noise sequence with an average switching rate γ, and σ is the triad of the Pauli matrices.

In this paper, we assume that our noise sources satisfy Poisson statistics and are independent of each other, and the correlation function of classical noise is

$$\langle S_i(t)S_{i'}(t')\rangle = \delta_{ii'}g^2\exp(-2\gamma|t-t'|). \tag{2}$$

The power spectrum is obtained by using the Fourier transform

$$S(\omega) = \frac{1}{\pi}\frac{2\gamma g^2}{\omega^2+4\gamma^2}, \tag{3}$$

where the $\gamma > g$ case corresponds to the weak coupling between the qubits and the classical noise in the Markovian region, while $\gamma < g$ belongs to the strong coupling between the qubits and the classical noise in the non-Markovian region.

We aim to compute the qubit system density matrix $\rho_s(t)$, given any $\rho_s(0)$ for a fixed sequence $S(\omega)$

$$\rho_s(t) = \frac{1}{N_q}\left[I + \sum_{i=1}^{N_q^2-1} n_i(S,t)\lambda_i\right], \tag{4}$$

where I is the unit matrix, N_q is the dimensional number of the Hilbert space, $n_i(t) = \mathrm{Tr}[\rho_s(t)\lambda_i]$ is the evolution of a generalized Bloch vector and λ_i are the generators of SU(N). Here, we assume two noninteracting qubits coupling to their own independent noise, λ_i is labelled in the following way: $\lambda_i = \sigma_0 \otimes \sigma_i (i = 0,1,2,3)$; $\lambda_{i'} = \sigma_i \otimes \sigma_j (i = 1,2,3; j = 0,1,2,3)$, with σ_0 the 2×2 identity matrix.

The time evolution of the generalized Bloch vector of the qubit system in the presence of classical noise can be cast into a time-dependent transfer matrix $T(t)$ acting on the generalized Bloch vector[26]

$$\boldsymbol{n}(t) = T(t)\boldsymbol{n}(0), \tag{5}$$

where $T(t) = R_A(t)\otimes R_B(t)$ are the qubit transfer matrices. To simplify, we consider that each qubit is subject to the same classical random noise. Therefore, the single qubit is subject to classical random noise, and the transfer matrix can be written as

$$R_{A/B}(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \xi(t)\cos(B_0t) & \xi(t)\sin(B_0t) & 0 \\ 0 & -\xi(t)\sin(B_0t) & \xi(t)\cos(B_0t) & 0 \\ 0 & 0 & 0 & \exp(-\Gamma t) \end{pmatrix}, \tag{6}$$

where $\xi(t)$ is the dephasing factor of the qubits, and Γ_1 is the longitudinal relaxation rate of the classical noise. These two parameters have an effect on the dynamics of the qubit system.

From Eq. (6), the two types of classical noise[26] are separated by the criterion $\gamma = g\cos\theta$. This is one case $\gamma > g\cos\theta$ for Markovian classical noise, and corresponds to the dephasing function given by

$$\xi(t) = \exp\left[-\left(\frac{\gamma g^2\sin^2\theta}{4\gamma^2+B_0^2} + \frac{g^2\cos^2\theta}{2\gamma}\right)t\right]. \tag{7}$$

The longitudinal relaxation rate is

$$\Gamma = \frac{\gamma g^2\sin^2\theta}{4\gamma^2+B_0^2}, \tag{8}$$

and in the other case $\gamma < g\cos\theta$ for non-Markovian classical noise, the dephasing function is given by

$$\xi(t) = \exp(-\gamma t)\left\{[\cos(g\cos\theta)t] + \frac{\gamma}{g\cos\theta}\sin[(g\cos\theta)t]\right\}, \tag{9}$$

and the longitudinal relaxation rate is

$$\Gamma = \frac{2\gamma g^2\sin^2\theta}{B_0^2}. \tag{10}$$

3. The QD expression of two qubits for the X-state

Let us start with the definition of QD for any bipartite quantum X-state

$$\rho_{AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{11}$$

then, the classical correlation $C(\rho_{AB})$ is defined as

$$C(\rho_{AB}) = S(\rho_A) - \min_{\{\prod_k^B\}} S(\rho_{A|B}), \tag{12}$$

where $S(\rho) = -\mathrm{Tr}(\rho\log_2\rho)$ is the von Neumann entropy. Moreover, the minimum is taken over the set of the positive operator valued measurement $\{\prod_k^B\}$ on subsystem B, and $S(\rho_{A|B})$ is the conditional entropy for subsystem A. QD can be simply obtained by subtracting $C(\rho_{AB})$ from the total amount of correlation

$$Q(\rho_{AB}) = I(\rho_{A:B}) - C(\rho_{AB}), \tag{13}$$

where the total correlation is quantified by the quantum mutual information

$$I(\rho_{A:B}) = S(\rho_A) + S(\rho_B) + \sum_{i=1}^{4}\varepsilon_i\log_2\varepsilon_i, \tag{14}$$

where $\rho_A(\rho_B)$ is the reduced matrix of ρ_{AB} by tracing out B (A), and

$$S(\rho_A) = -(\rho_{11}+\rho_{22})\log_2(\rho_{11}+\rho_{22}) - (\rho_{33}+\rho_{44})\log_2(\rho_{33}+\rho_{44}), \tag{15}$$

$$S(\rho_B) = -(\rho_{11}+\rho_{33})\log_2(\rho_{11}+\rho_{33}) - (\rho_{22}+\rho_{44})\log_2(\rho_{22}+\rho_{44}). \tag{16}$$

The eigenvalues of the density matrix ρ_{AB} in Eq. (11) are given by

$$\begin{aligned}
\varepsilon_1 &= \frac{1}{2}[(\rho_{11}+\rho_{44})+\sqrt{(\rho_{11}-\rho_{44})^2+4|\rho_{14}|^2}],\\
\varepsilon_2 &= \frac{1}{2}[(\rho_{11}+\rho_{44})-\sqrt{(\rho_{11}-\rho_{44})^2+4|\rho_{14}|^2}],\\
\varepsilon_3 &= \frac{1}{2}[(\rho_{22}+\rho_{33})+\sqrt{(\rho_{22}-\rho_{33})^2+4|\rho_{23}|^2}],\\
\varepsilon_4 &= \frac{1}{2}[(\rho_{22}+\rho_{33})-\sqrt{(\rho_{22}-\rho_{33})^2+4|\rho_{23}|^2}].
\end{aligned} \tag{17}$$

It is difficult to calculate the QD by analytical solutions, while for the bipartite quantum X-state described by the density matrix ρ_{AB}, we can derive an expression of the QD[24]

$$Q(\rho_{AB}) = \min(Q_1, Q_2), \tag{18}$$

where

$$\begin{aligned}
&Q_j = H(\rho_{11}+\rho_{33}) + \textstyle\sum_{i=1}^{4} \varepsilon_i \log_2 \varepsilon_i + D_j (j \equiv 1,2),\\
&D_1 = H\left(\frac{1+\sqrt{[1-2(\rho_{33}+\rho_{44})]^2+4(|\rho_{14}|+|\rho_{23}|)^2}}{2}\right),\\
&D_2 = -\textstyle\sum_i \rho_{ii} \log_2 \rho_{ii} - H(\rho_{11}+\rho_{33}),\\
&H(x) = -x\log_2 x - (1-x)\log_2(1-x).
\end{aligned}$$

4. The dynamics of QD for two qubits under classical noise

In this section, we will discuss the QD for the two independent qubits being prepared initially in the generalized Bell states under the classical noise

$$|\Psi\rangle = \alpha|00\rangle + \beta\, \mathrm{e}^{\mathrm{i}\phi}|11\rangle, \tag{19}$$

where $|\alpha|^2 + |\beta\, \mathrm{e}^{\mathrm{i}\phi}|^2 = 1$. To simplify, we assume that each qubit is subject to the identical classical noise, and hence the transfer matrix of the two qubits driven by the same classical random noise can be written as $T(t) = R_A(t) \otimes R_B(t)$.

After some calculations, the time evolution of the density matrix $\rho_{AB}(t)$ for the initial state $|\Psi\rangle$, we can obtain

$$\rho_{AB}(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & 0 & 0 \\ 0 & 0 & \rho_{33}(t) & 0 \\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix}, \tag{20}$$

where

$$\begin{aligned}
\rho_{11}(t) &= \frac{1}{4}[2(2\alpha^2-1)+(1+\mathrm{e}^{-2\Gamma t})],\\
\rho_{22}(t) &= \frac{1}{4}(1-\mathrm{e}^{-2\Gamma t}),\\
\rho_{33}(t) &= \frac{1}{4}(1-\mathrm{e}^{-2\Gamma t}),\\
\rho_{44}(t) &= \frac{1}{4}[(1+\mathrm{e}^{-2\Gamma t})-2(2\alpha^2-1)],\\
\rho_{14}(t) &= \rho_{41}^*(t) = \alpha\beta\xi^2(t)\,\mathrm{e}^{\mathrm{i}\phi}\,\mathrm{e}^{\mathrm{i}2B_0 t}.
\end{aligned}$$

Combining Eqs. (18)–(20), we can compute the QD. Figure 1 shows the QD as a function of t with $g\cos\theta = 0.5$ for different γ in the weak coupling region ($\gamma > g\cos\theta$) for the initial state $|\Psi\rangle = |00\rangle/\sqrt{2} + |11\rangle/\sqrt{2}$. It is obvious that the dynamics of QD exhibit exponent decay and no revival occurs. In Fig. 1(a), we plot the case $\gamma = 5$, $g = 1$, and $\theta = \pi/3$, and the dynamics of QD exponentially decay to zero quickly. When the average switching rate γ of classical noise increases (e.g., $\gamma = 50$ in Fig. 1(b) and $\gamma = 500$ in Fig. 1(c)), it can be seen that the dynamics of QD exponentially decay much more slowly, which means that the QD is long-lived. This can be explained as follows: in the weak coupling region $\gamma > g\cos\theta$, with the average switching rate γ increasing, the dephasing factor $\xi(t)$ of qubits and the longitudinal relation rate Γ decay more slowly, which leads to the QD of the qubits decreasing slowly. Therefore, in the weak coupling region, increasing the average switching rate γ can suppress the dissipation of QD. It is worth pointing out that our results provide a new method of protecting the QD by increasing the average switching rate γ of the classical noise in the weak coupling region.

We plotted the dynamics of QD with $g\cos\theta = 0.5$ for different γ in the strong coupling region ($\gamma < g\cos\theta$) for the initial state given by Eq. (19), as shown in Fig. 2. It is clearly shown that the dynamics of the QD experience damped oscillations and revivals. In Fig. 2(a), we have the case $\gamma = 0.05$, $g = 1$, and $\theta = \pi/3$, and the oscillations amplitude of the QD undergoes a decay after each revival. However, with the decrease in the average switching rate γ (e.g., $\gamma = 0.005$ in Fig. 2(b) and $\gamma = 0.0005$ in Fig. 2(c)), the dynamics of the QD display

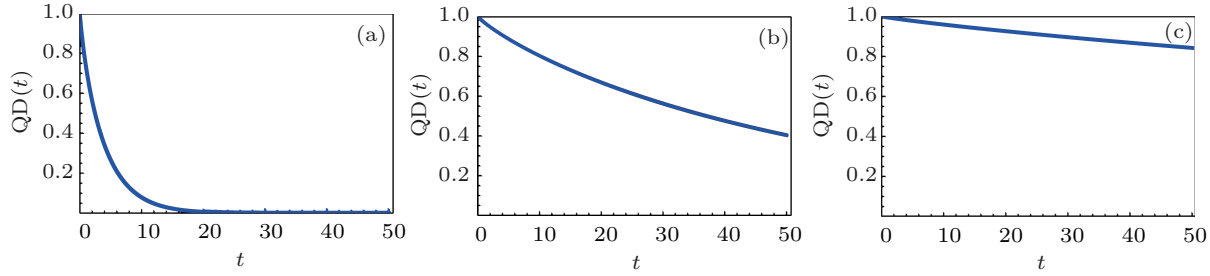


Fig. 1. The dynamics of quantum discord for the initial Bell states given by Eq. (19) with $\phi = \pi/2$, $\alpha = 1/\sqrt{2}$, and $\beta = 1/\sqrt{2}$ in the weak coupling region $g\cos\theta = 0.5$, $B_0 = 1$: (a) $\gamma = 5$; (b) $\gamma = 50$; (c) $\gamma = 500$.

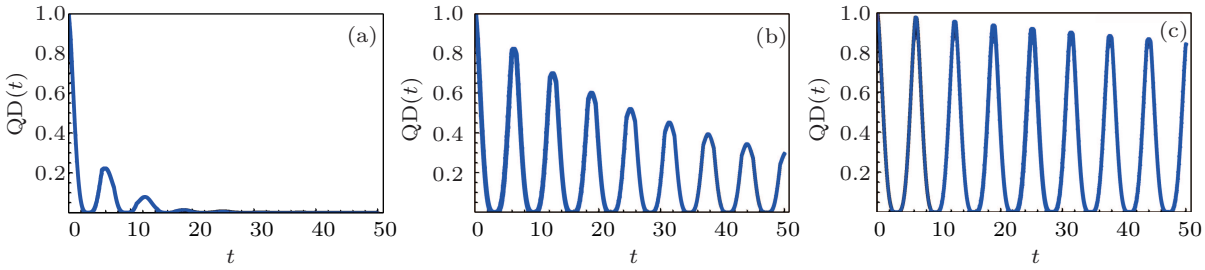


Fig. 2. (color online) The dynamics of quantum discord for the initial Bell states given by Eq. (19) with $\phi = \pi/2$, $\alpha = 1/\sqrt{2}$, and $\beta = 1/\sqrt{2}$ in the strong coupling region $g\cos\theta = 0.5$, $B_0 = 1$: (a) $\gamma = 0.05$; (b) $\gamma = 0.005$; (c) $\gamma = 0.0005$.

slowly decayed oscillations with quick revival. The above results can be interpreted as follows: in the strong coupling region $\gamma < g\cos\theta$, as the average switching rate γ decreases, the decays in the dephasing factor $\xi(t)$ of the qubits and the longitudinal relation rate Γ obviously slow down, the dissipation of energy between the system and the environment decreases, and hence the decays in the oscillations amplitude of QD become slow. Meanwhile, the strong coupling between the qubits and classical noise results in the speeding up of information exchange between the qubit system and classical noise, which leads to the quick revival of QD.

5. Conclusion

We studied the QD dynamics of two noninteracting qubits that are, respectively, subject to classical noise. Our results show that the coupling between the qubits and classical noise, and the average switching rate of the noise play an important role in controlling the QD dynamics in a two-qubit system. We can conclude the following. First, in the weak-coupling Markovian region, QD can be well protected by increasing the average switching rate of the classical noise. Second, in the strong-coupling non-Markovian region, QD exhibits slowly decayed oscillations with quick revival by decreasing the average switching rate of the classical noise. Our results presented a new method of protecting the QD in a two-qubit system by controlling the coupling between the qubits and classical noise, and the average switching rate of the classical noise. This will be helpful in quantum information processing.

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