Entanglement of two two-level atoms trapped in coupled cavities with a Kerr medium*

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In this paper, the entanglement dynamics of two two-level atoms trapped in coupled cavities with a Kerr medium is investigated. We find that the phenomena of entanglement sudden death (ESD) and entanglement sudden birth (ESB) appear during the evolution process. The influences of initial atomic states, Kerr medium, and cavity–cavity hopping rate on the atom–atom entanglement are discussed. The results obtained by the numerical method show that the atom–atom entanglement is strengthened and even prevented from ESD with increasing cavity–cavity hopping rate and Kerr nonlinearity.

Keywords: entanglement sudden death, concurrence, coupled cavities, Kerr medium

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1. Introduction

Quantum entanglement, one of the most striking consequences of the nonlocal quantum correlation, is important in quantum physics for understanding the nonlocality of quantum mechanics[1] and quantum computation and communications.[2] However, the entanglement would undergo decoherence due to the unavoidable interaction with the environment. Hence, it is of great importance to study the quantum entanglement dynamics. Yu and Eberly have showed that the entanglement may decrease abruptly to zero in a finite time due to the influence of quantum noise, this striking phenomenon is the so-called entanglement sudden death (ESD).[3,4] Opposite to the currently extensively discussed ESD, the entanglement sudden birth (ESB) is the creation of the entanglement, in which initially unentangled qubits can be entangled after a finite evolution time.[5] Recently, an explicit expression of the ESD time for various entangled states has been presented in Ref. [6]. In particular, the ESD has been demonstrated experimentally.[7,8]

As an available candidate system, the cavity quantum electrodynamics (CQED) with atoms (or ions, atomic ensembles, etc) interacting with quantized cavity modes has been proved to be an excellent device to produce entanglement.[9,10] Based on the CQED, there are several proposals about the research of quantum dynamics via the J–C models and the generalized J–C models.[11–15] However, in most of those schemes, the investigated atoms are usually trapped in one cavity or two isolated cavities. Recently, there has been growing interest in coupled-cavity arrays,[16–19] for which some potential technologies have been demonstrated in experiments.[19,20] This system has potential applications in quantum information processing as well,[21] since it can control and measure individual sites and provide almost lossless guiding and coupling of light pulses at slow group velocities. Schemes have been proposed[22–25] to realize quantum communication, deterministic entanglement, and phase gates between two atomic qubits trapped in separate optical cavities, which are coupled by an optical fiber via coherent dynamics. In such systems, the atoms are usually used for the storage of quantum information and the cavity modes act as the carrying qubits. Therefore, it is important to study the entanglement dynamics for atoms in the coupled cavities.[26–31] However, the cavities in the previous works do not contain a Kerr medium. In this paper, we consider a system composed of two two-level atoms resonantly interacting with two coupled cavities filled with a Kerr medium, and study the influences of the initial atomic state, the Kerr medium, and the cavity–cavity hopping rate on the entanglement dynamics of the two atoms. It shows that the phenomena of ESD and ESB appear simultaneously and are sensitive to the initial state as well as the system parameters. Moreover, the ESD effect can be weakened by enhancing the cavity–cavity hopping strength and the Kerr nonlinearity.

2. Model

The system under consideration is depicted in Fig. 1. The system consists of two coupled single-mode cavities (denoted

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as a and b) filled with a nonlinear Kerr medium. Each cavity contains a two-level atom (labeled as A and B). The excited and the ground states of each atom are denoted by $|e\rangle$ and $|g\rangle$, respectively. Atom A resonantly interacts only with field a and similarly for atom B and field b. The two cavities are coupled so that photons may hop between them.

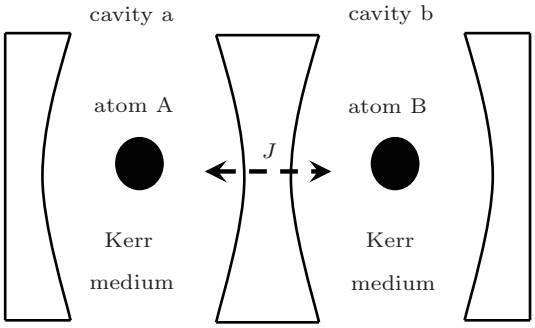


Fig. 1. Schematic diagram of the coupled-cavity system.

The interaction Hamiltonian of the considered system with the rotating-wave approximation in the interaction picture can be written as ($\hbar = 1$)

$$\begin{aligned} H_I &= g_1(aS_{\rm A}^+ + a^\dagger S_{\rm A}^-) + g_2(bS_{\rm B}^+ + b^\dagger S_{\rm B}^-) \\ &\quad + \chi_1 a^\dagger a a^\dagger a + \chi_2 b^\dagger b b^\dagger b + J(ab^\dagger + a^\dagger b), \end{aligned} \tag{1}$$

where $S_i^\pm$ ($i = {\rm A}, {\rm B}$) are the raising and lowering operators of the i-th two-level atom, a (b) and $a^\dagger$ ($b^\dagger$) are the annihilation and creation operators of the field in cavity a (b). The coupling strength g_i between the cavity and the atom is obtained by changing the location of the atom embedded in the cavity. In Eq. (1), χ_j ($j = 1, 2$) is the dispersive part of the third-order nonlinearity of the Kerr medium in the corresponding cavity, and J is the cavity–cavity hopping rate.

We take the two atoms initially in the Φ-type Bell state, and both cavities are in the vacuum state. Thus the initial state of the system can be expressed as

$$|\Phi(0)\rangle = \left\{ \cos\left(\frac{\theta}{2}\right)|ee\rangle + \sin\left(\frac{\theta}{2}\right)|gg\rangle \right\} \otimes |00\rangle, \tag{2}$$

and the state of the system at any time t can be written as

$$\begin{aligned} |\Phi(t)\rangle &= y_1|ee00\rangle + y_2|gg00\rangle + y_3|eg01\rangle \\ &\quad + y_4|ge10\rangle + y_5|eg10\rangle + y_6|ge01\rangle \\ &\quad + y_7|gg11\rangle + y_8|gg02\rangle + y_9|gg20\rangle, \end{aligned} \tag{3}$$

where $|\alpha\beta\eta\xi\rangle$ is the abbreviation of state $|\alpha\rangle_{\rm A}|\beta\rangle_{\rm B}|\eta\rangle_{\rm a}|\xi\rangle_{\rm b}$ of atoms A, B and cavities a, b. Substituting state $|\Phi(t)\rangle$ into the Schrödinger equation in the interaction picture, one can obtain the equations of amplitudes

$$\begin{aligned} {\rm i}\dot{y}_1 &= g_1 y_4 + g_2 y_3, \\ {\rm i}\dot{y}_2 &= 0, \\ {\rm i}\dot{y}_3 &= g_1 y_7 + g_2 y_1 + \chi_2 y_3 + J y_5, \\ {\rm i}\dot{y}_4 &= g_1 y_1 + g_2 y_7 + \chi_1 y_4 + J y_6, \\ {\rm i}\dot{y}_5 &= \sqrt{2} g_1 y_9 + \chi_1 y_5 + J y_3, \\ {\rm i}\dot{y}_6 &= \sqrt{2} g_2 y_8 + \chi_2 y_6 + J y_4, \\ {\rm i}\dot{y}_7 &= g_1 y_3 + g_2 y_4 + (\chi_1 + \chi_2) y_7 + \sqrt{2} J (y_8 + y_9), \\ {\rm i}\dot{y}_8 &= \sqrt{2} g_2 y_6 + 4\chi_2 y_8 + \sqrt{2} J y_7, \\ {\rm i}\dot{y}_9 &= \sqrt{2} g_1 y_5 + 4\chi_1 y_9 + \sqrt{2} J y_7. \end{aligned} \tag{4}$$

Since the explicit expressions of these parameters are very complicated, here the results are given in terms of numerical calculations. In the atomic basis $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$, the atom–atom density operator $\rho_{\rm AB}$ is written as

$$\rho_I^{\rm AB}(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{5}$$

where

$$\begin{aligned} &\rho_{11} = |y_1|^2, \quad \rho_{14} = y_1 y_2^*, \quad \rho_{22} = |y_3|^2 + |y_5|^2, \\ &\rho_{23} = y_3 y_6^* + y_5 y_4^*, \quad \rho_{32} = y_6 y_3^* + y_4 y_5^*, \\ &\rho_{33} = |y_4|^2 + |y_6|^2, \quad \rho_{41} = y_2 y_1^*, \\ &\rho_{44} = |y_2|^2 + |y_7|^2 + |y_8|^2 + |y_9|^2. \end{aligned}$$

3. The entanglement between the two atoms

Several different measures have been proposed to identify the entanglement between two qubits, and we choose the Wootters entanglement measure,[32] the concurrence C, defined as

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \tag{6}$$

where $\lambda_1, \ldots, \lambda_4$ are the eigenvalues of matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. Here ρ is the density matrix representing the quantum state, and the matrix elements are taken with respect to the basis $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$. The range of the concurrence is from 0 to 1. For unentangled qubits, $C = 0$, whereas $C = 1$ for the maximally entangled qubits.

We mainly study the entanglement dynamics of the bipartite system in the symmetrical case where the two atom–cavity coupling rates are equal ($g_1 = g_2 = g$), since we find that the properties of the atom–atom entanglement in the unsymmetrical case ($g_1 \neq g_2$) are similar to those in the symmetrical one. As a result, the concurrence of the density matrix (5) is expressed as

$$C^{\rm AB}(t) = 2\max\{0, |\rho_{32}| - \sqrt{\rho_{11}\rho_{44}}, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}\}. \tag{7}$$

Firstly, the evolution of the concurrence for various initial atomic states (different parameters θ) can be investigated via numerical simulation. For simplicity, we firstly assume $\chi_1 = \chi_2 = \chi$. The time evolution of the concurrence $C_{\rm AB}$ is plotted in Fig. 2, where we have set $\chi = J = 0.5g$ and $\theta =$

0, π, $\pi/8$, $\pi/4$, $\pi/2$. The following results are obtained. (i) The separate state $|gg\rangle$ ($\theta = \pi$) can never be entangled at any time, while the separate two atoms, initially in the two-excitation state $|ee\rangle$ ($\theta = 0$), can be entangled with a delayed time, i.e., the ESB effect occurs (Fig. 2(a)). This interesting phenomenon does not appear in Ref. [31]. (ii) For the initial entangled states, as shown in Fig. 2(b), the entanglement of the two atoms can fall abruptly to zero and will remain zero for a period of time before it recovers, which is the so-called ESD. Comparing the solid ($\theta = \pi/8$) and the dashed ($\theta = \pi/4$) lines with the dashed-dotted line ($\theta = \pi/2$), we can find that the larger the initial entanglement is, the shorter the state will be in the disentangled separable state.

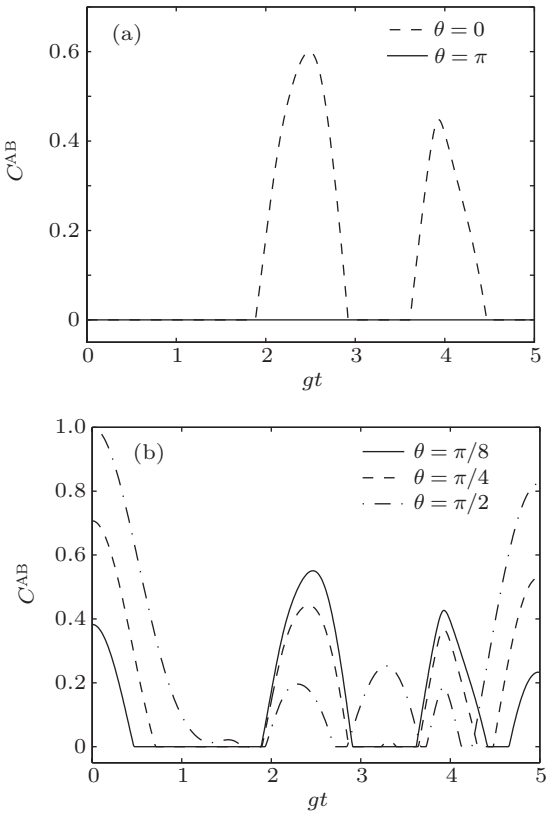


Fig. 2. Influence of the initial atomic state on the atom–atom entanglement with $\chi = J = 0.5g$ for various initial atomic states ((a) and (b)).

Secondly, the influence of the Kerr medium on the atom–atom entanglement is taken into account for different initial atomic states, where we have set $J = 0.5g$ and $\theta = 0$ (Fig. 3), $\pi/2$ (Fig. 4). In the case of $\theta = 0$, the atoms are disentangled in the whole evolution time when neither of the coupled cavities is filled with the Kerr medium ($\chi_1 = \chi_2 = 0$). However, the phenomena of ESB and ESD occur when one of the cavities is filled with the Kerr medium ($\chi_1 = g$, $\chi_2 = 0$) or both cavities are filled with the Kerr medium ($\chi_1 = \chi_2 = g$), as seen from Fig. 3(a). In order to further discuss the entanglement dynamics of the system influenced by the Kerr medium, we assume $\chi_1 = \chi_2 = \chi$. The concurrence versus the scale time gt and the Kerr nonlinearity χ/g is plotted in Fig. 3(b). It is apparent that the stronger the Kerr nonlinearity is, the earlier the entanglement sudden birth appears. In order to study the dependence of the maximum entanglement obtained during the time evolution on the Kerr nonlinearity χ, we extract the maximum value on the time scale $(0, 100/g)$, which is large enough for our model. As shown in Fig. 3(c), the maximum value of the entanglement increases first and goes through a peak at about $\chi = g$, then drops close to zero. It implies that we can control the occurrence time of the ESB and obtain a larger degree of entanglement through selecting the Kerr nonlinearity appropriately.

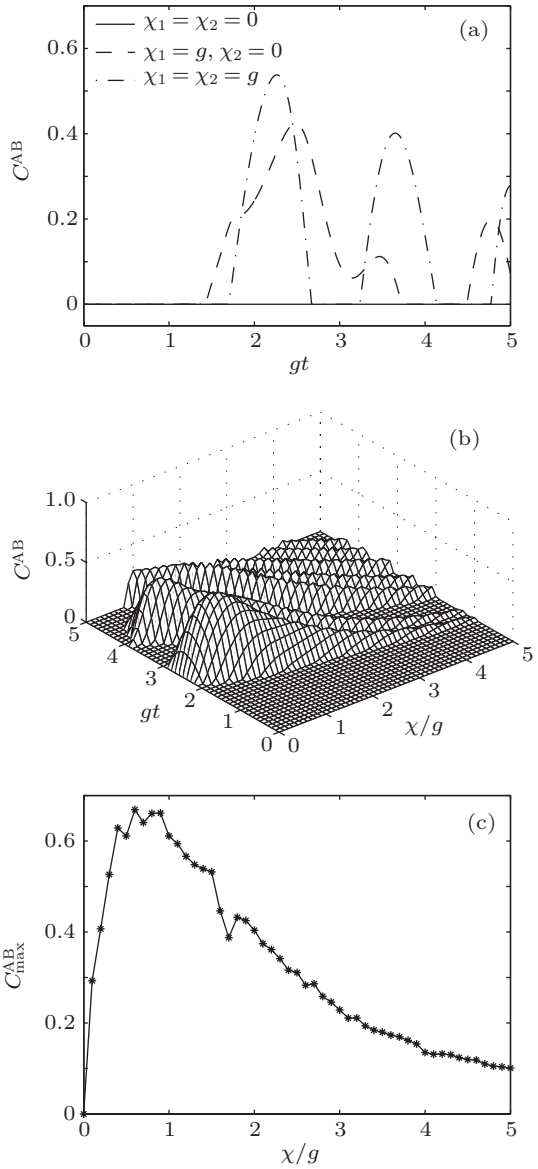


Fig. 3. Influence of the Kerr medium on the atom–atom entanglement with $J/g = 0.5$ and $\theta = 0$: (a) concurrence versus scale time gt, (b) concurrence versus scale time gt and Kerr nonlinearity χ/g, (c) the maximum value of the concurrence versus Kerr nonlinearity χ/g.

While for the initial maximally entangled state ($\theta = \pi/2$), the ESD effect always takes place no matter whether the Kerr medium is considered or not, as seen in Fig. 4(a). Compar-

ing the solid and the dashed lines with the dashed-dotted line, we find that the more cavities containing the Kerr medium, the more slowly the entanglement decreases to zero, and the shorter the atoms remain in the disentangled separable state, which indicates that the Kerr effect is more obvious when both cavities have the Kerr medium. With the increase of the Kerr nonlinearity, the ESD effect completely disappears, and the atom–atom entanglement is enhanced gradually, which can be seen from Fig. 4(b). This implies that the strong Kerr nonlinearity can prevent the two atoms maximally entangled initially from disentangling. Figure 4(c) shows the minimum values of entanglement for different values of χ, from which we can see that the ESD effect is eliminated absolutely when $\chi > 1.9g$.

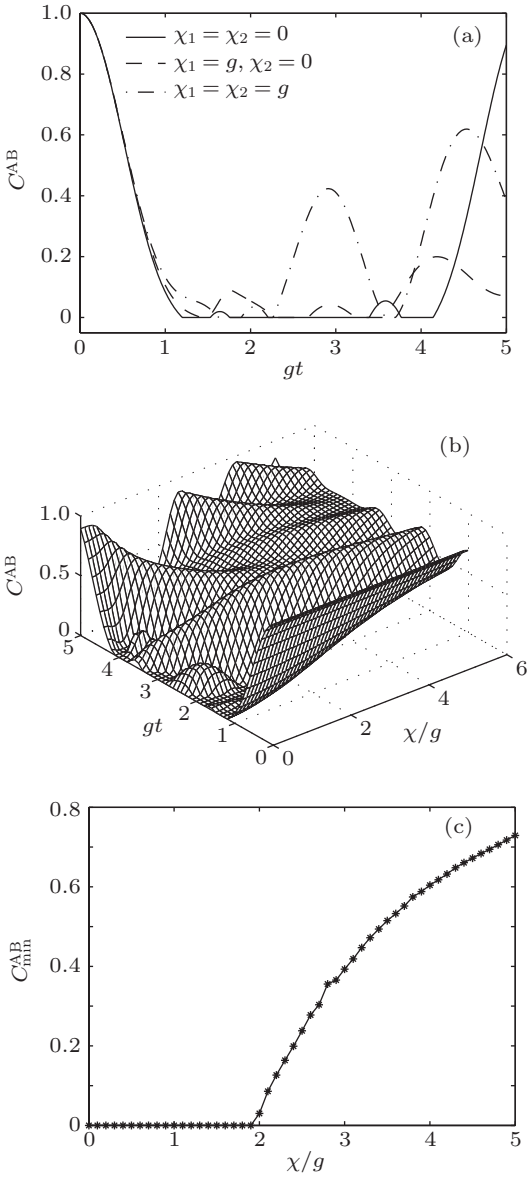


Fig. 4. Influence of the Kerr medium on the atom–atom entanglement with $J/g = 0.5$ and $\theta = \pi/2$: (a) concurrence versus scale time gt, (b) concurrence versus scale time gt and Kerr nonlinearity χ/g, (c) the minimum value of the concurrence versus Kerr nonlinearity χ/g.

Finally, the concurrence C_{AB} versus the scale time gt and the ratio J/g is shown in Fig. 5, where we choose $\chi = 0.5g$ and $\theta = 0, \pi/8, \pi/4, \pi/2$. To do so in experiment, the hopping rate J can be controlled by controlling the length of the cavity in principle, which can be considered as an adjustable parameter. The numerical results demonstrate the following results. (i) The initial separable state ($\theta = 0$) can be entangled if $J > 0$, i.e., the ESB effect takes place. (ii) For the initial entangled states, the results are shown in Figs. 5(b)–5(d), where each plot is divided into two regions with the boundary $J/g = \Gamma$. In the region of $J/g < \Gamma$, the ESD effect always occurs, while

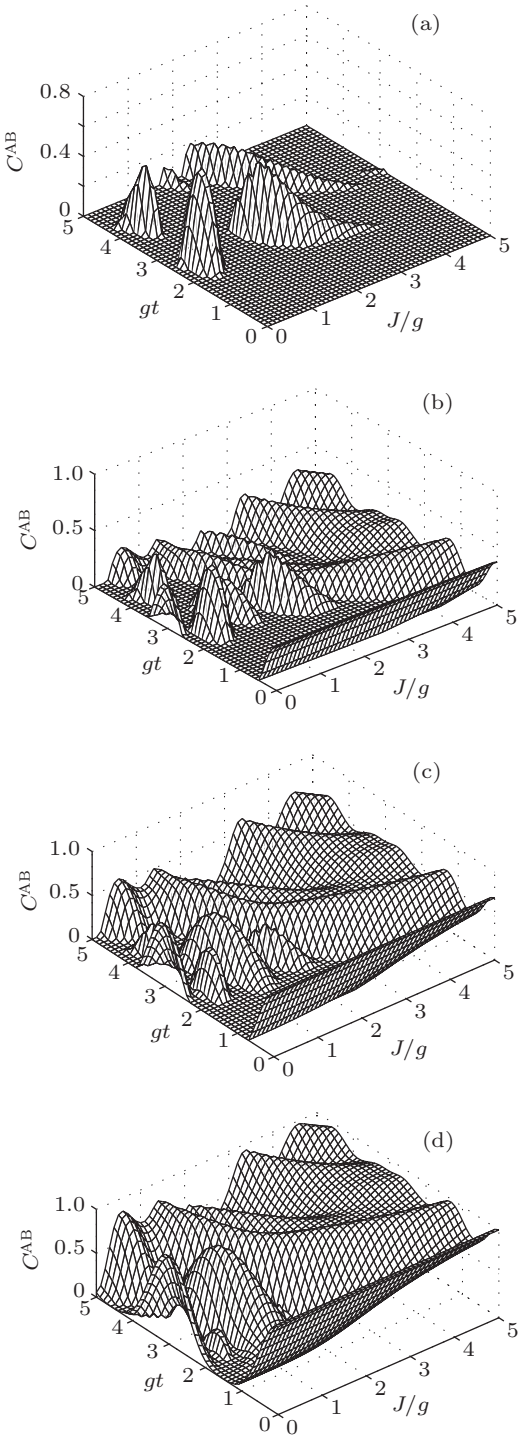


Fig. 5. Influence of the cavity–cavity hopping rate on the atom–atom entanglement with $\chi = 0.5g$ and (a) $\theta = 0$, (b) $\theta = \pi/8$, (c) $\theta = \pi/4$, (d) $\theta = \pi/2$.

for the case of $J/g > \Gamma$, the ESD effect disappears and the atom–atom entanglement is enhanced as J/g increases, which indicates that the ESD effect can be eliminated completely by a strong hopping strength between two cavities. The Γ is dependent on the initial state. The larger the initial degree of entanglement, the smaller the boundary of the cavity–cavity hopping rate, as seen in Fig. 6. The $\Gamma = 4.7, 3.5, 2.7$ correspond to the cases of $\theta = \pi/8, \pi/4, \pi/2$, respectively. (iii) The maximal entanglement can exist only when the two atoms are initially entangled maximally.

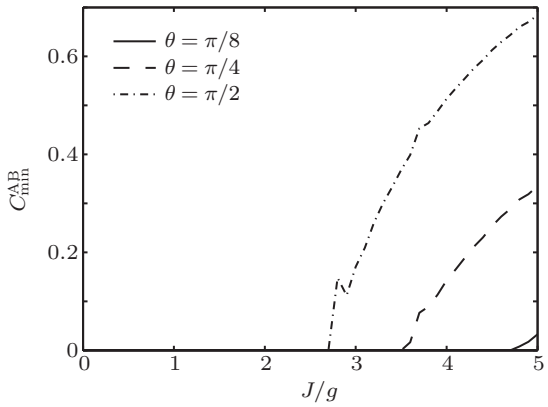


Fig. 6. The minimum concurrence C^{AB}_{min} versus J/g with $\chi = 0.5g$.

4. Conclusion

We have investigated the entanglement dynamics between two atoms trapped in two coupled cavities filled with a Kerr medium by using the entanglement degree concurrence. The entanglement dynamics shows distinctive features for different initial atomic states, different Kerr nonlinear coefficients, and different hopping rates. Based on the numerical calculation, we find that the phenomena of ESD and ESB appear during the evolution process. By selecting different Kerr nonlinear coefficients and different cavity–cavity hopping rates, we can not only stabilize the entanglement between the two atoms, but also effectively suppress the appearance of the ESD. Physically, this phenomenon originates from the fact that the nonlocal exchange interaction happens between the two atoms when they interact with the coupled cavities. This may be useful for quantum information processing based on the entanglement. The two coupled cavities can be achieved either by using adjacent cavities or optical fiber connected cavities. The cavity with the Kerr medium can be realized if the cavity contains two different species of Rydberg atoms: one behaves like the two-level atom, and the other behaves like the Kerr medium.[33] In practice, the Kerr nonlinear coefficient is usually very small. Fortunately, the Kerr nonlinearities can be enhanced by using electromagnetically induced transparency.[34]

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[2] Bennett C H and Divincenzo D P 2000 *Nature* **404** 247
[3] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404
[4] Yu T and Eberly J H 2006 *Phys. Rev. Lett.* **97** 140403
[5] Ficek Z and Tanas R 2008 *Phys. Rev. A* **77** 054301
[6] Ikram M, Li Fl and Zubairy M S 2007 *Phys. Rev. A* **75** 062336
[7] Almeida M P, de Melo F, Hor-Merll M, Salles A, Walborn S P, Ribeiro P H S and Davidovich L 2007 *Science* **316** 579
[8] Xu J S, Li C F, Gong M, Zou X B, Shi C H, Chen G and Guo G C 2010 *Phys. Rev. Lett.* **104** 100502
[9] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
[10] Kimble H J 2008 *Nature* **453** 1023
[11] Yönac M and Eberly J H 2008 *Opt. Lett.* **33** 270
[12] Yang Q, Yang M and Gao Z L 2008 *Chin. Phys. Lett.* **25** 825
[13] Yang R C, Li J, Wang J M and Zhang T C 2011 *Commun. Theor. Phys* **56** 429
[14] Ouyang X C, Fang M F, Kang G D, Deng X J and Huang L Y 2010 *Chin. Phys. B* **19** 030309
[15] Shan C J and Xia Y J 2006 *Acta Phys. Sin.* **55** 1585 (in Chinese)
[16] Cho J, Angelakis D G and Bose S 2008 *Phys. Rev. A* **78** 022323
[17] Ogden C D, Irish E K and Kim M S 2008 *Phys. Rev. A* **78** 063805
[18] Pado S and Antonio Q 2012 *Phys. Rev. A* **86** 043811
[19] Hartmann M J, Brandão F G S L and Plenio M B 2006 *Nature Phys.* **2** 849
[20] Hartmann M J, Brandão F G S L and Plenio M B 2008 *Laser Photonics Rev.* **2** 527
[21] Lin G W, Zou X B, Lin X M and Guo G C 2009 *Appl. Phys. Lett.* **95** 224102
[22] Cirac J I, Zoller P, Kimble H J and Mabuchi H 1997 *Phys. Rev. Lett.* **78** 3221
[23] Serafini A, Mancini S and Bose S 2006 *Phys. Rev. Lett.* **96** 010503
[24] Yin Z Q and Li F L 2007 *Phys. Rev. A* **75** 012324
[25] Zheng S B 2009 *Appl. Phys. Lett. A* **94** 154101
[26] Zheng S B, Yang Z B and Xia Y 2010 *Phys. Rev. A* **81** 015804
[27] Song J, Sun X D, Xia Y and Song H S 2011 *Phys. Rev. A* **83** 052309
[28] Schwendimann P and Quattropani A 2012 *Phys. Rev. A* **86** 043811
[29] Peng J, Wu Y W and Li X J 2012 *Chin. Phys. B* **21** 060302
[30] Pan G X, Xiao R J and Zhou L 2013 *Chin. Phys. B* **22** 010307
[31] Yang R C, Zhang P F, Guo Y Q and Zhang T C 2012 *Commun. Theor. Phys.* **57** 195
[32] Wootters K 1998 *Phys. Rev. Lett.* **80** 2245
[33] Amitabh J and Puri R R 1992 *Phys. Rev. A* **45** 5056
[34] Lukin M D and Imamoğlu A 2001 *Nature* **413** 273