A control method applied to mixed traffic flow for the coupled-map car-following model*

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In light of previous work [*Phys. Rev. E* **60** 4000 (1999)], a modified coupled-map car-following model is proposed by considering the headways of two successive vehicles in front of a considered vehicle described by the optimal velocity function. The non-jam conditions are given on the basis of control theory. Through simulation, we find that our model can exhibit a better effect as $p = 0.65$, which is a parameter in the optimal velocity function. The control scheme, which was proposed by Zhao and Gao, is introduced into the modified model and the feedback gain range is determined. In addition, a modified control method is applied to a mixed traffic system that consists of two types of vehicle. The range of gains is also obtained by theoretical analysis. Comparisons between our method and that of Zhao and Gao are carried out, and the corresponding numerical simulation results demonstrate that the temporal behavior of traffic flow obtained using our method is better than that proposed by Zhao and Gao in mixed traffic systems.

Keywords: traffic flow, coupled-map car-following model, optimal velocity function, feedback control scheme system

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1. Introduction

In recent years, traffic problems have been widely investigated. A variety of approaches have been found to describe the properties of traffic flow jams, and many traffic models have been developed, including macroscopic models where traffic flow is viewed as a compressible fluid formed by vehicles, and microscopic models where an individual vehicle is represented by a particle and the vehicle traffic is treated as a system of interacting particles driven far from equilibrium.[1–10] In 1961, Newell[11] presented a car-following model described by a differential first-order equation, and first defined the optimal velocity (OV) function. In 1995, Bando *et al.*[12] presented a traffic model called the optimal velocity car-following model (OVM). Since then, much research has been carried out on this model.

It is noteworthy that Konishi *et al.*[13] utilized a dynamic version of the decentralized delayed-feedback control scheme to suppress the congestion described by the KKH model in 1999. In 2006, Zhao and Gao[14] presented another simple method to suppress the congested state in traffic systems. A control signal that incorporates the effect of the velocity difference between the preceding and the considered vehicle is presented. In the model analyzed by Han,[15] the control signal is proportional to the velocity difference between the $(i-1)$-th vehicle in front and the i-th vehicle. The simulation results are in good agreement with the theoretical analysis. Research on the control signal has been carried out recently.[15–17] In 2012, the control method was applied to the car-following model.[18,19]

Until now, modifications to the coupled-map (CM) car-following model were mainly on the delayed-feedback control signals, but we pay attention to the modification of the OV function in the model. In particular, a modified control method applied to a mixed traffic system that consists of two types of vehicle is presented. In our control scheme, the effect of the velocity difference between the following and the considered vehicle is also considered.

The paper is organized as follows. In Section 2, the modified CM car-following traffic model is presented, and its stability analyzed. The value of the p parameter is also derived. In Section 3, two kinds of delayed-feedback control schemes (the modified control method and the method proposed by Zhao and Gao) are introduced to suppress the traffic jam in the model, and comparisons are carried out. Finally, conclusions are drawn in Section 4.

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2. The CM car-following model

2.1. The model

Our investigations are based on the KKH model. The leading vehicle is described as follows:

$$x_0(n+1) = v_0 T + x_0(n), \tag{1}$$

where $x_0(n) > 0$ is the position of the leading vehicle at time $t = nT$, $v_0 > 0$ is its velocity, which is a constant, $T > 0$ is the time interval, and $x_0(0) > 0$ is its initial position.

The following vehicles are given by

$$x_i(n+1) = v_i(n)T + x_i(n), \ \ (i = 1,2...,N), \tag{2}$$

where $x_i(n) > 0$ is the position of the i-th vehicle, $v_i(n) > 0$ is its velocity, and N is the total number of vehicles. The speed of the following vehicles is obtained by

$$v_i(n+1) = \alpha_i[V_i^{\rm op}(y_{i-1}(n), y_i(n)) - v_i(n)]T + v_i(n), \tag{3}$$

where $\alpha_i > 0$ is the sensitivity of the i-th vehicle driver, and $V_i^{\rm op}(y_{i-1}(n), y_i(n))$ is the OV function, which depends on the headway distances $y_{i-1}(n)$ and $y_i(n)$

$$y_{i-1}(n) = x_{i-2}(n) - x_{i-1}(n), \ \ y_i(n) = x_{i-1}(n) - x_i(n). \tag{4}$$

The OV function is described as

$$\begin{aligned} & V_i^{\rm op}(y_{i-1}(n), y_i(n)) \\ = \ & \frac{v_i^{\max}}{2}\left[1 + \bar{H}_{\rm sat}\left(2\frac{(1-p)y_{i-1}(n) + p y_i(n) - \eta_i}{\zeta_i}\right)\right], \end{aligned} \tag{5}$$

where p $(p > 0.5)$ is a parameter that can be determined by later computer simulations. The traditional OV function

$$V_i^{\rm op}(y_i(n)) = \frac{v_i^{\max}}{2}\left[1 + \bar{H}_{\rm sat}\left(2\frac{y_i(n) - \eta_i}{\zeta_i}\right)\right] \tag{6}$$

is replaced by Eq. (5). The saturation function $\bar{H}_{\rm sat}$ is

$$\bar{H}_{\rm sat} = \begin{cases} +1, \ \text{if } \rho > +1, \\ \rho, \quad \text{if } -1 \le \rho \le +1, \\ -1, \ \text{if } \rho < -1, \end{cases} \tag{7}$$

where $v_i^{\max} > 0$ is the maximum velocity, $\eta_i > 0$ is the neutral headway distance, and $\zeta_i > 0$ is the parameter. We assume that v_0 is less than the maximum speed of all the following vehicles, i.e.,

$$v_0 < v_i^{\max}, \ \ (i = 1,2,...,N). \tag{8}$$

This assumption guarantees the existence of a vehicle group.

To avoid collisions and backward motions, all the vehicles adopt the full-braking action if $(1-p)y_{i-1}(n) + p y_i(n) < y_i^{\min}$, then $x_i(n+1) = x_i(n)$, and $v_i(n+1) = 0$. This action means that the i-th vehicle stops suddenly when the whole headway distance is less than the safety distance $y_i^{\min}$.

2.2. The non-jam condition

The dynamics of the i-th vehicle can be given as

$$v_i(n+1) = \alpha_i[V_i^{\rm op}(y_{i-1}(n), y_i(n)) - v_i(n)]T + v_i(n), \tag{9}$$

$$y_i(n+1) = v_{i-1}(n)T - v_i(n)T + y_i(n). \tag{10}$$

The steady state of systems (9) and (10) is

$$\begin{aligned} [v_i^*, y_i^*]^{\rm T} \ = \ & \left[v_0, \left(1 - \frac{1}{p}\right)^{i-1}\left(y_1 - \frac{v_0}{r_i} + \frac{\zeta_i}{2} - \eta_i\right)\right. \\ & \left. + \frac{v_0}{r_i} - \frac{\zeta_i}{2} + \eta_i\right]^{\rm T}, \end{aligned} \tag{11}$$

where $r_i = v_i^{\max}/\zeta_i$. Obviously, the necessary and sufficient condition for the existence of the steady state (11) is condition (8). Let us consider an error system around steady state (11), that is, $\delta v_i(n) = v_i(n) - v_i^*$, $\delta y_i(n) = y_i(n) - y_i^*$, we have the following system:

$$\boldsymbol{q}_i(n+1) = \boldsymbol{A}_i \boldsymbol{q}_i(n) + \boldsymbol{B}_i \boldsymbol{q}_{i-1}(n), \tag{12}$$

$$\delta v_i(n) = \boldsymbol{C}\boldsymbol{q}_i(n), \tag{13}$$

where

$$\boldsymbol{q}_i = \begin{pmatrix} \delta v_i(n) \\ \delta y_i(n) \end{pmatrix}, \ \ \boldsymbol{A}_i = \begin{pmatrix} 1 - \alpha_i T & p\alpha_i r_i T \\ -T & 1 \end{pmatrix},$$

$$\boldsymbol{B}_i = \begin{pmatrix} 0 & (1-p)\alpha_i r_i T \\ T & 0 \end{pmatrix}, \ \ \boldsymbol{C} = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

The characteristic polynomial of $\boldsymbol{A}_i$ is

$$P_i(z) = z^2 + a_i z + b_i, \tag{14}$$

where $a_i = \alpha_i T - 2$ and $b_i = 1 - \alpha_i T + p\alpha_i r_i T^2$. The transfer function from $q_{i-1}(n)$ to $\delta v_i(n)$ is described by

$$\begin{aligned} \boldsymbol{G}_i(z) \ &= \ \boldsymbol{C}(z\boldsymbol{I}_{2\times 2} - \boldsymbol{A}_i)^{-1}\boldsymbol{B}_i \\ &= \ \frac{\begin{pmatrix} p\alpha_i r_i T^2 & (z-1)(1-p)\alpha_i r_i T \end{pmatrix}}{P_i(z)}. \end{aligned} \tag{15}$$

According to the condition of no traffic jams obtained in the KKH model, we need the following conditions: (a) $P_i(z)$ is stable; (b) $\max_{|z|=1}|\boldsymbol{G}_i(z)| \le 1$, $(i = 1,2,...,N)$.

Then, we give a simple, sufficient and necessary condition of no traffic jams.

Theorem 1 Assuming that all the vehicles satisfy the following conditions, there are no traffic jams in systems (12) and (13).

$$0 < r_i \le \min\left\{\frac{pT(2 - \alpha_i T) + |2 - \alpha_i T|\sqrt{p^2T^2 + 4(1-p)^2}}{2(1-p)^2\alpha_i T}, \frac{1}{pT}\right\}, \tag{16}$$

$$0 < \alpha_i < \frac{4}{(2 - pr_iT)T}. \tag{17}$$

Proof See Appendix A.

This condition implies that the traffic system is robust. The robustness enhances the stability of a system, under which the amplitude of the transient disturbance decreases with time even if the structure and parameters of the system are variational. We confirm that the traffic jam never occurs when all the vehicles satisfy conditions (16) and (17). The next subsection will determine parameter p.

2.3. Numerical simulations

Our simulations are based on the CM car-following model under open boundary conditions. For simplicity, we assume that all vehicles have the same parameters.

The initial conditions are set as follows:

$$x_i(0) = \sum_{j=i+1}^{N} y_j^*, \;\; y_i(0) = y_i^*, \;\; v_i(0) = v_i^*, \;\; (i = 1, 2, ..., N).$$

To guarantee the steady state for our model, we define $y_1 =$ 8 m. Considering a situation where the lead vehicle stops suddenly for a short time,

$$x_0(n) = 0, \;\; \text{for} \;\; 100\ \text{s} \le nT \le 102\ \text{s}.$$

Figure 1 shows the speed of the first, the 25th and the 50th vehicle with four different p parameters ($p = 0.60, 0.65, 0.70, 0.75$) in our OV function. From these simulations, we can see that the amplitude of speed fluctuation increases with the increase in p. However, to guarantee that the vehicle group can run with larger velocities, we define $p = 0.65$ in our model.

From the simulations above, we find that our model can obtain a better effect as $p = 0.65$. The next section will consider how to suppress the traffic jam by two types of control method: the modified control method and the method proposed by Zhao and Gao.

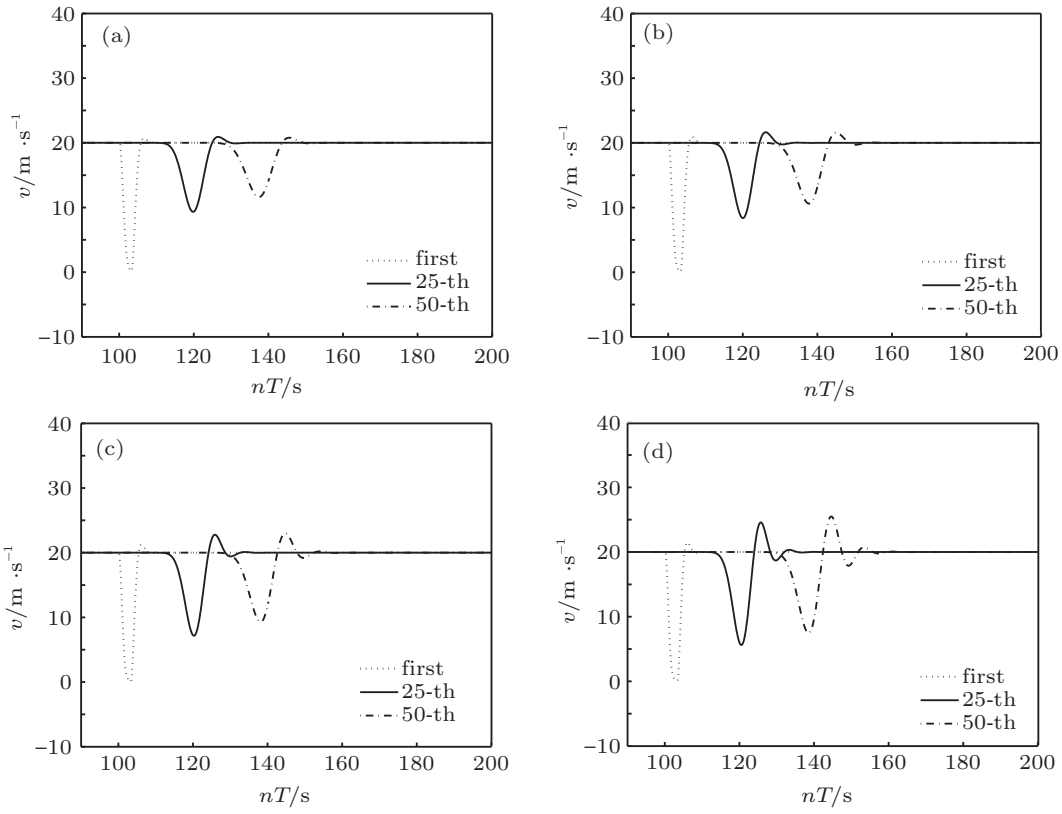


Fig. 1. The velocity of the first, the 25th, and the 50th vehicle of the running traffic model. Our model with (a) $p = 0.60$, (b) $p = 0.65$, (c) $p = 0.70$, and (d) $p = 0.75$.

3. The control scheme of the traffic model

3.1. The control scheme proposed by Zhao and Gao

The control signal $u_i(n)$ proposed by Zhao and Gao is described as follows:

$$u_i(n) = k[v_{i-1}(n) - v_i(n)], \tag{18}$$

where k ($k > 0$) is the feedback gain, which can be adjusted. The control signal term is added to Eq. (9)

$$\begin{aligned} v_i(n+1) \;=\; & \alpha_i[V_i^{\text{opt}}(y_{i-1}(n), y_i(n)) - v_i(n)]T \\ & + v_i(n) + u_i(n). \end{aligned} \tag{19}$$

Around the steady state (11), the system can be described by

$$\boldsymbol{q}_i(n+1) = \bar{\boldsymbol{A}}_i q_i(n) + \bar{\boldsymbol{B}}_i q_{i-1}(n), \tag{20}$$

$$\delta v_i(n) = \boldsymbol{C}\boldsymbol{q}_i(n), \tag{21}$$

where

$$\bar{\boldsymbol{A}}_i = \begin{pmatrix} 1-\alpha_i T - k & p\alpha_i r_i T \\ -T & 1 \end{pmatrix}, \quad \bar{\boldsymbol{B}}_i = \begin{pmatrix} k & (1-p)\alpha_i r_i T \\ T & 0 \end{pmatrix}.$$

The characteristic polynomial of $\bar{\boldsymbol{A}}_i$ is

$$\bar{P}_i(z) = z^2 + \bar{a}_i z + \bar{b}_i, \tag{22}$$

where $\bar{a}_i = \alpha_i T - 2 + k$, and $\bar{b}_i = 1 - \alpha_i T + p\alpha_i r_i T^2 - k$. The transfer function from $q_{i-1}(n)$ to $\delta v_i(n)$ is described by

$$\begin{aligned} \bar{\boldsymbol{G}}_i(z) &= \boldsymbol{C}(z\boldsymbol{I}_{2\times 2} - \bar{\boldsymbol{A}}_i)^{-1}\bar{\boldsymbol{B}}_i \\ &= \frac{(p\alpha_i r_i T^2 + k(z-1), (z-1)(1-p)\alpha_i r_i T)}{\bar{P}_i(z)}. \end{aligned} \tag{23}$$

According to the condition of no traffic jams, we also need the following conditions: ($\bar{a}$) $\bar{P}_i(z)$ is stable; ($\bar{b}$) $\max_{|z|=1}|\bar{\boldsymbol{G}}_i(z)| \le 1$, $(i = 1, 2, ..., N)$.

Theorem 2 Assuming that all the vehicles satisfy the following conditions, there are no traffic jams in the system described by Eqs. (20) and (21).

If $\alpha_i T - 2 < 0$, $0 < p\alpha_i r_i T^2 < 2$ then

$$\begin{aligned} &\max\{0, p\alpha_i r_i T^2 - \alpha_i T\} \\ &< k \le \min\left\{K, \frac{4 - 2\alpha_i T + p\alpha_i r_i T^2}{2}\right\}. \end{aligned} \tag{24}$$

If $\alpha_i T - 2 \ge 0$ and $0 < p\alpha_i r_i T^2 < 2$, then

$$\max\{0, p\alpha_i r_i T^2 - \alpha_i T, K\} \le k < \frac{4 - 2\alpha_i T + p\alpha_i r_i T^2}{2}, \tag{25}$$

where

$$\begin{aligned} K = {} & [(1-p)^2\alpha_i^2 r_i^2 T^2 - 4 - 4\alpha_i^2 T^2 - 2p\alpha_i r_i T^2 \\ & + p\alpha_i^2 r_i T^3 + 4\alpha_i T][2(\alpha_i T - 2)]^{-1}. \end{aligned}$$

Proof It is similar to the proof of Theorem 1.

Theorem 2 provides us with the ability to design the gain k. Substituting the parameter values into Eq. (24), we obtain $0 < k \le 0.9065$.

3.2. The modified control scheme

The modified control scheme is described as follows:

$$\bar{u}_i(n) = k_1 \Delta v_i(n) + k_2 H(-\Delta v_{i+1}(n))|\Delta v_{i+1}(n)|, \tag{26}$$

where

$$H(-\Delta v_{i+1}(n)) = \begin{cases} 0, & \text{if } \Delta v_{i+1}(n) \ge 0, \\ 1, & \text{if } \Delta v_{i+1}(n) < 0, \end{cases} \tag{27}$$

$\Delta v_i(n) = v_{i-1}(n) - v_i(n)$, $k_1, k_2 > 0$ are the feedback gains, which can be adjusted. This means that if $\Delta v_{i+1}(n) \ge 0$, then $\bar{u}_i(n) = u_i(n)$, and our method reduces to the Zhao–Gao scheme. If $\Delta v_{i+1}(n) < 0$, then

$$\begin{aligned} \bar{u}_i(n) &= k_1 \Delta v_i(n) + k_2 |\Delta v_{i+1}(n)| \\ &= k_1 v_{i-1}(n) - (k_1 + k_2) v_i(n) + k_2 v_{i+1}(n). \end{aligned} \tag{28}$$

The control signal term is added to Eq. (9) as follows:

$$\begin{aligned} v_i(n+1) = {} & \alpha_i [V_i^{\text{opt}}(y_{i-1}(n), y_i(n)) - v_i(n)]T \\ & + v_i(n) + \bar{u}_i(n). \end{aligned} \tag{29}$$

Around the steady state (11), the system can be described by

$$\boldsymbol{q}_i(n+1) = \widetilde{\boldsymbol{A}}_i \boldsymbol{q}_i(n) + \widetilde{\boldsymbol{B}}_i \boldsymbol{Q}(n), \tag{30}$$

$$\delta \boldsymbol{v}_i(n) = \boldsymbol{C}\boldsymbol{q}_i(n), \tag{31}$$

where

$$\widetilde{\boldsymbol{A}}_i = \begin{pmatrix} 1 - \alpha_i T - k_1 - k_2 & p\alpha_i r_i T \\ -T & 1 \end{pmatrix},$$

$$\widetilde{\boldsymbol{B}}_i = \begin{pmatrix} k_1 & k_2 & (1-p)\alpha_i r_i T \\ T & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{Q}(n) = \begin{pmatrix} \delta v_{i-1}(n) \\ \delta v_{i+1}(n) \\ \delta y_{i-1}(n) \end{pmatrix}.$$

The characteristic polynomial of $\widetilde{\boldsymbol{A}}_i$ is

$$\widetilde{P}_i(z) = z^2 + \widetilde{a}_i z + \widetilde{b}_i, \tag{32}$$

where $\widetilde{a}_i = \alpha_i T - 2 + k_1 + k_2$, and $\widetilde{b}_i = 1 - \alpha_i T + p\alpha_i r_i T^2 - k_1 - k_2$. The transfer function from $Q(n)$ to $\delta v_i(n)$ is described by

$$\begin{aligned} &\widetilde{\boldsymbol{G}}_i(z) \\ &= \boldsymbol{C}(z\boldsymbol{I}_{2\times 2} - \widetilde{\boldsymbol{A}}_i)^{-1}\widetilde{\boldsymbol{B}}_i \\ &= \frac{(k_1(z-1) + p\alpha_i r_i T^2, k_2(z-1), (z-1)(1-p)\alpha_i r_i T)}{\widetilde{P}_i(z)}. \end{aligned} \tag{33}$$

According to the condition of no traffic jams, we also need the following conditions: (ã) $\widetilde{P}_i(z)$ is stable; (b̃) $\max_{|z|=1}|\widetilde{\boldsymbol{G}}_i(z)| \le 1$, $(i = 1, 2, ..., N)$.

Theorem 3 Assuming that all the vehicles satisfy the following conditions, there are no traffic jams in the system described by Eqs. (30) and (31).

If $2\alpha_i T - 4 + 2k_1 - p\alpha_i r_i T^2 > 0$, $0 < p\alpha_i r_i T^2 < 2$, then

$$\begin{aligned} &\max\{0, p\alpha_i r_i T^2 - \alpha_i T - k_1, K_2\} \\ &\le k_2 < \frac{4 - 2\alpha_i T + p\alpha_i r_i T^2}{2} - k_1. \end{aligned} \tag{34}$$

If $2\alpha_i T - 4 + 2k_1 - p\alpha_i r_i T^2 < 0$, $0 < p\alpha_i r_i T^2 < 2$, then

$$\begin{aligned} &\max\{0, p\alpha_i r_i T^2 - \alpha_i T - k_1\} \\ &< k_2 \le \min\left\{K_2, \frac{4 - 2\alpha_i T + p\alpha_i r_i T^2}{2} - k_1\right\}, \end{aligned} \tag{35}$$

where

$$\begin{aligned} K_2 = {} & [4(k_1 - 1) - 2p\alpha_i r_i T^2 - \alpha_i^2 T^2 \\ & + p\alpha_i^2 r_i T^3 + (1-p)^2\alpha_i^2 r_i^2 T^2 + (4 - 2k_1)\alpha_i T] \\ & \times [2\alpha_i T - 4 + 2k_1 - p\alpha_i r_i T^2]^{-1}. \end{aligned}$$

Proof It is similar to the proof of Theorem 1.

3.3. Numerical simulations

In this part, we consider a situation where the lead vehicle stops suddenly four times: $x_0(n) = 0$, for $nT = 100$–103 s, 120–123 s, 140–143 s, 160–163 s. The other parameters are not altered.

Figure 2 shows the simulation results for the traffic model with the control method in the Zhao–Gao model. Substituting the parameter values into Eq. (24), we obtain $0 < k \leq 0.9065$. In this paper, we shall define $k = 0.82$. As we can see, the traffic system has no traffic jams.

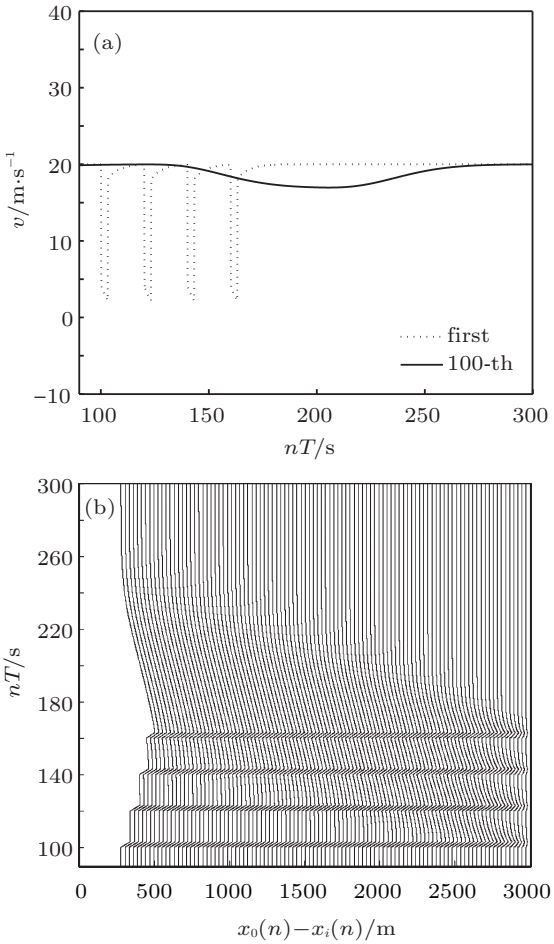


Fig. 2. Simulations for the controlled traffic model with $k = 0.82$. (a) The temporal speed behavior of two vehicles. (b) A space–time plot of the controlled traffic model.

In order to compare with the control method proposed by Zhao and Gao, we shall define $k_1 = 0.82$ in our method. Substituting the parameter values into Eq. (35), we obtain $0 < k_2 \leq 0.1237$ and define $k_2 = 0.1$ for simplicity. Figure 3 shows the simulation result for the traffic model with our control scheme. Figure 3(a) shows the temporal speed behavior of two vehicles, and figure 3(b) shows the space–time plot of the controlled traffic model.

From these simulations, we find that there is little difference between the controlled system with our method and the method in the Zhao–Gao model. It is interesting that our method is no better than the method proposed by Zhao and Gao, although we add the effect of the following vehicle into the control scheme. By analyzing, we find that the condition $\Delta v_{i+1}(n) < 0$ (i.e., the velocity of the following vehicle is larger than that of the considered vehicle) hardly appears under the assumption that all vehicles have the same parameters in our model. Therefore, the effect of velocity difference between the following and the considered vehicle is almost zero. In this case, our method is almost equivalent to that of Zhao and Gao.

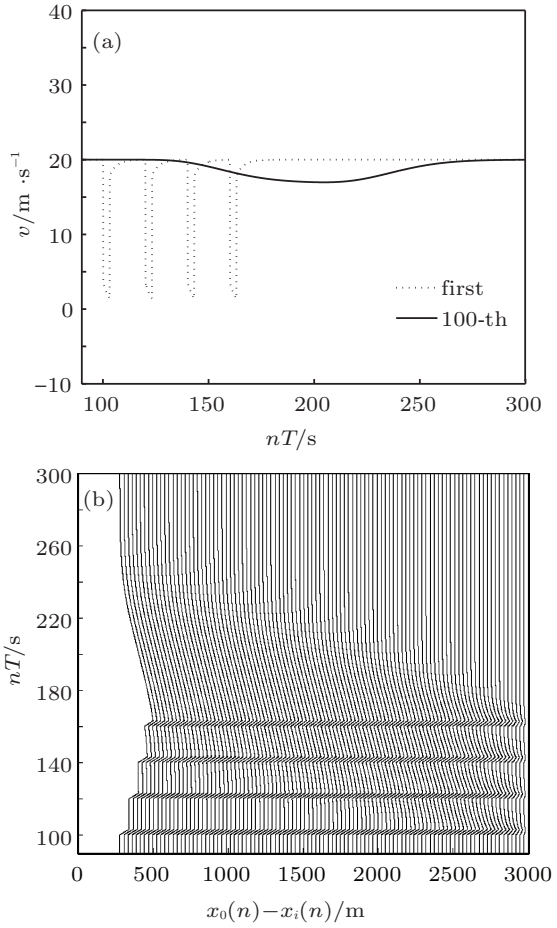


Fig. 3. Simulations for the controlled system with $k_1 = 0.82$, $k_2 = 0.1$. (a) The temporal speed behavior of two vehicles. (b) A space–time plot of the controlled traffic model.

Next, let us introduce two types of vehicle, as shown in Table 2. They are different in $v^{\max}$, α, and $y^{\min}$. The other parameters ζ, T, and η are the same as in Table 1. We consider 100 vehicles of two different types. Among them, the probability of the second type of vehicle is a quarter. For each vehicle, the type is selected randomly. We also consider the situation where the lead vehicle stops suddenly four times. This short stop can be considered as a kind of external disturbance for our traffic model.

Because of the introduction of the second type of vehicle, the value of gain k_2 will play a role in the control scheme in suppressing traffic congestion. Hence, let us first determine

the gain k_2. Figure 4 shows the simulation of the velocity of the first and the 100th vehicle of the running traffic model with $k_1 = 0.82$. Figures 4(a)–4(d) show four types of parameter k_2 ($k_2 = 0.03, 0.06, 0.09, 0.12$) in our control scheme. From these simulations, we can see that the amplitude of speed fluctuation decreases with the increase in k_2. Thus, we define $k_2 = 0.12$ in the mixed traffic system.

Table 1. The numerical simulation parameters.

	$v^{\max}$	α	$y^{\min}$	η	T	ζ
value	33.6	2.0	7.02	25	0.1	23.3
unit	m	s^{-1}	m	m	s	m

Table 2. The two types of vehicle.

Type	$v^{\max}$	α	$y^{\min}$
I	33.3	2.0	7.02
II	26	2.2	7.5

Figure 5 shows the simulation result of the mixed traffic system. Figure 5(a) shows the speed of the first and 100th vehicles of the running traffic model without control. Figure 5(b) is a space–time plot of the running traffic model corresponding to Fig. 5(a). The horizontal axis represents the distance between the lead vehicle and the following vehicle. The vertical axis indicates the time development. The lead vehicle suddenly stops four times. As we can see, the stop disturbances propagate backwards and the traffic jam occurs without any control signal. In Figs. 5(d) and 5(f), it is observed that there is no oscillating behavior and traffic jam when the control signals are introduced into the system. All the vehicles are running smoothly without full braking in the two controlled systems. Figures 5(c) and 5(e) are the temporal velocity of the first and the 100th vehicles, corresponding to Figs. 5(d) and 5(f), respectively. It should be noted that there are smaller velocity amplitudes in the system controlled by our method, compared with the system controlled by the method of Zhao and Gao.

From the simulation results above, it can be seen that there is no oscillating behavior and no traffic jam with the two kinds of control signal in the system. It is observed that the simulation results indicate that the amplitude of the velocity fluctuation increases less than that in the Zhao–Gao model. Comparison demonstrates that both our method and that of Zhao and Gao can be used to suppress the traffic jam, and the traffic system exhibits better temporal behavior by using our method.

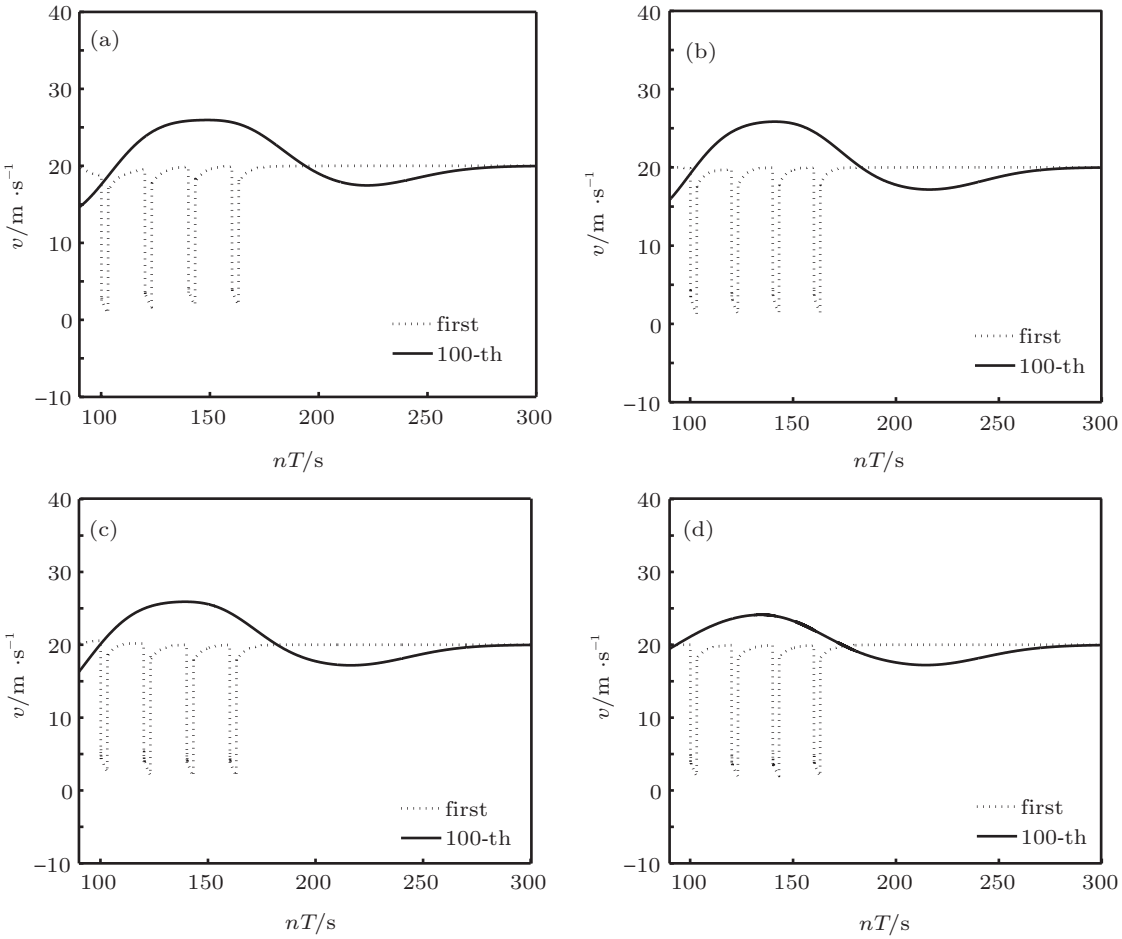


Fig. 4. The velocity of the first and 100th vehicle of the running traffic model with $k_1 = 0.82$. Our model with (a) $k_2 = 0.03$, (b) $k_2 = 0.06$, (c) $k_2 = 0.09$, and (d) $k_2 = 0.03$.

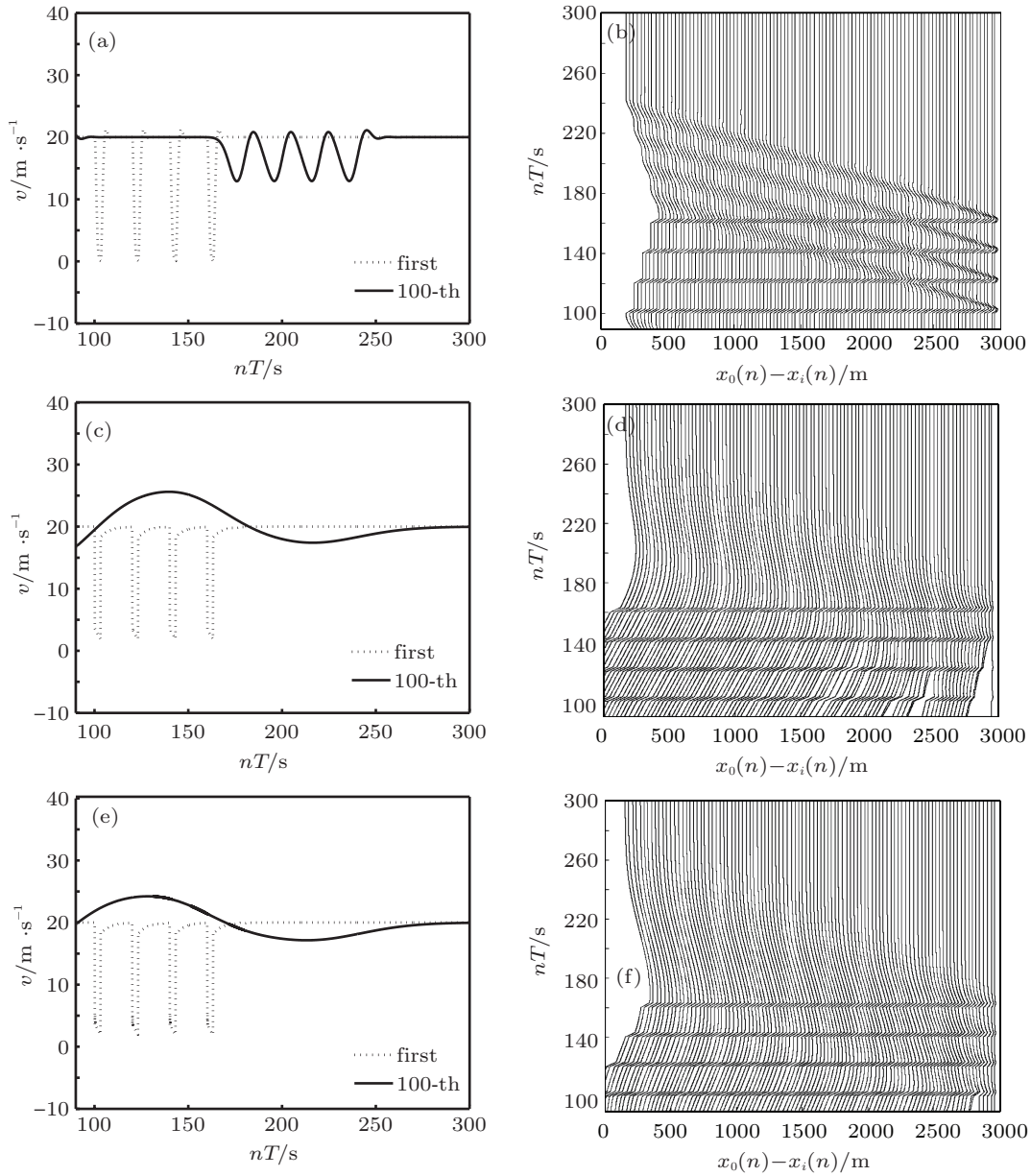


Fig. 5. The traffic model consisting of two types of vehicle. (a), (b) The uncontrolled system. (c)–(f) The system controlled by the method in the Zhao–Gao model and our model.

4. Summary

We proposed a modified CM car-following model, in which the headways of two successive vehicles in front of the considered vehicle is incorporated into the OV function. Through simulation, we found that our model can achieve a better effect as $p = 0.65$. In addition, a modified control method applied in a mixed traffic system, which consists of two types of vehicle, is presented on the basis of the method proposed by Zhao and Gao. The ranges of the feedback gains k_1 and k_2 are determined through theoretical analysis. Comparisons between our method and that of Zhao and Gao are carried out. The corresponding numerical simulation results indicate that both our method and that of Zhao and Gao can be used to suppress traffic jams, and that the traffic system exhibits better temporal behavior by using our method.

Appendix A

Firstly, according to the stability criterion, $P_i(z)$ is stable if

$$\begin{cases} 1+a_i+b_i>0, \\ 1-b_i>0, \\ 1-a_i+b_i>0. \end{cases}$$

We obtain

$$0<r_i<\frac{1}{pT},\ \ 0<\alpha_i<\frac{4}{(2-pr_iT)T}. \tag{A1}$$

Then we consider $\max_{|z|=1}|\boldsymbol{G}_i(z)|\le 1$.

The absolute of the transfer function $\boldsymbol{G}_i(z)$ can be described as

$$|\boldsymbol{G}_i(z)|=\sqrt{\boldsymbol{G}_i(z)\boldsymbol{G}_i(\bar{z})}.$$

Let $z = \cos\theta + \mathrm{i}\sin\theta$, $\theta \in [0, 2\pi]$. We can obtain

$$|\boldsymbol{G}_i(z)| = \sqrt{\frac{p^2\alpha_i^2 r_i^2 T^4 + 2(1-\cos\theta)(1-p)^2\alpha_i^2 r_i^2 T^2}{4b_i cos^2\theta + 2a_i(1+b_i)cos\theta + (b_i-1)^2 + a_i^2}}.$$

It is obvious that if the conditions: (1) $|\boldsymbol{G}_i(1)| \leq 1$; (2) $|\boldsymbol{G}_i(-1)| \leq 1$ are satisfied, then condition (b) holds. (i) It is easy to confirm that $|\boldsymbol{G}_i(1)| = 1$ for any parameters. (ii) Condition (2) can be reduced to

$$(1-p)^2\alpha_i^2 r_i^2 T^2 + (p\alpha_i^2 T^3 - 2p\alpha_i T^2)r_i - (\alpha_i T - 2)^2 \leq 0.$$

Then we have

$$R_{11} \leq r_i \leq R_{12},$$

where

$$R_{11} = \frac{pT(2-\alpha_i T) - |2-\alpha_i T|\sqrt{p^2T^2 + 4(1-p)^2}}{2(1-p)^2\alpha_i T},$$

$$R_{12} = \frac{pT(2-\alpha_i T) + |2-\alpha_i T|\sqrt{p^2T^2 + 4(1-p)^2}}{2(1-p)^2\alpha_i T}.$$

Considering condition (A1), we obtain expressions (16) and (17).

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