Periodic synchronization of community networks with non-identical nodes uncertain parameters and adaptive coupling strength*

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In this paper, we propose a novel approach for simultaneously identifying unknown parameters and synchronizing time-delayed complex community networks with nonidentical nodes. Based on the LaSalle's invariance principle, a criterion is established by constructing an effective control identification scheme and adjusting automatically the adaptive coupling strength. The proposed control law is applied to a complex community network which is periodically synchronized with different chaotic states. Numerical simulations are conducted to demonstrate the feasibility of the proposed method.

Keywords: community networks, periodic synchronization, adaptive coupling strength, uncertain parameters

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1. Introduction

In the early 1960s, Erdös and Rényi[1] proposed a random-graph model, which laid a solid foundation of modern network theory. In a random different network, each pair of nodes is connected with a certain probability. To design a transition from a regular network to a random network, Watts and Strogatz[2] proposed a novel small-world network model. Then, Newman and Watts[3] modified it to generate another variant of the small-world model. Later, Barabsi and Albert[4] proposed a scale-free network model. Since then, complex networks have received more and more attention due to their wide applications in many fields. A recent introduction to networks has been given by Newman.[5] A feature of many real networks is the community structure. Generally, a community refers to groups of nodes which probably share common properties within the networks. Subsequently, a large amount of work has been devoted to the study of community networks.[6–10] Zhang *et al.*[11] proposed the synchronization speed of identical oscillators in community networks. Some new advances have been reported in the synchronization of complex networks with nonidentical nodes,[12–23] which laid a very good foundation for subsequent work. Song *et al.*[24] investigated two effective control schemes to synchronize a complex network with nonidentical nodes. On the whole, the study on the synchronization of complex community networks with nonidentical nodes has been carried out very little.

The obtained results laid an important step forward in the investigation of community networks. Most of the above methods only synchronize with a synchronization state in the whole time series. However, it is not always practical to assume that the synchronization state has only one state since the synchronization process of some real-world complex networks may consist of different synchronization states. On the other hand, the previous methods have studied community networks with fully known parameters. While, in practice, it is hard to exactly determine the values of the network parameters in advance.[25–30] Therefore, these unknown parameters should be first considered in order to simulate more realistic community networks.

Motivated by the above discussion, in this paper, we investigate the synchronization of a community network with periodic states.[31] The main contribution of this paper lies in the following aspects. Firstly, a simple and useful control scheme is designed for a complex community network with nonidentical nodes and unknown parameters. Secondly, some synchronization criteria are obtained by using the LaSalle's invariance principle and adjusting time-varying adaptive coupling strengths.[32–34] It should be noted that most of the above-mentioned studies on synchronization of community networks mainly focus on networks where the couplings are constant in time. Finally, the developed techniques are applied to a community network which is periodically synchronized with the different synchronization states in the whole time series. Further, representative numerical simulations are provided to show the feasibility of the proposed methods.

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2. Mathematical preliminaries

Consider a complex community network consisting of N dynamical nodes with v communities and each node is an n-dimensional dynamical system with unknown parameters. Then, the controlled complex network is characterized by

$$\begin{aligned}\dot{x}_i(t) &= f_r(t,\theta,x_i(t),x_i(t-\tau_r(t))) \\ &\quad + c_g(t)\sum_{j=1}^{N} b_{ij}\Gamma x_j(t) + u_i, \quad i \in G_r. \end{aligned} \tag{1}$$

We can rewrite system (1) in the following form:

$$\begin{aligned}\dot{x}_i(t) &= f_{r_1}(t,x_i(t),x_i(t-\tau_r(t))) \\ &\quad + f_{r_2}(t,x_i(t),x_i(t-\tau_r(t)))\theta_i \\ &\quad + c_g(t)\sum_{j=1}^{N} b_{ij}\Gamma x_j(t) + u_i, \quad i \in G_r, \end{aligned} \tag{2}$$

where $f_{r_1}: R^n \to R^n$ is a continuous vector function; $f_{r_2}: R^n \to R^{n\times m}$ is a continuous matrix function; $\theta_i \in R^m$ is the unknown parameter vectors; $i \in \{1,2,\ldots,N\}$; $r \in \{1,2,\ldots,v\}$; $G_1 = \{1,2,\ldots,N_1\}$; $G_2 = \{N_1+1, N_1+2,\ldots,N_1+N_2\},\ldots$; $G_v = \{N_1+N_2+\cdots+N_{v-1}+1,\ldots, N_1+N_2+\cdots+N_v\}$, with $1 \le v \le N$ and $1 \le N_r \le N$ satisfying $\sum_{r=1}^{v} N_r = N$ for $r \in \{1,2,\ldots,v\}$; $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\mathrm{T}} \in R^n$ is the state vector of the i-th dynamical node; $\tau_r(t)$ is a time-varying delay, with $0 \le \tau_r(t) \le \tau_r$ for each $r \in \{1,2,\ldots,v\}$; u_i is a control action applied to node i; $c_g(t) > 0$ is a time-varying coupling strength; $\Gamma \in R^{n\times n}$ is an inner coupling matrix; $B = (b_{ij})_{N\times N}$ is the coupling configuration matrix representing the underlying topological structure of the network which is defined as follows: if there is a connection between node i and node j ($j \ne i$), then $b_{ij} = b_{ji} > 0$, otherwise, $b_{ij} = b_{ji} = 0$ ($j \ne i$), with the diagonal elements of matrix B defined by

$$b_{ii} = -\sum_{j=1, j\ne i}^{N} b_{ij}.$$

The initial condition of Eq. (2) is given by

$$x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0], \quad i \in \{1,2,\ldots,N\}, \tag{3}$$

where $\tau = \max\{\tau_i\}$, $\varphi_i(s) = (\varphi_{i1}(s), \ldots, \varphi_{in}(s))^{\mathrm{T}} \in D([-\tau,0], R^n)$.

The main aim in this paper is to impose the simple controller $u_i(t)$ on system (2) such that the network synchronizes with any smooth dynamics $s_g(t) \in R^n$ periodically. In order to obtain the main results, the following assumptions and definitions are necessary.

Assumption 1 For the vector-valued functions $f_r(t,x(t), x(t-\tau_r(t)))$, the uniform semi-Lipschitz conditions with respect to time t are assumed to hold, that is, there exist positive constants $\delta_r > 0$ and $\rho_r > 0$ such that

$$\begin{aligned}&[x(t)-y(t)]^{\mathrm{T}}\left[f_r(t,x(t),x(t-\tau_r(t))) - f_r(t,y(t),y(t-\tau_r(t)))\right] \\ &\le \delta_r[x(t-\tau_r(t))-y(t-\tau_r(t))]^{\mathrm{T}}[x(t-\tau_r(t))-y(t-\tau_r(t))] \\ &\quad + \rho_r[x(t)-y(t)]^{\mathrm{T}}[x(t)-y(t)]\end{aligned}$$

for any $x(t), y(t) \in R^n$ and $r \in \{1, 2, \ldots, v\}$.

Assumption 2 For each $r \in \{1,2,\ldots,v\}$, $\tau_r(t)$ is assumed to be a differential function with $0 < \dot{\tau}_r(t) < \varepsilon < 1$ and the inner coupling matrix Γ is assumed to be a positive definite matrix.

Definition 1 Assume that $s_g(t) \in R^n$ is any smooth dynamics. The controlled complex community network (2) is said to be globally periodically synchronized with the different synchronization states $s_g(t)$ in the whole time series if the solution satisfies

$$\lim_{t\to\infty}\|x_i(t) - s_g(t)\| = 0, \quad t \in [mT, (m+1)T)$$

for any initial conditions satisfying (3) and $i \in \{1, 2, \ldots, N\}$, $g \in \{1, 2, \ldots, n_1\}$, where T is the control period and $m = 0, 1, 2, \ldots$

Definition 2 Suppose that $B = (b_{ij}) \in R^{p\times p}$. If $b_{ij} = b_{ji} \ge 0$ for $i \ne j$ and $b_{ii} = -\sum_{i=1, j\ne i}^{p} b_{ij}$ for $i = 1, 2, \ldots, p$, and B is irreducible, then we say that $B \in B_1$.

Lemma 1 (See Ref. [35].) For any matrix $B_{N\times N} \in B_1$, its eigenvalues are all real and can be sorted as

$$0 = \lambda_1(B) > \lambda_2(B) \ge \lambda_3(B) \ge \cdots \ge \lambda_N(B).$$

Moreover, $(1, 1, \ldots, 1)^{\mathrm{T}}$ is the right eigenvector of B corresponding to eigenvalue 0 with multiplicity 1.

Lemma 2 (See Ref. [35].) If $B \in B_1$, then all eigenvalues of the matrix B–Λ are negative, where $\Lambda = \mathrm{diag}(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p)$, ε_i are nonnegative constants satisfying $\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_p > 0$.

3. Main results

In this section, a control and identification scheme is developed to periodically synchronize a delayed complex network with nonidentical nodes with any dynamics $s_g(t)$. In order to achieve the aim, the controller $u_i(t)$ is designed into the following form:

$$\begin{aligned}u_i(t) = &\; F_r(t) - k_i c_g(t)[x_i(t) - s_g(t)] \\ &- f_{r_2}(t,x_i(t),x_i(t-\tau_r(t)))\hat{\theta}_i \\ &+ f_{r_2}(t,s_g(t),s_g(t-\tau_r(t)))\hat{\theta}_i, \end{aligned} \tag{4}$$

where $F_r(t) = \dot{s}_g(t) - f_{r_1}(t,s_g(t),s_g(t-\tau_r(t))) - f_{r_2}(t,s_g(t),s_g(t-\tau_r(t)))\theta_i$, with $s_g(t)$ being any given smooth dynamics; $k_i > 0$ for each $i \in l_1 = \{N_1, N_1+N_2, \ldots, N\}$ and $k_i = 0$ for $i \notin l_1$.

Let $\hat{\theta}_i$ denote the parameter estimation vectors of θ_i. Define the error vector between system (2) and $s_g(t)$ as $e_i(t) = x_i(t) - s_g(t)$ and the corresponding parameter error vector as

$\tilde{\theta}_i = \hat{\theta}_i - \theta_i$. According to system (2) and the control law (4), the error dynamical system can be derived as

$$\begin{aligned}\dot{e}_i(t) &= \hat{f}_{r_1}(t) + \hat{f}_{r_2}(t)\tilde{\theta}_i + c_g(t)\sum_{j=1}^{N} b_{ij}\Gamma e_j(t) - k_i c_g(t) e_i(t), \\ &\quad i \in G_r,\ \ i \in l_1, \qquad (5)\\ \dot{e}_i(t) &= \hat{f}_{r_1}(t) + \hat{f}_{r_2}(t)\tilde{\theta}_i + c_g(t)\sum_{j=1}^{N} b_{ij}\Gamma e_j(t), \\ &\quad i \in G_r,\ \ i \notin l_1, \qquad (6)\end{aligned}$$

and the corresponding adaptive parameter updating rule as follows:

$$\dot{\tilde{\theta}}_i = -\hat{f}_{r_2}^{\mathrm{T}}(t, e_i(t), e_i(t-\tau_r(t)))\, e_i(t) - q_i\tilde{\theta}_i, \qquad (7)$$

where

$$\begin{aligned}\hat{f}_{r_1}(t) &= f_{r_1}(t, x_i(t), x_i(t-\tau_r(t))) \\ &\quad - f_{r_1}(t, s_g(t), s_g(t-\tau_r(t))), \\ \hat{f}_{r_2}(t) &= f_{r_2}(t, s_g(t), s_g(t-\tau_r(t))) \\ &\quad - f_{r_2}(t, x_i(t), x_i(t-\tau_r(t))),\end{aligned}$$

and q_i are arbitrary positive constants. It is easy to see that the globally asymptotical synchronization of the controlled complex network (2) is achieved if the error converges to zero periodically and asymptotically which is ensured by the following theorem.

Theorem 1 Suppose Assumptions 1–2 hold and $B \in \boldsymbol{B}_1$, then the periodically adaptive coupling strength $c_g(t)$ will have the following form:

$$c_g(t) = \varphi_g(t) + \omega, \qquad t \in [mT, (m+1)T), \qquad (8)$$

where $\omega > 0$ and $\varphi_g(0) \geq 0$. If $\varphi_g(t)$ satisfies the following law:

$$\dot{\varphi}_g(t) = d\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t), \quad t \in [mT, (m+1)T), \qquad (9)$$

where $d > 0$ is an arbitrary positive constant, then the controlled dynamical network (2) is globally periodically and synchronized to $s_g(t)$, $g \in \{1, 2, \ldots, n_1\}$.

Proof Firstly, let $e(t) = \left(e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \ldots, e_N^{\mathrm{T}}(t)\right)^{\mathrm{T}}$. Consider the following Lyapunov function:

$$\begin{aligned}V(t) &= \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) + \frac{1}{2d\alpha}\left(\rho + \frac{\delta}{1-\varepsilon} - \alpha\varphi_g(t)\right)^2 \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \sum_{r=1}^{v}\sum_{i\in G_r}\frac{\delta_r}{1-\varepsilon}\int_{t-\tau_r(t)}^{t} e_i^{\mathrm{T}}(s) e(s)\,\mathrm{d}s, \qquad (10)\end{aligned}$$

where $\rho = \max\{\rho_i\}$, $\delta = \max\{\delta_i\}$, and α is a positive constant determined later.

In virtue of Assumptions 1 and 2, the time derivative of $V(t)$ is derived as

$$\begin{aligned}\dot{V}(t) &= \sum_{r=1}^{v}\sum_{i\in G_r} e_i^{\mathrm{T}}(t)\hat{f}_{r_1}(t, e_i(t), e_i(t-\tau_r(t))) \\ &\quad + \sum_{r=1}^{v}\sum_{i\in G_r} e_i^{\mathrm{T}}(t)\hat{f}_{r_2}(t, e_i(t), e_i(t-\tau_r(t)))\tilde{\theta}_i \\ &\quad + c_g(t)\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^{\mathrm{T}}(t) b_{ij}\Gamma e_j(t) - c_g(t)\sum_{i\in l_1} k_i e_i^{\mathrm{T}}(t) e_i(t) \\ &\quad - \left(\rho + \frac{\delta}{1-\varepsilon} - \alpha\varphi_g(t)\right)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \\ &\quad + \sum_{i=1}^{N}\tilde{\theta}_i^{\mathrm{T}}\left[-\hat{f}_{r_2}^{\mathrm{T}}(t, e_i(t), e_i(t-\tau_r(t))) e_i(t) - q_i\tilde{\theta}_i\right] \\ &\quad + \sum_{r=1}^{v}\sum_{i\in G_r}\frac{\delta_r}{1-\varepsilon}\Big\{e_i^{\mathrm{T}}(t) e_i(t) - [1-\dot{\tau}_r(t)] \\ &\quad \times e_i^{\mathrm{T}}(t-\tau_r(t)) e_i(t-\tau_r(t))\Big\} \\ &\leq \sum_{r=1}^{v}\sum_{i\in G_r}\left[\rho_r e_i^{\mathrm{T}}(t) e_i(t) + \delta_r e_i^{\mathrm{T}}(t-\tau_r(t)) e_i(t-\tau_r(t))\right] \\ &\quad + c_g(t)\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^{\mathrm{T}}(t) b_{ij}\Gamma e_j(t) \\ &\quad - c_g(t)\sum_{i\in l_1} k_i e_i^{\mathrm{T}}(t) e_i(t) - \sum_{i=1}^{N} q_i\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i \\ &\quad - \left(\rho + \frac{\delta}{1-\varepsilon} - \alpha\varphi_g(t)\right)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \\ &\quad + \sum_{r=1}^{v}\sum_{i\in G_r}\left[\frac{\delta_r}{1-\varepsilon} e_i^{\mathrm{T}}(t) e_i(t)\right. \\ &\quad \left. - \delta_r e_i^{\mathrm{T}}(t-\tau_r(t)) e_i(t-\tau_r(t))\right] \\ &\leq c_g(t)\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^{\mathrm{T}}(t) b_{ij}\Gamma e_j(t) - c_g(t)\sum_{i\in l_1} k_i e_i^{\mathrm{T}}(t) e_i(t) \\ &\quad + \alpha\varphi_g(t)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) - \sum_{i=1}^{N} q_i\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i \\ &= c_g(t) e^{\mathrm{T}}(t)(B\otimes\Gamma) e(t) - \eta c_g(t) e^{\mathrm{T}}(t)(\zeta\otimes I_n) e(t) \\ &\quad + \varphi_g(t) e^{\mathrm{T}}(t)(\alpha I_N\otimes I_n) e(t) - \tilde{\theta}^{\mathrm{T}}(Q\otimes I_n)\tilde{\theta}, \qquad (11)\end{aligned}$$

where

$$\zeta = \mathrm{diag}\left(0, \ldots, 0, \frac{k_1}{\eta}, 0, \ldots, 0, \frac{k_2}{\eta} 0, \ldots, 0, \frac{k_{v-1}}{\eta}, 0, \ldots, 0, \frac{k_v}{\eta}\right),$$

with $\eta > 0$ being the smallest eigenvalue of Γ.

In virtue of Assumption 2 and Lemma 1, we obtain

$$\begin{aligned}\dot{V}(t) &\leq \eta c_g(t) e^{\mathrm{T}}(t)[(B-\zeta)\otimes I_n] e(t) \\ &\quad + \varphi_g(t) e^{\mathrm{T}}(t)(\alpha I_N\otimes I_n) e(t) - \tilde{\theta}^{\mathrm{T}}(Q\otimes I_n)\tilde{\theta} \\ &= \eta\varphi_g(t) e^{\mathrm{T}}(t)\left[\left(A + \frac{\alpha}{\eta} I_N\right)\otimes I_n\right] e(t) \\ &\quad + \eta\omega e^{\mathrm{T}}(t)(A\otimes I_n) e(t) - \tilde{\theta}^{\mathrm{T}}(Q\otimes I_n)\tilde{\theta}, \qquad (12)\end{aligned}$$

where $A = B - \zeta$. Since A is a real symmetric matrix, there exists a unitary matrix $W = (w_1, w_2, \ldots, w_N)$ satisfying $W^{\mathrm{T}}W = I$ such that $A = W\Lambda W^{\mathrm{T}}$, where $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ and $0 > \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N$ according to Lemma 2. Evidently, the diagonal elements λ_i are all the eigenvalues of A. Choose $0 < \alpha < -\eta\lambda_1$. It is easy to know that $\varphi_g(t) \geq 0$, which, together with inequality (11) and Assumption 2, gives

$$\dot{V}(t) \leq \eta\omega e^{\mathrm{T}}(t)[A \otimes I_n]e(t) - \tilde{\theta}^{\mathrm{T}}(Q \otimes I_n)\tilde{\theta} \leq 0, \quad (13)$$

which implies that system (2) is asymptotically and periodically synchronized with the given dynamics $s_g(t)$ and the unknown parameters can also be identified by using the control law and the adaptive updating law. The proof is completed.

4. Numerical results

In this section, we present numerical results to verify the analytical predictions obtained in the previous section. Consider a complex community network with two clusters consisting of fifty time-delayed Lorenz systems and fifty time-delayed Coullet systems, which are described by

$$\begin{aligned}\dot{x}_i(t) &= f_{1_1}(t, x_i(t), x_i(t-\tau_1(t))) \\ &\quad + f_{1_2}(t, x_i(t), x_i(t-\tau_1(t)))\theta_i + c_g(t)\sum_{j=1}^{100} b_{ij}\Gamma x_j(t), \\ &\quad i \in G_1 = (1, 2, \ldots, 50), \quad (14)\\ \dot{x}_i(t) &= f_{2_1}(t, x_i(t), x_i(t-\tau_2(t))) \\ &\quad + f_{2_2}(t, x_i(t), x_i(t-\tau_2(t)))\theta_i + c_g(t)\sum_{j=1}^{100} b_{ij}\Gamma x_j(t), \\ &\quad i \in G_2 = (51, 52, \ldots, 100), \quad (15)\end{aligned}$$

where the time delays are randomly chosen to be $\tau_1 = \tau_2 = 1$, $\Gamma = \mathrm{diag}\{1, 1, 1\}$, $b_{ii} = -99$, $b_{ij}b_{ji} = 1$

$$f_{1_1} = \begin{pmatrix} 0.1x_{i1}(t-\tau_1(t)) \\ -x_{i2}(t) - x_{i1}(t)x_{i3}(t) \\ x_{i1}(t)x_{i2}(t) \end{pmatrix},$$

$$f_{1_2} = \begin{pmatrix} -x_{i1}(t) + x_{i2}(t) & 0 & 0 \\ 0 & x_{i1}(t) & 0 \\ 0 & 0 & -x_{i3}(t) \end{pmatrix} \begin{pmatrix} \theta_1^{i1} \\ \theta_1^{i2} \\ \theta_1^{i3} \end{pmatrix},$$

$$f_{2_1} = \begin{pmatrix} x_{i2}(t) \\ x_{i3}(t) + 0.1\cos(0.1x_{i2}(t-\tau_2(t))) \\ -x_{i1}^3(t) \end{pmatrix},$$

$$f_{2_2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x_{i1}(t) & -x_{i2}(t) & -x_{i3}(t) \end{pmatrix} \begin{pmatrix} \theta_2^{i1} \\ \theta_2^{i2} \\ \theta_2^{i3} \end{pmatrix},$$

where θ_1^{i1}, θ_1^{i2}, and θ_1^{i3} are the unknown parameters of the G_1 community network to be estimated. When $\theta_1^{i1} = 10$, $\theta_1^{i2} = 28$, and $\theta_1^{i3} = 8/3$, system (13) exhibits chaotic behavior. θ_2^{i1}, θ_2^{i2}, and θ_2^{i3} are unknown parameters of the G_2 community network which need to be identified. In particular, system (14) has a chaotic attractor as $\theta_2^{i1} = 0.8$, $\theta_2^{i2} = 1.1$, and $\theta_2^{i3} = 0.45$. The chaotic behaviors of fifty time-delayed Lorenz systems and the Coullet systems are shown in Fig. 1.

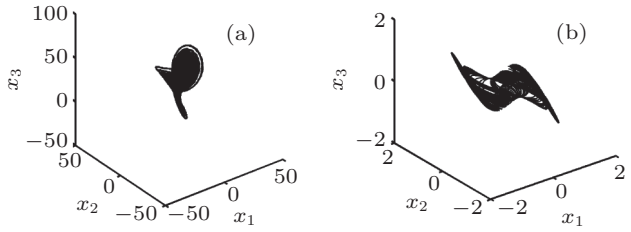


Fig. 1. Chaotic behaviors of (a) the Lorenz network and (b) the Coullet network.

In the following, the Lorenz system and Rössler system are chosen for the purpose of synchronization, which are given as

$$\begin{cases} \dot{s}_{11} = 10(-s_{11} + s_{12}), \quad \dot{s}_{21} = s_{22}, \\ \dot{s}_{12} = 28s_{11} - s_{12} - s_{11}s_{13}, \\ \dot{s}_{22} = s_{23}, \quad t \in [m\mathrm{T}, (m+1)\mathrm{T}), \\ \dot{s}_{13} = s_{11}s_{12} - \dfrac{8}{3}s_{13}, \\ \dot{s}_{23} = -6s_{21} - 2.9s_{22} - 1.2s_{23} + s_{23}^3, \end{cases} \quad (16)$$

where $T = 8$, $n_1 = 2$ and $m = 0, 1$. The chaotic behaviors of the Lorenz system and Rössler system are shown in Fig. 2.

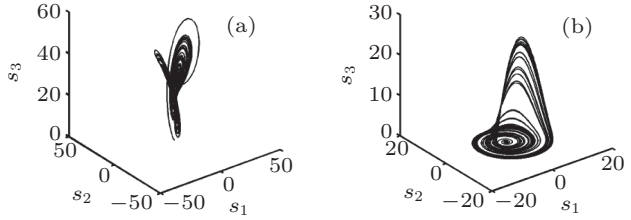


Fig. 2. Chaotic behaviors of (a) the Lorenz system and (b) the Rösler system.

The initial states of the G_1 community network (13) and the G_2 community network (14) are taken randomly as $(x_{i1}(0), x_{i2}(0), x_{i3}(0)) = [2, 3]$ and [2,4], respectively. The initial values of the unknown parameters $(\theta_{i1}(0), \theta_{i2}(0), \theta_{i3}(0))$ are chosen arbitrarily from the interval ([−10, 20], [10, 40], [−10, 10]) and ([0, 3], [−1, 2], [−1.5, 1]) for G_1 and G_2, respectively. It is easy to see that $l_1 = \{50, 50\}$. In the controller (3), $k_1 = k_2 = 30$, 50 are chosen to be $m = 0$, 1. In addition, $\omega = 0.1$ and 0.2, $d = 0.002$ and 0.05, $Q = \mathrm{diag}\{50, 50, \ldots, 50\}$ are adopted.

Figures 3 and 4 display the estimated values of the unknown parameters converging to $\theta_1^{i1} = 10$, $\theta_1^{i2} = 28$, $\theta_1^{i3} = 8/3$, $\theta_2^{i1} = 0.8$, $\theta_2^{i2} = 1.1$, and $\theta_2^{i3} = 0.45$ as $i \in G_1$ and G_2, respectively. As shown in the Figs. 3 and 4, all the unknown parameters are identified successfully by using the controller and the parameter updating rule. It can be seen from Figs. 5 and 6 that the state variable trajectories of the $x_i(t)$ and $s_g(t)$ are

exactly the same after adding controllers, $i = (1,2,\ldots,100)$. The values of coupling strength $c_g(t)$ tend to a positive constant when the synchronization is realized from Theorem 1, which are shown in Fig. 7. It can be seen from Fig. 8 that all the error signals approach to zero smoothly and rapidly in a short series of time once the design controller has been added.

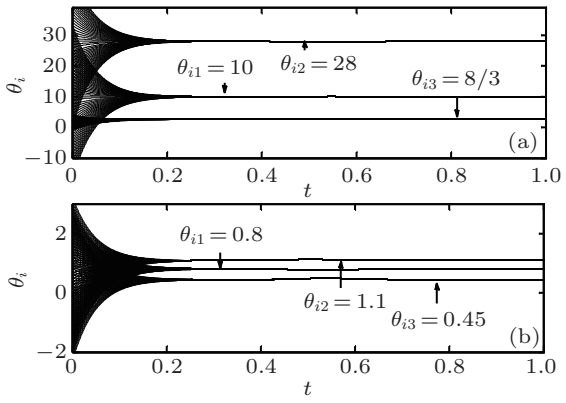


Fig. 3. Estimated values of the unknown parameters θ_{i1}–θ_{i3} as $m = 0$ for (a) G_1 and (b) G_2.

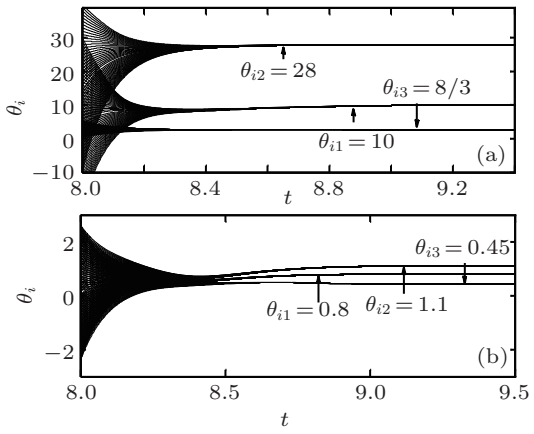


Fig. 4. Estimated values of the unknown parameters θ_{i1}–θ_{i3} as $m = 1$ for (a) G_1 and (b) G_2.

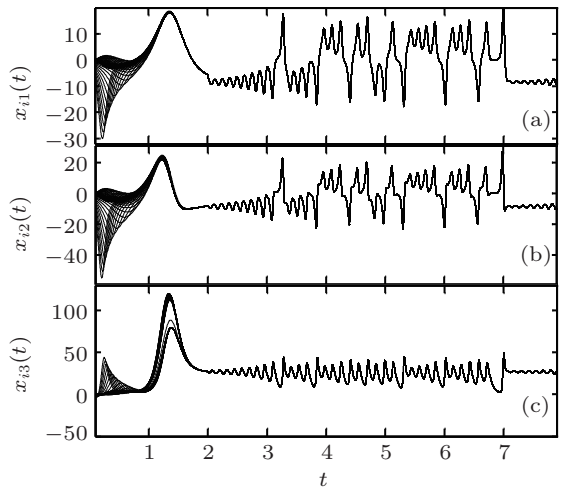


Fig. 5. State trajectories of the $x_i \to s_1$ with adding control as $m = 0$ for (a) x_{i1}, (b) x_{i2}, and (c) x_{i3}.

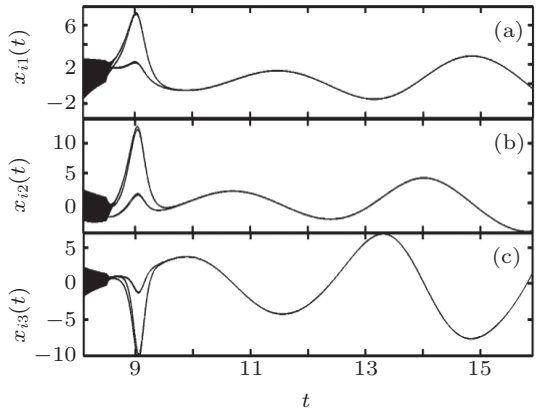


Fig. 6. Sate trajectories of the $x_i \to s_2$ with adding control as $m = 1$ for (a) x_{i1}, (b) x_{i2}, and (c) x_{i3}.

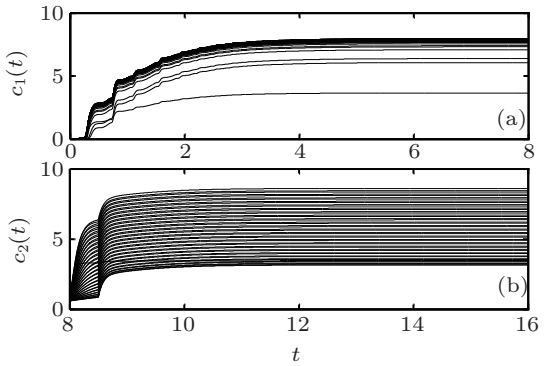


Fig. 7. Time evolutions of coupling strength $c_g(t)$ as (a) $m = 0$ and (b) $m = 1$.

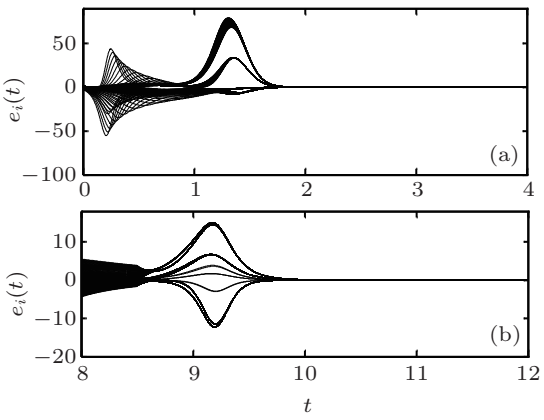


Fig. 8. Time evolutions of synchronization error as (a) $m = 0$ and (b) $m = 1$.

5. Conclusions

A new scheme, periodic synchronization, is introduced in this paper. It forms the states of many nonidentical chaotic systems with fully unknown parameters in a community network asymptotically synchronized with nonidentical synchronization states in the whole time series. A criterion is derived by adding a simple controller and adjusting the adaptive coupling strength. These synchronization and identification conditions are applicable to community networks with different topologies, huge sizes and unknown parameters. These results

not only present the possible reason that real community networks to change into more complex networks with uncertainty but also provide useful methods to demonstrate the synchronization of community networks for potential applications.

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