Quantum entanglement of an entangled coherent state: Role of particle losses*

Liu Pan(刘 盼), Feng Xiao-Min(冯晓敏), and Jin Guang-Ri(金光日)†

Department of Physics, Beijing Jiaotong University, Beijing 100044, *China*

(Received 28 June 2013; revised manuscript received 6 August 2013; published online 12 February 2014)

We analyze entanglement properties of entangled coherent state (ECS), $|\alpha,0\rangle_{1,2}+|0,\alpha\rangle_{1,2}$, with and without photon losses. By separating the coherent state into $|\alpha\rangle = c_0|0\rangle + \sqrt{1-c_0^2}|\tilde{\alpha}\rangle$, we derive exact results of the logarithmic negativity $E_{\mathcal{N}}$, which quantifies the degree of entanglement between the two bosonic modes. Without particle losses, $E_{\mathcal{N}} = 1$ for the N00N state; while for the ECS, $E_{\mathcal{N}}$ increases from 0 to 1 as $|\alpha|^2 \to \infty$. In the presence of photon losses, we find that the ECS with large enough photon number is more robust than that of the N00N state. An optimal ECS is obtained by maximizing $E_{\mathcal{N}}$ with respect to $|\alpha|^2$.

Keywords: quantum entanglement, entangled coherent state, photon losses

PACS: 03.67.Mn, 03.65.Ud, 42.50.–p **DOI:** 10.1088/1674-1056/23/3/030310

1. Introduction

Continuous variable entanglement is of significant importance for quantum information processing[1] and quantum metrology.[2–8] For instance, long-distance quantum communications with continuous variable entanglement have advantages over those relying upon discrete variable systems.[9–12] In particular, entangled coherent states (ECS) have attracted much attention.[13–18] As non-Gaussian states of light, the ECS can be produced by means of a Kerr medium combined with a 50:50 beam splitter (BS).[13,17–20]

Under various decoherence channels, the loss of entanglement for two-qubit systems has been studied intensively after the pioneering work by Yu and Eberly.[21–24] It is interesting to ask how the degree of entanglement evolves in other quantum systems, such as continuous variable quantum systems.[25–29] Here, we focus on a special case of the ECS-like state, $|\alpha,0\rangle_{1,2}+|0,\alpha\rangle_{1,2}$, where $|\alpha\rangle$ is a coherent state of the light. Such a kind of entangled state is indeed a superposition of the N00N states and is experimentally feasible with current technology (see Ref. [8], and references therein).

The main purpose of our work is to answer how much degree of entanglement is left for the ECS under the particle losses.[26–29] To quantify the degree of entanglement, we adopt the logarithmic negativity $E_{\mathcal{N}}$.[30,31] Without photon losses, $E_{\mathcal{N}} = 1$ for the N00N state; while for the ECS, $E_{\mathcal{N}}$ increases from 0 to 1 as $|\alpha|^2 \to \infty$. In the presence of photon losses, we find that the ECS with large enough mean photon number shows a greater robustness than that of the N00N state. For a given loss rate, we find that there exists an optimal ECS with maximal value of $E_{\mathcal{N}}$.

This paper is arranged as follows. In Section 2, we calculate the logarithmic negativity $E_{\mathcal{N}}$ without photon losses. By separating the coherent state into $|\alpha\rangle = c_0|0\rangle + \sqrt{1-c_0^2}|\tilde{\alpha}\rangle$, we derive the exact result of $E_{\mathcal{N}}$ of the ECS. In Section 3, we consider the role of photon losses on the entanglement of the N00N and the ECS states. The losses can be modeled as a linear interaction between the modes of interest and two lossy modes that are initially in the vacuum.[26] Exact results of $E_{\mathcal{N}}$ are obtained by using the above method. Finally, we conclude in Section 4 with the main results of our work.

2. Logarithmic negativity of the entangled states

The logarithmic negativity $E_{\mathcal{N}}$ has been introduced to quantify the degree of entanglement; this quantity is a rigorous entanglement monotone, and is zero for separable states. According to Refs. [30] and [31], the logarithmic negativity is defined as

$$E_{\mathcal{N}} = \log_2 ||\rho^{\mathrm{PT}_1}|| = \log_2 [1+2\mathcal{N}(\rho)], \tag{1}$$

where the negativity $\mathcal{N}(\rho) = |\sum_i \lambda_i|$ is the sum of the negative eigenvalues λ_i of ρ^{PT_1}. Here, ρ^{PT_1} denotes the partial transpose of ρ with respect to the mode 1. For instance, let us firstly consider the N00N state[32–34]

$$|\psi\rangle_{1,2} = \frac{1}{\sqrt{2}}\left(|N,0\rangle_{1,2}+|0,N\rangle_{1,2}\right), \tag{2}$$

*Project supported by the National Natural Science Foundation of China (Grant No. 11174028), the Fundamental Research Funds for the Central Universities of China (Grant No. 2011JBZ013), and the Program for New Century Excellent Talents in University of China (Grant No. NCET-11-0564).

†Corresponding author. E-mail: grjin@bjtu.edu.cn

© 2014 Chinese Physical Society and IOP Publishing Ltd

where N is the total number of photons. The density matrix of the N00N state can be written as

$$\begin{aligned}\boldsymbol{\rho} &= |\psi\rangle_{1,2}\langle\psi|_{1,2} \\ &= \frac{1}{2}(|N,0\rangle_{1,2}\langle N,0|_{1,2}+|0,N\rangle_{1,2}\langle 0,N|_{1,2} \\ &\quad + |N,0\rangle_{1,2}\langle 0,N|_{1,2}+|0,N\rangle_{1,2}\langle N,0|_{1,2}),\end{aligned}$$

and its partial transpose is

$$\begin{aligned}\boldsymbol{\rho}^{\mathrm{PT}_1} &= \frac{1}{2}\Big(|N,0\rangle_{1,2}\langle N,0|_{1,2}+|0,N\rangle_{1,2}\langle 0,N|_{1,2} \\ &\quad +|\eta_+\rangle_{1,2}\langle\eta_+|_{1,2}-|\eta_-\rangle_{1,2}\langle\eta_-|_{1,2}\Big), \qquad (3)\end{aligned}$$

where we introduce two orthogonal states[35]

$$|\eta_\pm\rangle_{1,2} = \frac{1}{\sqrt{2}}\Big(|N,N\rangle_{1,2}\pm|0,0\rangle_{1,2}\Big). \qquad (4)$$

From Eq. (3), we can see that the partial transpose of $\boldsymbol{\rho}$ has a negative eigenvalue $-1/2$, which yields the negativity $\mathcal{N}(\boldsymbol{\rho}) = 1/2$ and hence the logarithmic negativity of the N00N state[35]

$$E_{\mathcal{N}} = \log_2(2) = 1. \qquad (5)$$

To proceed, we calculate the logarithmic negativity of the entangled coherent state[8]

$$\begin{aligned}|\psi\rangle_{1,2} &= N_\alpha(|\alpha,0\rangle_{1,2}+|0,\alpha\rangle_{1,2}) \\ &= N_\alpha\sum_{N=0}^{\infty}c_N(|N,0\rangle_{1,2}+|0,N\rangle_{1,2}), \qquad (6)\end{aligned}$$

where the normalized factor N_α and the probability amplitude c_N are given by

$$N_\alpha = \frac{1}{\sqrt{2(1+\mathrm{e}^{-|\alpha|^2})}}, \qquad (7)$$

$$c_N \equiv \langle N|\alpha\rangle = \frac{\alpha^N}{\sqrt{N!}}\mathrm{e}^{-|\alpha|^2/2}. \qquad (8)$$

For the ECS, we obtain the mean photon number as

$$\bar{N} = 2N_\alpha^2\sum_{N=0}^{\infty}N|c_N|^2 = 2N_\alpha^2|\alpha|^2. \qquad (9)$$

Since the mean photon number can be rewritten as $\bar{N} = |\alpha|^2/(1+\mathrm{e}^{-|\alpha|^2})$, we obtain

$$|\alpha|^2 = \bar{N}+w(\bar{N}\mathrm{e}^{-\bar{N}}), \qquad (10)$$

where $w(z)$ denotes the Lambert W function, also called the product logarithm, which gives the principal solution for w in $z = w\mathrm{e}^w$. For large $|\alpha|$, $w(\bar{N}\mathrm{e}^{-\bar{N}})\to 0$, and thereby $\bar{N}\simeq|\alpha|^2$. It is interesting to note that the ECS can be rewritten as

$$|\psi\rangle_{1,2} = N_\alpha[2c_0|0,0\rangle_{1,2}+d_0|0,\tilde{\alpha}\rangle_{1,2}+d_0|\tilde{\alpha},0\rangle_{1,2}], \qquad (11)$$

where we separate the coherent state into

$$|\alpha\rangle = \sum_{N=0}^{\infty}c_N|N\rangle = c_0|0\rangle+d_0|\tilde{\alpha}\rangle, \qquad (12)$$

with $c_0 = \exp(-|\alpha|^2/2)$ and $d_0 = \sqrt{1-c_0^2}$. Note that the introduced state $|\tilde{\alpha}\rangle$ contains no vacuum, thereby $\langle 0|\tilde{\alpha}\rangle = (\langle\tilde{\alpha}|0\rangle)^* = 0$. In addition, as $|\alpha|^2\to\infty$ and the amplitude $c_0\to 0$, we have

$$|\psi\rangle_{1,2}\to\frac{1}{\sqrt{2}}\Big(|\tilde{\alpha},0\rangle_{1,2}+|0,\tilde{\alpha}\rangle_{1,2}\Big), \qquad (13)$$

which is a maximally entangled state due to $\langle 0|\tilde{\alpha}\rangle = 0$. Using Eq. (11), the density matrix of the ECS becomes

$$\begin{aligned}\boldsymbol{\rho} &= |\psi\rangle_{1,2}\langle\psi|_{1,2} \\ &= (N_\alpha)^2\Big\{\Big(2c_0|0,0\rangle_{1,2}+d_0|0,\tilde{\alpha}\rangle_{1,2}\Big) \\ &\quad\times(2c_0\langle 0,0|_{1,2}+d_0\langle 0,\tilde{\alpha}|_{1,2})+d_0^2|\tilde{\alpha},0\rangle_{1,2}\langle\tilde{\alpha},0|_{1,2} \\ &\quad+\Big[d_0\Big(2c_0|0,0\rangle_{1,2}+d_0|0,\tilde{\alpha}\rangle_{1,2}\Big)\langle\tilde{\alpha},0|_{1,2}+\mathrm{h.c.}\Big]\Big\}.\end{aligned}$$

Again, since $\langle 0|\tilde{\alpha}\rangle = \langle\tilde{\alpha}|0\rangle = 0$, we can choose the basis as $\{|0,0\rangle_{1,2}, |0,\tilde{\alpha}\rangle_{1,2}, |\tilde{\alpha},0\rangle_{1,2}, |\tilde{\alpha},\tilde{\alpha}\rangle_{1,2}\}$, then the partial transpose of $\boldsymbol{\rho}$ can be written in the following matrix form:

$$\boldsymbol{\rho}^{\mathrm{PT}_1} = (N_\alpha)^2\begin{pmatrix}4c_0^2 & 2c_0d_0 & 2c_0d_0 & d_0^2 \\ 2c_0d_0 & d_0^2 & 0 & 0 \\ 2c_0d_0 & 0 & d_0^2 & 0 \\ d_0^2 & 0 & 0 & 0\end{pmatrix}. \qquad (14)$$

After straightforward calculations, we obtain the eigenvalues of the above matrix as

$$\lambda_{1,2} = \pm\frac{1}{2}\tanh\left(\frac{|\alpha|^2}{2}\right), \qquad (15)$$

$$\lambda_{3,4} = \frac{1}{2}\pm\frac{\mathrm{e}^{|\alpha|^2/2}}{\mathrm{e}^{|\alpha|^2}+1} > 0. \qquad (16)$$

Obviously, the negativity is given by

$$\mathcal{N}(\boldsymbol{\rho}) = |\lambda_1| = \frac{1}{2}\tanh\left(\frac{|\alpha|^2}{2}\right), \qquad (17)$$

and hence the logarithmic negativity for the ECS is

$$E_{\mathcal{N}} = \log_2\left[\frac{2}{\mathrm{e}^{-|\alpha|^2}+1}\right]. \qquad (18)$$

As shown in Fig. 1(a), we find that $E_{\mathcal{N}}$ increases continuously from 0 for $\bar{N} = 0$, to 1 for $\bar{N} > 6$. This is because the ECS approaches the maximally entangled state (see Eq. (13)) as $\bar{N}\to\infty$ (i.e., $|\alpha|^2\to\infty$), which behaves like the N00N state, yielding $E_{\mathcal{N}} = 1$.

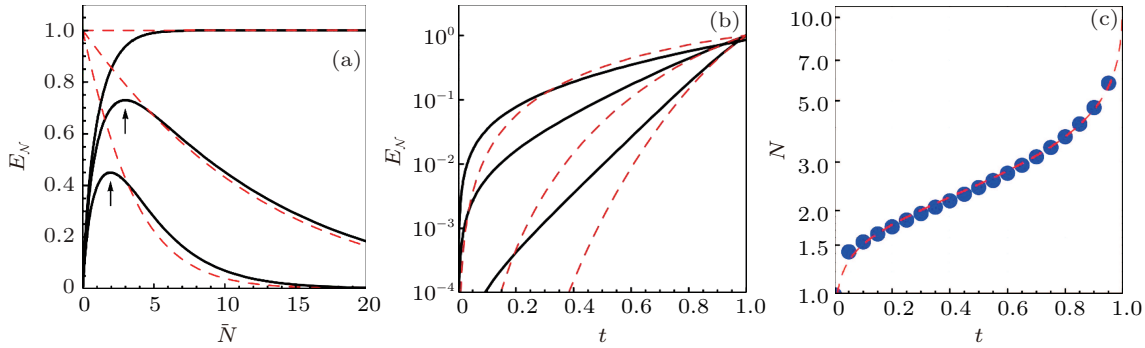


Fig. 1. (color online) Logarithmic negativity $E_{\mathcal{N}}$ against the number of photons $\bar{N}$ (a) and the transmission rate t (b) for the ECS (solid lines) and the N00N (red dashed lines) states, as well as numerical results of the crossover of $E_{\mathcal{N}}$ between the two states (c). In panel (a), the transmission rates $t = 1$, 0.9, and 0.7 (from top to bottom); in panel (b), $E_{\mathcal{N}}$ versus t for the photon numbers $\bar{N} = 2$, 5, and 10 (from top to bottom). For a fixed t (< 1), $E_{\mathcal{N}}$ reaches its maximal value for an optimal ECS with $\bar{N}_0 = 2N_{\alpha_0}^2|\alpha_0|^2$, as indicated by the arrows in panel (a). Moreover, a crossover of $E_{\mathcal{N}}$ between the ECS and the N00N states occurs when $\bar{N} \sim 3t^{-1/3}\text{arcsech}(1-t)/2$, as shown by the red dashed line of panel (c).

3. Role of particle losses on the degree of entanglement

We now consider the particle losses, which can be modeled by introducing two fictitious beam splitters with the same transmission coefficient t.[26] Firstly, let us consider the N00N state. In the presence of photon losses, the total state can be written as

$$|\psi\rangle_{\rm T} = \hat{B}_{1,3}\hat{B}_{2,4}\frac{|N,0\rangle_{1,2}+|0,N\rangle_{1,2}}{\sqrt{2}}|0\rangle_3|0\rangle_4, \tag{19}$$

where $|0\rangle_3$ and $|0\rangle_4$ are photon vacuums of two lossy modes. The action of the BS's can be described by unitary operators $\hat{B}_{ij} = \mathrm{e}^{\mathrm{i}\theta(\hat{J}_x)_{i,j}}$, with $(\hat{J}_x)_{i,j} = (\hat{a}_i^\dagger\hat{a}_j + \hat{a}_j^\dagger\hat{a}_i)/2$ for the subscript $i = 1$ or 2 and $j = 3$ or 4. For instance, we have

$$\hat{B}_{1,3}|N\rangle_1|0\rangle_3 = \sum_{m=-J}^{J}\mathrm{e}^{\mathrm{i}\frac{\pi}{2}(J-m)}d_{m,J}^J(\theta)|J,m\rangle, \tag{20}$$

where $J = N/2$ and angular-momentum states $|J,m\rangle = |J+m\rangle_1|J-m\rangle_3$, as well as Wigner's D-function

$$\begin{aligned} d_{m,J}^J(\theta) &= \langle J,m|\mathrm{e}^{-\mathrm{i}\theta(\hat{J}_y)_{1,3}}|J,J\rangle \\ &= \binom{2J}{J+m}^{1/2}t^{\frac{J+m}{2}}(1-t)^{\frac{J-m}{2}}. \end{aligned} \tag{21}$$

Here, $t = \cos^2(\theta/2)$ denotes the transmission rate between the mode 1 and the lossy mode 3. Specially, $t = 1$ means no particle losses and $t = 0$ corresponds to the case that all the photons are lost. Similarly, we can also obtain

$$\hat{B}_{2,4}|N\rangle_2|0\rangle_4 = \sum_{m=-J}^{J}\mathrm{e}^{\mathrm{i}\frac{\pi}{2}(J-m)}d_{m,J}^J(\theta)|J,m\rangle, \tag{22}$$

where $|J,m\rangle = |J+m\rangle_2|J-m\rangle_4$. Therefore, the total state can be obtained as

$$\begin{aligned} |\psi\rangle_{\rm T} = \frac{1}{\sqrt{2}}\sum_{m=-J}^{J}\mathrm{e}^{\mathrm{i}\frac{\pi}{2}(J-m)}d_{m,J}^J(\theta)\Big[&|J+m,0\rangle_{1,2} \\ &\times|J-m\rangle_3|0\rangle_4 + |0,J+m\rangle_{1,2}|0\rangle_3|J-m\rangle_4\Big]. \end{aligned}$$

Tracing over the lossy modes, we obtain the reduced density matrix as

$$\begin{aligned} \rho &= \mathrm{Tr}_{3,4}\left(|\psi\rangle_{\rm T}\langle\psi|_{\rm T}\right) \\ &= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\langle k|_3\langle l|_4|\psi\rangle_{\rm T}\langle\psi|_{\rm T}|k\rangle_3|l\rangle_4, \end{aligned} \tag{23}$$

where $|k\rangle_3$ and $|l\rangle_4$ are Fock states of the lossy modes. Partial transpose of the reduced density matrix can be easily obtained as

$$\begin{aligned} \rho^{\mathrm{PT}_1} = &\sum_{k=0}^{2J}\frac{\left[d_{J-k,J}^J(\theta)\right]^2}{2}\Big\{|2J-k,0\rangle_{1,2}\langle 2J-k,0|_{1,2} \\ &+|0,2J-k\rangle_{1,2}\langle 0,2J-k|_{1,2}\Big\} \\ &+\frac{\left[d_{J,J}^J(\theta)\right]^2}{2}\Big\{|0,0\rangle_{1,2}\langle 2J,2J|_{1,2}+\mathrm{h.c.}\Big\}, \end{aligned} \tag{24}$$

with $2J = N$. Note that all the terms in $\sum_k\{\cdots\}$ have been written in diagonalized form with positive eigenvalues $[d_{J-k,J}^J(\theta)]^2/2$. The last term can be diagonalized by introducing $|\eta_\pm\rangle_{1,2}$ (see Eq. (4)). Then we obtain the negativity $\mathcal{N}(\rho) = [d_{J,J}^J(\theta)]^2/2 = t^N/2$ and hence the logarithmic negativity of the N00N state as

$$E_{\mathcal{N}} = \log_2(1+t^N). \tag{25}$$

In Fig. 1, we plot $E_{\mathcal{N}}$ as a function of the photon number (left panel) and the transmission rate (middle panel). As shown by the red dashed curves, we find that $E_{\mathcal{N}}$ degrades rapidly with the decrease in t.

Next, we consider the case of the ECS. The total state can be obtained as

$$\begin{aligned} |\psi\rangle_{\rm T} &= N_\alpha\hat{B}_{1,3}\hat{B}_{2,4}\left(|\alpha,0\rangle_{1,2}+|0,\alpha\rangle_{1,2}\right)|0\rangle_3|0\rangle_4 \\ &= N_\alpha\Big(|\alpha\sqrt{t},0\rangle_{1,2}|\mathrm{i}\alpha\sqrt{1-t}\rangle_3|0\rangle_4 \\ &\quad+|0,\alpha\sqrt{t}\rangle_{1,2}|0\rangle_3|\mathrm{i}\alpha\sqrt{1-t}\rangle_4\Big), \end{aligned} \tag{26}$$

where we have used

$$\hat{B}_{i,j}(t)|\alpha\rangle_i|0\rangle_j = |\alpha\sqrt{t}\rangle_i|\mathrm{i}\alpha\sqrt{1-t}\rangle_j, \tag{27}$$

for $i = 1,2$ and $j = 3,4$. Tracing over the lossy modes, from Eq. (26) we obtain the reduced density matrix and its partial transpose as

$$\rho^{\mathrm{PT}_1} = N_\alpha^2 \{|\beta,0\rangle_{1,2}\langle\beta,0|_{1,2} + |0,\beta\rangle_{1,2}\langle 0,\beta|_{1,2} + \mathrm{e}^{-|\alpha|^2(1-t)}\,(|0,0\rangle_{1,2}\langle\beta,\beta|_{1,2} + \mathrm{h.c.})\}, \quad (28)$$

with $\beta = \alpha\sqrt{t}$. Similar to Eq. (12), we separate the coherent state into $|\beta\rangle = c_0|0\rangle + d_0|\tilde{\beta}\rangle$, where $c_0 = \exp(-t|\alpha|^2/2)$ and $d_0 = \sqrt{1-c_0^2}$. By choosing the basis $\{|0,0\rangle_{1,2}, |0,\tilde{\beta}\rangle_{1,2}, |\tilde{\beta},0\rangle_{1,2}$, and $|\tilde{\beta},\tilde{\beta}\rangle_{1,2}\}$, the partial transpose of ρ can be rewritten in a matrix form, like Eq. (14). Only one negative eigenvalue of ρ^{PT_1} can be obtained, $\lambda_1 = -(\mathrm{e}^{t|\alpha|^2} - 1)/2(\mathrm{e}^{|\alpha|^2} + 1)$, which in turn gives the logarithmic negativity

$$E_{\mathcal{N}} = \log_2\left[\frac{\mathrm{e}^{-(1-t)|\alpha|^2}+1}{\mathrm{e}^{-|\alpha|^2}+1}\right], \quad (29)$$

where $|\alpha|^2$ is given by Eq. (9). Without particle losses, i.e., $t = 1$, our result reduces to Eq. (18).

In Fig. 1(a), we plot $E_{\mathcal{N}}$ against the mean photon number $\bar{N}$ for given transmission rates $t = 1$, 0.9, and 0.7 (from top to bottom). It is found that for a fixed t, $E_{\mathcal{N}}$ reaches its maximum at an optimal value of the mean photon number $\bar{N}_0$. To solve it, we maximize $f(x) = (x^{1-t}+1)/(x+1)$ with respect to $x\,(= \mathrm{e}^{-|\alpha|^2})$ and find a transcendental equation

$$x_0^t + tx_0 + t - 1 = 0, \quad (30)$$

which in turn determines $|\alpha_0|^2 = -\ln(x_0)$ and hence $\bar{N}_0 = 2N_{\alpha_0}^2|\alpha_0|^2$. In Fig. 2, contour plot of $E_{\mathcal{N}}$ against t and $\bar{N}$ is depicted for the ECS case. It indicates that to keep the ECS with large $E_{\mathcal{N}}$ (say, > 0.7), the transmission rate of the photons t should be at least larger than 0.9. The green crosses of Fig. 2 give an optimal ECS

$$|\psi\rangle_{\mathrm{op}} = N_{\alpha_0}(|\alpha_0,0\rangle_{1,2} + |0,\alpha_0\rangle_{1,2}), \quad (31)$$

which has maximal $E_{\mathcal{N}}$ for a given t. Here, N_{α_0} is given by Eq. (7) and $|\alpha_0|^2$ is determined by the transcendental equation (Eq. (30)). For instance, $t = 0.7$ and 0.9, the entangled coherent states with $|\alpha_0|^2 = 2.165$ (i.e., $\bar{N}_0 = 1.942$) and 3.121 ($\bar{N}_0 = 2.989$) show maximal $E_{\mathcal{N}}$, as indicated by the arrows of Fig. 1(a).

In Fig. 1(b), we plot $E_{\mathcal{N}}$ against the transmission rate t for the ECS state (the solid lines) and the N00N state (the red dashed lines) with different photon numbers $\bar{N} = 2$, 5, and 10 (from top to bottom). It is found that $E_{\mathcal{N}}$ degrades with the decrease in t. Compared with the N00N state, however, we find that loss of entanglement is slower for the ECS state with large enough photon number $\bar{N}$. Moreover, we note that for a given t (< 1) there exists a crossover of the entanglement between the ECS and the N00N states; only the ECS with $\bar{N} \gtrsim 3t^{-1/3}\mathrm{arcsech}(1-t)/2$ shows larger $E_{\mathcal{N}}$, as depicted in Fig. 1(c).

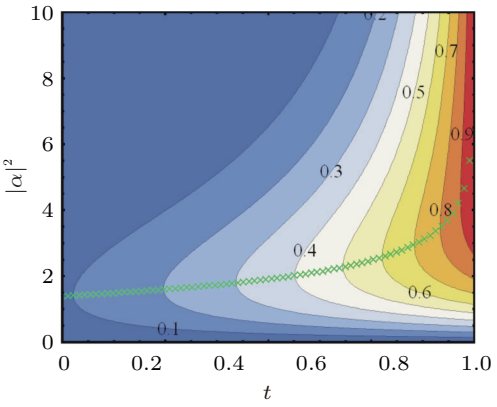


Fig. 2. (color online) Contour plot of $E_{\mathcal{N}}$ against the transmission rate t and square of the amplitude $|\alpha|^2$ for the ECS. To keep large enough entanglement, say $E_{\mathcal{N}} > 0.7$, it is required that the transmission rate of the photons $t > 0.9$. The green crosses indicate that for a given t, there exists an optimal ECS: $N_{\alpha_0}(|\alpha_0,0\rangle_{1,2} + |0,\alpha_0\rangle_{1,2})$, with $|\alpha_0|^2 = -\ln(x_0)$ and x_0 determined by the transcendental equation (Eq. (30)).

4. Discussion and conclusion

By separating the coherent state into $|\alpha\rangle = c_0|0\rangle + \sqrt{1-c_0^2}|\tilde{\alpha}\rangle$, we have derived exact solutions of the logarithmic negativity $E_{\mathcal{N}}$ for the entangled coherent state. With such a simple method, we can solve the lower bound of phase sensitivity (i.e., quantum Cramer–Rao bound) for an optical March–Zenhder interferometer felt with the ECS. In fact, for a single-shot measurement, the bound $\delta\phi_{\min} = 1/\sqrt{F_{\mathrm{Q}}}$ is determined by the quantum Fisher information[36–39]

$$F_{\mathrm{Q}} = \sum_{i,j}\frac{2}{\lambda_i+\lambda_j}\left|\left\langle\lambda_i\left|\frac{\partial\rho(\phi)}{\partial\phi}\right|\lambda_j\right\rangle\right|^2, \quad (32)$$

where λ_i and $|\lambda_i\rangle$ are the eigenvalues and the eigenvectors of the outcome state after phase accumulation $\rho(\phi)$. In a previous work,[8] only numerical results of F_{Q} have been obtained. Using the method mentioned above, we can obtain the exact solution of F_{Q} and analyze in detail the role of the losses on the achievable phase sensitivity. More interestingly, we can also investigate quantum entanglement and quantum non-locality for the squeezed superposition of coherent states under various lossy channels (see, e.g., Refs. [40] and [41]).

In summary, we have examined the role of photon losses on the logarithmic negativity $E_{\mathcal{N}}$ for the entangled coherent state. By diagonalizing partial transpose of the density matrix, we present exact results of $E_{\mathcal{N}}$ for both the N00N state and the ECS. Without particle losses, we find that $E_{\mathcal{N}}$ of the ECS increases from 0 to 1 as $|\alpha|^2 \to \infty$. In the presence of photon losses, $E_{\mathcal{N}}$ degrades for both the N00N and the ECS states. However, the ECS with large enough mean photon number shows a greater robustness than the particle losses. By maximizing $E_{\mathcal{N}}$ for a given loss rate, we find an optimal ECS

$\propto (|\alpha_0, 0\rangle_{1,2} + |0, \alpha_0\rangle_{1,2})$, with $|\alpha_0|^2$ determined by a transcendental equation.

Acknowledgment

Authors acknowledge Dr. D. L. Zhou for helpful discussion.

References

[1] Braunstein S L and van Loock P 2005 *Rev. Mod. Phys.* **77** 513
[2] Caves C M 1981 *Phys. Rev. D* **23** 1963
[3] Yurke B, MacCall S L and Klauder J R 1986 *Phys. Rev. A* **33** 4033
[4] Giovannetti V, Lloyd S and Maccone L 2011 *Nat. Photon.* **5** 222
[5] Genoni M G, Olivares S and Paris M G A 2011 *Phys. Rev. Lett.* **106** 153603
[6] Escher B M, Davidovich L, Zagury N and de Matos Filho R L 2012 *Phys. Rev. Lett.* **109** 190404
[7] Dorner U *et al.* 2009 *Phys. Rev. Lett.* **102** 040403
[8] Joo J, Munro W J and Spiller T P 2011 *Phys. Rev. Lett.* **107** 083601
[9] Leverrier A and Grangier P 2009 *Phys. Rev. Lett.* **102** 180504
[10] Grosshans F, van Assche G, Wenger J, Brouri R, Cerf N J and Grangier P 2003 *Nature* **421** 238
[11] Lance A M, Symul T, Sharma V, Weedbrook C, Ralph T C and Lam P K 2005 *Phys. Rev. Lett.* **95** 180503
[12] Ralph T C and Lam P K 2009 *Nature* **3** 671
[13] Sanders B C 1992 *Phys. Rev. A* **45** 6811
[14] Filip R, Rehácek J and Dusek M 2001 *J. Opt. B: Quantum Semiclass. Opt.* **3** 341
[15] Jeong H, Kim M S and Lee J 2001 *Phys. Rev. A* **64** 052308
[16] Wang X 2002 *J. Phys. A* **35** 165
[17] Tara K, Agarwal G S and Chaturvedi S 1993 *Phys. Rev. A* **47** 5024
[18] van Enk S J 2003 *Phys. Rev. Lett.* **91** 017902
[19] Paternostro M, Kim M S and Ham B S 2003 *Phys. Rev. A* **67** 023811
[20] Kuang L M, Chen Z B and Pan J W 2007 *Phys. Rev. A* **76** 052324
[21] Yu T and Eberly J H 2009 *Science* **323** 598
[22] Al-Qasimi A and James D F V 2009 *Opt. Lett.* **34** 268
[23] Ma J, Huang Y, Wang X and Sun C P 2011 *Phys. Rev. A* **84** 022302
[24] Liu Q G and Ji X 2012 *Acta Phys. Sin.* **61** 230303 (in Chinese)
[25] Gerry C C and Knight P L 2005 *Introductory Quantum Optics* (Cambridge: Cambridge University Press)
[26] van Enk S J 2005 *Phys. Rev. A* **72** 022308
[27] Wickert R, Bernardes N K and van Loock 2010 *Phys. Rev. A* **81** 062344
[28] El Allati A, Hassouni Y and Metwally N 2011 *Chin. Phys. B* **20** 110303
[29] Lastra F, Romero G, Lopez C E, Zagury N and Retamal J C 2010 *Opt. Commun.* **283** 3825
[30] Vidal G and Werner R F 2002 *Phys. Rev. A* **65** 032314
[31] Adesso G, Serafini A and Illuminati F 2004 *Phys. Rev. A* **70** 022318
[32] Boto A N, Kok P, Abrams D S, Braunstein S L, Williams C P and Dowling J P 2000 *Phys. Rev. Lett.* **85** 2733
[33] Mitchell M W, Lundeen J S and Steinberg A M 2004 *Nature* **429** 161
[34] Zhang D H, Zhou D L and Fan H 2011 *Chin. Phys. Lett.* **27** 90306
[35] Agarwal G S, Chaturvedi S and Rai A 2010 *Phys. Rev. A* **81** 043843
[36] Helstrom C W 1976 *Quantum Detection and Estimation Theory* (New York: Academic Press)
[37] Holevo A S 1982 *Probabilistic and Statistical Aspect of Quantum Theory* (Pisa: North-Holland Press)
[38] Braunstein S L and Caves C M 1994 *Phys. Rev. Lett.* **72** 3439
[39] Pezzé L and Smerzi A 2009 *Phys. Rev. Lett.* **102** 100401
[40] Zhou B Y, Deng L, Duan Y F, Yu L and Li G X 2012 *Chin. Phys. B* **21** 090302
[41] Obada A S F, Hessian H A, Mohamed A B A and Hashem M 2012 *Chin. Phys. B* **21** 100310