Subcooled pool boiling heat transfer in fractal nanofluids: A novel analytical model*

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A novel analytical model to determine the heat flux of subcooled pool boiling in fractal nanofluids is developed. The model considers the fractal character of nanofluids in terms of the fractal dimension of nanoparticles and the fractal dimension of active cavities on the heated surfaces; it also takes into account the effect of the Brownian motion of nanoparticles, which has no empirical constant but has parameters with physical meanings. The proposed model is expressed as a function of the subcooling of fluids and the wall superheat. The fractal analytical model is verified by a reasonable agreement with the experimental data and the results obtained from existing models.

Keywords: subcooled pool boiling, Brownian motion, fractal nanofluids

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1. Introduction

Nanofluids appeared with the development of nanoscience and nanotechnology.[1–5] Nanofluids are solid–liquid composite materials consisting of solid nanoparticles or nanofibers, with sizes typically on the order of 1 nm–100 nm, suspended in liquids. In other words, nanofluids are dispersions, or colloidal suspensions, of nanoscale particles in base liquids. Since nanoparticles offer an extremely large ratio between volume and surface area, nanofluids' thermal properties are superior to those of conventional heat transfer fluids and fluids containing micrometer-sized particles. The transport properties of nanofluids have been a topic of intense research due to their prospective technological applications in cooling and heat transfer for electronics. A number of studies have been carried out to understand and describe the different behaviors of nanofluids, such as effective thermal conductivity under static conditions, convective heat transfer associated with the flow, and transfer of fluids and subcooled boiling in cooling systems. Additionally, such cooling systems have been widely investigated and explored in recent years due to their great potential for applications in many fields, including space communication, microprocessors, and electronic devices. For instance, to cool high-power and high-density computer chips, a cooling system must keep the chip temperature below 85 °C. Liquid cooling with or without phase change is proposed to meet such a requirement. However, only a few previous studies describe subcooled boiling of nanofluids. It is a challenge to determine the subcooled pool boiling of nanofluids analytically due to the extremely complicated mechanisms of heat transfer as well as the interrelationship between heat flux and the nanoparticle concentration and subcooling of fluids.

The theory of fractals aids in analyzing the response of practical systems besides examining the complex geometries of objects. The fractal technique has received much attention due to its wide applicability in heat and mass transfer.[6–10] The aim of this study is to investigate the heat flux of subcooled pool boiling in fractal nanofluids, considering the effects of the subcooling of fluids, the concentration of nanoparticles and the average diameter of nanoparticles. In this work, the model predictions are compared with existing experimental data and good agreement is found. The validity of the fractal model for subcooled pool boiling heat transfer of nanofluids is thus verified.

The rest of this paper is organized as follows. In Section 2, we carry out the fractal analysis of nanoparticles in nanofluids. We propose a novel analytical model for subcooled pool boiling heat transfer of nanofluids in Section 3. In Section 4, results and discussion are presented. Some concluding remarks are given in Section 5.

2. Fractal analysis of nanoparticles in nanofluids

Euclidean geometry can be used to depict ordered objects such as curves, surfaces, and cubes, using integer dimensions

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1, 2, and 3, respectively. However, it is found that numerous objects are disordered and irregular in nature, such as coastlines, rough surfaces, mountains, rivers, lakes, and islands. They do not follow the Euclidean description since it employs scale-dependent measures of length, area, and volume. Such objects are called fractals and are described using a non-integral dimension called the fractal dimension.[11] The fractal object is related to the length scale by a power law:[11]

$$M(\lambda) \sim \lambda^{d_f}, \tag{1}$$

where d_f is the fractal dimension of an object , $M(\lambda)$ can be a quantity, volume or area of the object and λ is the length scale.

The fractal dimension (d_f) of nanoparticles is an important parameter for the fractal analysis of nanofluids. The relation between the fractal dimension of nanoparticles and nanoparticle concentration can be expressed as[2,5,12]

$$c = \beta^{2-d_f}, \tag{2}$$

where c is nanoparticle concentration, $D_{min}/D_{max} = \beta$, D_{max}, and D_{min} are the maximum and minimum diameters of nanoparticles, respectively.

Moreover, fractal math is highly important for describing the number of fractal nanoparticles. For fractal nanofluids, the number of nanoparticles whose sizes are within the infinitesimal range from D to $D + \mathrm{d}D$ can be expressed as[2,5,12]

$$-\mathrm{d}N = d_f D_{max}^{d_f} D^{-(d_f+1)}\,\mathrm{d}D. \tag{3}$$

The negative sign in Eq. (3) implies that the number of nanoparticles decreases with an increase of nanoparticle size and $-\mathrm{d}N > 0$.

The probability density function of nanoparticles is given by[12]

$$f(D) = d_f D_{min}^{d_f} D^{-(d_f+1)}. \tag{4}$$

Patterned after the probability theory of nanoparticles, the probability density function $f(\lambda)$ should satisfy the following normalization relationship or the total cumulative probability:

$$\int_{D_{min}}^{D_{max}} f(D)\,\mathrm{d}D = 1 - \beta^{d_f} \equiv 1. \tag{5}$$

In general, most fractal objects have $\beta \leq 10^{-2}$.[2,5,7,13] Thus equations (2)–(5) approximately hold. Thus, the fractal theory and technique can be used to analyze the characteristics of nanoparticles.

The above equation holds if and only if the following equation is satisfied:

$$\beta^{d_f} \cong 0. \tag{6}$$

Equation (6) can be considered as a criterion to indicate whether nanoparticles can be characterized by fractal theory and technique.

The average diameter of nanoparticles can be found with the aid of Eq. (3)

$$\begin{aligned} D_m &= \int_{D_{min}}^{D_{max}} Df(D)\,\mathrm{d}D \\ &= \int_{D_{min}}^{D_{max}} D[d_f D_{min}^{d_f} D^{-(d_f+1)}]\,\mathrm{d}D \\ &= \frac{d_f}{d_f - 1} D_{min}[1 - \beta^{d_f-1}]. \end{aligned} \tag{7a}$$

So, given β, d_f, and the average diameter D_m of nanoparticles, the minimum and maximum diameters of nanoparticles D_{min}, D_{max} can be obtained from Eq. (7a) as

$$D_{min} = \frac{(d_f - 1)D_m}{d_f[1 - \beta^{d_f-1}]}, \tag{7b}$$

$$D_{max} = \frac{(d_f - 1)D_m}{d_f\beta[1 - \beta^{d_f-1}]}. \tag{7c}$$

Equations (2)–(7) form the basis for the analysis of nanofluids and will be employed to derive the fractal subcooled pool boiling heat transfer model in the following subsections.

3. Modeling heat flux of subcooled pool boiling in fractal nanofluids

The gravity, Brownian force, and friction force between the fluids and nanoparticles may coexist in the nanofluids.[14] Besides, it is shown that energy transport due to Brownian diffusion is smaller than the energy transport due to the convection caused by the Brownian movement of the nanoparticles.[15,16] Experimental observations have shown that the addition of nanoparticles enhances single-phase convection heat transfer and decreased the incipient boiling superheat.[17–20] Hence, there are two main mechanisms contributing to subcooled pool boiling heat transfer of nanofluids: the heat flux (q_{nc}) from all nanoparticles moving in liquid and the other (q_{sb}) from subcooled pool boiling of the base fluids. Thus the heat flux (q_{sn}) of subcooled pool boiling heat transfer of nanofluids can be expressed as

$$q_{sn} = q_{nc} + q_{sb}. \tag{8}$$

The moving nanoparticles will carry energy and the heat exchange may occur between hot and cold regions, reducing the local temperature gradient for a given heat flux compared with the case of pure liquid. Thus, the heat transferred by convection for a single nanoparticle moving in fluids can be expressed as

$$Q = h_n A_n \Delta T_w, \tag{9}$$

where h_n is the average heat transfer coefficient caused by the Brownian motion of nanoparticles in fluids, A_n is the surface area of a nanoparticle, ΔT_w is the wall superheat ($T_w - T_s$), and T_w and T_s are the temperatures of the wall temperature and the

saturation temperature of fluids, respectively. In general, the temperature of nanofluids is moderate, not high, for subcooled pool boiling. So we can consider that the temperature difference between the nanoparticles and the base fluids is approximately equal to the temperature difference between the wall and the fluids. Similar methods were used in several reported studies.[2,21,22]

The root-mean-square velocity (V) of a Brownian nanoparticle can be calculated as[14,23]

$$V = \sqrt{\frac{3k_{\rm B}T}{m}} = \frac{1}{D}\sqrt{\frac{18k_{\rm B}T}{\pi\rho_{\rm n}D}}, \tag{10}$$

where $k_{\rm B} = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant, T is temperature, m is the mass of nanoparticles, and $\rho_{\rm n}$ is the density of nanoparticles.

For convection in the Stokes region, the average heat transfer coefficient $h_{\rm n}$ for an isothermal moving particle is given by[23,24]

$$h_{\rm n} = \frac{Nu \cdot k_{\rm f}}{D} = \frac{(2+0.5RePr)k_{\rm f}}{D}, \tag{11}$$

where $k_{\rm f}$ is the thermal conductivity of base fluids, D is the diameter of nanoparticles, $Re(=V \cdot D/\upsilon)$ is the Reynolds number, and $Pr(=\upsilon/\alpha)$ is the Prandtl number of fluids. Hence, $h_{\rm n}$ can be further reduced to

$$h_{\rm n} = D^{-3/2}\sqrt{\frac{4.5k_{\rm B}k_{\rm f}^2T}{\pi\rho_{\rm n}\alpha^2}} + 2D^{-1}k_{\rm f}. \tag{12}$$

The surface area of a Brownian nanoparticle can be expressed as

$$A_{\rm n} = \pi D^2. \tag{13}$$

So the total quantity of heat caused by all nanoparticles moving in liquids from $D_{\rm min}$ to $D_{\rm max}$ can be obtained from Eq. (3) as

$$\begin{aligned} Q_{\rm T} &= \int_{D_{\rm min}}^{D_{\rm max}} Q(-{\rm d}N) \\ &= \Delta T_{\rm w}\int_{D_{\rm min}}^{D_{\rm max}} h_{\rm n}A_{\rm n}(-{\rm d}N), \end{aligned} \tag{14}$$

where $h_{\rm n}$ is given by Eq. (12), $A_{\rm n}$ is given by Eq. (13), and $(-{\rm d}N)$ is given by Eq. (3).

Inserting Eqs. (3), (12), and (13) into Eq. (14), Eq. (14) can now be integrated as

$$\begin{aligned} Q_{\rm T} &= \int_{D_{\rm min}}^{D_{\rm max}} \frac{k_{\rm f}}{D}\left(D^{-3/2}k_{\rm f}\sqrt{\frac{4.5k_{\rm B}k_{\rm f}^2T}{\pi\rho_{\rm n}\alpha^2}} + 2D^{-1}k_{\rm f}\right) \\ &\quad \times \pi D^2[d_{\rm f}D_{\rm max}^{d_{\rm f}}D^{-(d_{\rm f}+1)}{\rm d}D](\Delta T_{\rm w}) \\ &= \Delta T_{\rm w}\left\{\frac{3}{\alpha}\sqrt{\frac{2\pi k_{\rm B}T}{\rho_{\rm n}}}\frac{[D_{\rm min}^{(1/2-d_{\rm f})} - D_{\rm max}^{(1/2-d_{\rm f})}]}{(2d_{\rm f}-1)}\right. \\ &\quad \left. + \frac{2\pi(D_{\rm min}^{(1-d_{\rm f})} - D_{\rm max}^{(1-d_{\rm f})})}{(d_{\rm f}-1)}\right\}k_{\rm f}d_{\rm f}D_{\rm max}^{d_{\rm f}}. \end{aligned} \tag{15}$$

Due to

$$c = \beta^{2-d_{\rm f}} \Rightarrow d_{\rm f} = 2 - \log_\beta^c. \tag{16}$$

By inserting Eq. (16) into Eq. (15), equation (15) can be further reduced to

$$\begin{aligned} Q_{\rm T} &= \left\{\frac{3}{\alpha}\sqrt{\frac{2\pi k_{\rm B}T}{\rho_{\rm n}}}\frac{\left[D_{\rm min}^{(\log_\beta^c-1.5)} - D_{\rm max}^{(\log_\beta^c-1.5)}\right]}{(3-2\log_\beta^c)}\right. \\ &\quad \left. + \frac{2\pi\left(D_{\rm min}^{\log_\beta^c-1} - D_{\rm max}^{\log_\beta^c-1}\right)}{(1-\log_\beta^c)}\right\} \\ &\quad \times k_{\rm f}(2-\log_\beta^c)D_{\rm max}^{(2-\log_\beta^c)}(\Delta T_{\rm w}). \end{aligned} \tag{17}$$

With the aid of Eq. (3), the total surface area of all nanoparticles can be obtained from Eq. (13) as

$$\begin{aligned} A_{\rm T} &= \int_{D_{\rm min}}^{D_{\rm max}} A_{\rm n}(-{\rm d}N) = \pi(2\log_c^\beta - 1) \\ &\quad \times \left(D_{\rm max}^{\log_\beta^c} - D_{\rm min}^{\log_\beta^c}\right)D_{\rm max}^{(2-\log_\beta^c)}. \end{aligned} \tag{18}$$

So the heat flux $q_{\rm nc}$ can be calculated as

$$\begin{aligned} q_{\rm nc} &= \frac{Q_{\rm T}}{A_{\rm T}} = \left\{\frac{3}{\alpha}\sqrt{\frac{2k_{\rm B}T}{\pi\rho_{\rm n}}}\frac{[\beta^{(\log_\beta^c-1.5)}-1]}{(3-2\log_\beta^c)}D_{\rm max}^{-1/2}\right. \\ &\quad \left. + \frac{2(\beta^{\log_\beta^c-1}-1)}{(1-\log_\beta^c)}\right\}\frac{k_{\rm f}\log_\beta^c D_{\rm max}^{-1}(\Delta T_{\rm w})}{(1-\beta^{\log_\beta^c})}. \end{aligned} \tag{19}$$

Equation (19) indicates that $q_{\rm nc}$ is related to the nanoparticle concentration (c), the maximum diameter ($D_{\rm max}$) of nanoparticles, and the wall superheat ($\Delta T_{\rm w}$), which is consistent with the physical phenomena.

By inserting Eq. (7c) into Eq. (19), the heat flux ($q_{\rm nc}$) can be further modified as

$$\begin{aligned} q_{\rm nc} &= \left\{\frac{3}{\alpha}\sqrt{\frac{2k_{\rm B}T}{\pi\rho_{\rm n}}}\frac{[\beta^{(\log_\beta^c-1.5)}-1]}{(3-2\log_\beta^c)}\right. \\ &\quad \times \left[\frac{\beta(2-\log_\beta^c)(1-\beta^{(1-\log_\beta^c)})}{(1-\log_\beta^c)}\right]^{3/2}D_{\rm m}^{-1/2} \\ &\quad \left. + \frac{2\beta(2-\log_\beta^c)[\beta^{(\log_\beta^c-1)}+\beta^{(1-\log_\beta^c)}-2]}{(1-\log_\beta^c)^2}\right\} \\ &\quad \times \frac{k_{\rm f}\log_\beta^c}{(1-\beta^{\log_\beta^c})}D_{\rm m}^{-1}(\Delta T_{\rm w}). \end{aligned} \tag{20}$$

Equation (20) shows that the heat flux ($q_{\rm nc}$) increases as the average diameter of nanoparticles ($D_{\rm m}$) decreases. This is expected because the small diameter of nanoparticles may cause an increase of nanoparticle velocity, leading to more heat being transferred by nanoparticles moving in nanofluids. Equation (20) depicts the heat flux ($q_{\rm nc}$) as a function of the average

diameter of nanoparticles (D_m), the nanoparticle concentration (c), and the wall superheat (ΔT_w).

Forster and Greif,[25] and Lin[26] assumed that the bubble ruptures when the bubble grows into a hemisphere and the radius of the bubble reaches R_b (the bubble departure radius) for subcooled boiling in base fluids. For the heat transfer of subcooled boiling, based on the work by Forster and Greif,[25] Lin[26] derived the heat flux (q_{sb}) as

$$q_{sb} = c_p \rho_f \left(\frac{2}{3}\pi R_b^3\right)\left(\frac{T_w + T_\infty}{2} - T_\infty\right) f N_a, \tag{21}$$

where c_p is the specific heat at a constant pressure, ρ_f is the density of the base fluids, R_b is the bubble departure radius, f is the bubble departure frequency and N_a is the number of active cavities per unit area of heated surfaces.

It is well known that subcooled pool boiling heat transfer is characterized by the formation of vapor bubbles that nucleate, grow and subsequently detach from the locations of active cavities. So it is important to characterize the distribution of active cavities for subcooled pool boiling. Similarly, the number of active cavities whose sizes are within the infinitesimal range from D_a to $D_a + dD_a$ can be expressed as[5,27,28]

$$-dN_a = D_f D_{a,max}^{D_f} D_a^{-(D_f+1)} dD_a, \tag{22}$$

where D_f is the fractal dimension of an active cavity on the heated surfaces, $dD_a > 0$, and $-dN_a > 0$, which means that the number of active cavities decreases with an increase of the diameter of active cavities.

Since the size distribution of active cavities is found to be fractal,[5,27,28] a fractal analytical model for base fluids in subcooled pool boiling from $D_{a,min}$ to the $D_{a,max}$ can be obtained by modifying Eq. (21) as

$$\begin{aligned} q_{sb} &= \int dq_{sb} = \int_{D_{a,min}}^{D_{a,max}} c_p \rho_f \left(\frac{2}{3}\pi R_b^3\right) \\ &\quad \times \left(\frac{T_w + T_\infty}{2} - T_\infty\right) f(-dN_a) \\ &= \frac{\pi}{3} c_p \rho_f (\Delta T_w + \Delta T_{sub}) \int_{D_{a,min}}^{D_{a,max}} R_b^3 f(-dN_a). \end{aligned} \tag{23}$$

Equation (23) can be integrated if f and R_b are expressed in terms of D_a. The volume V_b of single bubble at departure is given by Van der Geld[29] as

$$V_b = \frac{\pi D_a^3}{Eo}, \tag{24a}$$

where Eo is the Eotvos number, which is given by Mori[30] as

$$Eo = \frac{g(\rho_f - \rho_g) D_a^2}{\sigma}, \tag{24b}$$

where g is the gravitational acceleration, ρ_g is the vapor density, and σ is the surface tension of the fluid.

If Eq. (24b) is substituted into Eq. (24a), equation (24a) can be further modified as

$$V_b = \frac{\pi\sigma}{g(\rho_f - \rho_g)} D_a. \tag{24c}$$

We note that the volume of a single bubble at departure, V_b, is usually expressed as

$$V_b = \frac{4}{3}\pi R_b^3. \tag{25}$$

Comparing Eq. (24c) with Eq. (25), we can obtain R_b^3 as

$$R_b^3 = \frac{3\sigma}{4g(\rho_f - \rho_g)} D_a. \tag{26}$$

To this end, we note that the bubble departure frequency (f) is usually expressed as

$$f = \frac{1}{t_w + t_g}, \tag{27}$$

where t_w is the bubble waiting time and t_g is the bubble growth time. In pure liquids, Van Stralen *et al.*[31] assumed that the waiting time is related to the growth time by

$$t_w = 3t_g. \tag{28a}$$

The bubble waiting time is given by[27]

$$t_w = \frac{9}{4\pi\alpha}\left[\frac{(T_w - T_\infty) D_a}{2(T_w - T_s)}\right]^2. \tag{28b}$$

Due to $T_w - T_\infty = \Delta T_{sub} + \Delta T_w$ and $T_w - T_s = \Delta T_w$ for subcooled pool boiling, Eq. (28b) can be written as

$$t_w = \frac{9 D_a^2 (\Delta T_w + \Delta T_{sub})^2}{16\pi\alpha(\Delta T_w)^2}. \tag{28c}$$

Substituting Eqs. (28c) and (28a) into Eq. (27), we can see that the bubble departure frequency, f, is related to the sizes of the active cavities as

$$f = \frac{4\pi\alpha(\Delta T_w)^2}{3(\Delta T_w + \Delta T_{sub})^2} D_a^{-2}. \tag{29}$$

From Eqs. (26) and (29) we see that both R_b^3 and f are related to the sizes of the active cavities (D_a). So equation (23) can now be integrated to give

$$\begin{aligned} q_{sb} &= \frac{\pi}{3} c_p \rho_f (\Delta T_w + \Delta T_{sub}) \int_{D_{a,min}}^{D_{a,max}} R_b^3 f(-dN_a) \\ &= \frac{\pi}{3} c_p \rho_f (\Delta T_w + \Delta T_{sub}) \int_{D_{a,min}}^{D_{a,max}} \left[\frac{3\sigma}{4g(\rho_f - \rho_g)} D_a\right] \\ &\quad \times \left[\frac{4\pi\alpha(\Delta T_w)^2}{3(\Delta T_w + \Delta T_{sub})^2} D_a^{-2}\right] (D_f D_{a,max}^{D_f} D_a^{-(D_f+1)} dD_a) \\ &= \frac{\pi^2 \alpha c_p \rho_f \sigma}{3g(\rho_f - \rho_g)} \frac{D_f}{D_f + 1} D_{a,max}^{-1} \\ &\quad \times [\gamma^{-(D_f+1)} - 1] \frac{(\Delta T_w)^2}{(\Delta T_w + \Delta T_{sub})}, \end{aligned} \tag{30}$$

where $\gamma = D_{a,min}/D_{a,max}$. Equation (30) denotes that the heat flux (q_{sb}) is a function of the wall superheat, subcooling of fluids, the fractal dimension of active cavities, and physical properties of fluids. Feng[7] and Cai *et al.*[32,33] found that the ratio between minimum and maximum pore size is 0.001, which best fits the experimental data in a fractal porous medium. In this work, we consider that the active cavities formed on the heated surfaces are analogous to pores in porous media. Besides, the number and size of active cavities obey the fractal scaling law if and only if $\gamma < 10^{-2}$.[27,28] Additionally, Yang *et al.*[34] investigated the active cavity density and size for pool boiling using an experimental system of stainless steel and water. The experimental results[34] show that the ratio between minimum and maximum active cavity size could reach 0.001. So the ratio $\gamma = 10^{-3}$ is used for subcooled pool boiling heat transfer of fractal nanofluids in our work.

Due to $1 < D_f < 2$, $-3 < -D_f - 1 < -2$, $\gamma^{-(D_f+1)} \gg 1$, equation (30) can be further reduced to

$$q_{sb} = \frac{\pi^2 \alpha c_p \rho_f \sigma}{3g(\rho_f - \rho_g)} \frac{D_f \gamma^{-D_f}}{D_f + 1} D_{a,min}^{-1} \frac{(\Delta T_w)^2}{(\Delta T_w + \Delta T_{sub})}. \quad (31)$$

The minimum diameter of an active cavity is given by Griffith[35,36] as

$$D_{a,min} = \frac{4\sigma T_s}{\rho_g h_{fg} \Delta T_w}, \quad (32)$$

where h_{fg} is the latent heat of the evaporation of fluids.

Inserting Eq. (32) into Eq. (31), equation (31) can be further modified as

$$q_{sb} = \frac{\pi^2 \alpha c_p \rho_f \rho_g h_{fg}}{12g(\rho_f - \rho_g)T_s} \frac{D_f \gamma^{-D_f}}{D_f + 1} \frac{(\Delta T_w)^3}{(\Delta T_w + \Delta T_{sub})}. \quad (33)$$

In pool boiling of the base fluids, the fractal dimension D_f of an active cavity on the heated surfaces is given by[27]

$$D_f = \ln[\bar{D}_{a,max}/(\sqrt{2} D_{a,min})]^2 / \ln \gamma^{-1}, \quad (34)$$

where $\bar{D}_{a,max}$ is the averaged value over all the maximum active cavities.[27]

Inserting Eqs. (20) and (34) into Eq. (8), a fractal analytical model for subcooled pool boiling of nanofluids can be obtained as

$$\begin{aligned} q_{sn} &= q_{nc} + q_{sb} \\ &= \left\{ \frac{3}{\alpha} \sqrt{\frac{2k_B T}{\pi \rho_n}} \frac{[\beta^{(\log_\beta^c - 1.5)} - 1]}{(3 - 2\log_\beta^c)} \right. \\ &\quad \times \left[\frac{\beta(2 - \log_\beta^c)(1 - \beta^{(1-\log_\beta^c)})}{(1 - \log_\beta^c)} \right]^{3/2} D_m^{-1/2} \\ &\quad \left. + \frac{2\beta(2 - \log_\beta^c)[\beta^{(\log_\beta^c - 1)} + \beta^{(1-\log_\beta^c)} - 2]}{(1 - \log_\beta^c)^2} \right\} \\ &\quad \times \frac{k_f \log_\beta^c}{(1 - \beta^{\log_\beta^c})} D_m^{-1} (\Delta T_w) \\ &\quad + \frac{\pi^2 \alpha c_p \rho_f \rho_g h_{fg}}{12g(\rho_f - \rho_g)T_s} \frac{D_f \gamma^{-D_f}}{D_f + 1} \frac{(\Delta T_w)^3}{(\Delta T_w + \Delta T_{sub})}. \end{aligned} \quad (35)$$

Equation (35) indicates that the heat flux of subcooled pool boiling heat transfer in nanofluids is explicitly related to the nanoparticle concentration (c), the average diameter of nanoparticles (D_m), the fractal dimension (D_f) of an active cavity on the heated surfaces, the wall superheat and the subcooling of fluids. No additional/new empirical constant is introduced in this model. It is expected that Eq. (35) has fewer empirical constants than conventional models and that every parameter in Eq. (35) has clear physical meaning. It is expected that this fractal model can reveal the detailed mechanisms of the subcooled pool boiling heat transfer of nanofluids.

4. Results and discussion

Heat transfer characteristics of $CaCO_3$ and Cu nanofluids with and without acoustic cavitation have been investigated experimentally.[17,18] The effects of such factors as the acoustical parameters, nanoparticle concentration, and fluids subcooling on heat transfer enhancement around a heated horizontal copper tube were discussed in detail.[17,18] Figure 1 compares the heat flux of subcooled pool boiling heat transfer from this model (calculated using Eq. (35)) with that from reported experiments[17] at $D_m = 90$ nm, $c = 0.267$ g/l and $\Delta T_{sub} = 25$ K for $CaCO_3$ nanofluids. It is seen that there is a fair agreement between the model prediction and the experimental data. The solid line in Fig. 1 represents the predictions of this model. Figure 2 shows a comparison of the heat flux of subcooled pool boiling heat transfer predicted by this model (by Eq. (35)) and that from reported experiments[17] at $D_m = 80$ nm, $c = 0.267$ g/l and $\Delta T_{sub} = 14$ K for Cu nanofluids. Note that a reasonable agreement between the model predictions and the experimental data is found again. In Figs. 1 and 2, the obvious deviations between theoretical and experimental data spotted at large ΔT_w can be observed. Generally, there are some normal deviations between model predictions and experimental data. Besides, there are many parameters, such as subcooling of fluids, nanoparticle concentrations, size of nanoparticles, surface roughness, surface orientation, and contact angle, which can affect the heat transfer of nanofluids. Actually, although the subcooling of fluids and nanoparticle concentrations are the same in the literature,[17] the experimental data[17] are different when sound source distances between the vibrator head and the central horizontal plane are changed. Figure 3 shows that the fractal dimension (by Eq. (16)) of nanoparticles increases as the volumetric nanoparticle concentration (c) increases. This can be explained in that the high c causes an increase of the number of nanoparticles, leading to an increase of the fractal dimension of nanoparticles. Figure 4 is a plot of the fractal dimension (by Eq. (34)) of

active cavities on the heated surfaces versus the wall superheat. According to the fractal geometry theory, the fractal dimension of active cavities should be in the range of $1 < D_f < 2$ in two dimensions. Figure 4 shows that the fractal dimension of an active cavity increases with the wall superheat. This means that D_f is a function of the wall superheat. All of these characteristics are as expected and are consistent with the practical physical phenomena. Therefore, this fractal model can reveal the mechanisms of subcooled pool boiling heat transfer.

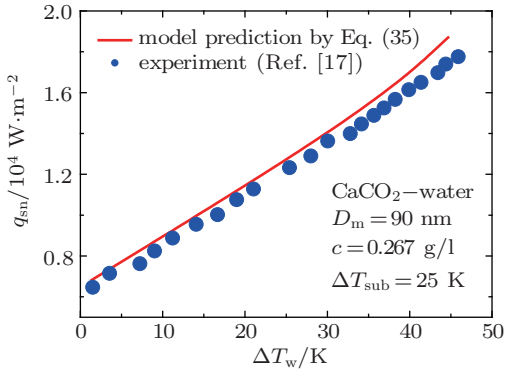


Fig. 1. (color online) Comparison between our model predictions and the experimental data at $D_m = 90$ nm, $c = 0.267$ g/l, and $\Delta T_{sub} = 25$ K for $CaCO_3$ nanofluids.

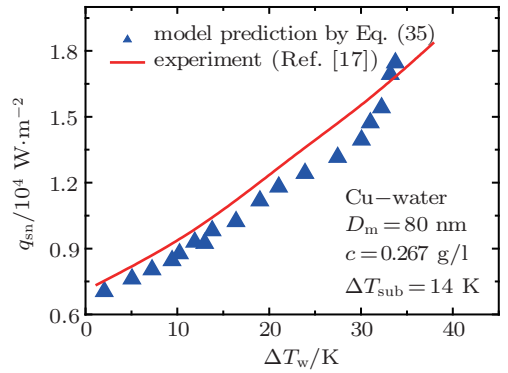


Fig. 2. (color online) Comparison between our model predictions and the experimental data at $D_m = 80$ nm, $c = 0.267$ g/l and $\Delta T_{sub} = 14$ K for Cu nanofluids.

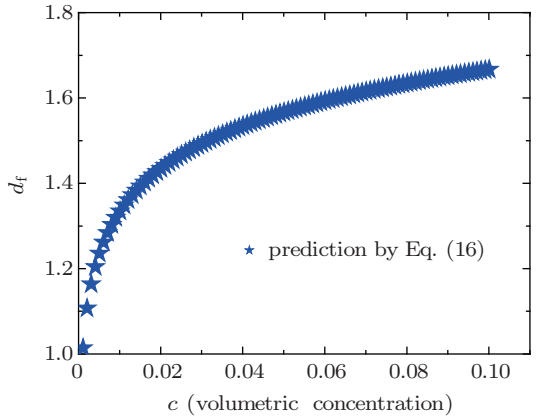


Fig. 3. (color online) Fractal dimension of nanoparticles versus volumetric concentration of nanoparticles.

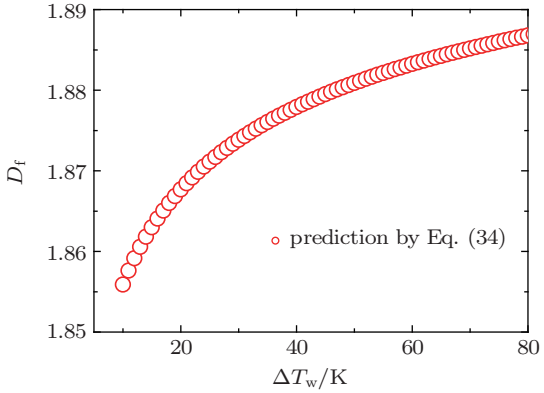


Fig. 4. (color online) The fractal dimension of active cavities on the heated surfaces versus the wall superheat.

5. Conclusions

In this work, we propose a novel fractal model for subcooled pool boiling heat transfer of nanofluids based on the fractal distribution of nanoparticles and active cavities on the heated surfaces. The formula for calculating the heat flux of a subcooled pool boiling is obtained considering the effect of nanoparticles moving in the fluids. In this approach, the proposed model is explicitly related to the subcooling of fluids and the wall superheat. Every parameter of these formulas has a clear physical meaning. It is shown that the fractal model is effectual, as there is reasonable agreement between the model predictions and experimental data. The validity of the proposed model is thus verified. It can reveal the physical mechanisms of subcooled pool boiling heat transfer of nanofluids. Moreover, the proposed model has potential uses in the analysis of other transport properties, such as the optical and electrical properties of nanofluids.

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