Parametric instabilities in single-walled carbon nanotubes*

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Parametric instabilities induced by the coupling excitation between the high frequency quantum Langmuir waves and the low frequency quantum ion-acoustic waves in single-walled carbon nanotubes are studied with a quantum Zakharov model. By linearizing the quantum hydrodynamic equations, we get the dispersion relations for the high frequency quantum Langmuir wave and the low frequency quantum ion-acoustic wave. Using two-time scale method, we obtain the quantum Zaharov model in the cylindrical coordinates. Decay instability and four-wave instability are discussed in detail. It is shown that the carbon nanotube's radius, the equilibrium discrete azimuthal quantum number, the perturbed discrete azimuthal quantum number, and the quantum parameter all play a crucial role in the instabilities.

Keywords: parametric instabilities, quantum Langmuir waves and ion-acoustic waves, quantum Zakharov model, single-walled carbon nanotubes

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1. Introduction

There has been a growing interest in the field of carbon nanotubes[1–14] since they were discovered by Iijima,[15] because of their unique properties and great potential applications. Their most advanced applications include nanoelectronics, hydrogen storage, field-emitters, biological and chemical gas sensors, medicine, nanomaterials, etc. The metallic or semiconducting properties of carbon nanotubes depend on its radius and the geometric angle, as well as their electron and ion components, which can be regarded as a two-species quantum plasmas. In order to well understand the characteristics and interactions of two-species components, particular attention has recently been extensively devoted to their collection excitations. On the one hand, different approaches (e.g., the classical hydrodynamic model[16,17] and the quantum dielectric-response model with the random-phase approximation[18,19]) have been used to study the high-frequency electron excitation in carbon nanotubes. On the other hand, recently, the low-frequency ion-acoustic excitation in single-walled carbon nanotubes has been further discussed[20] with the quantum hydrodynamic model (QHD) that was developed by Haas *et al.*[21] However, the coupling excitation between the high-frequency electron wave and the low-frequency ion-acoustic wave is not still very clear in the single-walled carbon nanotubes.

It is well known that the parametric instabilities[22] (such as decay instability, four-wave instability) induced by coupling excitations between high frequency quantum Langmuir waves and low frequency quantum ion-acoustic waves are closely related to the two-species quantum plasma excitation. Therefore, we will further study the parametric instabilities in single-walled carbon nanotubes in this context. The Zakharov equations[22–26] are an effective model to investigate the parametric instabilities. So, we start with the quantum hydrodynamical equations (QHE) to discuss the dynamics of the two-species quantum plasma in the single-wall carbon nanotubes in cylindrical coordinates, and get the dispersion relations for the high frequency quantum Langmuir wave and the low frequency quantum ion-acoustic wave, respectively. Then, using the two-time scale method, we obtain the quantum cylindrical Zakharov model describing the coupling of high frequency quantum Langmuir waves and the low frequency quantum ion-acoustic waves in single-walled carbon nanotubes. Furthermore, parametric instabilities in the single-walled carbon nanotubes are discussed in detail. Interestingly, it is found that parameter instabilities depend on the carbon nanotube's radius, the discrete azimuthal quantum number, and the quantum parameter which is related to the Planck constant $\hbar$. Lager discrete azimuthal quantum number tends to enhance the decay instability, while the larger nanotube's radius suppresses the decay instability. However, for the general four-wave instability, the influence of the tubule's radius and the perturbed discrete azimuthal quantum number on the instability depends on the ratio of the perturbed discrete azimuthal quantum number to the equilibrium discrete azimuthal quantum number. But the quantum parameter suppresses instabilities.

This paper is organized as follows. Linearizing the QHE

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for two-species particles, the dispersion relations for the high frequency quantum Langmuir wave, and the low frequency quantum ion-acoustic wave are given respectively. Taking two-time scale method, the quantum Zakharov model in the cylindrical coordinates is obtained in Section 2. In Section 3, we discuss the parametric instabilities in detail. Finally, our main results are briefly summarized in Section 4.

2. Model

2.1. QHE

In single-walled carbon nanotubes, we assume that electrons and ions are distributed uniformly over the cylindrical surface, and consider the single-walled carbon nanotubes as an infinitesimally thin and infinitely long cylindrical shell with a radius a. The QHE for the two-species plasmas in the particular cylindrical coordinates $\boldsymbol{r} = (a, \phi, z)$ can be described by the continuity and momentum-balance equations[20,22,24,27–29]

$$\frac{\partial n_\mathrm{e}}{\partial t} + \nabla_\parallel \cdot (n_\mathrm{e} \boldsymbol{u}_\mathrm{e}) = 0, \tag{1}$$

$$\frac{\partial n_\mathrm{i}}{\partial t} + \nabla_\parallel \cdot (n_\mathrm{i} \boldsymbol{u}_\mathrm{i}) = 0, \tag{2}$$

$$\frac{\partial \boldsymbol{u}_\mathrm{e}}{\partial t} + (\boldsymbol{u}_\mathrm{e} \cdot \nabla_\parallel)\boldsymbol{u}_\mathrm{e} = -\frac{e}{m_\mathrm{e}}\boldsymbol{E} - \frac{1}{m_\mathrm{e} n_\mathrm{e}}\nabla_\parallel P_\mathrm{e} + \frac{\hbar^2}{2m_\mathrm{e}^2}\nabla_\parallel\left(\frac{1}{\sqrt{n_\mathrm{e}}}\nabla_\parallel^2\sqrt{n_\mathrm{e}}\right), \tag{3}$$

$$\frac{\partial \boldsymbol{u}_\mathrm{i}}{\partial t} + (\boldsymbol{u}_\mathrm{i} \cdot \nabla_\parallel)\boldsymbol{u}_\mathrm{i} = \frac{e}{m_\mathrm{i}}\boldsymbol{E}, \tag{4}$$

where subscripts e and i stand for electrons and ions, respectively. $m_{\mathrm{e,i}}$, $n_{\mathrm{e,i}}$, and $\boldsymbol{u}_{\mathrm{e,i}}$ represent the mass, density, and fluid velocity of electrons and ions, respectively.

$$\nabla_\parallel = \frac{1}{a}\frac{\partial}{\partial\phi}\hat{\boldsymbol{e}}_\phi + \frac{\partial}{\partial z}\hat{\boldsymbol{e}}_z,$$

$\boldsymbol{E}$ is the self-consistent electric field, $\hbar$ is the Planck constant divided by 2π. The pressure term P_e appearing in Eq. (3) must be related to the electron density n_e via an equation of state (EOS) in order to close our system of electron fluid, we take

$$P_\mathrm{e} = \frac{1}{\gamma}n_0 k_\mathrm{B} T_\mathrm{e}(n_\mathrm{e}/n_0)^\gamma$$

for the homogeneous electron gas. Here, k_B is the Boltzmann constant, T_e is the electron temperature, n_0 is the equilibrium electron density, $\gamma = (2+d)/d$ represents the polytropic exponent, and $d = 2$ is the relevant dimensions. In addition, the third term on the right-hand side of Eq. (3) is the so-called Bohm potential due to the quantum diffraction effects. In general, we assume ions to be pressureless and neglect quantum diffraction in view of their large mass. The self-consistent electric field $\boldsymbol{E}$ satisfies the Poisson equation $\nabla_\parallel \cdot \boldsymbol{E} = \rho/\varepsilon_0$, where ε_0 is the vacuum dielectric. Since we have assumed that electrons and ions are distributed uniformly over the cylindrical surface, when the carbon nanotubes's radius a is infinitesimally small we can write the volume charge density (per unit length of the tubes) $\rho = 2\pi a e(n_\mathrm{i} - n_\mathrm{e})/\pi a^2$. So, we have the Poisson equation in the cylindrical coordinates $\boldsymbol{r} = (a, \phi, z)$

$$\nabla_\parallel \cdot \boldsymbol{E} = \frac{2e}{\varepsilon_0 a}(n_\mathrm{i} - n_\mathrm{e}). \tag{5}$$

As is usually done in linear disturbance theory, around the homogeneous equilibrium $n_\mathrm{e} = n_\mathrm{i} = n_0$, $\boldsymbol{u}_\mathrm{e} = 0$, and $\boldsymbol{E} = 0$, every relevant physical quantity is divided into its equilibrium and perturbed components $n_\mathrm{e} = n_0 + n_{\mathrm{e}1}$, $\boldsymbol{u}_\mathrm{e} = \boldsymbol{u}_{\mathrm{e}1}$, and $\boldsymbol{E} = \boldsymbol{E}_1$. We linearize Eqs. (1), (3), and (5), further apply the Fourier transform $A = \hat{A}\,\mathrm{e}^{\mathrm{i}kz + \mathrm{i}m\phi - \mathrm{i}\omega t}$, here A represents any of the above-list perturbed components. As a result, one obtains the following dispersion relation for the high frequency quantum Langmuir wave

$$\omega^2 = \omega_\mathrm{p}^2 + v_\mathrm{e}^2\left(k^2 + \frac{m^2}{a^2}\right) + \frac{\hbar^2}{4m_\mathrm{e}^2}\left(k^4 + \frac{m^4}{a^4}\right), \tag{6}$$

where k is the longitudinal wave number, m is the discrete azimuthal quantum number, ω is the Langmuir wave frequency, $\omega_\mathrm{p} = (2n_0 e^2/(m_\mathrm{e}\varepsilon_0 a))^{1/2}$ is the electron plasma frequency, and $v_\mathrm{e} = (\kappa_\mathrm{B} T_\mathrm{e}/m_\mathrm{e})^{1/2}$ is the electron thermal velocity. Then, following the steps of Ref. [25], we get the dispersion relation for the low frequency quantum ion-acoustic wave

$$\Omega^2 = \frac{v_\mathrm{i}^2\left(k^2 + \frac{m^2}{a^2}\right) + \frac{\hbar^2}{4m_\mathrm{i}m_\mathrm{e}}\left(k^4 + \frac{m^4}{a^4}\right)}{1 + \frac{v_\mathrm{i}^2}{\Omega_\mathrm{p}^2}\left(k^2 + \frac{m^2}{a^2}\right) + \frac{\hbar^2}{4m_\mathrm{i}m_\mathrm{e}\Omega_\mathrm{p}^2}\left(k^4 + \frac{m^4}{a^4}\right)}, \tag{7}$$

where, Ω is the ion-acoustic wave frequency, $\Omega_\mathrm{p} = (2n_0 e^2/(m_\mathrm{i}\varepsilon_0 a))^{1/2}$ is the ion plasma frequency, and $v_\mathrm{i} = (\kappa_\mathrm{B} T_\mathrm{e}/m_\mathrm{i})^{1/2}$ is the ion thermal velocity. Assuming the plasma system is a quasineutrality one, namely, $n_\mathrm{i} = n_\mathrm{e}$, the relation can be further simplified as

$$\Omega^2 = v_\mathrm{i}^2\left(k^2 + \frac{m^2}{a^2}\right) + \frac{\hbar^2}{4m_\mathrm{i}m_\mathrm{e}}\left(k^4 + \frac{m^4}{a^4}\right). \tag{8}$$

It can be seen that the tube's radius a and the azimuthal quantum number m modify the dispersion relations of the quantum Langmuir wave and the quantum ion-acoustic wave in single-walled carbon nanotubes.

2.2. Quantum cylindrical Zakharov model

Using two time scale theory, all quantities are separated into high frequency (subscript h) and low frequency (subscript l) components, $n_\mathrm{e} = n_0 + n_\mathrm{l} + n_\mathrm{h}$, $n_\mathrm{i} = n_0 + n_\mathrm{l}$, $\boldsymbol{u}_\mathrm{e} = \boldsymbol{u}_\mathrm{l} + \boldsymbol{u}_\mathrm{h}$, $\boldsymbol{u}_\mathrm{i} = \boldsymbol{u}_\mathrm{l}$, $\boldsymbol{E} = \boldsymbol{E}_\mathrm{l} + \boldsymbol{E}_\mathrm{h}$, the high frequency parts are neglected for ions, which is also due to the larger ion mass. As usually done,[22–24] writing the fast time scale electric field in the form $\boldsymbol{E}_\mathrm{h} = \widetilde{\boldsymbol{E}}[\exp(-\mathrm{i}\omega_\mathrm{p}t) + \exp(\mathrm{i}\omega_\mathrm{p}t)]/2$, here, $\widetilde{\boldsymbol{E}}$ is the slowly vary-

ing envelope of the high frequency electric field. We get the following equation about $\widetilde{\boldsymbol{E}}$ in the high frequency case

$$\mathrm{i}\frac{\partial \widetilde{\boldsymbol{E}}}{\partial t}+\frac{v_{\mathrm{e}}^2}{2\omega_{\mathrm{pe}}}\nabla_{\parallel}(\nabla_{\parallel}\cdot\widetilde{\boldsymbol{E}})-\frac{\hbar^2}{8m_{\mathrm{e}}^2\omega_{\mathrm{pe}}}\nabla_{\parallel}[\nabla_{\parallel}^2(\nabla_{\parallel}\cdot\widetilde{\boldsymbol{E}})] = \frac{\omega_{\mathrm{pe}}}{2}\frac{n_1}{n_0}\widetilde{\boldsymbol{E}}. \tag{9}$$

Taking a time-averaging method[22,23] to the high frequency quantities, one obtains the equation in the low frequency case

$$\frac{\partial^2 n_1}{\partial t^2}-v_{\mathrm{i}}^2\nabla_{\parallel}^2 n_1+\frac{\hbar^2}{4m_{\mathrm{i}}m_{\mathrm{e}}}\nabla_{\parallel}^4 n_1=\frac{\varepsilon_0 a}{8m_{\mathrm{i}}}\nabla_{\parallel}^2(|\widetilde{\boldsymbol{E}}|^2). \tag{10}$$

Equations (9) and (10) are the quantum Zakharov equations in the cylindrical coordinates $\boldsymbol{r}=(a,\phi,z)$. Normalizing Eqs. (9) and (10) with $\boldsymbol{r}\sim 2\sqrt{\mu}\boldsymbol{r}/\lambda_{\mathrm{e}}$, $t\sim 2\mu\omega_{\mathrm{p}}t$, $n\sim n_1/4\mu n_0$, and $\boldsymbol{E}\sim e\widetilde{\boldsymbol{E}}/4\sqrt{\mu}m_{\mathrm{e}}\omega_{\mathrm{p}}v_{\mathrm{e}}$, here, $\mu=m_{\mathrm{e}}/m_{\mathrm{i}}$, $\lambda_{\mathrm{e}}=v_{\mathrm{e}}/\omega_{\mathrm{p}}$, we get

$$\mathrm{i}\frac{\partial \boldsymbol{E}}{\partial t}+\nabla_{\parallel}(\nabla_{\parallel}\cdot\boldsymbol{E})-H^2\nabla_{\parallel}[\nabla_{\parallel}^2(\nabla_{\parallel}\cdot\boldsymbol{E})]=n\boldsymbol{E}, \tag{11}$$

$$\frac{\partial^2 n}{\partial t^2}-\nabla_{\parallel}^2 n+H^2\nabla_{\parallel}^4 n=\nabla_{\parallel}^2(|\boldsymbol{E}|^2), \tag{12}$$

where the dimensionless quantum parameter $H=\hbar\Omega_{\mathrm{p}}/\kappa_{\mathrm{B}}T_{\mathrm{e}}$. In general, the radius of the single-walled nanotubes ranges from 1 nm to 15 nm, surface electron density $n_0=4\times 38\ \mathrm{nm}^{-2}$.[27,30] Setting the nanotubes radius as 5 nm, electron temperature $T=10^4$ K, we roughly estimate $H=0.25$ and dimensionless radius $a=12$. We have obtained the effective models (11) and (12) for studying the coupling interaction between high frequency quantum Langmuir waves and low frequency ion-acoustic waves in the single-walled carbon nanotubes. Therefore, two parametric instabilities (i.e. the decay instability and the four-wave instability) in the single-walled carbon nanotubes need to be further discussed in the next section.

3. Parametric instabilities in single-walled carbon nanotubes

3.1. Decay instability

The decay instability generally involves the interaction between the primary Langmuir wave, its longitudinal perturbation, and one ion-acoustic wave. We assume the forms of the solution $E=E_0\exp[\mathrm{i}(k_0z+m\phi-\omega_0t)]+E_1(t)\exp[\mathrm{i}(k_1z+m_1\phi-\omega_1t)]$, and $n=n_1(t)\cos(Kz+M\phi-\Omega t)$. For the sake of simplicity, we use the notation $e_{0,1}=\exp[\mathrm{i}(k_{0,1}z+m_{0,1}\phi-\omega_{0,1}t)]$, $e_{\pm}=\exp[\pm\mathrm{i}(Kz+M\phi-\Omega t)]$, and $E_1(t)$ and $n_1(t)$ are the first-order quantities. According to quantum Zakharov Eqs. (11) and (12), we can obtain the dispersion relation of the quantum Langmuir wave and the quantum ion-acoustic wave, respectively

$$\omega_0=\left(k_0^2+\frac{m_0^2}{a^2}\right)+H^2\left(k_0^4+\frac{m_0^4}{a^4}\right), \tag{13}$$

$$\omega_1=\left(k_1^2+\frac{m_1^2}{a^2}\right)+H^2\left(k_1^4+\frac{m_1^4}{a^4}\right), \tag{14}$$

$$\Omega^2=\left(K^2+\frac{M^2}{a^2}\right)+H^2\left(K^4+\frac{M^4}{a^4}\right). \tag{15}$$

Using the usual parameter resonant conditions $k_0=k_1+K$, $m_0=m_1+M$, $\omega_0=\omega_1+\Omega$, which meet momentum, angular momentum, and energy conservation, respectively, the forms of three waves are inserted in Eqs. (11) and (12), taking the resonance part and $\dot{E}_1=\mathrm{i}\omega E_1$, we have the dispersion relation

$$\omega^3+2\Omega\omega^2+\left(K^2+\frac{M^2}{a^2}\right)|E_0|^2=0. \tag{16}$$

In Eq. (16), ω^3 can be neglected because of $\Omega\gg|\omega|$, letting $\omega=\mathrm{i}\gamma$, and using Eq. (15) we obtain the growth rate

$$\gamma=\frac{\sqrt[4]{K^2+\dfrac{M^2}{a^2}}|E_0|}{\sqrt{2}\left[1+H^2\left(K^4+\dfrac{M^4}{a^4}\right)\Big/\left(K^2+\dfrac{M^2}{a^2}\right)\right]^{1/4}}. \tag{17}$$

It can be seen from Eq. (17) that the growth rate of decay instability in single-walled carbon nanotubes depends on the tube's radius, the discrete azimuthal quantum number, and the quantum parameter. However, the maximum growth rate $\gamma_{\max}$ is not affected by the carbon nanotube's radius a and the discrete azimuthal quantum number, but by the quantum effects, because when $K\to\infty$, $\gamma=\gamma_{\max}=|E_0|/\sqrt{2H}$. Figure 1 shows the growth rate of the decay instability for different a, M, and H. We can clearly see that, for long-wavelength region (i.e., K is small), the instability growth rate decreases as the carbon nanotube's radius a increases, while it increases as the discrete azimuthal quantum number M increases. But the growth rate is suppressed by increasing the quantum parameter H. For fixed H, the carbon nanotubes's radius a and the discrete azimuthal quantum number M play important role for relatively long wave-length perturbation (i.e., with small K or $K\to 0$), but for medium-short length perturbation (i.e., with large K or $K\to\infty$) the influence of a and M on decay instability is evidently weakened.

3.2. Four-wave instability

The four-wave instability is yielded by the interaction between one equilibrium Langmuir wave, its longitudinal and transverse perturbations,[22,23] and one ion-acoustic wave. Taking $E=E_0\exp(\mathrm{i}k_0z+\mathrm{i}m_0\phi-\mathrm{i}\omega_0t)+E_+\exp[\mathrm{i}(k_0+k)z+\mathrm{i}(m_0+m)\phi-\mathrm{i}(\omega_0+\omega)t]+E_-\exp[\mathrm{i}(k_0-k)z+\mathrm{i}(m_0-m)\phi-\mathrm{i}(\omega_0-\omega^*)t]$, $n=\tilde{n}\exp(\mathrm{i}kz+\mathrm{i}m\phi-\mathrm{i}\omega t)+$ c.c., where c.c. denotes complex conjugate, following the related proposal for

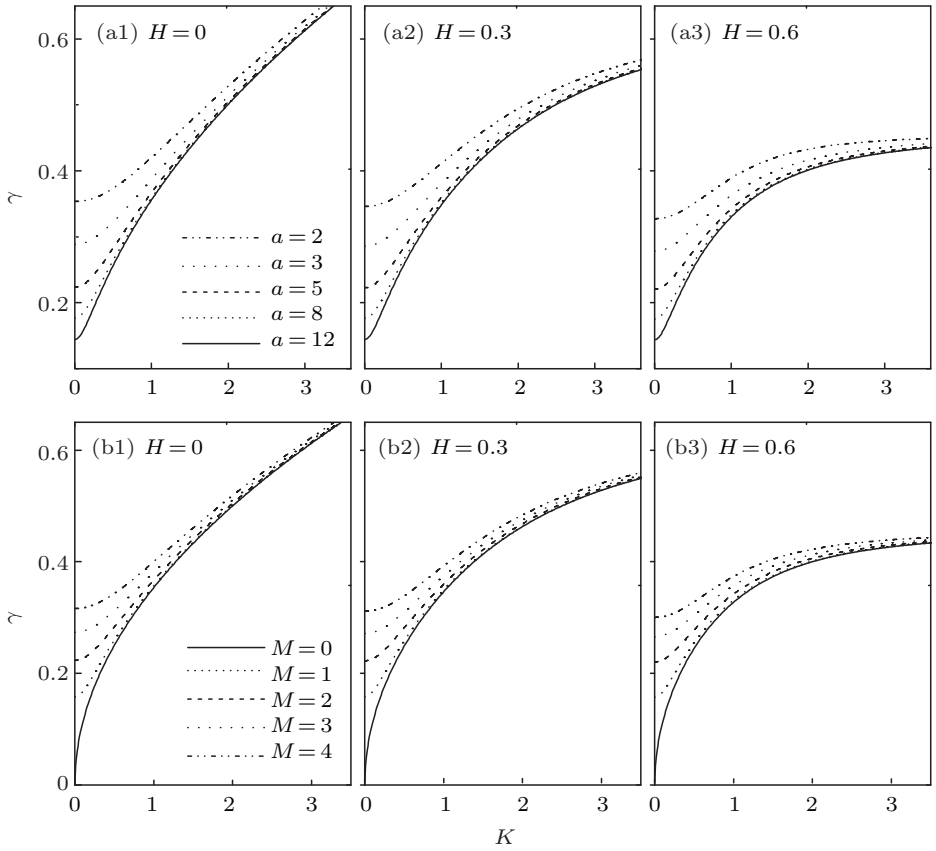


Fig. 1. The growth rate of the decay instability against K, $E_0 = 0.5$. Panels (al)–(a3) represent the different tubes' radius a and quantum parameter H for $M = 2$. Panels (b1)–(b3) represent different azimuthal quantum number M and H for $a = 5$.

the four-wave instability,[22,23] combining Eqs. (11) and (12) and

$$\omega_0 = \left(k_0^2 + \frac{m_0^2}{a^2}\right) + H^2\left(k_0^4 + \frac{m_0^4}{a^4}\right),$$

we have

$$D_s D_1 D_2 = \left(k^2 + \frac{m^2}{a^2}\right) E_0^2 (D_1 + D_2), \tag{18}$$

where,

$$\begin{aligned}
D_s &= \omega^2 - \left(k^2 + \frac{m^2}{a^2}\right) - H^2\left(k^4 + \frac{m^4}{a^4}\right), \\
D_1 &= \omega - (k^2 + 2k_0 k) - \frac{1}{a^2}(m^2 + 2m_0 m) \\
&\quad - H^2\Big[(k^4 + 4k_0k^3 + 6k_0^2k^2 + 4k_0^3k) \\
&\quad + \frac{1}{a^4}(m^4 + 4m_0m^3 + 6m_0^2m^2 + 4m_0^3m)\Big], \\
D_2 &= -\omega - (k^2 - 2k_0 k) - \frac{1}{a^2}(m^2 - 2m_0 m) \\
&\quad - H^2\Big[(k^4 - 4k_0k^3 + 6k_0^2k^2 - 4k_0^3k) \\
&\quad + \frac{1}{a^4}(m^4 - 4m_0m^3 + 6m_0^2m^2 - 4m_0^3m)\Big].
\end{aligned}$$

Equation (18) is the dispersion relation of four-wave instability in the single-walled carbon nanotubes, which is a fourth-order polynomial about ω and depends on the carbon radius a, the perturbed discrete azimuthal quantum number m, the equilibrium discrete azimuthal quantum number m_0, and the quantum parameter H. In order to further reveal their characteristics in the four-wave interaction process, equation (18) can be numerically analyzed to deduce the four-wave instability. Some results are shown in Figs. 2 and 3. It can be seen from the figures that the carbon radius a, the perturbed discrete azimuthal quantum number m, the equilibrium discrete azimuthal quantum number m_0, and the quantum parameter H have an obvious influence on the instability of the four-wave interaction in single-walled carbon nanotubes. Figure 2 shows the influences of m and m_0 on the instability for fixed radius a. It is important to note that, for fixed m_0 (i.e., for a fixed column), the instability region in k direction (as shown by dashed lines) up to its maximum when $m = m_0$ (Figs. 2(a1), 2(b2), 2(c3), 2(d4) for columns 1, 2, 3, and 4, respectively). When $m < m_0$, the instability region expands to a larger k region with increasing m, but contracts to a smaller k region when $m > m_0$. From the left column to the right column (i.e., with increasing m_0 for fixed m) we can find that the instability will appear in larger k region with increasing m_0. Figure 3 displays the influences of a on the instability for fixed m. It is found that the effects of the nanotube's radius on the instability depends on the ratio of the perturbed discrete azimuthal quantum number m to the equilibrium discrete azimuthal quantum number m_0,

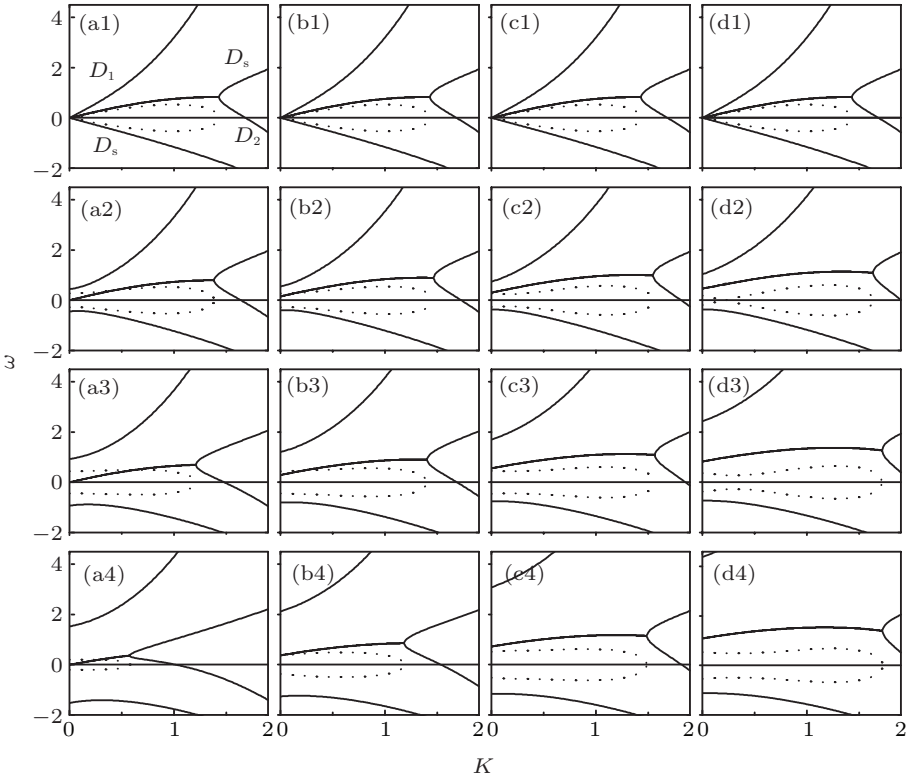


Fig. 2. The frequency ω (solid lines represent the real parts, dashed lines represent the imaginary parts) against k for different m and m_0. From the top row to the bottom one, $m = 0$, $m = 1$, $m = 2$, and $m = 3$. From the left column to the right one, $m_0 = 0$, $m_0 = 1$, $m_0 = 2$, and $m_0 = 3$. D_s, D_1, and D_2 indicate the branches of Eq. (18), respectively. $a = 3$, $E_0 = 0.8$, $k_0 = 0.7$, $H = 0.3$.

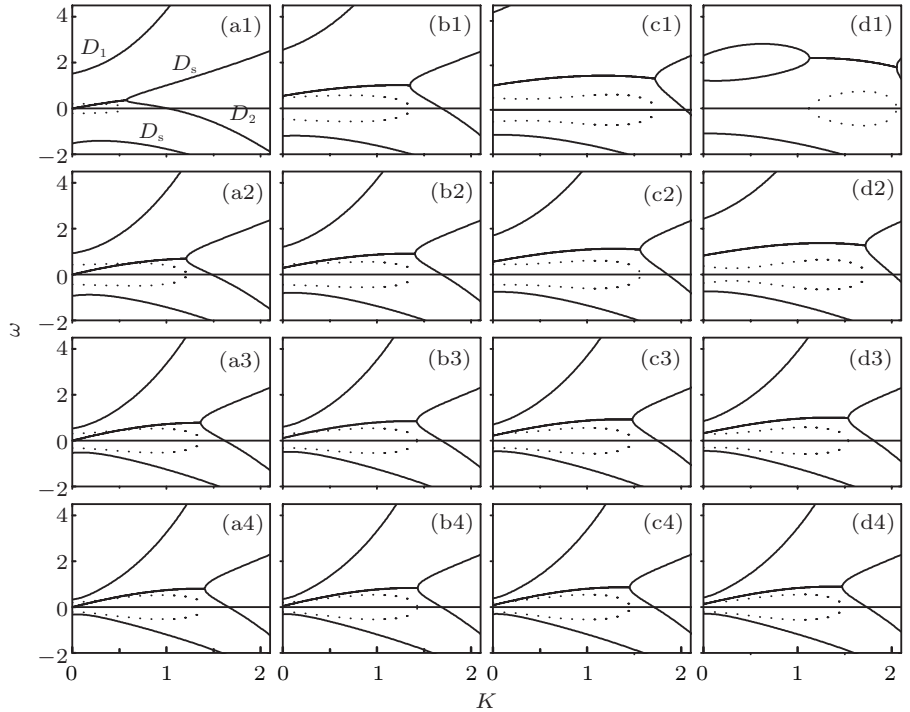


Fig. 3. The frequency ω against k for different a. From the top row to the bottom one, $a = 2$, $a = 3$, $a = 5$, and $a = 8$, from the left column to the right one $m_0 = 0$, $m_0 = 1$, $m_0 = 2$, $m_0 = 3$. We have $E_0 = 0.8$, $m = 2$, $k_0 = 0.7$, $H = 0.3$.

when $m > m_0$ the instability expands and appears in larger k region with increasing a, but contracts and appears in small k region when $m \leq m_0$. (These results differ from the results of the decay instability.)

Let us focus on the four-wave interaction for a simple case: $k_0 = 0$ and $m_0 = 0$. Setting $\omega = \mathrm{i}\gamma$, the dispersion relation (18) becomes

$$\left[\gamma^2 + \left(k^2 + \frac{m^2}{a^2}\right) + H^2\left(k^4 + \frac{m^4}{a^4}\right)\right]$$

$$\times \left\{\gamma^2+\left[\left(k^2+\frac{m^2}{a^2}\right)+H^2\left(k^4+\frac{m^4}{a^4}\right)\right]^2\right\}$$
$$= 2\left(k^2+\frac{m^2}{a^2}\right)E_0^2\left[\left(k^2+\frac{m^2}{a^2}\right)+H^2\left(k^4+\frac{m^4}{a^4}\right)\right]. \quad (19)$$

Solving γ^2, we get

$$\begin{aligned}\gamma^2 = & -\frac{1}{2}\left[\left(k^2+\frac{m^2}{a^2}\right)+H^2\left(k^4+\frac{m^4}{a^4}\right)\right]\\ & \times\left[1+\left(k^2+\frac{m^2}{a^2}\right)+H^2\left(k^4+\frac{m^4}{a^4}\right)\right]\\ & +\frac{1}{2}\left[\left(k^2+\frac{m^2}{a^2}\right)+H^2\left(k^4+\frac{m^4}{a^4}\right)\right]^{1/2}\\ & \times\left\{\left[\left(k^2+\frac{m^2}{a^2}\right)+H^2\left(k^4+\frac{m^4}{a^4}\right)\right]\right.\\ & \times\left[1-\left(k^2+\frac{m^2}{a^2}\right)-H^2\left(k^4+\frac{m^4}{a^4}\right)\right]^2\\ & \left.+8\left(k^2+\frac{m^2}{a^2}\right)E_0^2\right\}^{1/2}, \quad (20)\end{aligned}$$

the square of the growth rate $\gamma^2>0$, namely,

$$E_0>\frac{1}{2}\left[\left(k^2+\frac{m^2}{a^2}\right)+H^2\left(k^4+\frac{m^4}{a^4}\right)\right]^2\Big/\left(k^2+\frac{m^2}{a^2}\right),$$

the system is unstable because ω is imaginary. Figure 4 shows γ^2 against k^2 for different radius a, discrete azimuthal quantum number m, and quantum parameter H. We can find that, for fixed $m=1$ ($m>m_0$, Figs. 4(a1), 4(a2)), the instability region becomes wider and expands to a larger k region with increasing a, but it becomes narrower and contracts to a smaller k region with increasing m for fixed $a=5$ (Figs. 4(b1), 4(b2)). The maximum growth rate γ^2 also depends on a and m when $H\neq 0$. And the quantum parameter H decreases the maximum value of γ^2 and suppresses the instability.

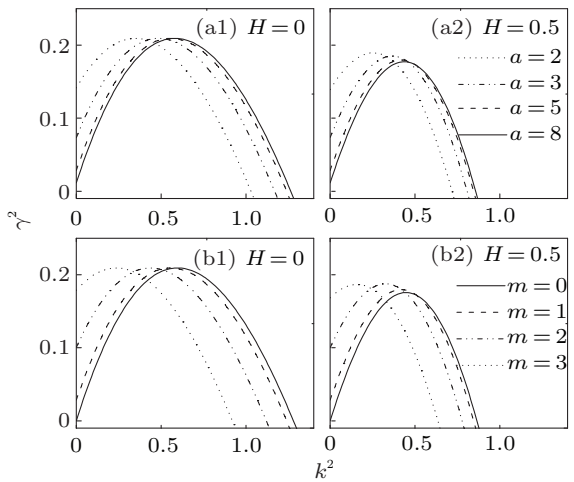


Fig. 4. γ^2 as a function of k^2 for the four-wave instability with $k_0=0$ and $m_0=0$. $E_0=0.8$. Panels 4(a1) and 4(a2) represent different tube's radius a and quantum parameter H for $m=1$. Panels 4(b1) and 4(b2) represent different discrete azimuthal quantum number m and H for $a=5$.

4. Conclusions

We have studied the parametric instabilities induced by the coupling excitation between high frequency quantum Langmuir waves and the low frequency quantum ion-acoustic waves in single-walled carbon nanotubes. The quantum Zakharov model in the cylindrical coordinates is obtained by using the quantum hydrodynamic equations and the two-time scale theory. We get the dispersion relations for the high frequency quantum Langmuir wave and the low frequency quantum ion-acoustic wave, respectively. Then, we obtain the dispersion relations of the decay instability and four-wave instability in the single-walled carbon nanotubes, and study the influence of the carbon nanotubes's radius, the discrete azimuthal quantum number, and the quantum parameter on the parametric instabilities. We have found that the carbon nanotubes' radius, the equilibrium discrete azimuthal quantum number, the perturbed discrete azimuthal quantum number, and the quantum effects all play a vital role in the instabilities.

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