On the modeling of synchronized flow in cellular automaton models*

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In this paper, we further analyze our cellular automaton (CA) traffic flow model. By changing some parameters, the characteristics of our model can be significantly varied, ranging from the features of phase transitions to the number of traffic phases. We also review the other CA models based on Kerner's three-phase traffic theory. By comparisons, we find that the core concepts for modeling the synchronized flow in these models are similar. Our model can be a good candidate for modeling the synchronized flow, since there is enough flexibility in our framework.

Keywords: traffic flow, cellular automaton, synchronized flow, three-phase traffic theory

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1. Introduction

The systematic investigation of traffic flow has quite a long history.[1–3] The most interesting and controversial topics in this field may be "synchronized flow" and "three-phase traffic theory". After the proposal of these ideas by Kerner and Rehborn[4–6] and Kerner,[7] the debate started and has not ended until now.

In Kerner's three-phase traffic theory, synchronized flow and wide-moving jam are different congested states. The latter one can propagate through bottlenecks while maintaining the mean velocity of the downstream jam front, while the downstream front of the former one is often fixed at the bottleneck. The phase transition from free flow to synchronized flow and the one from synchronized flow to wide-moving jams are both of first-order.

Helbing *et al.* did not think that the synchronized flow phase exists, but they thought that the transient states between free flows and jams come from many different factors, including the heterogeneity of traffic flow and the percentage of trucks,[8,9] errors of measurement in the local fundamental diagrams,[8,10] traffic accidents,[9,11] lane-changing behaviors,[11,12] bad weather conditions,[13] etc. They also thought that the widely-scattered appearance of data points in the local fundamental diagrams can be explained by many complex effects, including the memory effect,[14] time gap statistics,[15] effect of variance-driven time gaps,[16] boomerang effect,[11,12,17] etc.

Nevertheless, in recent years, more and more evidence can support Kerner's theory. The most important aspects of this evidence are as follows.

(i) In the vicinity of an isolated on-ramp, in the simulation results of Helbing's two-phase model, a low inflow rate leads to jams (the patterns named triggered stop-and-go waves (TSG) and oscillatory congested traffic (OCT)), while a high inflow rate leads to a special pattern named homogeneous congested traffic (HCT) without jams; in the simulation results of Kerner's three-phase model, the former leads to synchronized flow without jams, while the latter leads to jams. The empirical observations[3] show that Kerner's result is correct.

(ii) The single-vehicle data of HCT used in Helbing's papers are incorrect.[18] The unprocessed raw measured data are not homogeneous at all; on the contrary, they are similar to jams.

(iii) The definition of a moving synchronized flow pattern (MSP) and the differences between MSP and wide-moving jams are further clarified.[19]

(iv) We find that the first-order phase transition from free flow to synchronized flow can occur in a single-lane highway, i.e., it is unrelated to the lane-changing behaviors.[20]

Therefore, more and more researchers have gradually started to believe in the existence of a synchronized flow phase; the modeling of synchronized flow phase attracts much attention. Some traditional ways, including macroscopic models (hydrodynamic approach) and microscopic models (car-following approach) seem not to perform well in this field. On the contrary, the cellular automaton (CA) approach has become more and more popular in recent years. Since the proposal of the Kerner–Klenov–Wolf (KKW) model by Kerner *et al.*,[3,21,22] many different CA models have been proposed. They have different advantages and disadvantages.

But, which is the best model for synchronized flow? How is a "perfect" CA model established? More and more empirical studies of traffic flow have been done, and then we find that the empirical data from different sites are also quite different.

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Maybe one traffic flow model with all parameters fixed cannot simulate all the traffic flow phenomena around the world. Therefore, a better idea is to propose a model with variable rules and parameters, then we can adjust them according to the specific empirical data. We think that our model[23–25] could be the candidate. In the following, we will give a detailed description.

The rest of this paper is organized as follows. In Section 2, we review our model and point out its flexibility for different traffic situations. In Section 3, we further discuss the synchronized flow modeling. The conclusion and our future research plan are given in Section 4.

2. Rules and parameter settings of our model

First, we briefly recall our model. We design the velocity-updating rule with anticipated deceleration (AD) in the model, with which drivers prefer to decelerate when necessary. The anticipated safe velocity $V_{\text{anti}}(AD, gap)$ is the maximum of v satisfying the inequality:

$$B(v,AD) \le gap.$$

Here $B(v,AD)$ is the whole distance that a vehicle travels before stopping if it keeps decelerating with AD from an initial velocity. In cellular automaton models, function $B(v,AD)$ is often discredited. Suppose that $m = \text{int}(v/|AD|)$, then $B(v,AD)$ can be calculated from

$$\begin{aligned} B(v,AD) &= v + (v+AD) + (v+2AD) + \cdots + (v+mAD) \\ &= (2v+mAD)(m+1)/2. \end{aligned}$$

With the anticipated safe velocity $V_{\text{anti}}(AD, gap)$ calculated above, the model rules are executed on all vehicles in parallel, as follows:

i) determining the randomization probability (slow-to-start rule[26]):

$$p_n(v) = \begin{cases} p, & \text{when } v_n > 0, \\ p', & \text{when } v_n = 0; \end{cases}$$

ii) estimating the virtual velocity of the front vehicle (vehicle n–1):[27]

$$v'_{n-1} \to \min\{v_{\max} - A_n, \max\{0, V_{\text{anti}}(AD, gap_{n-1}) - A_n\}, v_{n-1}\};$$

iii) deterministic acceleration or deceleration:

$$v_n \to \begin{cases} \min\{v_n + A_n, v_{\max}\}, & \text{when } v_n < gap_n + v'_{n-1} \\ \text{(acceleration)}, & \\ V_{\text{anti}}(AD, gap_n + v'_{n-1}), & \text{when } v_n \ge gap_n + v'_{n-1} \\ \text{(deceleration)}; & \end{cases}$$

iv) randomization:

$$v_n \to \max\{v_n - A_n, 0\} \text{ with probability } p_n(v);$$

v) updating positions:

$$x_n(t+1) \to x_n(t) + v_n(t).$$

In the formulations above, $x_n(t)$ and $v_n(t)$ are the position and velocity of vehicle n at time t; vehicle $n-1$ precedes vehicle n; gap_n is the distance between vehicle n and vehicle $n-1$; $v_{\max}$ is the speed-limitation of the vehicles. In our simulation, each cell is set to be 1-m long, and each time step corresponds to 1 s. Each vehicle occupies eight cells. In the following, we set $v_{\max} = 32$ m/s, $A_n = 1$ m/s^2 and $p' = 0.4$.

The fundamental diagrams of our model with various parameters are shown in Fig. 1. Here the square curve starts from homogeneous configuration and the hollow circle curve starts from megajam. In our model, the values of AD and p should be noted. First, we consider the effects of AD. When $|AD|$ is small (-4 m/s^2 and -8 m/s^2), free flow with high rate can easily collapse, and first-order F→S phase transition can be observed. When $|AD|$ is larger (-16 m/s^2), both the flow rate and the velocity of the synchronized flow become higher.

With these AD values (-4 m/s^2, -8 m/s^2, and -16 m/s^2), the CA model is still within the framework of three-phase traffic theory. What will happen even when $|AD|$ is larger? We find it quite different. As shown in Fig. 1(d), the difference between free flow and synchronized flow is not obvious, and the transition between them cannot be considered to be first-ordered. We can even consider the data points of synchronized flow as free flow with lower velocities. In Fig. 1(e), when $|AD|$ equals the maximum velocity, the synchronized flow branch completely disappears, and only first-order F→J phase transition can be found. Now the rules and the simulation results of this model are both similar to those of the velocity-dependent randomization (VDR) model.[26]

Therefore, we think that our model can provide a unified framework for CA traffic flow modeling: with various AD values, the model can be a classical two-phase model or a three-phase model. The users can adjust the parameters to obtain different models as desired. At the same time, our framework can explain why there are three phases in real traffic: the normal AD value is between -4 m/s^2 and -8 m/s^2 for most drivers and vehicles. With such values, synchronized flow phase can exist. If the $|AD|$ can be larger, synchronized flow phase will disappear, but it is impossible in daily life! The classical two-phase models (e.g., VDR model and NS model) are based on abnormal AD values, thus the simulation results cannot be correct. Besides, in real life, the road capacity is usually between 2000 veh/h∼2500 veh/h (about 0.6 veh/s∼0.7 veh/s). Thus with normal AD values, we can obtain the capacities which are close to the empirical data, while abnormal AD values will lead to too high capacities. This point further confirms the importance of AD values in our model.

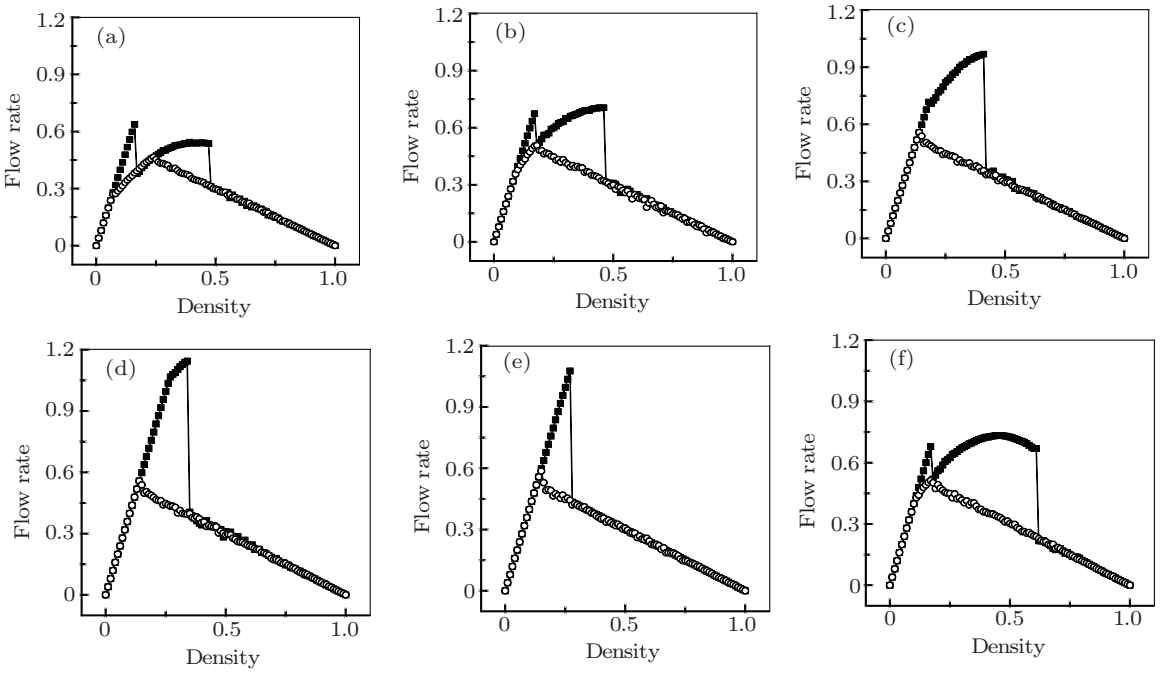


Fig. 1. Fundamental diagrams of our model. (a) $AD = -4$ m/s^2, $p = 0.1$; (b) $AD = -8$ m/s^2, $p = 0.1$; (c) $AD = -16$ m/s^2, $p = 0.1$; (d) $AD = -24$ m/s^2, $p = 0.1$; (e) $AD = -32$ m/s^2, $p = 0.1$; (f) $AD = -8$ m/s^2, $p = 0.01$.

The value of p is also very important. In Figs. 1(a)–1(e), we set $p = 0.1$ and it seems that the slope of the synchronized flow branch is always positive. However, if we set $p = 0.01$, the synchronized flow branch will become nonmonotonic (see Fig. 1(f)), and it could be divided into two parts: the left part with positive slopes and the right part with negative slopes. Actually, when $p = 0.1$ and the simulation time for periodic boundary condition is short, the data points corresponding to the right part with negative slopes still can exist. However, they are unstable and quick to collapse; S→J phase transition can occur very quickly due to large instabilities inside the synchronized flow.

The stability of the right part of the synchronized flow branch has a great effect on the features of our model. When it is stable, the moving synchronized flow pattern (MSP) can propagate both upstream and downstream. When it is unstable, MSFP could only propagate downstream, and the widening synchronized flow pattern (WSP) will also become unstable: it will be difficult to find a WSP upstream of an isolated on-ramp.

Different kinds of MSPs can be found in many empirical studies, including our latest work.[20] However, the number of typical examples for WSP is small in Kerner's empirical data[3,28] and cannot be found in our empirical data. It is very interesting that in Kerner's papers, there are many examples of general pattern (GP), dissolving general pattern (DGP), MSP and localized synchronized flow pattern (LSP), except WSP. In other words, the synchronized flow whose downstream front is fixed at the bottleneck cannot propagate upstream for a long distance. Kerner's explanation is that the formation of a WSP needs very constant traffic demand; small fluctuations of upstream traffic flow rates can make the WSP turn to GP quickly (e.g., when some upstream bottlenecks exist). This explanation implies that the WSP is usually unstable, which corresponds to the situation where $p = 0.1$ in our model.

Therefore, we think that one possibility is that in the ideal fundamental diagram for real traffic, the right part of synchronized flow branch is also unstable. Whether this hypothesis is right is out of the scope of this paper; we need more empirical data to check this. Nevertheless, we have presented a modeling framework that can be compatible to different situations, which is a good beginning for further study.

The features of phase transitions in our model can also be easily adjusted. In previous papers,[23,24] we introduced "α" to modify the basic rules. In order to avoid confusion, here we use γ_1 and γ_2 instead; the Rules ii) and iii) are changed into

ii)-a estimating the virtual velocity of the front vehicle (vehicle $n-1$):

$$\begin{aligned} v'_{n-1} \rightarrow\ & \gamma_1 * \min\{v_{\max} - A_n, \\ & \max\{0, V_{\text{anti}}(AD, gap_{n-1}) - A_n\}, v_{n-1}\}; \end{aligned}$$

iii)-a deterministic acceleration or deceleration:

$$v_n \rightarrow \begin{cases} \min\{v_n + A_n, v_{\max}\}, \\ \text{when } (1-\gamma_2) * v_n + \gamma_2 * B(v_n, AD) < gap_n + v'_{n-1}, \\ V_{\text{anti}}(AD, gap_n + v'_{n-1}), \\ \text{when } (1-\gamma_2) * v_n + \gamma_2 * B(v_n, AD) \geq gap_n + v'_{n-1}. \end{cases}$$

The situation where $\gamma_1 = 1$ and $\gamma_2 = 0$ has been presented in Fig. 1. The consideration of the virtual velocity of the front

vehicle can help to enhance the capacities of our model. If we set $\gamma_1 = 0.5$ or 0, the flow rates of synchronized flow will gradually decrease (see Figs. 2(a) and 2(b)). If we further set $p' = 0.1$, i.e., we remove the slow-to-start rule (Rule (i)) from our model, the first-order S→J phase transition will completely disappear (see Fig. 2(c)). The above two methods both can help to adjust the slope of Line J in our model, which corresponds to the propagation velocity of the downstream front of jams, i.e., the characteristics of Line J can be significantly varied by different parameters.

On the other hand, if we set $\gamma_2 = 0.5$, the synchronized flow branch will become more stable, while the free flow branch will be easier to lose stability (see Fig. 2(e)). When γ_2 =1, the first-order F→S phase transition will completely disappear (see Fig. 2(f)). In Fig. 2(e), we find the right part of the synchronized flow branch is also comparatively stable, just like that in Fig. 1(f). We can conclude that with more parameters introduced, there can be more methods obtained to enhance the stability of synchronized flow in our model.

It should be noted that the controversy over the features of F→S phase transition is still hot in this field. In the hypothesis of Kerner's three-phase traffic theory, in multi-lane traffic, the F→S phase transition is of first-order; while in single-lane traffic, it looks like being continuous:[3,28] in the local fundamental diagrams, there is no obvious gap between the data point of a free flow and that of a synchronized flow. However, in our latest work,[20] we find that the first-order F→S phase transition could also occur in single-lane traffic. In other words, at the same density, both the free flow and the synchronized flow can co-exist; they have different flow rates and average velocities. Which viewpoint is right is also out of the scope of this paper. Maybe more empirical data of single-lane traffic are needed to draw a conclusion. Nevertheless, our modeling framework can facilitate more candidates.

Besides, some even older models could also be simulated within our framework. For example, if we set $\gamma_1 = 0$, $\gamma_2 = 1$, $p' = 0.1$, our model will be similar to the Greenshields model that was proposed in 1930s. As shown in Fig. 2(d), the Greenshields model could be considered as a three-phase traffic flow model, in which F→S and S→J phase transitions are both continuous.

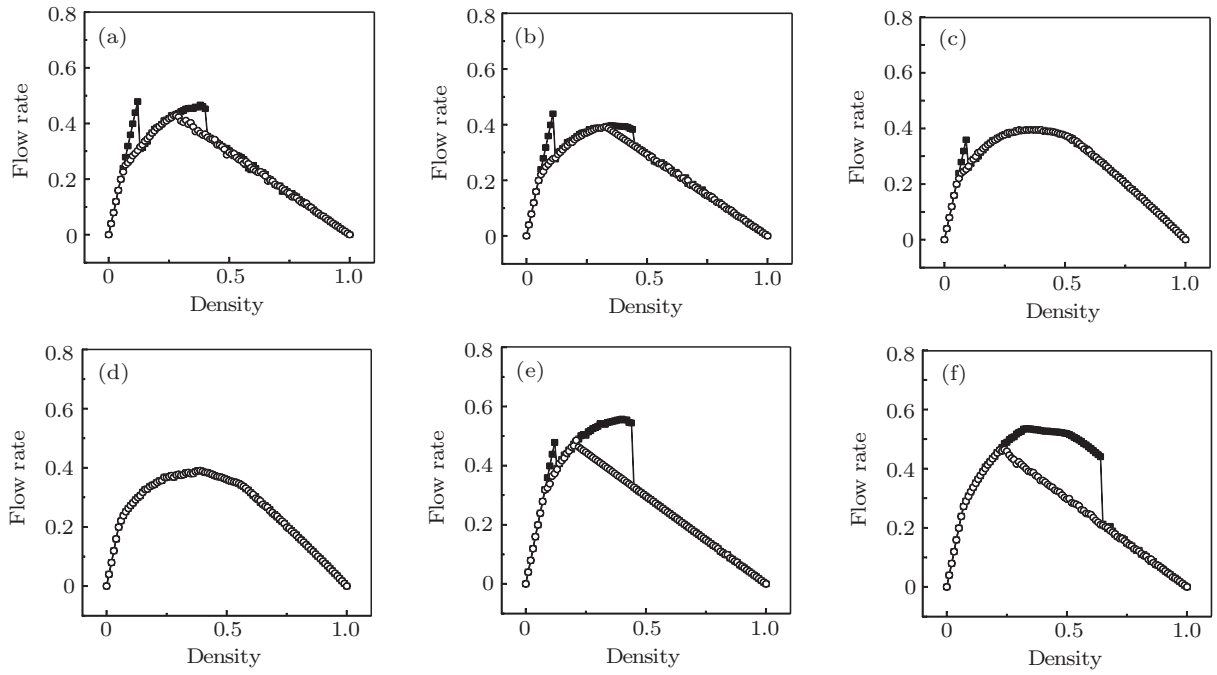


Fig. 2. Fundamental diagrams of our model when γ_1 and γ_2 are introduced, $p = 0.1$, $AD = -4$ m/s^2. (a) $\gamma_1 = 0.5$, $\gamma_2 = 0$, $p' = 0.4$; (b) $\gamma_1 = 0$, $\gamma_2 = 0$, $p' = 0.4$; (c) $\gamma_1 = 0$, $\gamma_2 = 0$, $p' = 0.1$; (d) $\gamma_1 = 0$, $\gamma_2 = 1$, $p' = 0.1$; (e) $\gamma_1 = 1$, $\gamma_2 = 0.5$, $p' = 0.4$; (f) $\gamma_1 = 1$, $\gamma_2 = 1$, $p' = 0.4$.

3. Further discussion of synchronized flow modeling

What is the key to introducing the synchronized flow into the CA traffic flow model? Different researchers may give different answers.

Kerner *et al.*[3,21,22] thought that the key is the competition between the opposing tendencies occurring within a random local disturbance: a) the tendency towards a synchronized flow due to the vehicle deceleration associated with a speed adaptation effect and b) the tendency towards an initial free flow due to the vehicle acceleration associated with an over-acceleration effect. In recent works, Kerner *et al.*[19] proposed a new model and thought that the competition could also oc-

cur between (i) overacceleration through lane changing to the faster lane and (ii) speed adaptation within the synchronization gap.

Knospe[29] thought that the key is the reaction to velocity changes of the next vehicle downstream, i.e. to "brake lights" (BL). Drivers adjust their velocity such that safe driving is possible. In the comparative research of CA modeling,[30] the effect of BL and the anticipation of the predecessor velocity were both emphasized. Jiang and Wu[31–33] and Gao *et al.*[34,35] further modified the rules and some parameters of BL model. The investigations by Tian *et al.*[36] and Zheng *et al.*[37] were also based on Jiang's model. Their basic thoughts are all similar to Knospe's.

Lee *et al.*[38] thought that the key is the mechanical restrictions on vehicles, realized by limited acceleration and deceleration capabilities. The human overreaction is also implemented.

Besides these classical models, in recent years, many new models were proposed. Zhao *et al.*[39] thought the key is the driver anticipation effect. Lárraga and Alvarez-lcaza[40] thought that the key is the driver reaction based on a safety analysis, which determines the most appropriate action for a vehicle to take. Kukobo *et al.*[41] thought that the key is the random braking effect, which is realized by considering its dependence on the velocity difference and heading distance with a preceding vehicle. Neto *et al.*[42] thought that the key is to reduce the driver velocity in response to the deceleration of the vehicle immediately in front of it whenever the distance is smaller than a security minimum. Jia *et al.*[43] thought that the key is that the randomization effect should be enhanced with the decrease of time gap and the long-time stopped vehicle should have a large randomization probability. Tian *et al.*[44,45] thought that the key is the anticipation rule, which takes into account the average space gap of multiple leading vehicles. There is a great deal of other relevant research[46–55] that contributes to this topic.

Although they use different modeling approaches, we think that their thoughts are all similar to ours: the action of drivers should depend on many complex factors, including the distance between it and its front vehicle, and the velocity of it and its front vehicle. The differences are only derived from the specific rules. Here we take four typical three-phase models for example: Kerner's KKW model,[21,22] LEE model,[38] Jiang's FMCD model,[31–33] and Gao's MVDE model.[34,35] Their fundamental diagrams are shown in Fig. 3. By comparisons, we find some points below.

(I) The basic properties of the models mentioned above are similar. For example, they all have two straight lines: the line for free flow (Line F) and the line for jams (Line J). The slope of Line F only depends on the setting of the maximum velocity in the simulation. The slope of Line J of each model seems similar to each other. The maximum flow rate of the free flow of each model is nearly equal to each other. So is that of the synchronized flow.

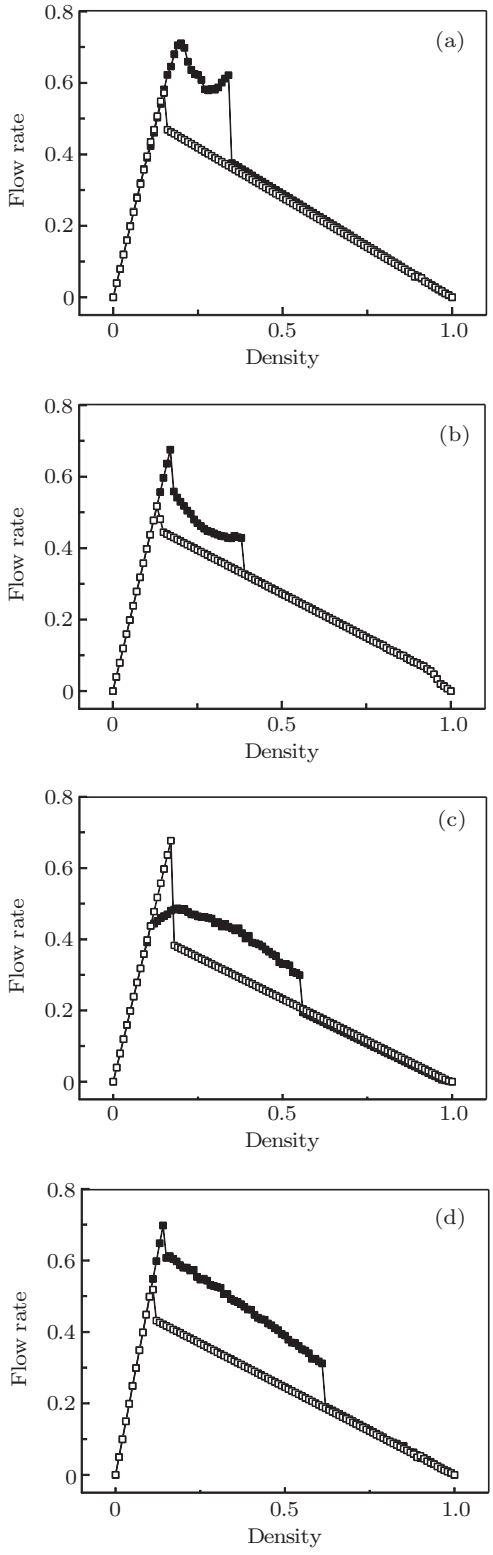


Fig. 3. Fundamental diagrams of four three-phase models. The parameter setting can be found in respective papers. (a) KKW model. (b) LEE model. (c) FMCD model. (d) MVDE model.

(II) The main differences between them are the features of synchronized flow. For example, we can find that in the LEE

model and the MVDE model, the synchronized flow branch only has a negative slope: it is always monotonic. However, in the KKW model and the FMCD model, part of the synchronized flow branch has a positive slope: it is nonmonotonic. The density ranges for synchronized flow in models are also different: they are small in the KKW model and the LEE model, but they are very large in the FMCD model and the MVDE model. Nevertheless, comparing Fig. 1 with Fig. 2, we can find that the differences between these models are not so obvious as the differences between various configurations of our model.

Therefore, we think that our model could be a good candidate for CA modeling. The three main reasons are as follows:

I) the rules are simple and easy to understand;

II) only two important parameters need to be considered: *AD* and *p*. *AD* represents the deterministic factors that can affect driving rules, while *p* reveals the probabilistic nature of traffic flow. They have independent meanings. By adjusting them, the model features can be easily and significantly varied, including the values of capacity and the number of traffic phases. Even classical VDR model and Greenshields model can be represented by our model with specific parameters.

III) Some rules corresponding to some specific phenomena can be added into the model or removed from the model as desired. For example, the removals of virtual velocity rule and slow-to-start rule can make the S→J phase transition continuous; the change of one parameter can make the F→S phase transition continuous.

Here reasons II) and III) are conducible to the model calibration and validation. In recent years, more and more researchers[56] believed that the idea of a "perfect traffic flow model" may be impossible: one traffic flow model with all parameters fixed cannot simulate all the traffic flow phenomena around the world. The traffic rules (including lane-changing, ramp-metering, etc.) and the preferences of driving style are quite different in different countries. The road geometries and weather conditions could also affect them. Thus, the best candidate may be that first we construct a framework for modeling and then, set the parameters according to the specific empirical data. We think that there is enough flexibility in our framework, which could not be found in previous models.

4. Conclusion and future plan

In this paper, we mainly focus on the characteristics of our CA model. By changing the parameters (including *AD*, p, p', γ_1, and γ_2), the capacities can be higher or lower; the stability of a synchronized flow branch can be strengthened or weakened; the F→S and S→J phase transitions can be continuous or of first-order; the model can be a two-phase model or a three-phase model, etc. Different characteristics can be presented as desired. Some older models can also be considered as special situations within our modeling framework. We think that the basic thoughts for synchronized flow modeling proposed by different researchers are nearly the same. Our model could be a good candidate since there is enough flexibility. The users can adjust the parameters according to the specific empirical data.

Nevertheless, the framework for synchronized flow modeling is just a beginning. In the future, many more complex problems, for example, how to do the "calibration and validation" for CA models, need to be solved. For car-following models, it is very easy; however, for the CA model, one important problem should be noted: how to deal with the randomization probability. It is difficult to calibrate it with single-vehicle data, thus only macroscopic statistical results can be considered. Then, what kinds of statistical results are suitable for calibration? Do they depend on some specific factors of the sites? These problems need to be further analyzed by more empirical data.

The empirical data we have are obtained by two methods. First, we obtained some video data from the monitoring center of the Nanjing Airport Highway. The cameras along the highway can help us learn more about real traffic flow. Both macroscopic results and microscopic single-vehicle data can be extracted from their recordings. At the same time, we can clearly see what has happened in the observation period. Using this method is much better than using those traditional loop detectors. On the other hand, we have conducted a car-following experiment in a suburban area of Hefei. We installed a high-precision global position system (GPS) device on each car to record the location and velocity of the car every 0.1 s. The leading vehicle is instructed to drive at a constant speed and other vehicles are instructed to follow each other safely. No overtaking is allowed. This experiment can remedy some minor disadvantages of video data, e.g., we still do not know what has happened during two consecutive cameras (usually about 1 km). With the help of GPS, the single-vehicle data can be much more accurate. By comparing the empirical traffic data with the experimental data, we find many interesting and important traffic flow phenomena, especially for synchronized flow. These findings will be presented in future papers.

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