Single photon transport properties in coupled cavity arrays nonlocally coupled to a two-level atom in the presence of dissipation*

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(Received 17 April 2013; revised manuscript received 11 June 2013; published online 10 December 2013)

We discuss the effects of dissipation on the behavior of single photon transport in a system of coupled cavity arrays, with the two nearest cavities nonlocally coupled to a two-level atom. The single photon transmission amplitude is solved exactly by employing the quasi-boson picture. We investigate two different situations of local and nonlocal couplings, respectively. Comparing the dissipative case with the nondissipative one reveals that the dissipation of the system increases the middle dip and lowers the peak of the single photon transmission amplitudes, broadening the line width of the transport spectrum. It should be noted that the influence of the cavity dissipation to the single photon transport spectrum is asymmetric. By comparing the nonlocal coupling with the local one, one can find that the enhancement of the middle dip of single photon transmission amplitudes is mostly caused by the atom dissipation and that the reduced peak is mainly caused by the cavity dissipation, no matter whether it is a nonlocal or local coupling case. Whereas in the nonlocal coupling case, when the coupling strength gets stronger, the cavity dissipation has a greater effect on the single photon transport spectrum and the atom dissipation affection becomes weak, so it can be ignored.

Keywords: single photon transport, dissipative coupled cavity arrays, dissipative two-level atom, local and nonlocal couplings

PACS: 42.50.Pq, 37.30.+i, 42.50.Ct **DOI:** 10.1088/1674-1056/23/2/024202

1. Introduction

Due to the potential applications for quantum information processing,[1–3] the single photon transport in coupled cavity arrays (CCAs)[4] or in waveguides have attracted much attention in both theoretical and experimental research. Many schemes have been put forward to study the photon transport properties in different kinds of models. For example, CCAs where one of the cavities is embedded with a two-level atom[5–8] or a Λ-type three-level atom have been studied,[9–13] respectively. Recently, Zhou *et al.*[14] considered the nonlocal coupling between the atom and the CCAs, and explained the destructive interference phenomenon of the transmission spectra according to the effect of which-way detection. In addition, Cheng *et al.*[15] regarded the single photon transport in a one-dimensional coupled-resonator waveguide with local and nonlocal couplings with a nanocavity containing a two-level system. Their consequences show that the photon transport can be modified strongly for the nonlocal coupling, which can be used in designing optical quantum devices.

Yet for all that, numerous issues still exist. A realistic system can be uncommonly isolated from its environment completely. The environment could induce an inelastic scattering as a result of the non-conservation of the energy of the incident photon when the single photon transports, which would influence the single photon transmission and reflection amplitudes and reduce the quantum switching in different ways.

Here, we use a new kind of quasi-boson method[16,17] to study the single photon propagating properties in a finite dissipative CCAs, with the two nearest cavities nonlocally coupling to a dissipative two-level atom. The coordinates of the baths would be eliminated, the localized mode and the interaction between cavities can be renormalized and the dissipative system Hamiltonian can be redescribed by an effective Hamiltonian, then an analytical result will be obtained. It can be found that the dissipation of the system increases the middle dip, lowers the peak of the single photon transmission amplitudes and broadens the line width of the transport spectrum. Moreover, the cavities' dissipation severely limits the transmittance when the scale increases, as reported in Ref. [18]. In addition, the important phenomenon is that the affection of the cavity dissipation influencing the single photon transport spectrum is asymmetric, which is different from the case of a two-level atom inside one of the cavities. By comparing the nonlocal coupling case with the local one, we can reach the conclusion that the enhancement of the middle dip is mainly caused by the atom dissipation, while the decrease of the peak is mainly due to the cavity dissipation, in spite of the local or nonlocal coupling case. However, in the nonlocal coupling case, with the increasing of the coupling strength, the single photon transport spectrum is mostly affected by the cavity dis-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10704031, 10874235, 11274148, and 10934010).

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sipation, so the infection of the atom dissipation can be neglected.

This paper is organized as follows. In Section 2, we describe the model for this system and derive the eigen-equation for propagating the single photon in the dissipative CCAs, then the transmission coefficient can be obtained. In Section 3, we investigate the effects of the dissipation resulting from atom and cavities to the single photon transmission spectrum. Finally, conclusions are presented in Section 4.

2. The model of the system and the solutions

We describe the system as a single photon propagating in a finite CCAs. The two-level atom is nonlocally coupled to the two nearest cavities with corresponding strengths g_0 and g_1. The quantum realization of the current model can be realized in experiment[14] by a nanoelectromechanical resonator array with the two nearest resonators coupled to a single spin by way of their attached magnetic tips. Its classical simulation is a waveguide of a coupled-defected cavity array with double couplings to a side-defected cavity. N cavities are arranged in a period L; the intercavity coupling constant is α. In experiment, the realistic system can rarely be isolated from its surrounding completely. We consider an ideal system coupling with a bath composed of an infinite set of harmonic oscillators, as shown in Fig. 1(a). Now we assume each resonator has the same leakage rate γ_c. By employing a quasi-boson picture,[16,17] the coupling to the bath renormalizes the localized mode from ω_c to $\omega_{\rm eff}$, which also renormalizes the interaction between cavities from $\alpha\omega_c$ to $\alpha\omega_{\rm eff}$, as shown in Fig. 1(b). Now, the effective Hamiltonian of the CCAs can be given by[14,15]

$$H_c = \omega_{\rm eff}\sum_j a_j^\dagger a_j - \alpha\omega_{\rm eff}\sum_{\langle j,j'\rangle} a_j^\dagger a_{j'}. \tag{1}$$

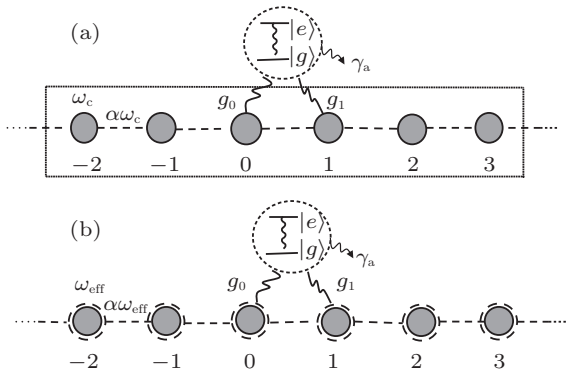


Fig. 1. Schematic configuration for the system. (a) The ideal system coupling with a bath. We assume that each cavity has the same leakage rate. The two-level atom is nonlocally coupled to two modes, supported by the two nearest resonators with corresponding strengths g_0 and g_1; when g_1=0, the two-level atom is locally coupled to one mode. The resonance frequency of the cavity is ω_c, the coupling parameter between cavities is α and γ_a is the decay rate of the atom. (b) Effective treatment in the quasi-boson picture, where the CCAs can be regarded as a chain of quasi-bosons. The effective frequency of the CCAs is $\omega_{\rm eff} = \omega_c - {\rm i}\gamma_c$.

Equation (1) describes a typical tight-binding boson model. The dispersion relation is given by expression $\omega_k = \omega_c - 2\alpha\omega_c\cos(k_nL)$,

$$k_n = \frac{n\pi}{N+1}\frac{1}{L}$$

for $n = 1$ to N. Here, the bosonic operator $a_j^\dagger$ (a_j) creates (annihilates) an excited state at the j-th cavity, $\sum_{\langle j,j'\rangle}$ sums all pairs of cavities that are nearest neighbors and $\omega_{\rm eff} = \omega_c - {\rm i}\gamma_c$ is the frequency of the CCAs. One can find that Eq. (1) does not have to mention the external degrees of freedom. The effective treatment would be of great conceptual and, moreover, computational advantage rather than the treatment of the general method, i.e. described by a constant but not by operators. The loss looks like a nonideal boundary instead of a field oscillation.

Under the rotating wave approximation, the interaction between the cavity field and the two-level decay atom is described by a Jaynes–Cummings Hamiltonian[5]

$$\begin{aligned} H_{\rm I} =\ & (\Omega - {\rm i}\gamma_a)|e\rangle\langle e| + g_0(a_0^\dagger|g\rangle\langle e| + |e\rangle\langle g|a_0) \\ & + g_1(a_1^\dagger|g\rangle\langle e| + |e\rangle\langle g|a_1), \end{aligned} \tag{2}$$

where Ω is the frequency of the two-level atom, γ_a is the decay rate of the two-level atom. The first term describes the free energy of the dissipative two-level atom; the second and the third terms are the interaction Hamitonian of the atom with the zero and the first cavities. When a single photon comes from the left with eigenenergy ω, the wave function can be expanded in a basis as

$$|\Psi\rangle = \sum_j e_j a_j^\dagger|0,g\rangle + e_a|0,e\rangle. \tag{3}$$

Here, $|0\rangle$ is the vacuum state of the cavity field, e_j is the probability amplitude of the j-th cavity in the excited state and e_a is the probability amplitude of the two-level atom in the excited state. The number of photons in each mode is restricted to zero or one. Solving the Schödinger equation, we get the discrete scattering equations

$$\begin{aligned} &\omega_{\rm eff}e_j - \alpha\omega_{\rm eff}(e_{j+1} + e_{j-1}) + g_0e_a\delta_{j0} + g_1e_a\delta_{j1} = \omega e_j, \\ &(\Omega - {\rm i}\gamma_a)e_a + g_0e_0 + g_1e_1 = \omega e_a. \end{aligned} \tag{4}$$

The scattering equations of a single photon coming from the left are[5,19]

$$\begin{aligned} e_{j^-} &= {\rm e}^{{\rm i}k_nsL} + r_j{\rm e}^{-{\rm i}k_nsL}, & s < j, \\ e_{j^+} &= t_j{\rm e}^{{\rm i}k_nsL}, & s > j, \end{aligned} \tag{5}$$

where $s = 1,\ldots,N$. t_j and r_j denote the local transmission amplitude and reflection amplitudes of the photon, respectively. Solving Eq. (4) by using the zeroth and first resonators with the above scattering equation, we can get the transmission amplitude of the zeroth and the first cavities to an incident photon

$$t_{j=0,1} = [({\rm e}^{{\rm i}k_nL} - {\rm e}^{-{\rm i}k_nL})(g_0g_1\varepsilon\xi + 2{\rm i}\alpha\gamma_c\xi - \xi^2)$$

$$+ \mathrm{i}\alpha\gamma_c\varepsilon g_0 g_1(1-\mathrm{e}^{\mathrm{i}k_nL})]/[(2g_0g_1\varepsilon\xi+4\mathrm{i}\alpha\gamma_c\xi)\mathrm{e}^{\mathrm{i}k_nL} - \xi^2(\mathrm{e}^{\mathrm{i}k_nL}-\mathrm{e}^{-\mathrm{i}k_nL})+(\varepsilon\xi-\mathrm{i}\gamma_c\mathrm{e}^{\mathrm{i}k_nL}+\mathrm{i}\alpha\gamma_c) \times (g_0^2+g_1^2)-2\mathrm{i}\gamma_c\xi]. \quad (6)$$

Here, $\xi = 2\alpha\omega_c$, $\varepsilon = 1/(\omega-\Omega+\mathrm{i}\gamma_a)$. When $j \neq 0, 1$, we can get the transmission amplitude of the j-th cavity to the incident photon

$$t_{j\neq 0,1} = \frac{(\xi-\mathrm{i}\kappa)\sin k_nL}{(\gamma_c+\xi\sin k_nL-\kappa\cos k_nL)-\mathrm{i}\kappa\sin k_nL}. \quad (7)$$

Then, the total transmittance of the total system can be written as

$$T = \prod_j |t_{j=0}|^2|t_{j=1}|^2|t_{j\neq 0,1}|^{2(N-2)}. \quad (8)$$

We can see from Eq. (8) that when N increases, the cavity dissipation severely limits the transmittance of the single photon. Differing from the other results,[14,15] we use an analytical method to study the dissipation effects on the single photon transport. Moreover, our result includes all the cavities' dissipation of the system.

3. The dissipation effects to the single-photon transport properties

In this section, we will investigate the dissipation effects of the atom and cavities on the single photon transmission spectrum. The dissipation has two main loss processes: the atom spontaneous emission from an excited state to the ground state and the photon leaking from the cavity mode. The unavoidable intrinsic dissipation will affect the single-photon transmission properties. In general, the dissipation influences both the free propagation of the single photon and the scattering process by broadening the line width.[6]

(i) Nonlocal coupling ($g_0 \neq 0$, $g_1 \neq 0$). When no dissipation is considered, we now discuss the following two cases: the two peaks in the spectrum are asymmetrical and the transmission generally vanishes at the band edges with $k = 0, \pi$. This result is the same as the one given in Ref. [14]. Considering the situation of the cavity dissipation, the two peaks of the transmission amplitude decrease as the cavity dissipation rate increases and the single photon transmission spectrum line width broadens, while the middle dip of the transmission amplitude shows no changes. The important phenomenon is that the cavity dissipation leads to the single photon transmission spectrum being more asymmetrical, as shown in Fig. 2(a). In the case of considering the dissipation of the atom only, as shown in Fig. 2(b), as the atom dissipation rate increases, the middle dip of the transmission amplitude can be enhanced and the single photon transmission spectrum line width broadens, while the two peaks of the transmission amplitude do not show obvious changes. As we consider the dissipation of the atom and cavity simultaneously, the single-photon transmission spectrum is shown in Fig. 2(c). The two peaks of the transmission amplitude decrease and the middle dip increases, the line width broadens. Comparing the dissipative cavity case with the dissipative atom case, we could see that the enhancement of the middle dip is mainly caused by the atom dissipation, while the decrease of the two peaks is mainly due to the cavity dissipation. Moreover, we investigated the dissipation effect on the single photon transport properties in the case of different coupling strengths, as shown in Figs. 3 and 4. When the dissipa-

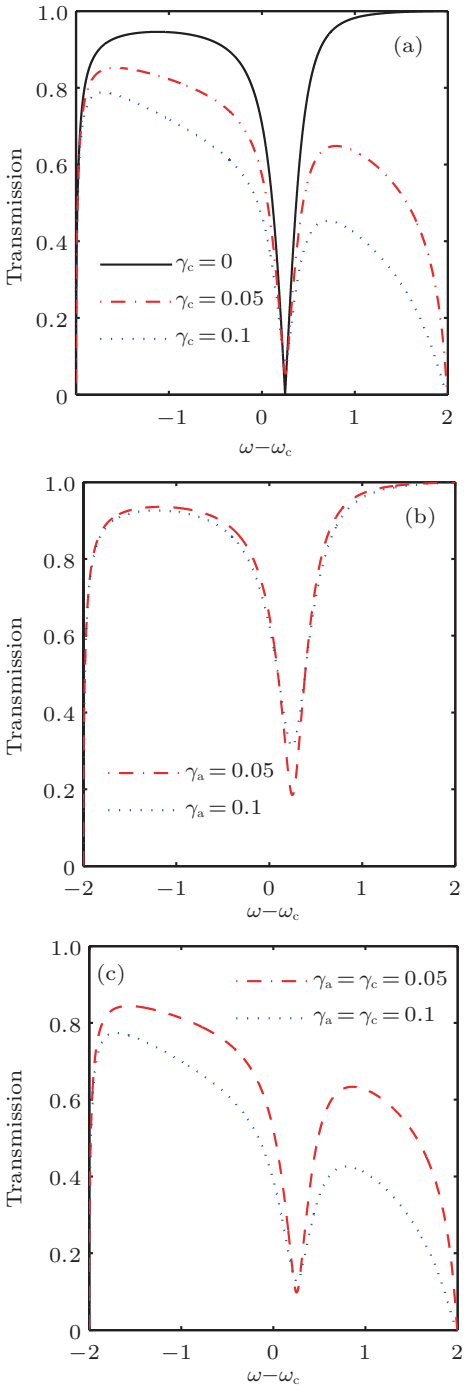


Fig. 2. (color online) The single photon transmission spectrum for the nonlocal coupling case. The parameters are set as follows: $\Omega = \omega_c = 2$, $\xi = 1$, $g_0 = 0.5$, $g_1 = 0.5$, $N = 10$. (a) Cavity dissipation case ($\gamma_a = 0$). (b) Atom dissipation case ($\gamma_c = 0$). (c) Cavity and atom dissipation case.

tion is not considered, with increasing the coupling strength, one peak is enhanced and the other is reduced. The middle dip moves to the right further and coincides with the right dip. When considering the dissipation, one can find that with the coupling strength increasing, the influence of the cavity dissipation effects on the single photon transport spectrum becomes great, while the atom dissipation impact changes to feeble and it can be ignored.

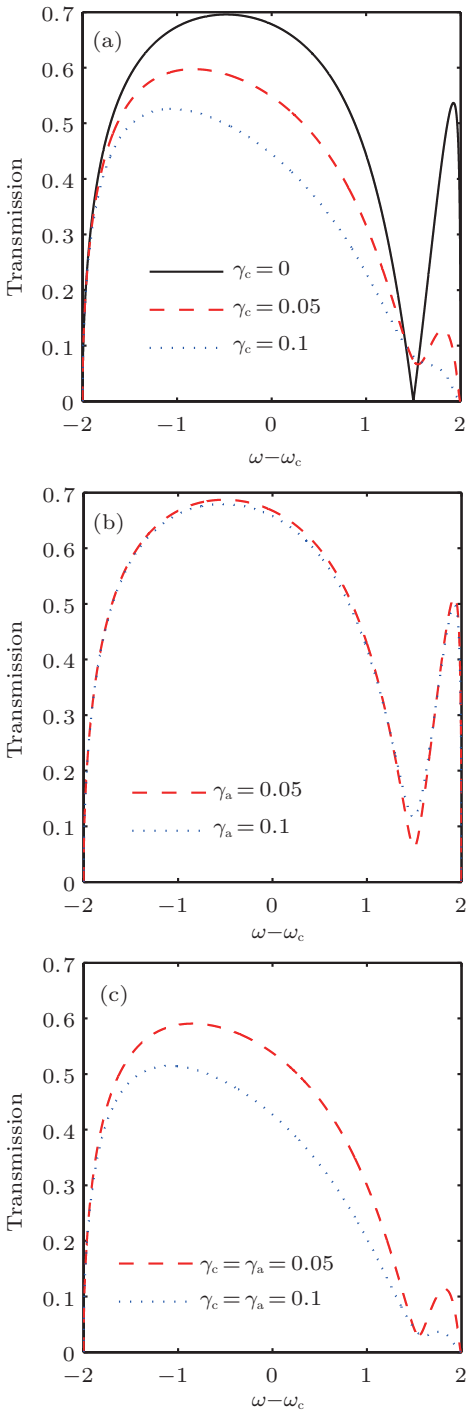


Fig. 3. (color online) The single photon transmission spectrum for the nonlocal coupling case. The parameters are set as follows: $\Omega = \omega_c = 2$, $\xi = 1$, $g_0 = 1$, $g_1 = 1.5$, $N = 10$. (a) Cavity dissipation case ($\gamma_a = 0$). (b) Atom dissipation case ($\gamma_c = 0$). (c) Cavity and atom dissipation case.

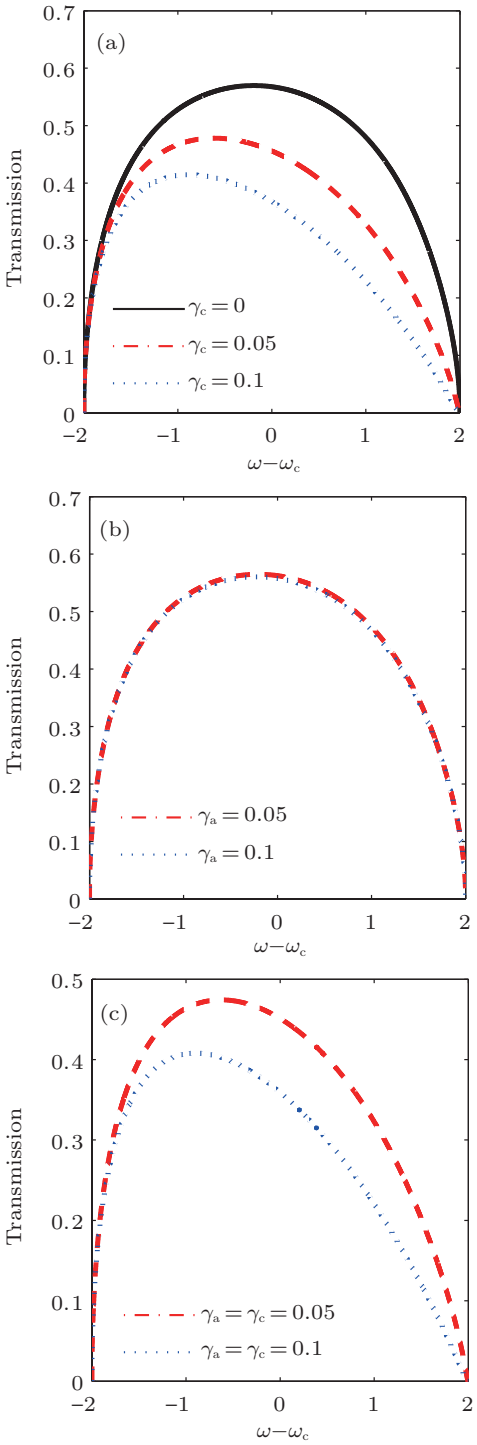


Fig. 4. (color online) The single photon transmission spectrum for the nonlocal coupling case. In order to investigate the dissipation effect on the single photon transport properties in the case of different coupling strengths, different from Fig. 3, we choose $g_1 = 2.5$. (a) Cavity dissipation case ($\gamma_a = 0$). (b) Atom dissipation case ($\gamma_c = 0$). (c) Cavity and atom dissipation case.

(ii) Local coupling ($g_0 \neq 0$, $g_1 = 0$). Firstly, we consider the no-dissipative case. The single photon transmission spectrum becomes symmetrical, as shown in Fig. 5(a). Similarly, we regard the case of the cavity dissipation, the atom dissipation and the atom and cavity dissipation simultaneously, respectively, as shown in Figs. 5(a)–5(c). The local coupling case is identical with the nonlocal coupling one. The middle dip of

the single photon transmission amplitude increases, which is mostly caused by the dissipation of the atom. The peak lowers, which is mainly caused by the cavity dissipation. The dissipation broadens the line width of the transport spectrum.

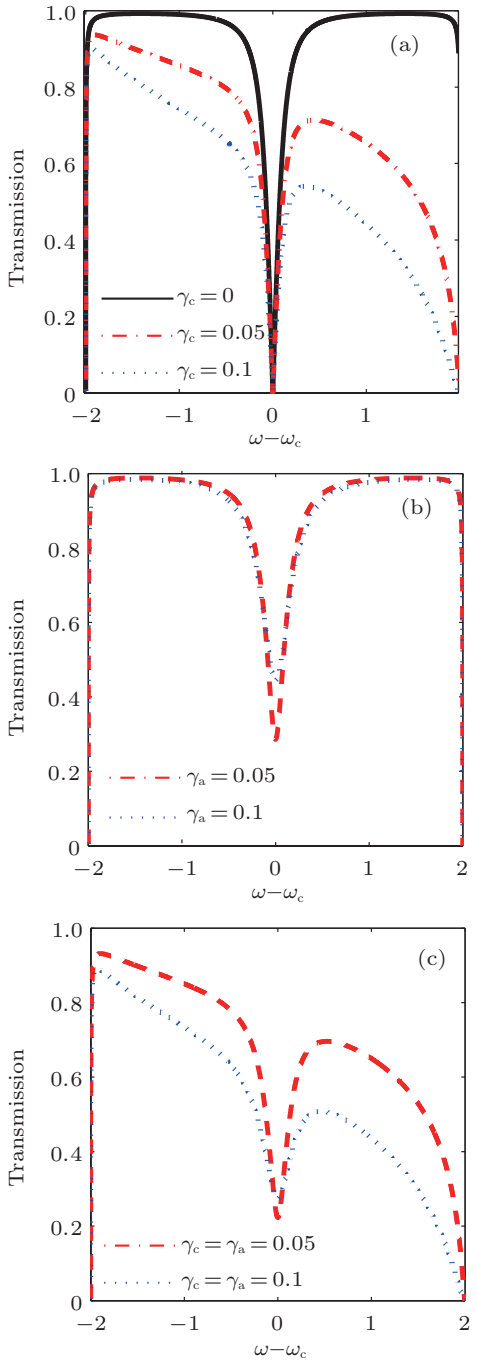


Fig. 5. (color online) The single photon transmission spectrum for the local coupling case. The parameters are set as follows: $\Omega = \omega_c = 2$, $\xi = 1$, $g_0 = 0.5$, $g_1 = 0$, $N = 10$. (a) Cavity dissipation case ($\gamma_a = 0$). (b) Atom dissipation case ($\gamma_c = 0$). (c) Cavity and atom dissipation case.

4. Conclusion

In summary, we have studied the dissipation effects on the single photon transport in the system of the coupled cavity arrays, in which two of the nearest cavities are nonlocally coupled to a two-level atom, by employing the new kind of quasi-boson picture. We solved the single photon transmission amplitude exactly. It is shown that the dissipation of the cavity and of the atom affects distinctively the transmission of photons in the coupled cavity arrays. The decrease of the single photon transmission amplitude peak is mostly caused by the cavity dissipation, but the increase of the middle dip is mostly caused by the atom dissipation, no matter the nonlocal or local coupling case. Both dissipations broaden the line width. Different from the case of the two-level atom inside one of the cavities, the affection of the cavity dissipation influencing the single photon transport spectrum is asymmetric. In the nonlocal coupling case, when the coupling gets stronger, the single photon transport spectrum is mostly influenced by the cavity dissipation, so the atom dissipation impact can be neglected.

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