Effect of normalized plasma frequency on electron phase-space orbits in a free-electron laser*

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Irregular phase-space orbits of the electrons are harmful to the electron-beam transport quality and hence deteriorate the performance of a free-electron laser (FEL). In previous literature, it was demonstrated that the irregularity of the electron phase-space orbits could be caused in several ways, such as varying the wiggler amplitude and inducing sidebands. Based on a Hamiltonian model with a set of self-consistent differential equations, it is shown in this paper that the electron-beam normalized plasma frequency functions not only couple the electron motion with the FEL wave, which results in the evolution of the FEL wave field and a possible power saturation at a large beam current, but also cause the irregularity of the electron phase-space orbits when the normalized plasma frequency has a sufficiently large value, even if the initial energy of the electron is equal to the synchronous energy or the FEL wave does not reach power saturation.

Keywords: free-electron laser, electron phase-space orbit, normalized plasma frequency, irregularity

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1. Introduction

The electron motion plays an important role in a free-electron laser (FEL), because the gained energy of the FEL wave comes from the kinetic energy loss of the electron beam. The electron dynamic behavior with irregular phase-space orbits induces a spread in energy distribution and hence deteriorates the electron-beam (e-beam) transport quality in a free-electron laser. So far, a number of papers have been devoted to the study of the electron motion.[1–19] Evidently, exploring the ways to irregulate phase-space orbits is of significance in improving the electron-beam transport quality and the FEL performance. Recent studies indicate that there are several ways to cause electron irregular motion and in consequence, what might be termed 'FEL chaos' has been found. An electron irregular motion was shown to be possible when the wiggler amplitude or the varying rate of the wiggler amplitude was large enough,[1,2] or when the wiggler amplitude had non-uniformity.[3] Stochastic electron de-trapping could be caused by inducing sidebands.[4] In many papers, the effect of the electron beam self-field on the electron's dynamic behavior and chaos has been investigated.[5–10] Chaotic velocity diffusion of a cold electron beam might be induced by the electrostatic wave.[11] Irregular and chaotic electron motion could occur in an FEL with an electromagnetic-wave wiggler, or in an FEL with an ion-channel guide field.[12–14]

In previous articles where the chaotic behaviors of electrons were discussed, the restriction was often made that the FEL wave field could be excluded, or regarded as constant or not self-consistent. Also, the effect of the beam plasma frequency on the electron motion was ignored. In this paper, utilizing a Hamiltonian model with a self-consistent FEL wave evolution taken into account, we study the effect of the e-beam normalized plasma frequency in an FEL. The rest of this paper is organized as follows. In Section 2, the theoretical model is described in terms of a set of self-consistent differential equations. In Section 3, nonlinear simulations are carried out for a general situation, as well as for the parameter domains of a self-amplified spontaneous emission (SASE) FEL and a laser-seeded (LS) FEL. Finally, a discussion is provided and conclusions are drawn in Section 4.

2. Physical model

We consider a two-wave FEL (referred to as a Compton-type FEL) with a vector potential of the FEL wave

$$\boldsymbol{A}_{\rm s} = A_{\rm s}(z)\left[\boldsymbol{e}_x\cos(k_{\rm s}z-\omega_{\rm s}t)-\boldsymbol{e}_y\sin(k_{\rm s}z-\omega_{\rm s}t)\right], \quad (1)$$

and a vector potential of the magneto-static wiggler pump wave

$$\boldsymbol{A}_{\rm w} = -A_{\rm w}\left[\boldsymbol{e}_x\cos(k_{\rm w}z)+\boldsymbol{e}_y\sin(k_{\rm w}z)\right], \quad (2)$$

where $k_{\rm s}$ and $\omega_{\rm s}$ are the wave-number and angular frequency of the FEL radiation, $A_{\rm s}$ is the amplitude of $\boldsymbol{A}_{\rm s}$, $k_{\rm w}$ is the wiggler wave-number, $A_{\rm w}$ is the amplitude of $\boldsymbol{A}_{\rm w}$, $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are the unit vectors in the x and y directions, respectively.

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Based on a Hamiltonian model introduced by Kroll, Morton and Rosenbluth (KMR),[20] the motion of the electrons and the FEL wave are coupled with each other by the following self-consistent differential equations of dimensionless quantities:

$$\frac{\mathrm{d}\Psi_i}{\mathrm{d}\bar{z}} = (1+\bar{k}_\mathrm{s}) - \frac{\bar{\omega}_\mathrm{s}\gamma_i}{\sqrt{\gamma_i^2-\mu^2+2a_\mathrm{w}a_\mathrm{s}\cos\Psi_i}}, \tag{3}$$

$$\frac{\mathrm{d}\gamma_i}{\mathrm{d}\bar{z}} = -\frac{\bar{\omega}_\mathrm{s}a_\mathrm{w}a_\mathrm{s}\sin\Psi_i}{\sqrt{\gamma_i^2-\mu^2+2a_\mathrm{w}a_\mathrm{s}\cos\Psi_i}}, \tag{4}$$

$$\frac{\mathrm{d}a_\mathrm{s}}{\mathrm{d}\bar{z}} = \frac{\bar{\omega}_\mathrm{p}^2 a_\mathrm{w}}{2\bar{\omega}_\mathrm{s}}\left\langle\frac{\sin\Psi_i}{\gamma_i}\right\rangle, \tag{5}$$

where

$$\mu^2 = 1+a_\mathrm{w}^2+a_\mathrm{s}^2, \tag{6}$$

$$\psi_i = (k_\mathrm{w}+k_\mathrm{s})z-\omega_\mathrm{s}t, \tag{7}$$

$$\bar{\omega}_\mathrm{p} = \frac{\sqrt{4\pi n_\mathrm{b}e^2/m}}{ck_\mathrm{w}}, \tag{8}$$

$$a_\mathrm{s} = \frac{eA_\mathrm{s}}{mc^2}, \tag{9}$$

$$a_\mathrm{w} = \frac{eA_\mathrm{s}}{mc^2}. \tag{10}$$

Here, c is the light speed in a vacuum, e and m are the electron charge and rest mass, n_b and $\bar{\omega}_\mathrm{p}$ denote the volume density and the normalized plasma frequency of the electron beam, $\bar{k}_\mathrm{s} = k_\mathrm{s}/k_\mathrm{w}$ and $\bar{\omega}_\mathrm{s} = \omega_\mathrm{s}/ck_\mathrm{w}$ are the normalized wave-number and angular frequency of the FEL wave, γ_i $(i = 1,2,\ldots,N)$ is the relativistic energy factor of the i-th individual electron, with N being the total number of electrons, and $\langle\cdots\rangle = \frac{1}{N}\sum_{i=1}^{N}$ denotes the quantity averaged over all the test electrons.

The motion of an arbitrary individual electron in the phase space (Ψ_i, γ_i) can be calculated by solving these three coupling differential Eqs. (3)–(5). In the following text, we concentrate our attention on analyzing the effect of the e-beam normalized plasma frequency $\bar{\omega}_\mathrm{p}$ on the electron motion. One should distinguish the terminology "effect of normalized plasma frequency" from the terminology "effect of plasma". As a matter of fact, the plasma effect in a three-wave FEL turns out to be a function of the space-charge wave, which is related only to the e-beam density, whereas the normalized plasma frequency defined by Eq. (8) is related not only to the e-beam density but also to the wiggler wave-number.

Concerning the simplified one-dimensional model described above, the following points should be clarified. First, although the KMR model does not provide a reliable description of the complicated problems associated with FEL engineering design (such as the beam energy emittance and spread, the initial detuning, etc.), it does make the results physically transparent and the entire process can be simply characterized with a few dimensionless parameters.[21] Second, equations (3)–(5) were derived without restriction of the FEL resonance condition [see the derivation of Eqs. (2.16) and (2.17) in Ref. [20]] and so they are valid for arbitrary electron motion in the non-resonance condition. Third, mathematically speaking, solutions of the above differential equations are affected by two factors: one is the initial values of the variables (i.e. the conditions at the initial time); the other is the parameters that appear in the equations. These two factors belong to different domains in mathematics and physics. When the parameters of the differential equations are fixed, the phase-space orbits (i.e. the solutions of the differential equations) will depend on the initial conditions, resulting in a phase-space diagram which contains a separatrix: outside the separatrix, the orbits are unstable and inside the separatrix, the orbits are stable. On the other hand, when the initial conditions are fixed, the solutions of the differential equations will be affected by the parameters of the differential equations. More specifically, the e-beam normalized plasma frequency $\bar{\omega}_\mathrm{p}$ considered here is one of the parameters that appear in the differential Eqs. (3)–(5), which is not time-dependent. In order to find its influence on the phase-space motion, we shall calculate the phase-space orbits for different values of $\bar{\omega}_\mathrm{p}$ by fixing the initial conditions of the variables ($\gamma|_{t=0} = \gamma_\mathrm{initial}$, $\psi|_{t=0} = \psi_\mathrm{initial}$, and $a_\mathrm{s}|_{t=0} = a_\mathrm{s,initial}$).

3. Electron phase-space orbits

First, we analyze a general situation of the normalized plasma frequency effect on the electron phase-space orbits by using a set of typical values; we choose the following parameters: $a_\mathrm{s} = 2.78\times10^{-5}$, $a_\mathrm{w} = 2$, $\bar{\omega}_\mathrm{s} = k_\mathrm{s}/k_\mathrm{w} = 250$, $\gamma_i = 25$, and the normalized wiggler length $\bar{L} = k_\mathrm{w}L = 1600$, where the electron has a synchronous energy $\gamma_\mathrm{r} \approx 25$ in accordance with the definition, Eq. (2.40), in the Kroll–Morton–Rosenbluth paper.[20] To achieve a good numerical accuracy, we adopt the normalized iteration step to be 0.0025 in our simulations. A phase-space diagram is plotted with the y axis denoting $\delta\gamma = \gamma - \gamma_\mathrm{initial}$ and the x axis denoting ψ (mod 2π). Attention should be paid to the fact that the variable γ is transferred to $\delta\gamma$ in terms of its initial value γ_initial. Figure 1 shows the vicissitude of the phase-space orbits of an individual test electron with the fixed initial values $\psi_\mathrm{initial} = \pi/3$ and $\gamma_\mathrm{initial} = 25$ for different values of the normalized plasma frequency parameter: $\bar{\omega}_p = 0$ (a), 0.015 (b), 1 (c), and 5 (d), respectively. Substantial changes of the electron phase-space orbits are evident as the normalized plasma frequency increases: (a) the phase-space orbit is periodic when $\bar{\omega}_p = 0$, (b) quasi-periodic when $\bar{\omega}_\mathrm{p} = 0.015$, (c) non-periodic when $\bar{\omega}_\mathrm{p} = 1$ and (d) irregular when $\bar{\omega}_\mathrm{p} = 5$. Therefore, we reasonably draw the conclusion that the effect of the normalized plasma frequency at sufficiently large values causes irregularity of the electron phase-space orbits, even if the initial energy of the electron is equal to the synchronous energy.

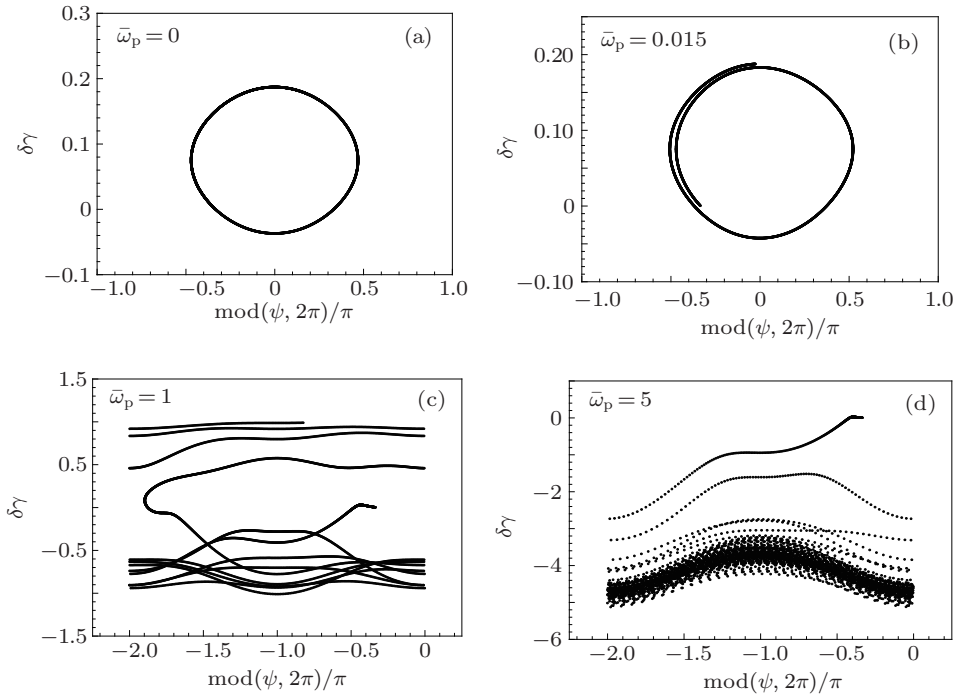


Fig. 1. Evolutions of the electron phase-space orbits with the increasing of beam normalized plasma frequency: (a) $\bar{\omega}_p = 0$, (b) 0.015, (c) 1, and (d) 5, where $a_s = 2.78 \times 10^{-5}$, $a_w = 2$, $\bar{\omega}_s = 250$, $\gamma_i = 25$, and $\psi_0 = \pi/3$. Here the phase-space orbit evolves from periodicity (a) to quasi-periodicity (b) to non-periodicity (c), and to irregularity (d) with the beam normalized plasma frequency increasing.

An issue should be clarified. Ignoring the evolution of the FEL wave field (i.e., letting $a_s(t) \equiv a_s|_{t=0} = a_{s,\mathrm{initial}}$) means decoupling Eq. (5) by setting the normalized plasma frequency to be zero ($\bar{\omega}_p = 0$). In this case, only the two differential Eqs. (3) and (4) survive and the system becomes integrable, where the electron executes oscillating motion and the phase-space orbit is periodic, as shown in Fig. 1(a). However, this does not mean that when ignoring the evolution of the FEL electromagnetic field is removed, the particle motions must become chaotic. As shown in Fig. 1(b), even if the FEL wave field is in evolution, the phase-space orbit is still quasi-periodic but not irregular. Only the normalized plasma frequency at sufficiently large value may cause irregular phase-space orbit, as shown in the sequence from Figs. 1(a)–1(d).

Now we come to examine the general conclusion above by employing a set of practical parameters in a self-amplified spontaneous-emission (SASE) FEL, conducted at the Los Alamos National Laboratory.[22] In that experiment, the wiggler had a period of 2.05 cm, an amplitude of 5430.5 Gs (1 Gs = 10^{-4} T) and a length of 250 cm, the electron beam had a current of 100 A, a transverse spot size of 0.014 cm and an energy of 18 MeV, and the FEL wavelength was 12 μm. By substituting these parameters into the Hamiltonian model, Eqs. (3)–(5), simulation results show a similar phenomenon: the phase-space orbits change from periodic to quasi-periodic, non-periodic and to an irregular behavior when the normalized plasma frequency increases from zero. Figure 2 shows the plots of the phase-space orbit with the normalized plasma frequency $\bar{\omega}_p = 3.5719$ (corresponding to a beam current of 100 A), which is irregular indeed.

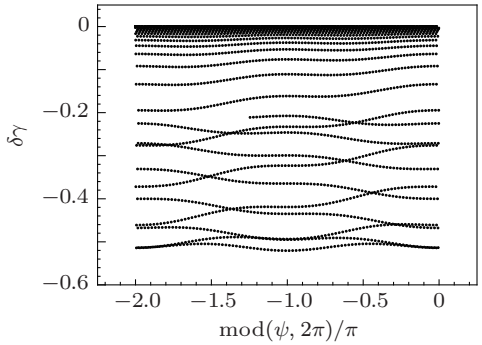


Fig. 2. Electron phase-space orbits for the parameter domain of a self-amplified spontaneous emission (SASE) FEL,[22] where the wiggler has a period of 2.05 cm, an amplitude of 5430.5 Gs and a length of 250 cm, the electron beam has a current of 100 A ($\bar{\omega}_p = 3.5719$), a transverse spot size of 0.014 cm and energy of 18 MeV, and the FEL wavelength is 12 μm.

Similar phenomena are found in the parameter domain of a laser-seeded FEL. An experiment was recently conducted at the Brookhaven National Laboratory (BNL),[23] where the wiggler had a period of 3.89 cm, a uniform amplitude of 3016 Gs with a length of 1000 cm (see Fig. 1 in Ref. [23]), the electron beam had a current of 350 A, a radius of 0.015 cm and an energy of 101.37 MeV. The FEL wavelength was 853 nm and a laser seed with 10 kW was injected. Figure 3 displays the simulation result obtained by use of these parameter values for the beam-wave interaction process from $z = 0$ to 1000 cm. It shows that the corresponding normalized plasma frequency ($\bar{\omega}_p = 11.8044$) is large enough to cause the electron phase-space orbit to become irregular. It should be emphasized that

the results are only theoretical in the sense of the test electron motion in phase space and not in the reliable reproduction of the experiments, because the KMR model was derived for the quasi-continuous electromagnetic wave, whereas the wave contains a chain of short laser pulses with pico-second duration in those two experiments.

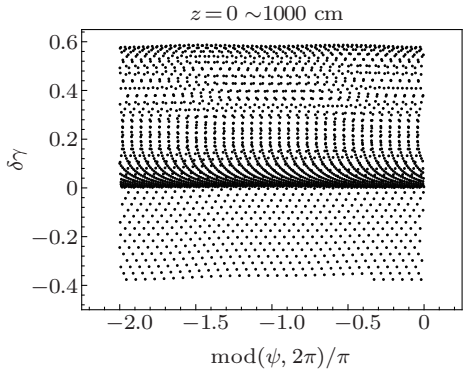


Fig. 3. Electron phase-space orbits in the beam-wave interaction process of $z = 0 \sim 1000$ cm for the parameters domain of a seeded configuration FEL,[23] where the wiggler has a period of 3.89 cm, a uniform amplitude of 3016 Gs with a length of 1000 cm (see Fig. 1 in Ref. [23]), the electron beam has a current of 350 A ($\bar{\omega}_p = 11.8044$), a radius of 0.015 cm and an energy of 101.37 MeV, the FEL wavelength is 853 nm; a laser seed has a power of 10 kW.

4. Discussion and conclusion

We now discuss why the evolution of the FEL wave field should be considered when we analyze the physical characteristics of the phase-space orbits. From a practical point of view, amplification of the FEL wave is based on the amplitude growing in the beam-wave interaction process and so the FEL wave field is certainly not constant. For instance, in the laser-seeded FEL experiment recently conducted at BNL, the energy of the FEL wave was increased from 0.04 μJ to 113 μJ in a uniform wiggler.[23] Numerical solution of the coupled differential Eqs. (3)–(5) are given in Fig. 4, where we plot the power evolution along the axial position for that parameter domain. The result shown in Fig. 4 is basically in agreement with the experimental measurement (see Fig. 1 in Ref. [23]). Clearly, the wave amplitude is evolving and reaches a saturation power of 2 MW. One can see that the evolution of the FEL wave field in the beam-wave coupling interaction is not negligible and consequently, the results with the FEL wave field evolution taken into account are closer to the practical situations in an FEL than the previous ones with the FEL wave field ignored, or regarded as constant or not self-consistent.

Surely, as electron beam current (consequently the normalized plasma frequency) increases up to a certain intensity, FEL gain also increases, then the wave evolution will reach a power saturation, where in such a high-gain regime, any electron does not show a regular circular motion in the phase space. For example, a current of 350 A (corresponding to a normalized plasma frequency of 11.8044) leads to the power saturation which begins at $z = 600$ cm in Fig. 4 and results in the irregular phase-space orbit shown in Fig. 3. That is to say, when the normalized plasma frequency has a sufficiently large value, both the FEL power saturation and the irregular phase-space orbit may simultaneously appear. However, it means neither that the power saturation of the FEL wave evolution is a necessary condition of the irregular phase-space orbits, nor that the dominant rule of the normalized plasma frequency effect on the irregular phase-space orbits is replaced by the power saturation. In other words, the FEL power saturation is one thing, while the irregular phase-space orbit is another thing: both are two different phenomena caused by the normalized plasma frequency. To demonstrate this point, in Fig. 5, we re-plot the phase-space orbit of Fig. 3 for the beam-wave interaction process only from $z = 0$ to 400 cm. It can be seen that in Fig. 4, the FEL wave gain is far from power saturation in the process from $z = 0$ to 400 cm, but the electron has an irregular motion in the phase space shown in Fig. 5.

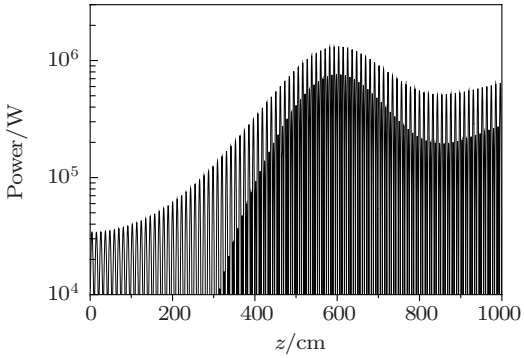


Fig. 4. Power evolution along the axial position simulated by coupling the beam-wave interaction in terms of the parameters in Fig. 3.

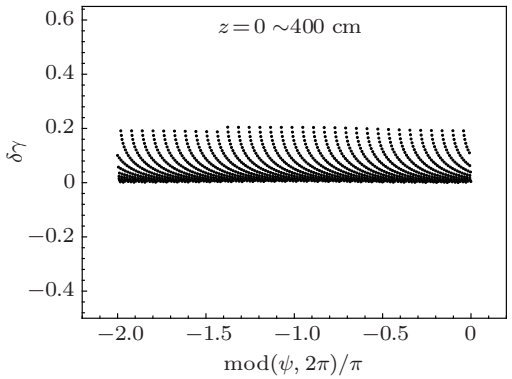


Fig. 5. Electron phase-space orbits in the beam-wave interaction process of $z = 0 \sim 400$ cm in which the FEL gain is far from the power saturation, where the parameters are the same as those in Fig. 3.

Hence, we conclude that the normalized plasma frequency of the electron beam has two consequences:

(i) by Eq. (5), the normalized plasma frequency couples the interaction between the electrons and the FEL wave, which results in the evolution of the FEL wave field $a_s = a_s(z)$ and a possible power saturation.

(ii) As shown in Figs. 1(d), 2 and 3, the normalized plasma frequency may cause the electron phase-space orbits to become irregular when it is at a sufficiently large value, even if the initial energy of the electron is equal to the synchronous energy or the FEL wave does not reach power saturation.

As is well known, a two-wave FEL has a low electron-beam current; its mechanism is based on the "individual-particle" interaction with the electromagnetic wave, where a loss of particle energy corresponds to a gain of electromagnetic field energy.[24] Since the normalized plasma frequency is related to both the e-beam current and the wiggler wavenumber and substantially affects the motion of individual electrons, the conclusions obtained in this paper are of significance in applications aiming to prevent the two-wave FEL performance from deteriorating.

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References

[1] Bonifacio R, Casagrande F, Casati G and Celi S 1984 *Coherence and Quantum Optics*, Vol. V, ed. Mandel L and Wolf E (New York: Plenum Publishing Corp.) p. 801
[2] Huo Y P 1982 *Acta Phys. Sin.* **31** 1347 (in Chinese)
[3] Spindler G and Renz G 1991 *Nucl. Instrum. Methods Phys. Res. A* **304** 492
[4] Riyopoulos S and Tang C 1988 *Phys. Fluids* **31** 1708
[5] Chen C and Davidson R 1991 *Phys. Rev. A* **43** 5541
[6] Zhang S C and Xu Y 1993 *Phys. Lett. A* **179** 311
[7] Zhang S C, Xu Y and Liu Q X 1993 *Phys. Rev. E* **48** 3952
[8] Bourdier B and Michel-Lours L 1994 *Nucl. Instrum. Methods Phys. Res. A* **341** 244
[9] Nam S and Kim K 2003 *Nucl. Instrum. Methods Phys. Res. A* **507** 69
[10] Soleyman A and Mehdian H 2006 *Acta Phys. Polonica A* **110** 459
[11] Ciraolo G, Chandre C, Lima R, Pettini M and Vittot M 2006 *Nucl. Instrum. Methods Phys. Res. A* **561** 244
[12] Taghavi A, Esmaeilzadeh M and Fallah M 2010 *Phys. Plasmas* **17** 093103
[13] Nasr N, Mehdian H and Hasanbeigi A 2011 *Phys. Plasmas* **18** 043104
[14] Esmaeilzadeh M and Taghavi A 2012 *Phys. Plasmas* **19** 113101
[15] Saviz S, Lashani E and Aghamir F 2012 *Chin. Phys. B* **21** 104104
[16] Huang X L, Wang S J, Xu Y G and Zhang S C 2012 *Phys. Rev. ST-Accel. Beams* **15** 120702
[17] Zhang S C 2013 *Phys. Lett. A* **377** 319
[18] Xu Y G, Wang S J, Ji Y P, Xu J Y, Lu H, Liu X X and Zhang S C 2013 *Acta Phys. Sin.* **62** 084104 (in Chinese)
[19] Wang S J, Xu Y G, Ji Y P, Xu J Y, Lu H, Liu X X and Zhang S C 2013 *Acta Phys. Sin.* **62** 144103 (in Chinese)
[20] Kroll N, Morton P and Rosenbluth M 1981 *IEEE J. Quantum Electron.* QE-**17** 1436
[21] Saldin E L, Chneidmiller E A and Yurkov M V 1999 *The Physics of Free Electron Lasers* (New York: Springer) p. 66
[22] Hogan M, Pellegrim C, Rosenzweig J, Anderson S, Frigola P, Tremaine A, Fortgang C, Nguyen D, Sheffield R, Kinross-Wright J, Varfolomeev A, Varfolomeev A A, Tolmachev S and Carr R 1998 *Phys. Rev. Lett.* **81** 4867
[23] Wang X, Freund H, Harder D, Miner W, Murphy J, Qian H, Shen Y and Yang X 2009 *Phys. Rev. Lett.* **103** 154801
[24] Marshall T C 1985 *Free Electron Lasers* (New York: Macmillan Publishing Company) p. 14