Effects of self-fields on gain in two-stream free electron laser with helical wiggler and an axial guiding magnetic field

Saviz S[a)†], Lashani E[a)], and Ashkarran A[b)]

[a)] Plasma Physics Research Center, Science and Research Branch, Islamic Azad University, Tehran, P. O. Box 14665-678, Iran
[b)] Department of Physics, Faculty of Basic Sciences, University of Mazandaran, Babolsar, Iran

(Received 24 March 2013; revised manuscript received 15 May 2013; published online 10 December 2013)

The theory for the two-stream free electron laser (FEL) consisting of a relativistic electron beam transporting along the axis of a helical wiggler in the presence of an axial guiding magnetic field is proposed and investigated. In the analysis, the effects of self-fields are taken into account. The electron trajectories and the small signal gain are derived. The characteristics of the linear-gain and the normalized maximum gain are studied numerically. The results show that there are seven stable groups of orbits in the presence of self-fields instead of two groups reported in the absence of the self-fields. It is also shown that the normalized gains of three groups decrease while the rest increase with the increasing of normalized cyclotron frequency Ω_0. Furthermore, it is found that the two-stream instability and the self-field lead to a decrease in the maximum gain except for group 3. The results show that the normalized maximum gain is enhanced in comparison with that of the single stream.

Keywords: two-stream free electron laser

PACS: 41.60.Cr

DOI: 10.1088/1674-1056/23/2/024102

1. Introduction

A free-electron laser (FEL) is a high-intensity, continuously tunable source of coherent electromagnetic radiation. The radiation is generated by a relativistic electron beam passing through a spatially periodic wiggler magnetic field.[1–6] A uniform axial magnetic field is often employed to collimate the relativistic electron beam in the transverse direction. The self-electric and self-magnetic fields are generated by charge and current densities of the electron beam and have significant effects on the operation of an FEL. Self-fields can produce chaos in FEL, particularly near gyro resonance.[7–10] The first-order self-magnetic field generated by the wiggler-induced transverse velocity has been calculated by Freund *et al.*[11] Recently, more exact calculations of self-fields and analyses of their effects on electron orbits have been published.[12,13] McNeil *et al.*[14] studied a model of the FEL amplifier operating with two electron beams of different energies. Bekefi and Jacobs[15] have shown that the amplitudes of the electromagnetic and space charge waves are considerably affected by the two-stream instability due to the prebunching of the electron beams. In addition, Liu *et al.*[16] and Mehdian and Abasi[17] have recently investigated the effects of two-stream instability on the growth rate of FEL with a helical wiggler. Finally, Liu *et al.*[16] have investigated the effects of two-stream instability on the growth rate of an FEL with a helical wiggler and an axial magnetic field. The effects of self-fields on gain in an FEL with an electromagnetic wave wiggler and axial magnetic field were investigated in Ref. [18].

The purpose of this paper is to investigate the effects of the self-fields on the gain in a two-stream FEL with a helical wiggler and an axial guiding field. The rest of this paper is organized as follows. In Section 2, the analyses of single-particle orbits of two-stream in the presence of axial guiding field and self-fields are given. In Section 3, the gain equation for the two-stream FEL is given. The numerical results and conclusions are given in Section 4.

2. Single particle orbits

The physical configuration consists of two-stream relativistic electron beams propagating through a helical wiggler in the presence of the axial guide magnetic field. The helical wiggler and axial guide field can be characterized by

$$\begin{aligned} \boldsymbol{B}_{\rm w} &= B_{\rm w}(\hat{e}_x \cos k_{\rm w} z + \hat{e}_y \sin k_{\rm w} z), \\ \boldsymbol{B} &= B_0 \hat{e}_z, \end{aligned} \tag{1}$$

where $B_{\rm w}$ is the wiggler amplitude and $\lambda_{\rm w} = 2\pi/k_{\rm w}$ is the wavelength, with $k_{\rm w}$ being the wave number of the wiggler.

The beam equilibrium self-electric and self-magnetic fields are

$$\boldsymbol{E}_{\rm si} = -2\pi \sum_{i=1}^{2} e n_{{\rm b}i}(x\hat{e}_x + y\hat{e}_y), \quad i = 1,2, \tag{2}$$

$$B_{\rm si} = 2\pi \sum_{i=1}^{2} e n_{{\rm b}i}\beta_{zi}(y\hat{e}_x - x\hat{e}_y). \tag{3}$$

The relativistic equations of motion for an electron with rest mass m and charge $-e$ in two-stream moving with velocity

†Corresponding author. E-mail: Shahrooz.saviz@srbiau.ac.ir

© 2014 Chinese Physical Society and IOP Publishing Ltd

$\boldsymbol{v}_i$ in these combined fields are given as

$$\frac{\mathrm{d}(m\gamma_i\boldsymbol{v}_i)}{\mathrm{d}t} = -e\Big[\boldsymbol{E}_{\mathrm{s}i} + \frac{1}{c}\boldsymbol{v}_i \times (\boldsymbol{B}_{\mathrm{w}} + B_0\hat{e}_z + \boldsymbol{B}_{\mathrm{s}i}\Big]. \quad (4)$$

With the aid of field Eqs. (1)–(3), the steady-state solution of Eq. (4) for the normalized electron velocity ($\boldsymbol{\beta}_i = \boldsymbol{v}_i/c$) is obtained as

$$\boldsymbol{\beta}_i = \boldsymbol{\beta}_{xi} + \boldsymbol{\beta}_{yi} + \boldsymbol{\beta}_{zi} = \beta_{\mathrm{w}i}(\hat{e}_x \cos k_{\mathrm{w}}z + \hat{e}_y \sin k_{\mathrm{w}}z) + \beta_{zi}\hat{e}_z, \quad (5)$$

where $\beta_{zi} = \text{const}$ is the magnitude of the normalized electron transverse velocity and

$$\beta_{\mathrm{w}i} = \frac{\Omega_{\mathrm{w}i}\beta_{zi}^2}{\Omega_{0i}\beta_{zi} + \Big(\sum_{i=1}^{2}\omega_{\mathrm{b}i}^2\beta_{zi}\Big)\beta_{zi} - \sum_{i=1}^{2}\omega_{\mathrm{b}i}^2 - \beta_{zi}^2} \quad (6)$$

is the magnitude of the normalized electron axial velocity. In Eq. (6),

$$\Omega_{\mathrm{w_i}} = \frac{eB_{\mathrm{w}}}{m\gamma_i k_{\mathrm{w}}c^2},\ \ \Omega_{0_i} = \frac{eB_0}{m\gamma_i k_{\mathrm{w}}c^2},\ \ \omega_{\mathrm{b}_i}^2 = \frac{2\pi e^2 n_{\mathrm{b}i}}{m\gamma_i k_{\mathrm{w}}^2 c^2}$$

denote the normalized wiggler frequency, normalized cyclotron frequency and normalized beam frequency respectively. Since the energy is a constant of motion, we require that $\beta_{\mathrm{w}i}$ should be related to β_{zi} by the following equation:

$$\gamma_i^{-2} = 1 - \beta_{xi}^2 - \beta_{yi}^2 - \beta_{zi}^2 = 1 - \beta_{\mathrm{w}i}^2 - \beta_{zi}^2, \quad i = 1, 2. \quad (7)$$

3. Small signal gain

Now consider electromagnetic radiation copropagating with the electron in the FEL interaction region. The electric and magnetic fields of radiation are written as

$$\boldsymbol{E}_{\mathrm{r}} = E_{\mathrm{r}}(\cos\xi\,\hat{e}_x - \sin\xi\,\hat{e}_y), \quad (8)$$

$$\boldsymbol{B}_{\mathrm{r}} = B_{\mathrm{r}}(\sin\xi\,\hat{e}_x + \cos\xi\,\hat{e}_y), \quad (9)$$

where $\xi = k_{\mathrm{r}}z - \omega_{\mathrm{r}}t + \phi k_{\mathrm{r}}$ is the wave number with ω_{r} being the angular frequency of the radiation wave and ϕ is a phase constant.

Substitute Eqs. (5) and (6) into Eq. (7) and then, after some straightforward algebra, an expression for the normalized axial velocity will be expressed as

$$\beta_{zi} = 1 - \frac{1}{2}\gamma_i^{-2}\Big(1 + \frac{a_{\mathrm{w}}\beta_{zi}^4}{h_i^2}\Big), \quad (10)$$

where $a_{\mathrm{w}} = eB_{\mathrm{w}}/(mk_{\mathrm{w}}c^2)$ and

$$h_i = \Omega_{0i}\beta_{zi} + \Big(\sum_{i=1}^{2}\omega_{\mathrm{b}i}^2\beta_{zi}\Big)\beta_{zi} - \sum_{i=1}^{2}\omega_{\mathrm{b}i}^2 - \beta_{zi}^2.$$

With the aid of Eqs. (5), (6) and (8), the energy exchange between an electron beam and the radiation field is given by

$$\dot{\gamma}_i = -\frac{eE_{\mathrm{r}}}{mc}\beta_{\mathrm{w_i}}\cos[\Omega_i t + \phi + (k_{\mathrm{r}} + k_{\mathrm{w}})D_i(\cos(\Omega_i t + \phi) - \cos\phi + \Omega_i t\sin\phi)], \quad (11)$$

where, $\Omega_i = (k_{\mathrm{r}} + k_{\mathrm{w}})c\beta_{zi} - \omega_{\mathrm{r}}$ and $D_i = k_i^2/(k_{\mathrm{r}} + k_{\mathrm{w}})\Omega_i^2$.

Defining the electron phase as

$$\psi = (k_{\mathrm{r}} + k_{\mathrm{w}})z - \omega_{\mathrm{r}}t + \frac{\pi}{2} + \phi, \quad (12)$$

differentiating it twice with respect to time and using Eqs. (10) and (11), we have

$$\ddot{\psi}_i = -k_i^2 \sin\psi, \quad (13)$$

where

$$k_i^2 = (k_{\mathrm{r}} + k_{\mathrm{w}})\frac{eE_{\mathrm{r}}a_{\mathrm{w}}\beta_{zi}^2}{m\gamma_i^4 h_i}\Big[1 - a_{\mathrm{w}}^2\Big(\frac{\beta_{zi}^2}{h_i}\Big)^3\Big].$$

Averaging Eq. (11) over all phases and integrating it over the electron transit time ($T = L/c\beta_{zi}$) through the wiggler interaction length (L), we obtain the average change in γ_i per electron as follows:

$$\langle\Delta\gamma_i\rangle_\phi = \int_0^{T=L/c\beta_{z_i}}\langle\dot{\gamma}_i\rangle\,\mathrm{d}t = \frac{eE_{\mathrm{r}}D_i\Omega_i^2T^3\beta_{\mathrm{w_i}}(k_{\mathrm{r}} + k_{\mathrm{w}})}{8mc}S_i(\theta_i),$$

$$S_i(\theta_i) = \frac{\mathrm{d}}{\mathrm{d}\theta_i}\Big(\frac{\sin\theta_i}{\theta_i}\Big)^2 = \frac{2\sin\theta_i\cos\theta_i}{\theta_i^2} - \frac{2\sin^2\theta_i}{\theta_i^3}, \quad (14)$$

where $S_i(\theta_i)$ is the spectral function and $\theta_i = -\Omega T/2$.

The gain equation can be obtained as

$$G = \frac{\delta P}{P} = \frac{8\pi}{cE^2\mathrm{Area}}\frac{-mc^2}{e}\Big[I_1\langle\Delta\gamma_1\rangle_\varphi + I_2\langle\Delta\gamma_2\rangle_\varphi\Big],$$

where $I_i = en_{\mathrm{b}i}c\beta_{zi}\pi r_{\mathrm{b}}^2$ is the average electron current. Finally by using Eq. (14), the gain equation in the presence of the self-fields becomes

$$G = -\frac{\pi e^2L^3(k_{\mathrm{r}} + k_{\mathrm{w}})}{2mc^2}\sum_{i=1}^{2}\frac{n_{\mathrm{b}i}\beta_{\mathrm{w}i}^2}{\beta_{zi}^2\gamma_i\gamma_{zi}^2}F_iS_i(\theta_i), \quad (15)$$

where $\gamma_{zi} = (1 - \beta_{zi}^2)^{-1/2}$.

$$F_i = 1 - a_{\mathrm{w}}^2\beta_{zi}^4[\Omega_{0i}\beta_{zi} - \omega_{\mathrm{b}i}^2(1 - \beta_{zi}^2)]/(h_i(h_i^2 + a_{\mathrm{w}}^2\beta_{zi}^4)),$$

with $L = 100\lambda_{\mathrm{w}}$, $f = Lk_{\mathrm{w}} = 200\pi$, and $w = k_{\mathrm{r}}/k_{\mathrm{w}}$ denoting the FEL interaction length, normalized wave number of wiggler, and normalized frequency respectively.

4. Numerical results and conclusions

A numerical study of the trajectories of two streams of steady-state relativistic electrons in the presence of self-fields, helical wiggler and axial guide field has been conducted. The typical dependence of the axial velocity on Ω_0 is obtained by solving Eqs. (6) and (7). The electron energies of two beams are taken to be $\gamma_1 = 4$ and $\gamma_2 = 4.2$; the normalized wiggler magnetic and normalized plasma frequencies of the two beams are chosen to be $\Omega_{\mathrm{w}} = 0.05$ and $\omega_{b1} = \omega_{b2} = 0.2$, respectively. Figures 1(a) and 1(b) show the plots of the normalized axial velocities of the two beams, β_{z1} and β_{z2}, each as a function of the normalized cyclotron frequency. It is evident from these figures that the electron trajectories in the presence of self-fields have seven distinguished group orbits instead of two groups reported in the absence of the self-fields.

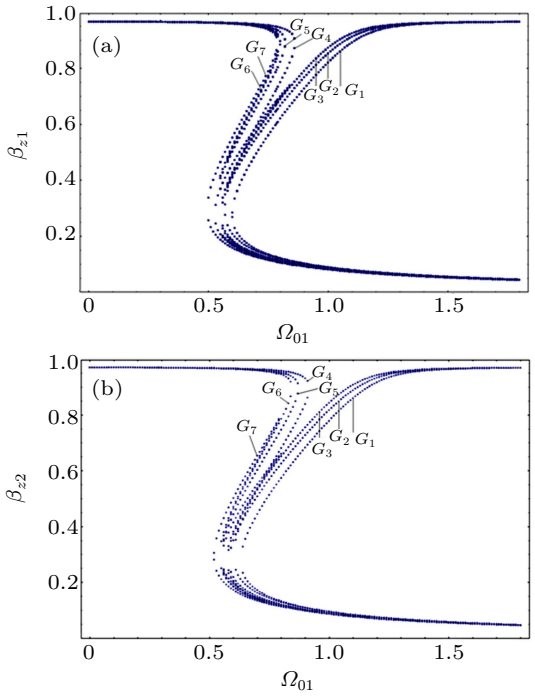


Fig. 1. (color online) Normalized axial velocities (a) β_{z1} and (b) β_{z2} of the steady-state orbits, each as a function of the normalized cyclotron frequency (Ω_0) in the case of the two-stream, with the parameters of $\gamma_1 = 4$, $\gamma_2 = 4.2$, $\Omega_{w_1} = 0.05$, and $\omega_{b1} = 0.2$ chosen.

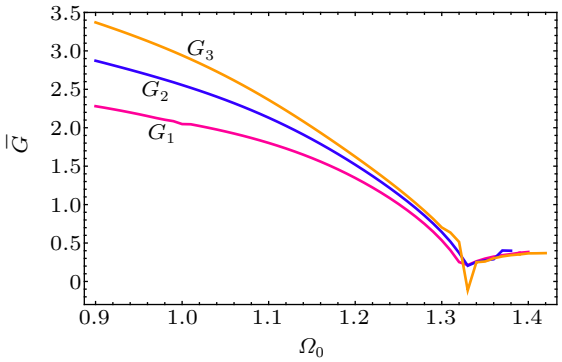


Fig. 2. (color online) Plots of normalized maximum gain ($\bar{G}$) versus cyclotron frequency (Ω_0) for groups 1–3.

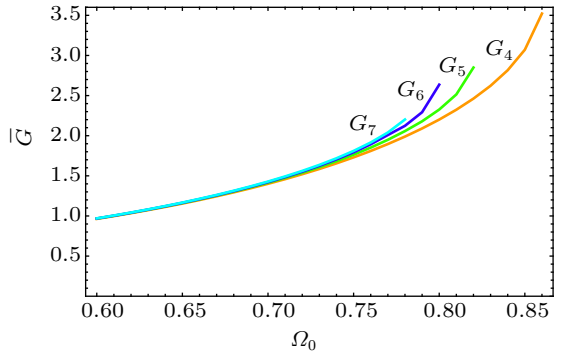


Fig. 3. (color online) Plots of normalized maximum gain ($\bar{G}$) versus the cyclotron frequency (Ω_0) for groups 4–7.

The plots of the normalized gain $\bar{G} = G_{\max}/G_{0\max}$ (the ratio of the maximum gain of the two-stream in the presence of the self-fields to maximum gain in the absence of the self-fields and axial magnetic field) versus normalized axial magnetic field, are shown in Figs. 2 and 3. Figure 2 shows that the gains of groups 1, 2, and 3 are similar to each other and decrease with the increasing of Ω_0 until a minimum is reached. The gains of three groups in the region $0.9 \leq \Omega_0 \leq 1.32$ are ordered as $\bar{G}_3 > \bar{G}_2 > \bar{G}_1$ ($\bar{G}_i$ indicate the normalized maximum gain for group i). On the contrary, the normalized gains of groups 4–7 presented in Fig. 3 follow an increasing trend with the increasing of Ω_0 until they reach their asymptotic regions. These normalized gains are ordered as $\bar{G}_7 > \bar{G}_6 > \bar{G}_5 > \bar{G}_4$ in the range of $0.6 \leq \Omega_0 \leq 0.85$. The plots of the normalized frequency $\hat{\omega}$ corresponding to the maximum gain versus Ω_0 are shown in Figs. 4(a) and 4(b). As shown in Fig. 4(a), the normalized frequencies are in the same order as the normalized maximum gains, i.e., $\hat{\omega}_1 > \hat{\omega}_2 > \hat{\omega}_3$ in the region $0.9 \leq \Omega_0 \leq 1.4$ ($\hat{\omega}_i$ denotes the normalized maximum frequency for group i). The normalized frequencies for groups 4–7, shown in Fig. 4(b), are ordered as $\hat{\omega}_4 > \hat{\omega}_5 > \hat{\omega}_6 > \hat{\omega}_7$. The comparisons of the normalized maximum gain for the two streams between with and without the self-fields for groups 1–3 and 4–7 are given in Figs. 5(a) and 5(b) respectively. The results show that the self-fields cause the maximum gains to decrease, with the exception of group 3. The comparisons of the normalized maximum gains between the two streams and single stream are given in Figs. 6(a) and 6(b). As shown in Fig. 6(a), the maximum gain for group 1 of the single-stream is smaller than the maximum gain for the two-stream in the region $1.11 \leq \Omega_0 \leq 1.325$. As shown in Fig. 6(b), the maximum

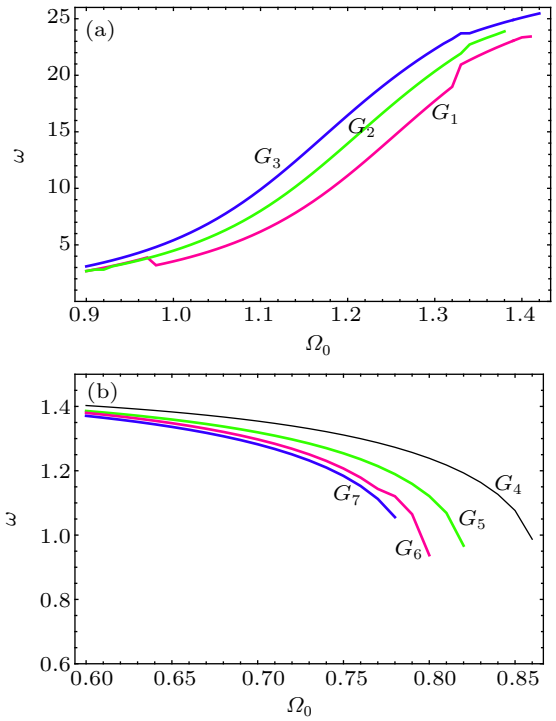


Fig. 4. (color online) Curves of normalized maximum frequency (ω) versus normalized cyclotron frequency Ω_0 for (a) groups 1–3 and (b) groups 4–7.

gain for group 2 of the single-stream is less than the maximum gain for the two-stream in the region $0.6 \leq \Omega_0 \leq 0.73$. The comparison of the normalized maximum gain between the two streams and single stream, corresponding to Figs. 6(a) and 6(b), are shown in Tables 1 and 2 for several values of the normalized cyclotron frequency Ω_0, respectively.

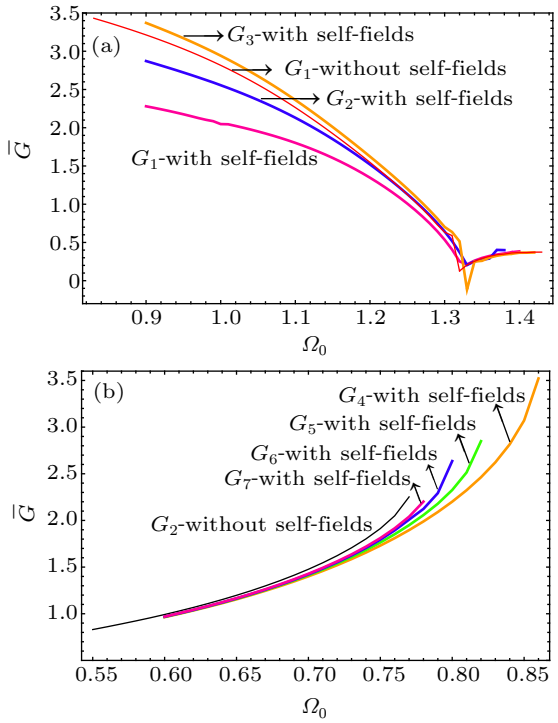


Fig. 5. (color online) Curves of normalized maximum gain ($\bar{G}$) of two-streams with and without self-fields versus cyclotron frequency (Ω_0) for (a) groups 1–3 and (b) groups 4–7.

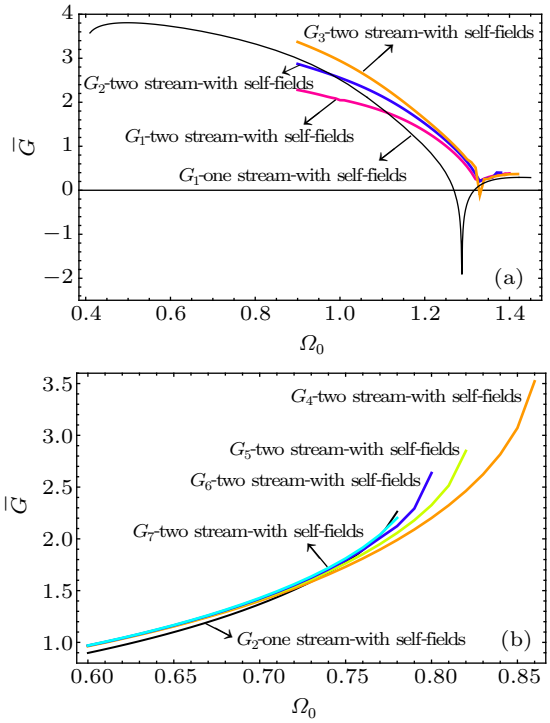


Fig. 6. (color online) Plots of normalized maximum gain $\bar{G}$ for two-stream and single stream versus normalized cyclotron frequency Ω_0 for (a) groups 1–3 and (b) groups 4–7.

Table 1. Comparisons of normalized maximum gain between two-stream ($\bar{G}_3$) and single stream ($\bar{G}_1$) for several values of the normalized cyclotron frequency Ω_0.

Ω_0	0.9	1.0	1.1	1.2	1.3	1.4
Normalized max gain of single-stream ($\bar{G}_1$) with self-fields	2346.28	874.54	230.359	41.9979	5.04247	2.31999
Normalized max gain of two-stream ($\bar{G}_3$) with self-fields	989.632	330.107	70.5663	9.90859	0.498975	1.93097

Table 2. Comparisons of normalized maximum gain between two-stream ($\bar{G}_7$) and single stream ($\bar{G}_2$) for several values of the normalized cyclotron frequency Ω_0.

Ω_0	0.6	0.64	0.68	0.7	0.74	0.78
Normalized max gain of two-stream with self-fields ($\bar{G}_7$)	9.3305	13.2992	20.6218	26.8464	52.3462	159.18
Normalized max gain of single-stream with self-fields ($\bar{G}_2$)	7.9033	11.3358	17.7914	23.3833	47.212	182.986

References

[1] Freund H P and Antonsen T M 1996 *Principles of Free-Electron Lasers* (London: Chapman and Hall) pp. 45–50
[2] Roberson C W and Sprangle P 1989 *Phys. Fluids B* **3** 1
[3] Jaber I and Demokan O 1996 *Phys. Plasmas* **3** 4190
[4] Freund H P and Sprangle P 1982 *Phys. Rev. A* **26** 2004
[5] Freund H P, Kehs R A and Granatstein V L 1985 *IEEE J. Quantum Electron.* **21** 1080
[6] Mehdian H, Jafari S and Hasanbeigi A 2008 *Phys. Plasmas* **15** 073102
[7] Chen C and Davidson R C 1991 *Phys. Rev. A* **43** 5541
[8] Michel L, Bourdier A and Buzzi J M 1991 *Nucl. Instrum. Methods Phys. Res. A* **304** 465
[9] Bourdier A and Michel L 1994 *Phys. Rev. E* **49** 3353
[10] Spindler G and Renz G 1991 *Nucl. Instrum. Methods Phys. Res. A* **304** 492
[11] Freund H P, Jackson R H and Pershing D E 1993 *Phys. Fluids B* **5** 2318
[12] Mirzanejhad S, Maraghechi B and Mohsenpour T 2004 *Phys. Plasmas* **11** 4777
[13] Esmaeilzadeh M and Willett J E 2007 *Phys. Plasmas* **14** 033102
[14] McNeil B W J, Robb G R M and Poole M W 2004 *Phys. Rev. E* **70** 035501
[15] Bekefi G and Jacobs K D 1982 *J. Appl. Phys.* **53** 4113
[16] Liu W X, Yang Z Q and Liang Z 2006 *J. Infrared Millim. Terahertz Waves* **27** 1073
[17] Mehdian H and Abasi N 2008 *Phys. Plasmas* **15** 013111
[18] Mehdian H and Saviz S 2010 *Chin. Phys. B* **19** 014214