Dynamical Casimir effect in superradiant light scattering by Bose–Einstein condensate in an optomechanical cavity

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We investigate the effects of dynamical Casimir effect in superradiant light scattering by Bose–Einstein condensate in an optomechanical cavity. The system is studied using both classical and quantized mirror motions. The cavity frequency is harmonically modulated in time for both the cases. The main quantity of interest is the number of intracavity scattered photons. The system has been investigated under the weak and strong modulations. It has been observed that the amplitude of the scattered photons is more for the classical mirror motion than the quantized mirror motion. Also, initially, the amplitude of scattered photons is high for lower modulation amplitude than higher modulation amplitude. We also found that the behavior of the plots are similar under strong and weak modulations for the quantized mirror motion.

Keywords: Bose–Einstein condensates, dynamical Casimir effect, superradiant light scattering, optomechanical cavity

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1. Introduction

The long-range coherence of Bose–Einstein condensate (BEC) offers the possibility to study the collective motion induced by external radiation beams.[1] Specifically, it has been observed that the coherent center-of-mass motion of atoms in a condensate driven by a highly detuned laser gives rise to the superradiant Rayleigh scattering and matter-wave amplification.[2–4] The matter-wave grating produced in these experiments is due to the coherent superposition of atomic momentum states, which is identical to the one formed in Bragg scattering experiments in which the condensate atoms are diffracted by a standing wave of light.[5,6] The collective atomic recoil lasing (CARL)[7,8] process shows the collective instability, which causes the spontaneous formation of a regular density grating in a BEC that was first demonstrated in superradiant Rayleigh scattering experiments[2] and matter-wave amplification.[4] The coherence in the atomic momentum states plays an important role in all these experiments. The CARL process can be suppressed by the effects such as spontaneous emission, inhomogeneous broadening, and collisions in the atomic sample which may ruin the coherences in the matter waves.[9] The atoms scatter a single laser photon and recoil with a momentum of $2\hbar k$ (k is the wave-vector of the incident photon) in the direction of the incident laser photon in the absence of Doppler broadening due to thermal motion of BEC atoms. The CARL instability is responsible for the exponentially enhancement in the scattered photon number and the amplitude of the density grating which arises due to the interference between the atomic wavepackets with momentum difference $2\hbar k$.[10–12] The quantum mechanical illustration of the center-of-mass motion of the condensate atoms is extended theoretically from the semiclassical model of CARL.[13] Experimentally and theoretically, the affect of motion of atoms on the superradiant scattering of light from a moving BEC was observed in Refs. [14] and [15]. The enhancement in the superradiant scattering process in the cavity of light field is possible due to the multiple reflections of the pump laser from the mirrors of the cavity which amplifies the coupling time of the atoms with the optical fields.[16,17] The effects of a movable mirror of an optical cavity on the superradiant light scattering from a BEC in an optical lattice has been investigated.[18]

The optomechanical cooling of a system to its quantum ground state is the main goal of the recent field of optomechanics. In recent years, there has been an outpouring of interest in the application of radiation forces to modify the center-of-mass motion of mechanical resonators involving a vast range from macroscopic mirrors in the Laser Interferometer Gravitational Observatory (LIGO) project[19,20] to nano- or micromechanical cantilevers,[21–26] vibrating microtoroids,[27,28] membranes,[29] and ultracold atoms.[30–36] The cooling of the motion of mechanical oscillators is possible with positive radiation pressure damping whereas parametric amplification of small forces is observed with negative damping.[27,37] In the fascinating work of Braginsky,[38,39] the mechanical damping due to radiation was first detected in the decay of an excited oscillator. Recently, the cooling of the center-of-mass motion of the mechanical oscillator (i.e., both the measurement and mechanical damping of the random thermal Brownian motion) was attained by using many techniques. These involve active feedback cooling[25,40,41] based on posi-

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tion measurements, intrinsic optomechanical cooling by radiation pressure[19,22,23,28,29] or photothermal forces.[21] A system composed of atoms in an optical cavity with moving end mirror was studied.[42] It was found that the motion of the mirror alters the dipole potential in which atoms move and the position becomes bistable.

Forty years ago, it was suggested[43] that the relativistic motion of the mirror could create real photons from virtual photons. This phenomenon was later termed as the dynamical Casimir effect (DCE).[44] This DCE has a common feature of creating quanta from vacuum due to the motion of macroscopic neutral boundaries or time-modulation of material properties of some macroscopic system. This fascinating phenomenon has attracted the attention of many theoreticians.[43–45] It has been observed that the DCE originates, so far, mostly due to the oscillation of a cavity boundary, the motion of a mirror in a vacuum, or the modulation of the dielectric properties of the medium inside the cavity which causes the generation of photon pairs from the electromagnetic vacuum. Recent experiments have observed the DCE which makes the problem of detecting photons generated from initial vacuum state realistic.[45–47] In view of recent progress, the so-called DCE has been observed with harmonic modulation of the cavity frequency in the presence of harmonic oscillators[48] and multi-level atoms.[49–54] A few years ago, dynamical Casimir photons have also been detected via superradiance amplification.[55,56] Recently, we have also examined the DCE and its acoustic analog in an optomechanical cavity consisting of a quantum well.[57]

Given the promising developments in the field of ultracold atoms, cavity optomechanics, and DCE, we propose a scheme to couple BEC to an optomechanical cavity to study the DCE. The optomechanical cavity has one cavity mirror fixed while the other mirror is moving. This system is studied by considering firstly the classical motion of the movable mirror and then the quantized mirror motion. The cavity field is harmonically time-modulated in both of the cases, thereby, making the system non-stationary. The main quantity of interest in the phenomenon of DCE is the number of photons produced inside the cavity which is studied here.

2. CARL-BEC model with classical mirror motion

The system investigated here consists of noninteracting bosonic two-level atoms inside an optomechanical cavity coupled to two radiation fields via the electric-dipole interaction as shown in Fig. 1. Specifically, we take an elongated cigar-shaped BEC consisting of N two-level atoms with mass m and frequency ω_a having transition $|F=1\rangle \rightarrow |F'=2\rangle$. When the frequency of the harmonic trap along the axial direction is smaller than the frequency along the transverse direction then an elongated cigar-shaped BEC is formed.

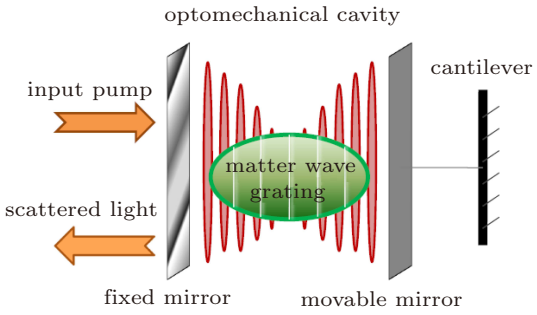


Fig. 1. (color online) Schematic figure of the system. The counter-propagating light fields make a periodic optical grating which further forms a periodic matter wave grating. The cavity has one mirror fixed and other movable. The fixed mirror is partially reflecting whereas the movable mirror is completely reflecting. The scattered light coming out of the fixed mirror is measured.

The bosonic atoms are confined in optomechanical cavity, whose mirror at one end is fixed while the mirror at the other end is movable. However, the mirror motion is considered to be classical. Furthermore, the cavity frequency is sinusoidally time-modulated; that is, $\omega_c(t) = \omega_c(1+\epsilon\sin(\Omega t))$. Here, ω_c is the unperturbed cavity frequency, ϵ is the modulation amplitude, and Ω is the modulation frequency. We consider an external pump laser with frequency ω_p and amplitude η' incident on one side mirror of the optical cavity. When the longitudnal mode spacing is assumed to be larger than the induced resonance frequency shift of the cavity, then one can take single longitudnal cavity mode. This approximation is considered here. The population of the atoms in the excited state is assumed to be negligible since the detuning of the pump laser and the atomic frequency is approximated to be too large as compared to the detuning of pump laser with cavity mode. Therefore, the spontaneous emission and two-body dipole–dipole interactions can be safely ignored. But the two photon transitions are allowed, which do not alter the atomic internal states. However, the center-of-mass motion of atom changes due to the recoil. An atom absorbs a photon from one mode and releases a photon in counter-propagating mode. Therefore, it experiences a recoil kick equal to the difference between the momenta of two photons (of the order of two optical momenta). The distinct momentum states of the atomic center-of mass motion interact with two-photon transition. Hence, the time-dependent Hamiltonian of the system in the dipole approximation is given as[49]

$$H_1 = H_1 + H_2 + H_3 + H_4 + H_5. \tag{1}$$

The free energy of photons inside the time-dependent cavity is given as

$$H_1 = \hbar\omega_c(t)a^\dagger a, \tag{2}$$

where $a(a^\dagger)$ is the annhilation (creation) operator, which obeys the commutation relation $[a, a^\dagger] = 1$. The free energy of atoms

is given by

$$H_2 = \int \mathrm{d}z\Big\{\psi_{\mathrm{g}}^{\dagger}(z)\Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\Big)\psi_{\mathrm{g}}(z) + \psi_{\mathrm{e}}^{\dagger}(z)\Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+\hbar\omega_{\mathrm{a}}\Big)\psi_{\mathrm{e}}(z)\Big\}, \quad (3)$$

where $\psi_{\mathrm{g}}(z)$, $\psi_{\mathrm{e}}(z)$ are the ground and excited field operators of the atoms, respectively. The third term in H_{I} represents the atom–photon coupling, which is given as

$$H_3 = \mathrm{i}\hbar g_0 a^{\dagger}\Big\{\int \mathrm{d}z\psi_{\mathrm{g}}^{\dagger}(z)\big(\mathrm{e}^{-\mathrm{i}kz}+\mathrm{e}^{\mathrm{i}kz}\big)\psi_{\mathrm{e}}(z)\Big\}+\mathrm{H.C.}, \quad (4)$$

where g_0 is the coupling constant. Here, the counter-propagating optical cavity field interacts with the condensate field. These counter-propagating optical cavity modes are directed towards $\pm z$ axes. Also, k represents the wavenumber of the cavity light mode. The atom field operators obey the commutation relation as follows:

$$\Big[\psi_j(z),\psi_{j'}^{\dagger}(z')\Big]=\delta_{j,j'}\delta(z,z'), \quad (5)$$

$$\Big[\psi_j(z),\psi_{j'}(z')\Big]=\Big[\psi_j^{\dagger}(z),\psi_{j'}^{\dagger}(z')\Big]=0, \quad (6)$$

where j, $j'=\mathrm{e}$, g. Now, the energy due to external laser pump is given by the fourth term of H_{I}; that is,

$$H_4 = -\mathrm{i}\hbar\eta'(a-a^{\dagger}), \quad (7)$$

with η' being the amplitude of the external laser beam. The fifth term (H_5) in the Hamiltonian (H_{I}) is the squeezing term[58] given as

$$H_5 = \mathrm{i}\hbar\chi(t)\big(a^{\dagger 2}\mathrm{e}^{-2\mathrm{i}\omega_{\mathrm{p}}t}-a^2\mathrm{e}^{2\mathrm{i}\omega_{\mathrm{p}}t}\big). \quad (8)$$

Due to degenerate parametric amplification of the field, this term helps in producing squeezed states of the field.[59] The photons are produced in the cavity from the vacuum state due to the fast motion of the cavity boundaries. This effect is known as the DCE.[44] The effect of DCE can possibly produce squeezed states.[58,60–63] Also, $\chi(t)$ is related to $\omega_{\mathrm{c}}(t)$ as[63]

$$\chi(t)=\frac{1}{4\omega_{\mathrm{c}}(t)}\frac{\mathrm{d}\omega_{\mathrm{c}}(t)}{\mathrm{d}t}. \quad (9)$$

Now we take the realistic case of small-amplitude time modulation; that is, $|\epsilon|\ll 1$. Therefore,

$$\chi(t)\approx\frac{\epsilon\Omega}{4}\cos(\Omega t)\approx 2\chi_0\cos(\Omega t), \quad (10)$$

where

$$\chi_0=\frac{\epsilon\Omega}{8}\ll 1. \quad (11)$$

Now, using the Heisenberg equation of motion ($\dot{\psi}_{\mathrm{e}}=(\mathrm{i}/\hbar)[H_{\mathrm{I}},\psi_{\mathrm{e}}]$), one can adiabatically eliminate the excited state of atoms as the pump–atom detuning ($\Delta=\omega_{\mathrm{p}}-\omega_{\mathrm{a}}$) is very large. This gives the Heisenberg equations of motion for the ground state field operator ($\psi_{\mathrm{g}}(z)$) and the cavity field annihilation operator (a) by taking into account all the dissipative processes:

$$\dot{\psi}_{\mathrm{g}}=\frac{\mathrm{i}\hbar}{2m}\frac{\partial^2\psi_{\mathrm{g}}}{\partial z^2}-\frac{2\mathrm{i}g_0^2a^{\dagger}a}{\Delta}(1+\cos(2kz))\psi_{\mathrm{g}}, \quad (12)$$

$$\dot{a}=\mathrm{i}\Delta_{\mathrm{c}}a-\mathrm{i}\omega_{\mathrm{c}}\epsilon\sin(\Omega t)a-\frac{2\mathrm{i}g_0^2aN}{\Delta} -\frac{2\mathrm{i}g_0^2a}{\Delta}\int \mathrm{d}z\psi_{\mathrm{g}}^{\dagger}\cos(2kz)\psi_{\mathrm{g}}+2\chi(t)a^{\dagger}+\eta-\kappa a, \quad (13)$$

where $\Delta_{\mathrm{c}}=\omega_{\mathrm{p}}-\omega_{\mathrm{c}}$, $N=\int \mathrm{d}z\psi_{\mathrm{g}}^{\dagger}(z)\psi_{\mathrm{g}}(z)$, and κ is the cavity photon decay rate. The proposed system is basically open as the photons leak from the cavity so the cavity field is damped. The second term on the right-hand side of the equality in Eq. (12) represents the self-consistent optical grating whose amplitude depends on time according to Eq. (13). Also, the fourth term on the right side of the equality in Eq. (13) gives the self-consistent matter wave grating.

The spatial periodic boundary conditions can be assumed if the density of the condensate atoms is homogeneous and the optical radiation wavelength is much shorter than the length of the condensate. This leads to the ground state wavefunction of the condensate atoms as

$$\psi_{\mathrm{g}}\Big(z,t\Big)=\sum_n C_n(t)\,\mathrm{e}^{2\mathrm{i}nkz}, \quad (14)$$

where, $\mathrm{e}^{2\mathrm{i}nkz}$ represents the momentum eigenfunctions with eigenvalues $p_z=n(2\hbar k)$. The atomic motion considered here is based on an equivalent assumption that the atoms in a BEC are not localized inside the length of the condensate and the uncertainity in the corresponding momentum state is negligibly small. Now, equations (12) and (13) can be reduced to the following set of ordinary differential equations

$$\dot{C}_n=-4\mathrm{i}\omega_r n^2C_n-\frac{\mathrm{i}g_0^2NA}{\Delta}(2C_n+C_{n+1}), \quad (15)$$

$$\dot{C}_{n+1}=-4\mathrm{i}\omega_r(n+1)^2C_{n+1}-\frac{\mathrm{i}g_0^2NA}{\Delta}(2C_{n+1}+C_n), \quad (16)$$

$$\dot{a}'=\mathrm{i}\Delta_{\mathrm{c}}a'-\mathrm{i}\omega_{\mathrm{c}}\epsilon\sin(\Omega t)a'-\frac{2\mathrm{i}g_0^2a'N}{\Delta} -\frac{2\mathrm{i}g_0^2a'N}{\Delta}C_{n+1}^{*}C_n+2\chi(t)a'^{\dagger}+\eta-\kappa a', \quad (17)$$

where, $A=a'^{\dagger}a'$, n is the initial momentum level, $n+1$ is the final momentum level, and $\omega_{\mathrm{r}}=\hbar k^2/2m$ is the recoil frequency. Also, here a' is the rescaled photon operator with $a\to a'\sqrt{N}$ and η is the rescaled external pump amplitude with $\eta\to\eta\sqrt{N}$.

Defining coherence $S=C_nC_{n+1}^{*}$ and the population difference between the two states as $W=|C_n|^2-|C_{n+1}|^2$, we obtain the following equations of motion

$$\dot{S}=4\mathrm{i}\omega_{\mathrm{r}}(1+2n)S+\frac{\mathrm{i}g_0^2NA}{\Delta}W-\gamma S, \quad (18)$$

$$\dot{W}=\frac{2\mathrm{i}g_0^2NA}{\Delta}(S-S^{*}), \quad (19)$$

$$\dot{a}' = \mathrm{i}\Delta_c a' - \mathrm{i}\omega_c \epsilon \sin(\Omega t) a' - \frac{2\mathrm{i}g_0^2 a' N}{\Delta} - \frac{2\mathrm{i}g_0^2 a' N}{\Delta} S + 2\chi(t) a'^{\dagger} + \eta - \kappa a'. \quad (20)$$

Here, we have introduced a damping term in Eq. (18) to account for the decay of the coherence between the two motional states n and $n+1$. Now, separating the above equations of motion into real and imaginary parts, we get

$$\dot{S}_r = -4\omega_r(1+2n)S_i - \gamma S_r, \quad (21)$$

$$\dot{S}_i = 4\omega_r(1+2n)S_r + g(a_r^2 + a_i^2)W - \gamma S_i, \quad (22)$$

$$\dot{W} = -4g(a_r^2 + a_i^2)S_i, \quad (23)$$

$$\dot{a}_r = -\Delta_c a_i + \omega_c \epsilon \sin(\Omega t) a_i + 2g a_i + 2g(a_i S_r + a_r S_i) + 2\chi(t) a_r + \eta - \kappa a_r, \quad (24)$$

$$\dot{a}_i = \Delta_c a_r - \omega_c \epsilon \sin(\Omega t) a_r - 2g a_r - 2g(a_r S_r - a_i S_i) - 2\chi(t) a_i - \kappa a_i, \quad (25)$$

where the renormalized coupling constant is given as $g = g_0^2 N/\Delta$. We investigate the system in the quantum superradiant regime $g/\omega_r\sqrt{N} < 2\sqrt{\kappa/\omega_r}$. In this regime, the condensate momentum alters by $2\hbar k$ as each atom scatters a single photon only. As soon as the pump power builds up in the ring cavity, the collective dynamics start. We solve the above coupled differential equations of motion (21)–(25) by making use of MATHEMATICA 9.0 to study this dynamics via the evolution of the power $A(t) = a'^{\dagger} a'$. We further identify two more interesting regimes under which the system is investigated. The first is strong modulation ($\chi_0 \gtrsim g$) and the other is weak modulation ($\chi_0 \ll g$).

Now, the transition from the Bragg scattering regime to the quantum superradiant regime has been investigated. We have considered the BEC to be at rest initially ($n = 0$). Figure 2 shows the evolution of power $A(t)$ versus scaled time ($\omega_r t$) by considering the classical motion of mirror. The plot in Fig. 2(a) corresponds to the weak modulation ($g = \omega_r$) with two modulation amplitudes $\epsilon = 0.2$ (thin line) and $\epsilon = 0.3$ (thick line). The plot shows damped periodic oscillations with time. These oscillations depict the typical Bragg resonances due to two-photon transitions between the momentum states. Under the strong modulation ($g = 0.005\omega_r$) as shown in Fig. 2(b), the Bragg oscillations vanish with time for higher modulation amplitude ($\epsilon = 0.3$). Therefore, one can switch between the pure superradiant regime and Bragg scattering regime by varying the modulation amplitude. Reducing the renormailzed coupling constant (g), increases the effect of DCE and decreases the Bragg scattering superradiant process. One more thing noticeable here is that the amplitude of oscillations is very high under the strong modulation as compared to weak modulation. This is because under the strong modulation the effect of DCE is dominant, which produces extra photons in the cavity from the vacuum state. It can be further observed that, initially, the amplitude of number of scattered photon is high for lower modulation amplitude as compared to higher modulation amplitude. This is due to the fact that as we are increasing the modulation amplitude, the dispersive effect increases. This reduces the matter wave grating, leading to less scattering of photons in the cavity. In the next section, we consider the same system with quantized motion of the movable mirror.

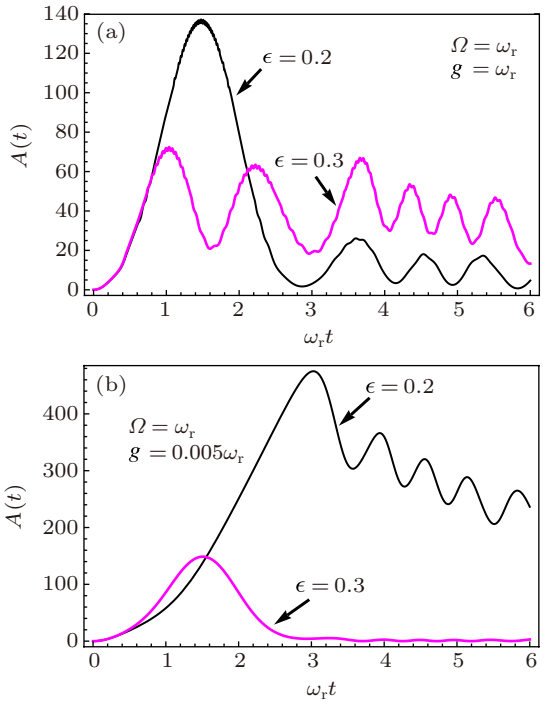


Fig. 2. (color online) Time signal of the scattered light power ($A(t) = \langle a^{\dagger} a \rangle$) with classical motion of the mirror using time modulated cavity frequency at resonant frequency ($\Omega = \omega_r$) for two modulation amplitudes $\epsilon = 0.2$ (thin line) and $\epsilon = 0.3$ (thick line). Parameters used are $n = 0$, $\gamma = 0.1\omega_r$, $\Delta_c = 5\omega_r$, $\omega_c = 28.8\omega_r$, $\eta = 10\omega_r$, and $\kappa = 0.1\omega_r$. Panel (a) for weak modulation ($g = \omega_r$) and panel (b) for strong modulation ($g = 0.005\omega_r$).

3. CARL-BEC model with quantized mirror motion

In this section, we consider the similar system with BEC confined in an optomechanical cavity. In addition, here we consider the quantized motion of the movable mirror of the cavity. The movable mirror is treated as a harmonic oscillator which oscillates with frequency ω_m and has mass m. Hence, once again, the cavity frequency is harmonically modulated due to the rapid motion of the mirror. A force proportional to the photon number acts on the movable mirror. As a result of this, various couplings of the system also modulate sinusoidally with time. The Hamiltonian with quantized motion of the mirror under rotating-wave and dipole approximation is given as

$$H_{II} = \hbar\omega_c a^{\dagger} a + \int \mathrm{d}z \left\{ \psi_g^{\dagger}(z) \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} \right) \psi_g(z) \right.$$

$$+\psi_{\rm e}^{\dagger}(z)\Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+\hbar\omega_{\rm a}\Big)\psi_{\rm e}(z)\Big\}$$
$$+\mathrm{i}\hbar g_0 a^{\dagger}\Big\{\int \mathrm{d}z\psi_{\rm g}^{\dagger}(z)\Big(\mathrm{e}^{-\mathrm{i}kz}+\mathrm{e}^{\mathrm{i}kz}\Big)\psi_{\rm e}(z)\Big\}$$
$$-\mathrm{i}\hbar g_0 a\Big\{\int \mathrm{d}z\psi_{\rm e}^{\dagger}(z)\Big(\mathrm{e}^{\mathrm{i}kz}+\mathrm{e}^{-\mathrm{i}kz}\Big)\psi_{\rm g}(z)\Big\}$$
$$-\mathrm{i}\hbar\eta'(a-a^{\dagger})+\mathrm{i}\hbar\chi(t)(a^{\dagger 2}\mathrm{e}^{-2\mathrm{i}\omega_{\rm p}t}-a^2\mathrm{e}^{2\mathrm{i}\omega_{\rm p}t})(b+b^{\dagger})$$
$$+\hbar\omega_{\rm m}b^{\dagger}b+\hbar k_0(t)a^{\dagger}a(b+b^{\dagger})$$
$$+\mathrm{i}\hbar G(t)a^{\dagger}(b+b^{\dagger})\Big\{\int \mathrm{d}z\psi_{\rm g}^{\dagger}(z)(\mathrm{e}^{-\mathrm{i}kz}+\mathrm{e}^{\mathrm{i}kz})\psi_{\rm e}(z)\Big\}$$
$$-\mathrm{i}\hbar G(t)a(b+b^{\dagger})\Big\{\int \mathrm{d}z\psi_{\rm e}^{\dagger}(z)(\mathrm{e}^{\mathrm{i}kz}+\mathrm{e}^{-\mathrm{i}kz})\psi_{\rm g}(z)\Big\}. \quad (26)$$

The above Hamiltonian is derived in Appendix A. In addition to the terms in Hamiltonian ($H_{\rm I}$), there are some extra terms in the Hamiltonian ($H_{\rm II}$) as the quantized motion of the mirror is considered here. Here the squeezing term (sixth term in Eq. (26)) is modified and is dependent on the coupling between the intracavity photons and position of the movable mirror. The annhilation (creation) operator of the mechanical mode is given by $b\,(b^{\dagger})$. The seventh term gives the free energy of the mechanical oscillator. The time-dependent coupling between the intensity of cavity light field and mechanical mode of the mirror is represented by the eighth term in Eq. (26) where $k_0(t)$ is the corresponding time-dependent coupling parameter which is defined as $k_0(t)=\omega_{\rm c}\epsilon\sin(\Omega t)$. The ninth term describes the three-body time-dependent interaction between the condensate mode, counter propagating cavity optical mode, and mechanical mode of the movable mirror. The last term in $H_{\rm II}$ is the hyper-conjugate of the ninth term. Here $G(t)$ is the time-dependent three-body coupling parameter which is given as $G(t)=g_0\epsilon\sin(\Omega t)/2$. Again, we eliminate the excited state of condensate atoms. Therefore, the Heisenberg equations of motion of the system for all the degrees of the system become

$$\dot{\psi}_{\rm g}=\frac{\mathrm{i}\hbar}{2m}\frac{\partial^2\psi_{\rm g}}{\partial z^2}-\frac{2\mathrm{i}g_0a^{\dagger}a}{\Delta}(g_0+G(t)(b+b^{\dagger}))\times(1+\cos(2kz))\psi_{\rm g}$$
$$-\frac{2\mathrm{i}G(t)a^{\dagger}a}{\Delta}(b+b^{\dagger})(g_0+G(t)(b+b^{\dagger}))\times(1+\cos(2kz))\psi_{\rm g}, \quad (27)$$

$$\dot{a}=\mathrm{i}\Delta_{\rm c}a-\mathrm{i}k_0(t)a(b+b^{\dagger})-\frac{2\mathrm{i}g_0aN}{\Delta}(g_0+G(t)(b+b^{\dagger}))$$
$$-\frac{2\mathrm{i}g_0a}{\Delta}(g_0+G(t)(b+b^{\dagger}))\int \mathrm{d}z\psi_{\rm g}^{\dagger}\cos(2kz)\psi_{\rm g}$$
$$+2\chi(t)a^{\dagger}(b+b^{\dagger})+\eta$$
$$-\frac{2\mathrm{i}G(t)aN}{\Delta}(b+b^{\dagger})(g_0+G(t)(b+b^{\dagger}))$$
$$-\frac{2\mathrm{i}G(t)a}{\Delta}(b+b^{\dagger})(g_0+G(t)(b+b^{\dagger}))\times\int \mathrm{d}z\psi_{\rm g}^{\dagger}\cos(2kz)\psi_{\rm g}, \quad (28)$$

$$\dot{b}=-\mathrm{i}\omega_{\rm m}b-\mathrm{i}k_0(t)a^{\dagger}a-\frac{2\mathrm{i}G(t)(1+2a^{\dagger}a)N}{\Delta}\times(g_0+G(t)(b+b^{\dagger}))$$
$$-\frac{2\mathrm{i}G(t)(1+2a^{\dagger}a)}{\Delta}(g_0+G(t)(b+b^{\dagger}))\times\int \mathrm{d}z\psi_{\rm g}^{\dagger}\cos(2kz)\psi_{\rm g}+\chi(t)(a^{\dagger 2}-a^2). \quad (29)$$

In Eq. (28), the second term represents the radiation pressure coupling of the cavity optical mode with the mirror displacement. As in the previous section, here we also use the ground state wavefunction of the condensate atom given by Eq. (14). Therefore, by considering all the dissipation processes of the system we get the following set of reduced differential equations of motion

$$\dot{C}_n=-4\mathrm{i}\omega_{\rm r}n^2C_n-\frac{\mathrm{i}NA}{\Delta}(g_0+G(t)x_{\rm m})^2(2C_n+C_{n+1}), \quad (30)$$

$$\dot{C}_{n+1}=-4\mathrm{i}\omega_{\rm r}(n+1)^2C_{n+1}-\frac{\mathrm{i}NA}{\Delta}(g_0+G(t)x_{\rm m})^2(2C_{n+1}+C_n), \quad (31)$$

$$\dot{a}'=(\mathrm{i}\Delta_{\rm c}-\mathrm{i}k_0(t)x_{\rm m})a'-\frac{2\mathrm{i}a'N}{\Delta}(g_0+G(t)x_{\rm m})^2-\frac{2\mathrm{i}a'N}{\Delta}(g_0+G(t)x_{\rm m})^2C_{n+1}^{*}C_n+\eta+2\chi(t)a'^{\dagger}x_{\rm m}-\kappa a', \quad (32)$$

$$\dot{x}_{\rm m}=\omega_{\rm m}p_{\rm m}, \quad (33)$$

$$\dot{p}_{\rm m}=-\omega_{\rm m}x_{\rm m}-2k_0(t)NA-2\mathrm{i}\chi(t)N(a'^{\dagger 2}-a'^2)-\gamma_{\rm m}p_{\rm m}-\frac{4G(t)N^2}{\Delta}(1+2A)(g_0+G(t)x_{\rm m})\times(1+C_n^{*}C_{n+1}+C_{n+1}^{*}C_n), \quad (34)$$

where $\gamma_{\rm m}$ is the dissipation rate of the mechanical mode, $x_{\rm m}=(b+b^{\dagger})$ is the position of the movable mirror and $p_{\rm m}=\mathrm{i}(b^{\dagger}-b)$ is the momentum of the movable mirror. All of the other assumptions are the same as in the previous case.

In Eq. (32), the first term on the right-hand side of the equality gives the renormalized cavity detuning. So, the cavity detuning is altering with time due to $k_0(t)$. One should note that when $k_0(t)=0$ (stationary mirror) and $\Delta_{\rm c}=0$ gives the Bragg condition of the scattering. This condition arises due to the energy and momentum conservation. We also noticed from Eq. (32) that the Bragg resonances are affected by dynamical dispersive effect of the mirror motion, which is proportional on the mirror displacement $x_{\rm m}$. In the linear regime, when $x_{\rm m}=x_0$ (some initial value) and a is still small, the Bragg condition becomes $\Delta_{\rm c}=k_0(t)$. As defined earlier, the equations of motion of the coherences and other equations in terms of coherences are given as

$$\dot{S}=4\mathrm{i}\omega_{\rm r}(1+2n)S+\frac{\mathrm{i}NA}{\Delta}(g_0+G(t)x_{\rm m})^2W-\gamma S, \quad (35)$$

$$\dot{W}=\frac{2\mathrm{i}NA}{\Delta}(g_0+G(t)x_{\rm m})^2(S-S^{*}), \quad (36)$$

$$\dot{a}' = \mathrm{i}\Delta_{\mathrm{c}} a' - \mathrm{i}k_0(t)a' x_{\mathrm{m}} - \frac{2\mathrm{i}a'N}{\Delta}(g_0 + G(t)x_{\mathrm{m}})^2 - \frac{2\mathrm{i}a'N}{\Delta}(g_0 + G(t)x_{\mathrm{m}})^2 S + \eta + 2\chi(t)a'^{\dagger} x_{\mathrm{m}} - \kappa a', \quad (37)$$

$$\dot{x}_{\mathrm{m}} = \omega_{\mathrm{m}} p_{\mathrm{m}}, \quad (38)$$

$$\dot{p}_{\mathrm{m}} = -\omega_{\mathrm{m}} x_{\mathrm{m}} - 2k_0(t)NA - 2\mathrm{i}\chi(t)N\left(a'^{\dagger 2} - a'^2\right) - \gamma_{\mathrm{m}} p_{\mathrm{m}} - \frac{4G(t)N^2}{\Delta}(1+2A)(g_0 + G(t)x_{\mathrm{m}})(1 + S + S^*). \quad (39)$$

Now again separating the above set of differential equations into real and imaginary parts, we obtain the following equations

$$\dot{S}_{\mathrm{r}} = -4\omega_{\mathrm{r}}(1+2n)S_{\mathrm{i}} - \gamma S_{\mathrm{r}}, \quad (40)$$

$$\dot{S}_{\mathrm{i}} = 4\omega_{\mathrm{r}}(1+2n)S_{\mathrm{r}} + \frac{NA}{\Delta}(g_0 + G(t)x_{\mathrm{m}})^2 W - \gamma S_{\mathrm{i}}, \quad (41)$$

$$\dot{W} = -\frac{4NA}{\Delta}(g_0 + G(t)x_{\mathrm{m}})^2 S_{\mathrm{i}}, \quad (42)$$

$$\dot{a}_{\mathrm{r}} = -\Delta_{\mathrm{c}} a_{\mathrm{i}} + k_0(t)a_{\mathrm{i}} x_{\mathrm{m}} + \frac{2a_{\mathrm{i}}N}{\Delta}(g_0 + G(t)x_{\mathrm{m}})^2 + \frac{2N}{\Delta}(g_0 + G(t)x_{\mathrm{m}})^2(a_{\mathrm{r}}S_{\mathrm{i}} + a_{\mathrm{i}}S_{\mathrm{r}}) + \eta + 2\chi(t)a_{\mathrm{r}}x_{\mathrm{m}} - \kappa a_{\mathrm{r}}, \quad (43)$$

$$\dot{a}_{\mathrm{i}} = \Delta_{\mathrm{c}} a_{\mathrm{r}} - k_0(t)a_{\mathrm{r}} x_{\mathrm{m}} - \frac{2a_{\mathrm{r}}N}{\Delta}(g_0 + G(t)x_{\mathrm{m}})^2 - \frac{2N}{\Delta}(g_0 + G(t)x_{\mathrm{m}})^2(a_{\mathrm{r}}S_{\mathrm{r}} - a_{\mathrm{i}}S_{\mathrm{i}}) - 2\chi(t)a_{\mathrm{i}}x_{\mathrm{m}} - \kappa a_{\mathrm{i}}, \quad (44)$$

$$\dot{x}_{\mathrm{m}} = \omega_{\mathrm{m}} p_{\mathrm{m}}, \quad (45)$$

$$\dot{p}_{\mathrm{m}} = -\omega_{\mathrm{m}} x_{\mathrm{m}} - 2k_0(t)NA - 2\mathrm{i}\chi(t)N(a_{\mathrm{r}}a_{\mathrm{i}} + a_{\mathrm{i}}a_{\mathrm{r}}) - \gamma_{\mathrm{m}} p_{\mathrm{m}} - \frac{4G(t)N^2}{\Delta}(1+2A)(g_0 + G(t)x_{\mathrm{m}})(1+2S_{\mathrm{r}}). \quad (46)$$

Now we renormalize $x_{\mathrm{m}} \to Nx_{\mathrm{m}}$ and $p_{\mathrm{m}} \to Np_{\mathrm{m}}$ and solve the above coupled differential equations of motion using Mathematica 9.0. Again, we investigate the system under the same quantum superradiant regime. Also, the system is studied under the same limits as in the previous section. Apart from the observations noticed in the previous section, here we note some additional features as described below.

Figures 3(a) and 3(b) show the time signal of the scattered light power ($A(t)$) and figures 3(c) and 3(d) show the corresponding position dynamics of the movable mirror ($x_{\mathrm{m}}(t)$) versus scaled time ($\omega_{\mathrm{r}}t$) using time-modulated cavity frequency at resonant modulating frequency ($\Omega = \omega_{\mathrm{r}}$) for two modulation amplitudes $\epsilon = 0.2$ (thin line) and $\epsilon = 0.3$ (thick line). The plots demonstrate an initial rise in the amplitude of scattered light (see Figs. 3(a) and 3(b)) while an initial dip in the position dynamics of the mirror (see Figs. 3(c) and 3(d)). As time passes, the oscillations of the scattered light is damped whereas the corresponding oscillations of the position dynamics of the movable mirror is enhanced. This clearly shows the exchange of energy between different degrees of freedom of the system. Also, as observed with the classical motion of the mirror, here the amplitude of the number of scattered photons is initially more for lower modulation amplitude as compared to higher modulation amplitude and the amplitude of the corresponding mirror displacement is less for lower modulation amplitude as compared to higher modulation amplitude.

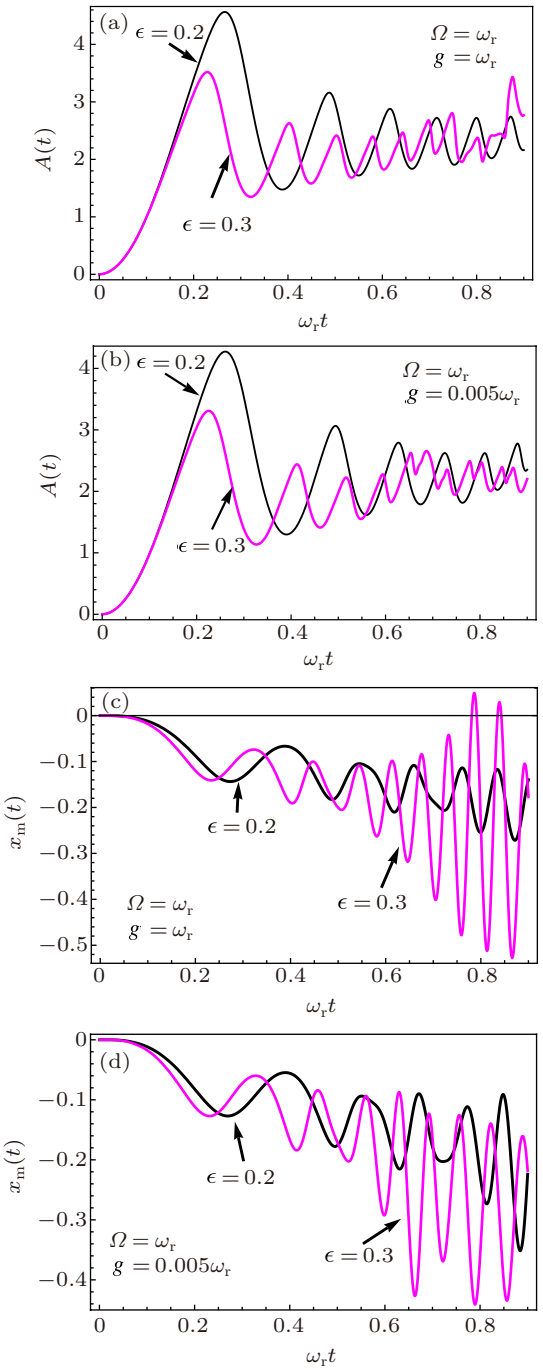


Fig. 3. (color online) Plots (a) and (b) show time signal of the scattered light power ($A(t) = \langle a^{\dagger}a \rangle$) and plots (c) and (d) show the corresponding position dynamics of the movable mirror ($x_{\mathrm{m}}(t)$) versus scaled time ($\omega_{\mathrm{r}}t$) with quantized motion of the mirror using time-modulated cavity frequency at resonant frequency ($\Omega = \omega_{\mathrm{r}}$) for two modulation amplitudes $\epsilon = 0.2$ (thin line) and $\epsilon = 0.3$ (thick line). Parameters used are $n = 0$, $\gamma = 0.1\omega_{\mathrm{r}}$, $\Delta_{\mathrm{c}} = 5\omega_{\mathrm{r}}$, $\omega_{\mathrm{c}} = 28.8\omega_{\mathrm{r}}$, $\eta = 10\omega_{\mathrm{r}}$, $\kappa = 0.1\omega_{\mathrm{r}}$, $\omega_{\mathrm{m}} = 100\omega_{\mathrm{r}}$, $N = 100$, and $\gamma_{\mathrm{m}} = \omega_{\mathrm{r}}$. Plots (a) and (c) for weak modulation ($g = \omega_{\mathrm{r}}$) and plots (b) and (d) for strong modulation ($g = 0.005\omega_{\mathrm{r}}$).

As compared to the case of classical mirror motion (see Fig. 2), here the amplitude of the number of scattered photons is less because here there is an additional degree of freedom (movable mirror). So, the exchange of energy takes place with an extra degree of freedom which decreases the scattered photons in the cavity. Also, in the present case, there is not much difference in the plots of number of scattered photons under the strong and weak modulations in contrast to the case of classical motion of the mirror. This means that the effect of modulating the cavity frequency does not much alter the number of scattered photons in the cavity by considering the quantized mirror motion.

Figure 4 shows the influence of cavity detuning on the time signal of the scattered light power $(A(t))$ and the corresponding position dynamics of the movable mirror at resonant modulating frequency $(\Omega = \omega_r)$ for two modulation amplitudes $\epsilon = 0.2$ (thin line) and $\epsilon = 0.3$ (thick line) with $\Delta_c = \omega_m$. Panels 4(a) and 4(b) illustrate amplification in the scattered light. This is due to the DCE which produces many photons inside the cavity from the vacuum state, leading to enhancement in the number of scattered photons in the cavity. Panels 4(c) and 4(d) show the corresponding suppression in the position dynamics of the movable mirror. This variation is again due to the balance of energy between different degrees of freedom. Figure 5 illustrates the same influence of cavity detuning as in Fig. 4 with $\Delta_c = -\omega_m$. Not much amplification is observed in the scattered light power. In both the Figs. 4 and 5, we observe that at a particular Δ_c, the plots for the scattered number of photons and the corresponding mirror displacement are similar; that is, they are not affected by the strong or weak modulation. Also, the amplitude of the scattered number of photons inside the cavity is higher for $\Delta_c = \omega_m$ than for $\Delta_c = -\omega_m$.

Now, we will describe the experimentally realizable parameters used in our calculations. A BEC consisting of 10^5 ^{87}Rb atoms[64] coupled to an optical field of an ultrahigh finesse Fabry–Perot cavity has the strength of the coherent coupling as $g_0 = 2\pi \times 10.9$ MHz,[65] ($2\pi \times 14.4$ MHz[31]). The kinetic and potential energy of the atoms is about $\nu = 35$ kHz[30] ($\nu = 49$ kHz[31]). Also, the coherent amplification or the damping of atomic motion is ignored as the condensate temperature $T_c \ll \hbar\gamma_c/k_B$. The detuning of the atom pump is $2\pi \times 32$ GHz. The atom field interaction is decreased due to decrease in the energy of the cavity mode because of the loss of photons through the cavity mirrors. By using high quality factor cavities, such loss of photons can be minimized. The cavity field can have damping rate $\kappa = 2\pi \times 8.75$ kHz[65] ($2\pi \times 0.66$ MHz[31]). The mechanical frequency of the movable mirror in an optomechanical system can be altered from $2\pi \times 100$ Hz,[66] $2\pi \times 10$ kHz,[67] to $2\pi \times 73.5$ MHz.[68] The corresponding decay rate of the movable mirror can thus be varied from $2\pi \times 10^{-3}$ Hz,[66] $2\pi \times 3.22$ Hz,[67] to $2\pi \times 1.3$ kHz.[68]

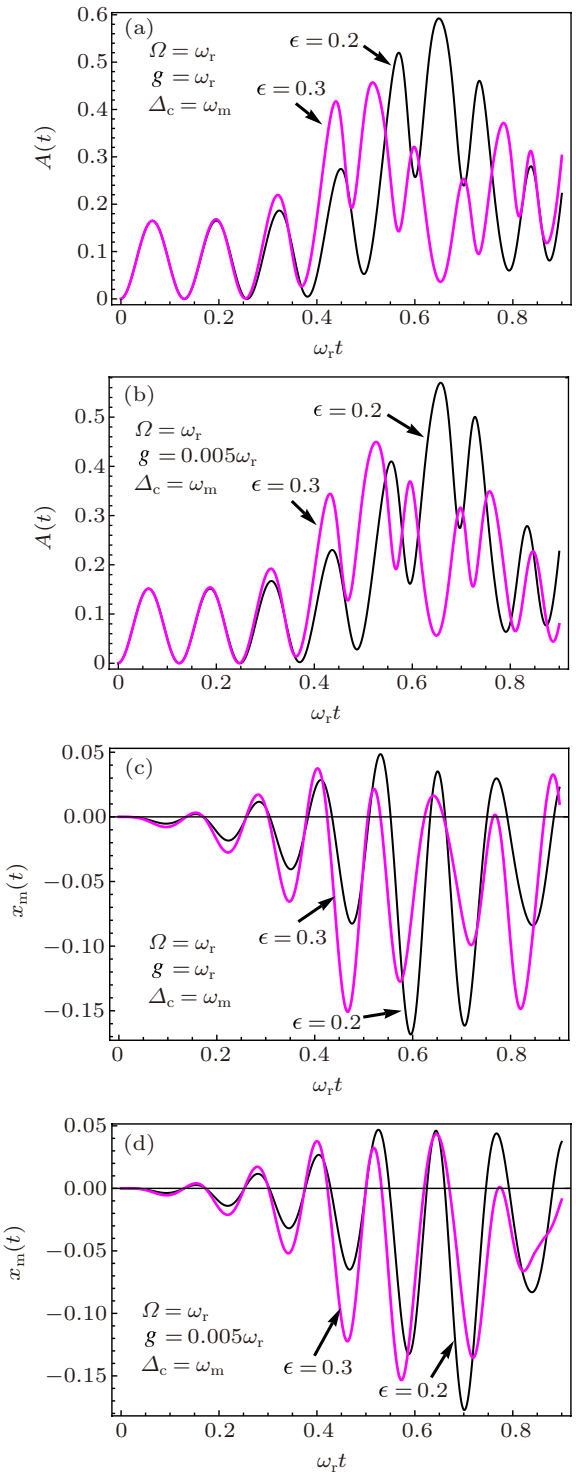


Fig. 4. (color online) Plots (a) and (b) show the influence of cavity detuning on the time signal of the scattered light power $(A(t) = \langle a^\dagger a \rangle)$ and plots (c) and (d) show the corresponding position dynamics of the movable mirror $(x_m(t))$ using the quantized motion of the movable mirror with time-modulated cavity frequency at resonant frequency $(\Omega = \omega_r)$ for two modulation amplitudes $\epsilon = 0.2$ (thin line) and $\epsilon = 0.3$ (thick line) with $\Delta_c = \omega_m$ where $\omega_m = 50\omega_r$. Plots (a) and (c) show the variation of $A(t)$ and $x_m(t)$ respectively under weak modulation $(g = \omega_r)$. Plots (b) and (d) show the same variation under strong modulation $(g = 0.005\omega_r)$. Other parameters used are same as those in Fig. 3.

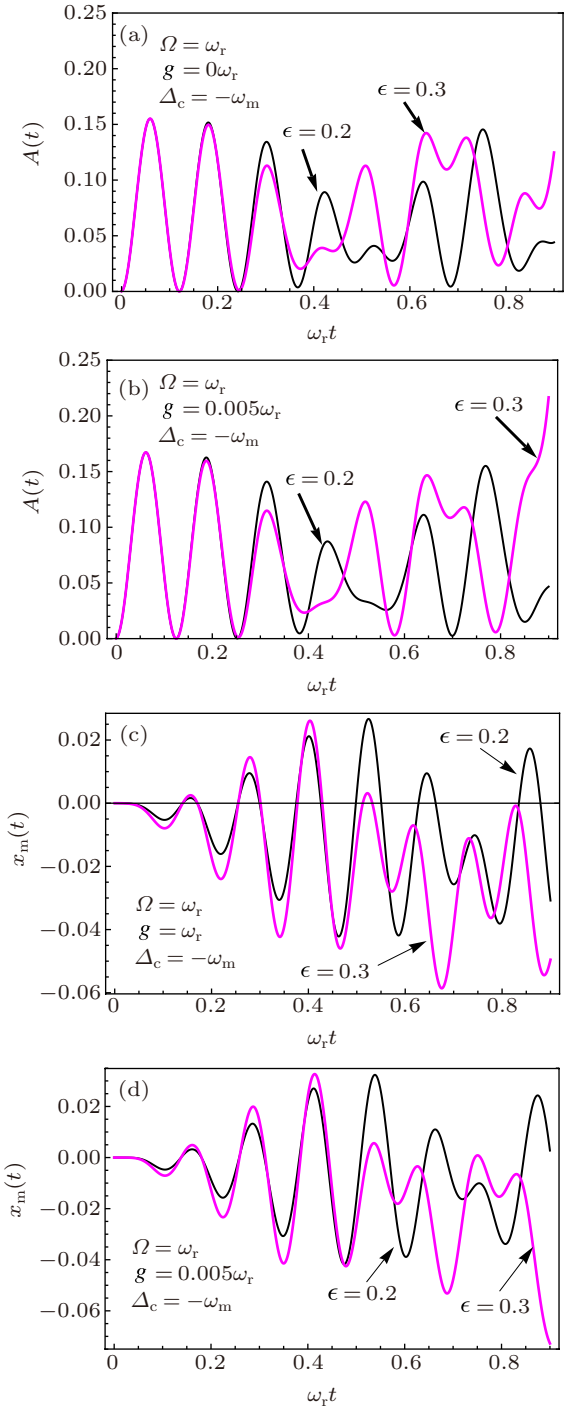


Fig. 5. (color online) Plots (a) and (b) show the influence of cavity detuning on the time signal of the scattered light power $(A(t) = \langle a^{\dagger}a \rangle)$ and plots (c) and (d) show the corresponding position dynamics of the movable mirror $(x_m(t))$ using the quantized motion of the movable mirror with time-modulated cavity frequency at resonant frequency $(\Omega = \omega_r)$ for two modulation amplitudes $\epsilon = 0.2$ (thin line) and $\epsilon = 0.3$ (thick line) with $\Delta_c = -\omega_m$. Plots (a) and (c) show the variation of $A(t)$ and $x_m(t)$ respectively under weak modulation $(g = \omega_r)$. Plots (b) and (d) show the same variation under strong modulation $(g = 0.005\omega_r)$. Other parameters used are same as those in Fig. 3.

4. Conclusion

In conclusion, we have analyzed how the superradiant light scattering from an elongated BEC in an optomechanical cavity in the quantum regime is altered in the presence of the DCE by considering the classical and quantized motion of the movable mirror. The cavity frequency is sinusoidally modulated with time in both mirror motions. The scattered light spectrum shows Bragg oscillations, modified by the dynamical Casimir effect, superimposed on the usual superradiant spectrum. The mirror's motion has a dispersive effect, leading to a higher amplitude of scattered photon for lower modulation amplitude at initial times. Also, the amplitude of the scattered photon number is much higher for classical mirror motion than the quantized mirror motion. The quantized motion of the movable mirror leads to similar behavior of the spectrum of scattered light for both weak and strong modulations. The DCE is found to be a new handle to coherently control and manipulate the superradiant light scattering process.

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Appendix A

In this appendix, we derive the time-dependent Hamiltonian for the system by considering the quantized motion of the movable mirror. As one knows, the cavity frequency is given as[49]

$$\omega_c = \frac{C_1}{L}, \tag{A1}$$

where C_1 is constant and L is the length of the cavity. Therefore, change in the cavity length modifies the cavity frequency. Hence, the time varying cavity frequency is given as

$$\omega_c(x,t) = \frac{C_1}{L - x(t)}. \tag{A2}$$

Since the perturbation in the cavity length is very small in comparison with its original length. Therefore, the time varying cavity frequency becomes

$$\omega_c(x,t) = \omega_c\left(1 + \frac{x(t)}{L}\right). \tag{A3}$$

Now one can take the time-modulated perturbation in cavity length as

$$x(t) = x'\epsilon' \sin(\Omega t). \tag{A4}$$

where $x' = (\Delta x)(b + b^{\dagger})$, x' is the quantized perturbation in cavity length, b is the annhilation operator of movable mirror, $(b + b^{\dagger})$ is the dimensionless position operator of the movable mirror, ϵ' is the modulation amplitude, and Ω is the modulation frequency. Using the time-modulated perturbation of

cavity length in Eq. (A3), we get

$$\omega_{\rm c}(x,t)=\omega_{\rm c}\Big(1+\frac{(\Delta x)(b+b^{\dagger})\epsilon'\sin(\Omega t)}{L}\Big). \tag{A5}$$

Now we take the normalized modulation amplitude as $\epsilon=((\Delta x)\epsilon')/L$, therefore, the time varying cavity frequency becomes

$$\omega_{\rm c}(t)=\omega_{\rm c}(1+(b+b^{\dagger})\epsilon\sin(\Omega t)). \tag{A6}$$

The effective frequency also changes in this case as it is dependent on time varying cavity frequency (see Eq. 9). Therefore, the modified effective frequency for the system with the quantized mirror motion becomes

$$\chi'(t)=\frac{(\epsilon'\Omega\cos(\Omega t))}{4}=\chi(t)(b+b^{\dagger}). \tag{A7}$$

The coupling constant for the photon–exciton coupling is given as[49]

$$g_0=\frac{C_2}{\sqrt{L}}, \tag{A8}$$

where, C_2 is a constant. Now the time varying coupling parameter becomes

$$g_0(x,t)=\frac{C_2}{\sqrt{L-x(t)}}. \tag{A9}$$

Under the small cavity length perturbation, the time varying coupling parameter becomes

$$g_0(x,t)=\frac{C_2}{\sqrt{L}}\Big(1+\frac{x(t)}{2L}\Big). \tag{A10}$$

Again, using the small quantized perturbation of the cavity length, we get

$$g_0(x,t)=g_0\Big(1+\frac{(\Delta x)(b+b^{\dagger})\epsilon'\sin(\Omega t)}{2L}\Big). \tag{A11}$$

Therefore, the time varying coupling parameter becomes

$$g_0(t)=g_0\Big(1+\frac{(b+b^{\dagger})\epsilon\sin(\Omega t)}{2}\Big). \tag{A12}$$

Hence, the Hamitonian for the system with the quantized mirror motion under rotating-wave and dipole approximation can be written as

$$\begin{aligned}H_{\rm II}=&\hbar\omega_{\rm c}(t)a^{\dagger}a+\int{\rm d}z\Big\{\psi_{\rm g}^{\dagger}(z)\Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\Big)\psi_{\rm g}(z)\\&+\psi_{\rm e}^{\dagger}(z)\Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+\hbar\omega_{\rm a}\Big)\psi_{\rm e}(z)\Big\}\\&+{\rm i}\hbar g_0(t)a^{\dagger}\Big\{\int{\rm d}z\psi_{\rm g}^{\dagger}(z)\big({\rm e}^{-{\rm i}kz}+{\rm e}^{{\rm i}kz}\big)\psi_{\rm e}(z)\Big\}\\&-{\rm i}\hbar g_0(t)a\Big\{\int{\rm d}z\psi_{\rm e}^{\dagger}(z)\big({\rm e}^{{\rm i}kz}+{\rm e}^{-{\rm i}kz}\big)\psi_{\rm g}(z)\Big\}\\&-{\rm i}\hbar\eta'\big(a-a^{\dagger}\big)+{\rm i}\hbar\chi'(t)\big(a^{\dagger2}{\rm e}^{-2{\rm i}\omega_{\rm p}t}-a^2{\rm e}^{2{\rm i}\omega_{\rm p}t}\big)(b+b^{\dagger})\\&+\hbar\omega_{\rm m}b^{\dagger}b.\end{aligned} \tag{A13}$$

In the above Hamiltonian, the free energy of the movable mirror is represented by $\hbar\omega_{\rm m}b^{\dagger}b$ where $\omega_{\rm m}$ is the frequency. Now, substituting the time varying cavity frequency Eq. (A6), time varying effective frequency Eq. (A7) and time varying coupling parameter Eq. (A12) in the above Hamiltonian Eq. (A13), the Hamiltonian becomes

$$\begin{aligned}H_{\rm II}=&\hbar\omega_{\rm c}a^{\dagger}a+\int{\rm d}z\Big\{\psi_{\rm g}^{\dagger}(z)\Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\Big)\psi_{\rm g}(z)\\&+\psi_{\rm e}^{\dagger}(z)\Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+\hbar\omega_{\rm a}\Big)\psi_{\rm e}(z)\Big\}\\&+{\rm i}\hbar g_0a^{\dagger}\Big\{\int{\rm d}z\psi_{\rm g}^{\dagger}(z)\big({\rm e}^{-{\rm i}kz}+{\rm e}^{{\rm i}kz}\big)\psi_{\rm e}(z)\Big\}\\&-{\rm i}\hbar g_0a\Big\{\int{\rm d}z\psi_{\rm e}^{\dagger}(z)\big({\rm e}^{{\rm i}kz}+{\rm e}^{-{\rm i}kz}\big)\psi_{\rm g}(z)\Big\}\\&-{\rm i}\hbar\eta'(a-a^{\dagger})+{\rm i}\hbar\chi(t)\big(a^{\dagger2}{\rm e}^{-2{\rm i}\omega_{\rm p}t}-a^2{\rm e}^{2{\rm i}\omega_{\rm p}t}\big)(b+b^{\dagger})\\&+\hbar\omega_{\rm m}b^{\dagger}b+\hbar k_0(t)a^{\dagger}a(b+b^{\dagger})\\&+{\rm i}\hbar G(t)a^{\dagger}(b+b^{\dagger})\Big\{\int{\rm d}z\psi_{\rm g}^{\dagger}(z)\big({\rm e}^{-{\rm i}kz}+{\rm e}^{{\rm i}kz}\big)\psi_{\rm e}(z)\Big\}\\&-{\rm i}\hbar G(t)a(b+b^{\dagger})\Big\{\int{\rm d}z\psi_{\rm e}^{\dagger}(z)\big({\rm e}^{{\rm i}kz}+{\rm e}^{-{\rm i}kz}\big)\psi_{\rm g}(z)\Big\}.\end{aligned} \tag{A14}$$

where $k_0(t)=\omega_{\rm c}\epsilon\sin(\Omega t)$ and $G(t)=g_0\epsilon\sin(\Omega t)/2$ are the time-dependent coupling parameters.

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