Electronic cluster state entanglement concentration based on charge detection*

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We propose an efficient entanglement concentration protocol (ECP) based on electron-spin cluster states assisted with single electrons. In the ECP, we adopt the electron polarization beam splitter (PBS) and the charge detector to construct the quantum nondemolition measurement. According to the result of the measurement of the charge detection, we can ultimately obtain the maximally entangled cluster states. Moreover, the discarded items can be reused in the next round to reach a high success probability. This ECP may be useful in current solid quantum computation.

Keywords: entanglement concentration, quantum computation, cluster states

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1. Introduction

Quantum entanglement is very important in both quantum communication and quantum computation. Quantum teleportation,[1–3] quantum secure direct communication,[4–10] quantum key distribution (QKD),[11–13] and other protocols in quantum communication all need entanglement.[14–18] Many splendid models in quantum computation, such as the standard circuit model, one-way quantum computation model, and so on,[19–24] are all based on entanglement. The one-way quantum computation model was first introduced by Briegel and Raussndorf.[19] It is shown that a one-way quantum computation can be built by the use of cluster states, and all the operations can be implemented with single-qubit measurement. Taking the features of the cluster states into consideration, many schemes for the realization of cluster states were presented. For example, in 2007, Chen *et al.* reported the experiment realization of two-photon four-qubit cluster states.[25] In 2008, Tokunaga *et al.* also reported a simple scheme for generating a four-photon entangled cluster states with fidelity over 0.860 ± 0.015.[26] There are a lot of theoretical works about cluster states, such as: the optical quantum computation using cluster states,[27] generation of atomic cluster states through the cavity input-output process,[28] reduce, reuse, recycle for robust cluster-state generation in hybrid optical system,[29] and so on.[30,31]

The conduction electron has become a good candidate for the realization of quantum computation, since Beenakker *et al.*[32] broke through the obstacle of the no-go theorem[33] in 2004. An electron system has both spin degree of freedom and charge degree of freedom. In addition, the spin of the electron and the charge of the electron can be operated independently; that is to say, the spin of the electron will be unaffected if one measures the charge of the electron quantum system by charge detector. With the charge detector,[34] the case of one electron can be distinguished from the case 0 or 2 electrons, but the cases 0 and 2 cannot be distinguished. Many works based on these electrons have been done, such as entanglement purification,[35–37] entanglement concentration,[38–40] and the generation of the cluster states.[41,42] Especially, the group of Chuang discussed the measurement-based quantum computation scheme from spin system to fermion systems.[43]

However, similar to the two-particle Bell state and the three-particle W state, the multipartite cluster states will also inevitably have interaction influences with environment, which will degrade the quality of the entanglement. The maximally entangled state will be degraded to a mixed entangled state or a pure less-entangled state. Entanglement concentration provides an effective way to recover the pure less-entangled state to the maximally entangled state with local operation and classical communication.[44] Bennett *et al.* proposed the first entanglement concentration protocol (ECP), which is known as the Schmidt projection method, in 1996.[44] The disadvantage of this protocol is that some measurements are difficult to operate in experiments. Later, ECPs based on

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entanglement swapping were proposed.[45,46] During the past few years, various ECPs have been proposed, such as ECPs based on linear optics,[47,48] the cross-Kerr nonlinearity,[49–62] quantum dot, and optical cavities systems,[63–66] atoms,[67] and so on.[68–72]

Inspired by the previous works of the cluster states and the protocols for entanglement concentration, we will propose an efficient ECP for the cluster states for electrons. Compared with the conventional ECPs, this has several advantages. First, we only use one pair of less-entangled cluster states and a single electron to complete the task, while most ECPs require two pairs of less-entangled states. Second, with the help of the quantum nondemolition detector constructed by the electronic PBS and charge detector, the concentrated entangled state can remain for further application, while it is always destroyed in linear optics because of the post-selection principle. Moreover, in this ECP, the discussed items can be reused to reach a high success probability. Therefore, this ECP is optimal. Since the solid electron is one of the good candidates for quantum computation, this ECP for cluster state may be useful in current one-way quantum computation. This paper is organized as follows. In Section 2, we will briefly explain our ECP with a simple example. In Section 3, we will redescribe this ECP and show that it can be repeated to obtain a high success probability. In Section 4, we generalize this ECP to the concentration of the K-qubit cluster states. In Section 5, we will present a discussion and summary.

2. Single-qubit assisted entanglement concentration for entangled electron-spin cluster states

Let us consider a four-qubit system of less-entangled electron-spin cluster states, which is shared originally by four parties, say Alice, Bob, Charlie, and Dick. The less-entangled state can be written as

$$|\varphi\rangle_{abcd} = \alpha(|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_d + |\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_d) + \beta(|\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_d - |\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_d), \quad (1)$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the spin-up state and spin-down state, respectively. Here α and β satisfy $2(|\alpha|^2+|\beta|^2)=1$. As shown in Fig. 1, the electrons owned by Alice, Bob, Charlie, and Dick are in the spatial modes a, b, c, and d, respectively. In the ECP, Dick first prepares an ancillary electron in the spatial mode e of the form

$$|\varphi\rangle_e = \frac{1}{\sqrt{\alpha^2+\beta^2}}(\beta|\uparrow\rangle_e + \alpha|\downarrow\rangle_e). \quad (2)$$

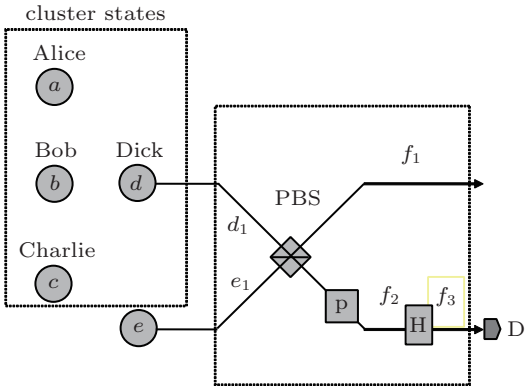


Fig. 1. (color online) The schematic diagram showing the principle of the proposed ECP for electron-spin cluster states with a single electron. Alice, Bob, Charlie, and Dick share the cluster states, and own the electron a, b, c, and d, respectively. The polarizing beam splitter (PBS), Hadamard operation, and charge detector denote as PBS, H, and P, respectively.

In this way, the initial nonmaximally entangled state of the five electrons can be written as

$$\begin{aligned}
&|\Psi\rangle_{abcde}\\
&= |\varphi\rangle_{abcd}\otimes|\varphi\rangle_e\\
&= \frac{1}{\sqrt{\alpha^2+\beta^2}}\{\alpha\beta[|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_d|\uparrow\rangle_e\\
&\quad + |\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_d|\uparrow\rangle_e\\
&\quad + |\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_d|\downarrow\rangle_e - |\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_d|\downarrow\rangle_e]\\
&\quad + \alpha^2[|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_d|\downarrow\rangle_e + |\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_d|\downarrow\rangle_e]\\
&\quad + \beta^2[|\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_d|\uparrow\rangle_e - |\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_d|\uparrow\rangle_e]\}. \quad (3)
\end{aligned}$$

Dick lets the two electrons in the modes d_1 and e_1 pass through the electronic PBS, where the $|\uparrow\rangle$ polarization electron will be fully transmitted and the $|\downarrow\rangle$ polarization electron will be fully reflected. The PBS can distinguish the states $|\uparrow\rangle|\uparrow\rangle$ and $|\downarrow\rangle|\downarrow\rangle$ from the states $|\uparrow\rangle|\downarrow\rangle$ and $|\downarrow\rangle|\uparrow\rangle$ according to different spatial modes. On the one hand, the states $|\uparrow\rangle_d|\uparrow\rangle_e$ and $|\downarrow\rangle_d|\downarrow\rangle_e$ will make both the spatial modes f_1 and f_2 contain one electron. On the other hand, the state $|\uparrow\rangle_d|\downarrow\rangle_e$ will make the spatial mode f_2 get two electrons and the state $|\downarrow\rangle_d|\uparrow\rangle_e$ will make the spatial mode f_1 get two electrons. Dick selects the items which make the charge detector detect exactly one electron. In this way, they can obtain the state

$$\begin{aligned}
|\varphi\rangle'_{abcf_1f_2} = \frac{1}{2}&(|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_1}|\uparrow\rangle_{f_2}\\
&+|\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_1}|\uparrow\rangle_{f_2}\\
&+|\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_1}|\downarrow\rangle_{f_2}\\
&-|\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_1}|\downarrow\rangle_{f_2}), \quad (4)
\end{aligned}$$

with the probability $P_1 = 8|\alpha\beta|^2$. It is easy to get a maximally entangled cluster states from the state described in Eq. (4). Dick first performs a Hadamard operation on the electron in the spatial mode f_2 — the Hadamard operation completes the

transformations

$$|\uparrow\rangle \longrightarrow (|\uparrow\rangle+|\downarrow\rangle)/\sqrt{2} \text{ and } |\downarrow\rangle \longrightarrow (|\uparrow\rangle-|\downarrow\rangle)/\sqrt{2}.$$

The state in Eq. (4) can be written as

$$\begin{aligned}|\varphi\rangle'_{abcf_1f_3} = \frac{1}{2}\Big[&|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_1}\frac{1}{\sqrt{2}}(|\uparrow\rangle_{f_3}+|\downarrow\rangle_{f_3})\\ &+|\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_1}\frac{1}{\sqrt{2}}(|\uparrow\rangle_{f_3}+|\downarrow\rangle_{f_3})\\ &+|\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_1}\frac{1}{\sqrt{2}}(|\uparrow\rangle_{f_3}-|\downarrow\rangle_{f_3})\\ &-|\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_1}\frac{1}{\sqrt{2}}(|\uparrow\rangle_{f_3}-|\downarrow\rangle_{f_3})\Big]. \quad (5)\end{aligned}$$

Finally, Dick measures the electron in spatial mode f_3. On the one hand, if the result of the measurement is $|\uparrow\rangle_{f_3}$, they will obtain the maximally entangled state, which can be written as

$$\begin{aligned}|\Phi\rangle_{abcf_1} = \frac{1}{2}&(|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_1}+|\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_1}\\ &+|\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_1}-|\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_1}). \quad (6)\end{aligned}$$

On the other hand, if the result of the measurement is $|\downarrow\rangle_{f_3}$, they will obtain

$$\begin{aligned}|\Phi\rangle'_{abcf_1} = \frac{1}{2}&(|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_1}+|\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_1}\\ &-|\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_1}+|\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_1}). \quad (7)\end{aligned}$$

Both equations (6) and (7) are maximally entangled cluster states. Equation (7) can be converted to Eq. (6) by the phase-flip operation. Therefore, they have completed the entanglement concentration.

3. Recycle concentration

In the above section, we have described the ECP with a simple example. One of the parties say Dick picks up the case that the charge detector (P) detects only one electron and discards the cases that it detects 0 or 2 electrons. Actually, if the result of the charge detector's measurement is 0 or 2, it can also be reused to obtain a higher success probability. In detail, as shown in Fig. 2, they first add another PBS, here named PBS2, to couple the $|\uparrow\rangle|\downarrow\rangle$ and $|\downarrow\rangle|\uparrow\rangle$ to ensure that each spatial modes f_3 and f_4 exactly contain one electron. The remained less-entangled state is

$$\begin{aligned}|\varphi_1\rangle''_{abcf_3f_4} = \frac{1}{\sqrt{2(\alpha^4+\beta^4)}}\{&\alpha^2[|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_3}|\downarrow\rangle_{f_4}\\ &+|\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_3}|\downarrow\rangle_{f_4}]\\ &+\beta^2[|\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_3}|\uparrow\rangle_{f_4}\\ &-|\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_3}|\uparrow\rangle_{f_4}]\}, \quad (8)\end{aligned}$$

with the probability $P'_1 = 4(|\alpha|^4+|\beta|^4)$.

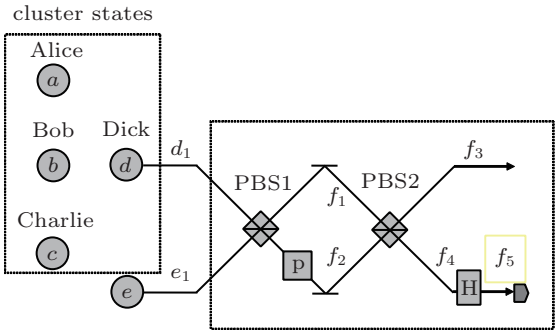


Fig. 2. (color online) The principle for reusing the discarded less-entanglement states. The PBS2 is used to combine to ensure the spatial modes f_3 and f_4 exactly contain one electron.

In Eq. (8), Dick first performs a Hadamard operation on electron in the spatial mode f_4, and finally measures it in $\{|\uparrow\rangle,|\downarrow\rangle\}$ basis. They will obtain

$$\begin{aligned}|\varphi_1\rangle_{abcf_3} = &\alpha'[|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_3}+|\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_3}]\\ &+\beta'[|\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_3}-|\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_3}]. \quad (9)\end{aligned}$$

if the result of the measurement is $|\uparrow\rangle_{f_5}$. Here

$$\alpha' = \frac{\alpha^2}{\sqrt{2(\alpha^4+\beta^4)}}, \quad \beta' = \frac{\beta^2}{\sqrt{2(\alpha^4+\beta^4)}}.$$

And they will obtain

$$\begin{aligned}|\varphi_1\rangle'_{abcf_3} = &-\alpha'[|\uparrow\rangle_a|\uparrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_3}+|\downarrow\rangle_a|\downarrow\rangle_b|\uparrow\rangle_c|\uparrow\rangle_{f_3}]\\ &+\beta'[|\uparrow\rangle_a|\uparrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_3}\\ &-|\downarrow\rangle_a|\downarrow\rangle_b|\downarrow\rangle_c|\downarrow\rangle_{f_3}], \quad (10)\end{aligned}$$

if the result is $|\downarrow\rangle_{f_5}$. One can perform a phase-flip operation to convert the state $|\varphi_1\rangle'_{abcf_3}$ to $|\varphi_1\rangle_{abcf_3}$. It is obvious that equation (9) has the same form with Eq. (1). Therefore, they can reuse the less-entanglement state shown in Eq. (9) by preparing another single electron state

$$|\varphi\rangle'_e = \frac{1}{\sqrt{\alpha'^2+\beta'^2}}(\beta'|\uparrow\rangle_e+\alpha'|\downarrow\rangle_e). \quad (11)$$

Following the same principle, they can restart the ECP to obtain the desired maximally entangled state. Certainly, if the ECP is still a failure in the second round, they can also repeat it in the third round. Therefore, by repeating this ECP, they can reach a high success probability. We can calculate the success probability in each round as

$$\begin{aligned}P_1 &= 8|\alpha|^2|\beta|^2,\\ P_2 &= 8\frac{|\alpha|^4|\beta|^4}{|\alpha|^4+|\beta|^4},\\ P_3 &= 8\frac{|\alpha|^8|\beta|^8}{(|\alpha|^4+|\beta|^4)(|\alpha|^8+|\beta|^8)},\\ &\cdots\\ P_n &= 8\frac{|\alpha|^{2^n}|\beta|^{2^n}}{(|\alpha|^4+|\beta|^4)(|\alpha|^8+|\beta|^8)\cdots(|\alpha|^{2^n}+|\beta|^{2^n})},\\ P_{\text{sum}} &= P_1+P_2+P_3+\cdots+P_n\end{aligned}$$

$$= 8\Big[|\alpha|^2|\beta|^2 + \frac{|\alpha|^4|\beta|^4}{|\alpha|^4+|\beta|^4} + \frac{|\alpha|^8|\beta|^8}{(|\alpha|^4+|\beta|^4)(|\alpha|^8+|\beta|^8)} + \cdots + \frac{|\alpha|^{2^n}|\beta|^{2^n}}{(|\alpha|^4+|\beta|^4)(|\alpha|^8+|\beta|^8)\cdots(|\alpha|^{2^n}+|\beta|^{2^n})}\Big]$$
$$= \sum_{n=1}^{n} \frac{4(|\alpha|^2|\beta|^2)^n}{\prod_{n=1}^{n}[(|\alpha|^2)^n+(|\beta|^2)^n]}. \quad (12)$$

The subscripts 1, 2, …, n represent the iteration number and the $P_{\rm sum}$ is the total success probability.

We show the total success probability P alters with the initial coefficient α in Fig. 3. Here, the coefficient $\alpha \in (0, \sqrt{2}/2)$. From Fig. 3, the P increases with the α when $\alpha \in (0, 1/2)$, and decreases with α when $\alpha \in [1/2, \sqrt{2}/2)$. It can reach the maximal value when $\alpha = 1/2$. From Eq. (12), it is shown that, if $\alpha = 1/2$, we can obtain $P = 1/2+1/4+1/8+\cdots = 1$.

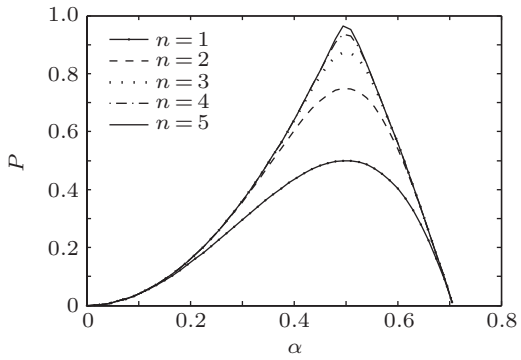


Fig. 3. The relationship between the coefficient α and the total success probability P. The variable n indicates the iteration number. We calculate the total success probability from $n = 1$ to $n = 5$. It is shown that P increases by repeating this ECP.

4. Single-qubit-assisted entanglement concentration of entangled electron-spin *K*-qubit cluster state

It is straightforward to generalize this ECP to the case for arbitrary K-qubit cluster state. The K-qubit cluster state of electrons is[73]

$$|\Phi_K\rangle_{123\cdots K} = \frac{1}{2^{K/2}} \otimes_{a=1}^{K} (|\uparrow\rangle_a \delta_z^{a+1} + |\downarrow\rangle_a), \quad (13)$$

where δ_z^{a+1} is a Pauli operator, with $\delta_z^{a+1}|\uparrow\rangle_{a+1} = |\uparrow\rangle_{a+1}$ and $\delta_z^{a+1}|\downarrow\rangle_{a+1} = -|\downarrow\rangle_{a+1}$, and $\delta_z^{K+1} \equiv 1$. Moreover, the minimum number of product terms in cluster states $|\Phi_K\rangle$ is $2^{\lfloor K/2 \rfloor}$. Here, $\lfloor K/2 \rfloor$ indicates the operation of rounding down to $K/2$.

From Eq. (13), the less-entangled K-qubit cluster state can be written as

$$|\varphi_K\rangle_{123\cdots K} = \alpha \otimes_{a=1}^{K-1} (|\uparrow\rangle_a \delta_z^{a+1} + |\downarrow\rangle_a)|\uparrow\rangle_K + \beta \otimes_{a=1}^{K-1} (|\uparrow\rangle_a \delta_z^{a+1} + |\downarrow\rangle_a)|\downarrow\rangle_K, \quad (14)$$

where $\otimes_{a=1}^{K-1}(|\uparrow\rangle_a \delta_z^{a+1} + |\downarrow\rangle_a)$ are states of $K-1$ electrons, which are owned by the other $K-1$ parties (not K), and the coefficients α and β satisfying with $2^{\lfloor K/2 \rfloor - 1}(|\alpha|^2+|\beta|^2) = 1$. With a similar principle, Dick will also introduce the ancillary electron of the form of

$$|\varphi\rangle_e = \frac{1}{\sqrt{\alpha^2+\beta^2}}(\beta|\uparrow\rangle_e + \alpha|\downarrow\rangle_e).$$

The whole system can be written as

$$\begin{aligned} &|\Psi\rangle_{123\cdots Ke} \\ &= |\varphi_K\rangle_{123\cdots K} \otimes |\varphi\rangle_e \\ &= \frac{1}{\sqrt{\alpha^2+\beta^2}}\{\alpha\beta[\otimes_{a=1}^{K-1}(|\uparrow\rangle_a \delta_z^{a+1} + |\downarrow\rangle_a)|\uparrow\rangle_K|\uparrow\rangle_e \\ &\quad + \otimes_{a=1}^{K-1}(|\uparrow\rangle_a \delta_z^{a+1} + |\downarrow\rangle_a)|\downarrow\rangle_K|\downarrow\rangle_e] \\ &\quad + \alpha^2 \otimes_{a=1}^{K-1}(|\uparrow\rangle_a \delta_z^{a+1} + |\downarrow\rangle_a)|\uparrow\rangle_K|\downarrow\rangle_e \\ &\quad + \beta^2 \otimes_{a=1}^{K-1}(|\uparrow\rangle_a \delta_z^{a+1} + |\downarrow\rangle_a)|\downarrow\rangle_K|\uparrow\rangle_e\}. \end{aligned} \quad (15)$$

Following the same principle, let the number K-th electron and the ancillary electron pass through the PBS. By picking up the cases that the charge detector only detects one electron, they will ultimately obtain the maximally entangled state. On the other hand, if the charge detector shows 0 or 2, they can also restart this ECP with another ancillary electron. Therefore, they can obtain a higher success probability similar with the four-electron cluster states.

For the multipartite cluster states, the total success probability can be written as

$$P_n^K = 2^{2\lfloor K/2 \rfloor - 1}\Big[|\alpha|^2|\beta|^2 + \frac{|\alpha|^4|\beta|^4}{|\alpha|^4+|\beta|^4} + \frac{|\alpha|^8|\beta|^8}{(|\alpha|^4+|\beta|^4)(|\alpha|^8+|\beta|^8)} + \cdots + \frac{|\alpha|^{2^n}|\beta|^{2^n}}{(|\alpha|^4+|\beta|^4)(|\alpha|^8+|\beta|^8)\cdots(|\alpha|^{2^n}+|\beta|^{2^n})}\Big]$$
$$= 2^{\lfloor K/2 \rfloor} \sum_{n=1}^{n} \frac{(|\alpha|^2|\beta|^2)^n}{\prod_{n=1}^{n}[(|\alpha|^2)^n+(|\beta|^2)^n]}. \quad (16)$$

Here, we choose the particle number being K. From Eq. (16), we can easily find $P_n^2 = P_n^3$, $P_n^4 = P_n^5$, …, $P_n^{2m} = P_n^{2m+1}$ ($m = 1, 2, 3, \ldots$). We can get the conclusion that for every even K, the success probability of K electrons cluster state is equal to the case of $K+1$ electrons in the condition of same iteration number n, respectively. Taking this conclusion into consideration, we will only discuss the case of an even particles cluster state. As shown in Fig. 4, we calculate the total success probability with $K = 4$, 6, 8, and 10. In each cluster state, we only perform the ECP for one time. In Fig. 5, we calculate the total success probability by repeating each ECP for n time, where $n = 1$, 2, 3, 4, and 5. It is shown that the total success probability increases rapidly.

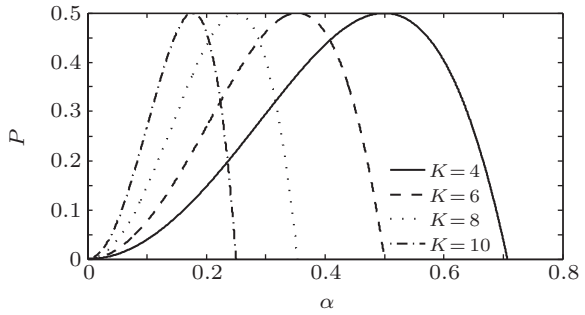


Fig. 4. The relationship between the coefficient α and the total success probability P for the K-qubit less-entangled cluster states. Here we let $K = 4$, 6, 8, and 10, and the ECP is performed for one time. We choose $\alpha \in (0, 1/\sqrt{2^{\lfloor K/2 \rfloor - 1}})$ in each case.

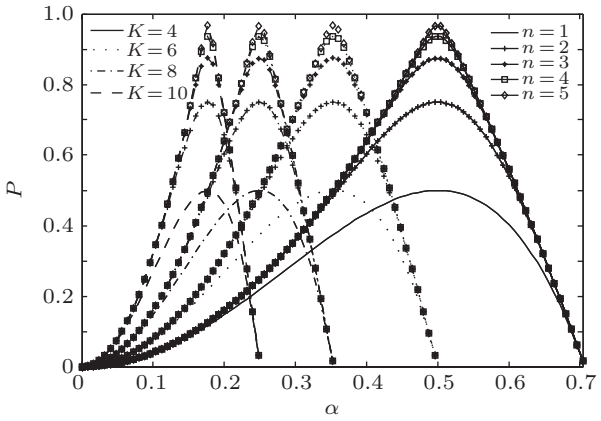


Fig. 5. The relationship between the coefficient α and the total success probability P for concentrating the K-qubit less-entangled cluster states. Here we let $K = 4$, 6, 8 and 10, and iteration number is $n = 1$, 2, 3, 4, 5. We choose $\alpha \in (0, 1/\sqrt{2^{\lfloor K/2 \rfloor - 1}})$ in each ECP.

5. Discussion and summary

We have briefly explained our ECP for less-entangled electron spin state. In contrast from the conventional ECP, only one pair of less-entangled state is required. During the whole ECP, we require the PBSs combined with the charge detector to detect the number of the electrons. Actually, they act the role of the parity check gate, and can distinguish the even parity state from the odd parity state according to the number of the electrons. On the other hand, the charge degree of freedom and the polarization degree of freedom commute, which means that the measurement on the charge degree of freedom does not influence the polarization freedom. Therefore, it is essentially the quantum nondemolition measurement. This advantage makes the concentrated maximally entangled state be remained. Moreover, if the whole process fails, it can also be reused in the next round. Recently, the ECPs for cluster states based on optical elements have been proposed. However, they can still be improved. The authors in Ref. [48] discussed the ECP for cluster states. First, their protocol can only be performed for one time and the concentrated cluster states will be destroyed for the post-selection principle. Second, the three-step concentration protocol requires the controlled-not (CONT) gate and three-qubit Toffoli gate, which makes it hard to realize in experiment.[61] Third, the ECP based on the cross-Kerr nonlinearity needs six different Kerr materials, to generate different phase shifts, which is also a controversial topic in current quantum optics.[62]

The key elements for realizing this ECP is the electronic PBS and the charge detector. In 2003, Ionicioiu and Amico designed a protocol for such a device.[74] They used nondispersive phases to separate spin-up and spin-down states, which carries into distinct outputs and make it analogous to a Stern–Gerlach apparatus. The charge detection has been realized in a two-dimensional electron gas. Our current experiment reported that currently achievable time resolution for charge detection is μs.[75] In a semiconductor, it has been shown that the resolution required for ballistic electrons is less than 5 ps.[76]

In summary, we proposed an efficient ECP based on electron-spin for less-entangled cluster states. We require the electronic PBS and a charge detector to work together to distinguish the parity of two electrons. The ECP has several advantages. First, it only requires one pair of less-entangled state. Second, the concentrated maximally entangled state can be remained. Third, it can be repeated to obtain a higher success probability. These advantages make our ECP have a bright prospect in current solid quantum computation.

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