Deterministic joint remote preparation of an arbitrary two-qubit state in the presence of noise*

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(Received 9 May 2013; revised manuscript received 8 July 2013; published online 20 December 2013)

Using two tripartite Greenberger–Horne–Zeilinger (GHZ) states as the shared channels, we investigate the noise effects on the deterministic joint remote preparation of an arbitrary two-qubit state. By unitary matrix decomposition procedure, we first construct the quantum logic circuit of the deterministic joint remote state preparation protocol. Then, we analytically derive the fidelity and the average fidelity for the deterministic joint remote preparation of an arbitrary two-qubit state and of four types of special two-qubit states under the influence of the Pauli noises. It is found that the fidelity depends on the noise types, the qubit-environment coupling strength, and the state to be remotely prepared. Moreover, even if the two GHZ channels are subject to the same environmental noises, the average fidelities for remotely preparing different two-qubit states display different time evolution behaviors. The remote preparation of the identical two-qubit states also shows that the average fidelities affected by different noisy environments exhibit different evolution actions.

Keywords: joint remote state preparation, GHZ states, noisy environments

PACS: 03.67.Hk, 03.65.Ud **DOI:** 10.1088/1674-1056/23/2/020312

1. Introduction

Quantum teleportation (QT), which was first proposed by Bennett *et al.*,[1] is one of the most important discoveries for remotely transmitting quantum states in quantum information processing. In QT, an unknown quantum state can be transmitted from a sender to a remote receiver via the previously shared entanglement channel with the assistance of some classical information. Subsequently, in contrast to QT, Lo,[2] Pati,[3] and Bennett *et al.*[4] have put forward another interesting quantum communication protocol called remote state preparation (RSP). In RSP the sender completely knows the information of the state but the receiver does not, while in QT neither the sender nor the receiver has any knowledge of the state to be transmitted. So, it is possible for RSP to have a trade-off between classical communication cost and entanglement resource. Due to this prominent advantage, RSP has attracted extensive attention and many extended protocols have been reported, both theoretically[5–17] and experimentally.[18–23] Recently, the authors of Refs. [24] and [25] have put forward a joint remote state preparation (JRSP) protocol, in which the information of the initial state is secretly shared by two or more senders, located at spatially separate sites, in such a way that none of the senders can know the full information of the initial state. In contrast to general RSP, in JRSP the receiver can remotely reconstruct the original state only under the mutual cooperation of all the shared senders. Since the first JRSP protocol for a single-qubit state,[24,25] there have been many generalized ones for two-, three-, and multi-qubit cases[26–32] by using different types of shared quantum resources. As a matter of fact, the above-mentioned JRSP protocols can be realized only with success probabilities of less than 100%; that is, they are probabilistic. Recently, the deterministic joint remote preparation of arbitrary two- and three-qubit states have been proposed by Xiao *et al.*,[33] who used the shared Greenberger–Horne–Zeilinger (GHZ) states. More recently, the deterministic joint remote preparation of arbitrary single-, two-, and three-qubit states have been suggested by Refs. [34]– [38], who used entangled resources consisting of Einstein–Padolsky–Rosen (EPR) pairs, or of EPR pairs and a GHZ state.

Generally, real quantum systems unavoidably interact with their surrounding environments. These unwanted interactions make the systems lose their mysterious quantum properties. Thus, from a realistic angle, it is of great significance to take the RSP protocols into account in the presence of noisy environments. We note that Liang *et al.*[39] have discussed the remote preparation of a qubit state and of a bipartite entangled state via a noisy GHZ-class channel. Zhang *et al.*[40] examined how to remotely prepare a general two-qubit state by using two EPR pairs within non-Markovian environment. However, the two noisy RSP protocols[39,40] are probabilistic and cannot achieve arbitrary pure states of two qubits. To the best of our knowledge, there are few studies involving the deterministic RSP protocols in noisy environments and the existing JRSP protocols[24–37] were accomplished in a closed system. In this paper we study the influence of Pauli noises on the protocol of deterministic joint remote preparation of an arbitrary two-qubit state with two GHZ states serving as quantum channels. After briefly describing the deterministic JRSP

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11174081, 11034002, 11104075, and 11134003), the National Basic Research Program of China (Grant Nos. 2011CB921602 and 2012CB821302), and the Open Fund from the SKLPS of ECNU.

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protocol,[33] we construct its corresponding quantum logic circuit with a set of single- and two-qubit logic gates. By solving the Lindblad master equation of the GHZ state subjected to Pauli noises, we obtain the fidelity and the corresponding average fidelity for deterministic JRSP of an arbitrary two-qubit state and of four special initial states of two qubits. Our results show that the fidelity depends on the state to be remotely prepared, the types of noises affecting the two GHZ states, and the qubit-environment coupling coefficient. Moreover, the average fidelities for joint remotely preparing different states of two qubits exhibit different time evolution behaviors, even if the two GHZ states are affected by the same environmental noises. In addition, the average fidelities for JRSP of the identical two-qubit states under the influence of different noisy environments also display different evolution actions.

This paper is outlined as follows. In Section 2, we construct the quantum circuit for implementing deterministic joint remote preparation of an arbitrary two-qubit state through two GHZ states, and then analytically derive the fidelity and the average fidelity for the JRSP protocol under the influence of noisy environments. In Section 3, we analyze the average fidelity for JRSP of four types of special two-qubit pure states. The conclusions are summarized in Section 4.

2. Deterministic joint remote preparation of an arbitrary state of two qubits in noisy environments

First of all, we briefly review the protocol[33] for deterministic joint remote preparation of an arbitrary two-qubit pure state with the collaboration of two senders by using two tripartite GHZ states as the entanglement resource. Suppose that two senders, Alice and Bob, located at two spatially separate sites, intend to help the receiver Charlie remotely prepare an arbitrary two-qubit state taking the form

$$|\Psi\rangle = \lambda_0|00\rangle + \lambda_1 e^{i\varphi_1}|01\rangle + \lambda_2 e^{i\varphi_2}|10\rangle + \lambda_3 e^{i\varphi_3}|11\rangle, \quad (1)$$

where the real parameters $\varphi_k \in [0, 2\pi]$ $(k = 1,2,3)$ and $\lambda_i \geq 0$ $(i = 0,1,2,3)$ with the normalization condition $\sum_{i=0}^{3}(\lambda_i)^2 = 1$. We also assume that Alice and Bob know the parameters λ_i and φ_k, respectively, and the two senders have the complete knowledge of $|\Psi\rangle$ only if they cooperate with each other. To achieve the task of JRSP, the two senders and the receiver are assumed to share the quantum channels consisting of two tripartite GHZ states given by

$$\begin{aligned} |\Phi\rangle_{135} &= \frac{1}{\sqrt{2}}(|000\rangle_{135} + |111\rangle_{135}), \\ |\Phi\rangle_{246} &= \frac{1}{\sqrt{2}}(|000\rangle_{246} + |111\rangle_{246}), \end{aligned} \quad (2)$$

in which particles 1 and 2 belong to Alice, particles 3 and 4 to Bob, and particles 5 and 6 to Charlie.

To help Charlie remotely reconstruct the initial state at his site, now Alice performs a collective orthogonal measurement on her particles 1 and 2 by choosing a set of complete orthonormal basis vectors, as follows:

$$|\eta_r\rangle_{12} = U_1(r)|\eta_1\rangle_{12}, \quad (r = 1,2,3,4), \quad (3)$$

with $U_1(1) = I_1 \otimes I_2$, $U_1(2) = I_1 \otimes (-i\sigma_y)_2$, $U_1(3) = (-i\sigma_y)_1 \otimes (\sigma_z)_2$, $U_1(4) = (-i\sigma_y)_1 \otimes (\sigma_x)_2$, and $|\eta_1\rangle_{12} = \lambda_0|00\rangle + \lambda_1|01\rangle + \lambda_2|10\rangle + \lambda_3|11\rangle$, where I is an identity operation, and $\sigma_x, \sigma_y, \sigma_z$ are Pauli operations. Then, Alice transmits her measurement results not only to Bob but also to Charlie through classical channels. After Bob gets the classical message from Alice, he makes the corresponding local unitary transformation on particles 3 and 4, conditioned on Alice's measurement results. Subsequently, Bob implements a collective projective measurement on his particles 3 and 4 under the following basis vectors

$$|\varepsilon_r\rangle_{34} = U_2(r)|\varepsilon_1\rangle_{34}, \quad (4)$$

with $U_2(1) = I_3 \otimes I_4$, $U_2(2) = I_3 \otimes (\sigma_z)_4$, $U_2(3) = (\sigma_z)_3 \otimes (I)_4$, $U_2(4) = (\sigma_z)_3 \otimes (\sigma_z)_4$, and $|\varepsilon_1\rangle_{34} = (|00\rangle + e^{-i\varphi_1}|01\rangle + e^{-i\varphi_2}|10\rangle + e^{-i\varphi_3}|11\rangle)/2$. Then, Bob transfers his measurement outcomes to Charlie with classical information. By performing the corresponding local unitary operators on particles 5 and 6 in accordance with the results of Alice's PM1 and Bob's PM2. Finally, Charlie can successfully restore the initial state with unit probability.

According to the decomposition procedure of arbitrarily unitary operations,[41,42] we construct the quantum circuit for implementing the deterministic JRSP protocol[33] in Fig. 1 by using the compositions of single-qubit logic gates and two-qubit controlled-rotation gates.[43] Here, the dot "•" denotes the control bit, while the cross "⊕", "$\boxed{S_1}$", "$\boxed{i\sigma_y}$", and "$\boxed{\sigma_z}$" denote the target bits. Moreover, H is the Hadamard operation, and the unitary operations R_m $(m = 1,2,y,z)$ and S_n $(n = 1,2,3)$ can be expressed, respectively, as

$$\begin{aligned} &R_1 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \ R_2 = R_1^+, R_y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}, \\ &R_z(\zeta) = \begin{pmatrix} e^{i\zeta} & 0 \\ 0 & e^{-i\zeta} \end{pmatrix}, \ S_1 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i(\varphi_3-\varphi_2-\varphi_1)} \end{pmatrix}, \\ &S_2 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi_2} \end{pmatrix}, \ S_3 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi_1} \end{pmatrix}, \end{aligned} \quad (5)$$

where $\tan^2(\theta/2) = (\lambda_2^2 + \lambda_3^2)/(\lambda_0^2 + \lambda_1^2)$, $\zeta = \theta_1 + \theta_2$ or $\theta_1 - \theta_2$ with $\tan(2\theta_1) = -\lambda_1/\lambda_0$ and $\tan(2\theta_2) = \lambda_2/\lambda_3$. In Fig. 1(a), the two top lines are owned by Alice, the third and fourth lines belong to Bob, and the two bottom ones belong to Charlie. It is worth mentioning that our constructed quantum circuit is not completely in accordance with the JRSP protocol.[33] In contrast to the protocol of Ref. [33], in our quantum circuit less unitary operations are required for Charlie due to Bob implementing different local unitary operations on particles 3 and 4.

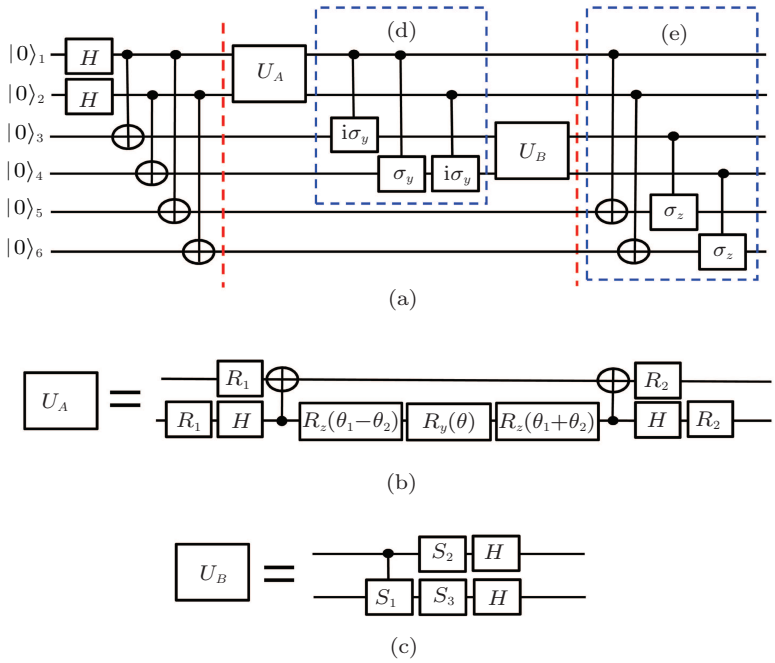


Fig. 1. (color online) (a) quantum logic network for deterministic joint remote preparation of an arbitrary two-qubit state via two GHZ states, in which Alice's unitary transformation U_A and Bob's unitary transformation U_B are decomposed with a combination of single- and two-qubit logic gates shown in panels (b) and (c), respectively. Here, the dashed boxes (d) and (e) denote Bob's and Charlie's local unitary operations acting on their respective particles.

In general, a real quantum system is inevitably exposed to its external environment and will interact with the surrounding world. Due to environmental noises, the coherent property of the system will diminish gradually. Under the assumption of Markov and Born approximations, and after tracing over the environmental degrees of freedom, the time evolution of an open quantum system can be described by a master equation for the density operator $\rho(t)$, which can be expressed in the Lindblad form as[42,44]

$$\frac{\partial\rho}{\partial t} = -\frac{\mathrm{i}}{\hbar}[H',\rho] + \sum_{j,\alpha}\left(L_{j,\alpha}\rho L_{j,\alpha}^{\dagger} - \frac{1}{2}L_{j,\alpha}^{\dagger}L_{j,\alpha}\rho - \frac{1}{2}\rho L_{j,\alpha}^{\dagger}L_{j,\alpha}\right), \tag{6}$$

where H' is the system's Hamiltonian. On the right-hand side of the master equation, the first term describes the unitary evolution of the quantum system, and the remaining terms describe the interaction between the system and the environment. The Lindblad operator $L_{j,\alpha} = \sqrt{\gamma_j}\sigma_\alpha^j$ $(\alpha = x,z)$ acts on the j-th qubit, with σ_α^j being the Pauli matrix of the j-th qubit, and γ_j describes the strength of the qubit j coupling with its environment.

In the following, we will study the above JRSP process in noisy environments. To be specific, we only consider the situation that the two GHZ entangled channels are subject to the Pauli noises simultaneously. Due to the fact that the two states $|\Phi\rangle_{135}$ and $|\Phi\rangle_{246}$ have an identical form, their time evolution under the action of the same noises will display the same behavior. It is not hard to learn that the initial density matrix elements of the GHZ state $\chi_{135} = |\Phi\rangle_{135}\langle\Phi|$ (or $\chi_{246} = |\Phi\rangle_{246}\langle\Phi|$) can have the form of ρ_{uv} $(u,v = 0,1,\ldots,7)$ with $\rho_{00} = \rho_{07} = \rho_{70} = \rho_{77} = 1/2$ and $\rho_{uv} = 0$ for the other values of u, v under the basis of $\{|000\rangle, |001\rangle, |010\rangle, \ldots, |111\rangle\}$. Now, we assume that the total density matrix for the time evolution of the two GHZ states is given by $\chi_{135}(t)\otimes\chi_{246}(t)$. Therefore, by means of Fig. 1, the density matrix of the remotely prepared state at Charlie's side through the noisy channels can be expressed as

$$\widetilde{\rho}_{56} = \mathrm{Tr}_{1234}\left\{U_{\mathrm{JRSP}}(\chi_{135}(t)\otimes\chi_{246}(t))U_{\mathrm{JRSP}}^{+}\right\}, \tag{7}$$

where Tr_{1234} is the partial trace over particles 1, 2, 3, and 4, and the unitary operator $U_{\mathrm{JRSP}} = (\sigma_z)_{4\to6}(\sigma_z)_{3\to5}(\sigma_x)_{2\to6}(\sigma_x)_{1\to5}(U_B)_{34}(\mathrm{i}\sigma_y)_{2\to4}(\sigma_z)_{1\to4}(\mathrm{i}\sigma_y)_{1\to3}(U_A)_{12}$ is the compositions of single- and two-qubit logic gates. Here, $(\sigma_w)_{p\to q}$ represents a two-qubit controlled-σ_w $(w = x,y,z)$ gates with qubit p controlling qubit q $(p = 1,2,3,4$ and $q = 3,4,5,6)$. Next, we analyze the influence of three types of the Pauli noises on the JRSP process one by one.

(i) Firstly, we consider the case when both of the two GHZ states are affected by the dephasing noise, whose Lindblad operator takes the form of $L_{j,z} = \sqrt{\gamma_j}\sigma_z^j$ $(j = 1,2,\ldots,6)$. For simplicity, we assume the Hamiltonian $H' = 0$ and $\gamma_j = \gamma$ throughout this paper. Through straightforward calculations from Eq. (6), the density matrix elements of $\chi_{135}^{zz}(t)$ (or $\chi_{246}^{zz}(t)$), depicting the time evolution of the GHZ state $|\Phi\rangle_{135}$ (or $|\Phi\rangle_{246}$), are given by

$$\rho_{00} = \rho_{77} = \frac{1}{2},\quad \rho_{07} = \rho_{70} = \frac{1}{2}\mathrm{e}^{-6\lambda t},$$

$$\rho_{uv} = 0 \text{ for the other values of } u, v. \tag{8}$$

Substituting Eq. (8) into Eq. (7), $\widetilde{\rho}_{56}$ can be derived as

$$\begin{aligned}\widetilde{\rho}_{56} = {} & \lambda_0^2 |00\rangle\langle 00| + \lambda_0\lambda_1 \mathrm{e}^{-\mathrm{i}\varphi_1} \mathrm{e}^{-6\gamma t} |00\rangle\langle 01| \\ & + \lambda_0\lambda_2 \mathrm{e}^{-\mathrm{i}\varphi_2} \mathrm{e}^{-6\gamma t} |00\rangle\langle 10| \\ & + \lambda_0\lambda_3 \mathrm{e}^{-\mathrm{i}\varphi_3} \mathrm{e}^{-12\gamma t} |00\rangle\langle 11| \\ & + \lambda_0\lambda_1 \mathrm{e}^{\mathrm{i}\varphi_1} \mathrm{e}^{-6\gamma t} |01\rangle\langle 00| + \lambda_1^2 |01\rangle\langle 01| \\ & + \lambda_1\lambda_2 \mathrm{e}^{\mathrm{i}(\varphi_1-\varphi_2)} \mathrm{e}^{-12\gamma t} |01\rangle\langle 10| \\ & + \lambda_1\lambda_3 \mathrm{e}^{\mathrm{i}(\varphi_1-\varphi_3)} \mathrm{e}^{-6\gamma t} |01\rangle\langle 11| \\ & + \lambda_0\lambda_2 \mathrm{e}^{\mathrm{i}\varphi_2} \mathrm{e}^{-6\gamma t} |10\rangle\langle 00| \\ & + \lambda_1\lambda_2 \mathrm{e}^{\mathrm{i}(\varphi_2-\varphi_1)} \mathrm{e}^{-12\gamma t} |10\rangle\langle 01| + \lambda_2^2 |10\rangle\langle 10| \\ & + \lambda_2\lambda_3 \mathrm{e}^{\mathrm{i}(\varphi_2-\varphi_3)} \mathrm{e}^{-6\gamma t} |10\rangle\langle 11| \\ & + \lambda_0\lambda_3 \mathrm{e}^{\mathrm{i}\varphi_3} \mathrm{e}^{-12\gamma t} |11\rangle\langle 00| \\ & + \lambda_1\lambda_3 \mathrm{e}^{\mathrm{i}(\varphi_3-\varphi_1)} \mathrm{e}^{-6\gamma t} |11\rangle\langle 01| \\ & + \lambda_2\lambda_3 \mathrm{e}^{\mathrm{i}(\varphi_3-\varphi_2)} \mathrm{e}^{-6\gamma t} |11\rangle\langle 10| + \lambda_3^2 |11\rangle\langle 11|. \end{aligned} \tag{9}$$

To describe how much quantum information is transmitted from the initial state to the remotely prepared state through the noisy channels, it is quite useful to calculate the fidelity defined by

$$F = \langle\Psi| \rho_{\text{out}} |\Psi\rangle, \tag{10}$$

where $\rho_{\text{out}} = \widetilde{\rho}_{56}$ is the final output state of the JRSP protocol at Charlie's side.

According to Eqs. (9) and (10), if the six qubits of the quantum channels are subject to the dephasing noise at the same time, we can obtain the fidelity

$$\begin{aligned} F^{zz} = {} & \lambda_0^4 + \lambda_1^4 + \lambda_2^4 + \lambda_3^4 \\ & + 2(\lambda_0^2\lambda_1^2 + \lambda_0^2\lambda_2^2 + \lambda_3^2\lambda_1^2 + \lambda_2^2\lambda_3^2)\mathrm{e}^{-6\gamma t} \\ & + 2(\lambda_0^2\lambda_3^2 + \lambda_2^2\lambda_1^2)\mathrm{e}^{-12\gamma t}. \end{aligned} \tag{11}$$

Clearly, the fidelity F^{zz} only depends on the amplitude information of the initial state to be remotely prepared, but is independent of the phase information of the state $|\Psi\rangle$. In general, since the input state to be remotely prepared is an arbitrary pure state of two qubits, it is more significant to analyze the average fidelity over all possible input states, which is defined by

$$\begin{aligned} F_{\text{av}} = {} & \left(\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi} \mathrm{d}\varphi_1\mathrm{d}\varphi_2\mathrm{d}\varphi_3 \int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\delta \right. \\ & \left. \times F \sin\alpha\sin\beta\sin\delta \right) \Big/ 64\pi^3, \end{aligned} \tag{12}$$

with the assumption of

$$\lambda_0 = \cos\frac{\alpha}{2}\cos\frac{\beta}{2}, \quad \lambda_1 = \cos\frac{\alpha}{2}\sin\frac{\beta}{2},$$

$$\lambda_2 = \sin\frac{\alpha}{2}\cos\frac{\delta}{2}, \quad \lambda_3 = \sin\frac{\alpha}{2}\sin\frac{\delta}{2},$$

and α, β, $\delta \in [0, \pi]$. Substituting Eq. (11) into Eq. (12), the corresponding average fidelity can be calculated as

$$F_{\text{av}}^{zz} = \frac{4}{9} + \frac{7}{18}\mathrm{e}^{-6\gamma t} + \frac{1}{6}\mathrm{e}^{-12\gamma t}. \tag{13}$$

It is easy to verify that $F^{zz} = F_{\text{av}}^{zz} = 1$ when the quantum channels are in the absence of the dephasing noise.

(ii) Next we consider the case when every qubit of the two GHZ states is subject to the bit-flip noise simultaneously. In this case, the Lindblad operator takes the form $L_{j,x} = \sqrt{\gamma}\sigma_x^j$. Based on Eq. (6), the density matrix elements of $\chi_{135}^{xx}(t)$ (or $\chi_{246}^{xx}(t)$), characterizing the time evolution of the GHZ state $|\Phi\rangle_{135}$ (or $|\Phi\rangle_{246}$), are derived as[39,45]

$$\begin{aligned} \rho_{00} &= \rho_{77} = \rho_{07} = \rho_{70} = \frac{1}{8} + \frac{3}{8}\mathrm{e}^{-4\gamma t}, \\ \rho_{11} &= \rho_{22} = \rho_{33} = \rho_{44} = \rho_{55} = \rho_{66} = \frac{1}{8} - \frac{1}{8}\mathrm{e}^{-4\gamma t}, \\ \rho_{16} &= \rho_{61} = \rho_{25} = \rho_{52} = \rho_{34} = \rho_{43} = \frac{1}{8} - \frac{1}{8}\mathrm{e}^{-4\gamma t}, \\ \rho_{uv} &= 0 \text{ for the other values of } u, v. \end{aligned} \tag{14}$$

Similar to case (i), the fidelity F^{xx} between the remotely prepared state and the initial state can be calculated as

$$\begin{aligned} F^{xx} = {} & M_{11}^2\{16\lambda_0\lambda_1\lambda_2\lambda_3[\cos(2\varphi_1) + \cos(2\varphi_2) + \cos(2\varphi_3) \\ & + \cos(2\varphi_1 - 2\varphi_3) + \cos(2\varphi_2 - 2\varphi_3) \\ & + \cos(2\varphi_1 - 2\varphi_2) + 1] + 4[6(\lambda_1\lambda_2 + \lambda_0\lambda_3)^2 \\ & - (\lambda_0^4 + \lambda_1^4 + \lambda_2^4 + \lambda_3^4)] + 16[(\lambda_0\lambda_1 + \lambda_2\lambda_3)^2 \\ & \times \cos(\varphi_1 - \varphi_2 + \varphi_3) + (\lambda_0\lambda_2 + \lambda_1\lambda_3)^2 \\ & \times \cos(\varphi_1 - \varphi_2 - \varphi_3)] + [8(\lambda_0\lambda_1 + \lambda_2\lambda_3)^2 \\ & + 6(\lambda_0\lambda_2 + \lambda_1\lambda_3)^2 + 4\lambda_0\lambda_1\lambda_2\lambda_3]\cos(\varphi_1 + \varphi_2 - \varphi_3)\} \\ & + M_{00}M_{11}\{8[(\lambda_0\lambda_1 + \lambda_2\lambda_3)^2 + (\lambda_0\lambda_2 + \lambda_1\lambda_3)^2] \\ & \times \cos(\varphi_1 + \varphi_2 - \varphi_3) + 16[\lambda_2^2\lambda_3^2\cos(2\varphi_3 - 2\varphi_2) \\ & + \lambda_1^2\lambda_3^2\cos(2\varphi_3 - 2\varphi_1) + \lambda_0^2\lambda_2^2\cos(2\varphi_2) \\ & + \lambda_0^2\lambda_1^2\cos(2\varphi_1)] + 8[(\lambda_0\lambda_2 + \lambda_1\lambda_3)^2 \\ & + (\lambda_0\lambda_1 + \lambda_2\lambda_3)^2 - 2(\lambda_0\lambda_3 - \lambda_1\lambda_2)^2]\} \\ & + 8(M_{11}^2 + M_{00}M_{11})(\lambda_1\lambda_2 + \lambda_0\lambda_3)^2 \\ & \times [\cos(\varphi_1 - \varphi_2 - \varphi_3) + \cos(\varphi_1 - \varphi_2 + \varphi_3)] \\ & + 4(M_{00}^2 + 2M_{11}^2 + 2M_{00}M_{11}), \end{aligned} \tag{15}$$

where $M_{00} = 1/8 + (3/8)\mathrm{e}^{-4\gamma t}$ and $M_{11} = 1/8 - (1/8)\mathrm{e}^{-4\gamma t}$. By substituting Eq. (15) into Eq. (12), the corresponding average fidelity can be analytically obtained as

$$\begin{aligned} F_{\text{av}}^{xx} = {} & \frac{23}{72} + \frac{\pi^2}{256} + \left(\frac{3}{8} + \frac{\pi^2}{384}\right)\mathrm{e}^{-4\gamma t} \\ & + \left(\frac{11}{36} - \frac{5\pi^2}{768}\right)\mathrm{e}^{-8\gamma t}. \end{aligned} \tag{16}$$

It is easy to see that the fidelity F^{xx} is the function of λ_i, φ_k, and γt, while the average fidelity $F_{\rm av}^{xx}$ only depends on γt.

(iii) Thirdly, we consider the case when one GHZ state is infected by the dephasing noise and the other GHZ state by the bit flip noise. In this case, the density matrix elements of the time evolution of the two GHZ states can be given by Eqs. (8) and (14), respectively. Using the similar method as the case (i), we obtain the following fidelity

$$\begin{aligned} F^{zx} &= 2M_{11}[\lambda_0^4+\lambda_1^4+\lambda_2^4+\lambda_3^4+6(\lambda_0^2\lambda_1^2+\lambda_2^2\lambda_3^2) \\ &\quad +4\lambda_2^2\lambda_3^2\cos(2\varphi_2-2\varphi_3)+4\lambda_1^2\lambda_0^2\cos(2\varphi_1)] \\ &\quad +8M'_{11}M_{11}[4\lambda_0\lambda_1\lambda_2\lambda_3+(\lambda_0\lambda_3+\lambda_2\lambda_1)^2 \\ &\quad \times\cos(\varphi_1-\varphi_2+\varphi_3)+(\lambda_2\lambda_0+\lambda_1\lambda_3)^2 \\ &\quad \times\cos(\varphi_1+\varphi_2-\varphi_3)]+2M_{00}[\lambda_0^4+\lambda_1^4+\lambda_2^4+\lambda_3^4 \\ &\quad +2(\lambda_0^2\lambda_1^2+\lambda_2^2\lambda_3^2)]+8M'_{11}M_{00}(\lambda_0^2+\lambda_1^2) \\ &\quad \times(\lambda_2^2+\lambda_3^2) \end{aligned} \tag{17}$$

with $M'_{11}=\mathrm{e}^{-6\gamma t}/2$. Substituting Eq. (16) into Eq. (12), the corresponding average fidelity $F_{\rm av}^{zx}$ reads

$$\begin{aligned} F_{\rm av}^{zx} &= \frac{4}{9}+\frac{2}{9}\mathrm{e}^{-4\gamma t}+\frac{1}{576}\{(48+3\pi^2)\mathrm{e}^{-6\gamma t} \\ &\quad +(144-3\pi^2)\mathrm{e}^{-10\gamma t}\}. \end{aligned} \tag{18}$$

Summarizing cases (i), (ii), and (iii), we find that the fidelities F^{zz}, F^{xx}, and F^{zx} depend on the state to be remotely prepared, the noise types, and the qubit-environment coupling strength, while F^{zz} does not depend on the phase information of the arbitrary two-qubit state. Moreover, it is readily verified from Eqs. (11), (15), and (17) that the fidelities $F^{zz}=F^{xx}=F^{zx}=1$ in the absence of noisy environments. In Fig. 2 we plot the average fidelity of the JRSP protocol against γt under the influence of three types of different Pauli noises. It can easily be seen that different noises have different impacts on the average fidelity. When $\gamma t=0$, all the average fidelities have the same value $F_{\rm av}^{zz}=F_{\rm av}^{xx}=F_{\rm av}^{zx}=1$. Furthermore, for the limit of $\gamma t\to\infty$, the values of the average fidelities $F_{\rm av}^{zz}$, $F_{\rm av}^{zx}$, and $F_{\rm av}^{xx}$ approach to 4/9, 4/9, and 23/72, respectively. Therefore, in the long-time limit, the effect on the initial state transmission for the case of the two GHZ states affected by the bit-flip noise is relatively stronger than that for the two GHZ states infected by the dephasing noise or for one GHZ state infected by the dephasing noise and the other by the bit-flip noise.

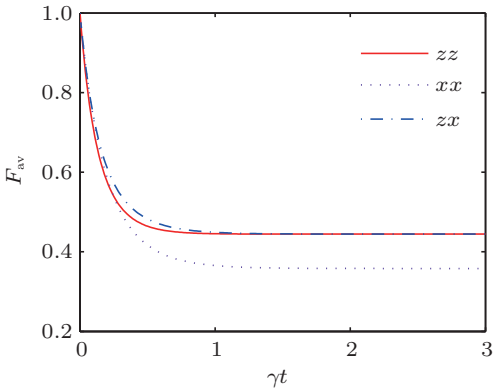


Fig. 2. (color online) The average fidelity for JRSP of an arbitrary two-qubit state is plotted as a function of γt. Here, the solid and dotted lines denote the two GHZ states infected by the dephasing noise and the bit-flip noise, respectively, while the dash–dotted line denotes the situation of one GHZ state infected by the dephasing noise and the other by the bit-flip noise.

3. Deterministic joint remote preparation of four types of special two-qubit states in noisy environments

In the previous section, we obtained the fidelity and the average fidelity for the deterministic joint remote preparation of an arbitrary two-qubit pure state in noisy environments. Generally speaking, for different input states to be remotely prepared, the formulas of the average fidelity are also different. In this section, we turn our attention to investigate the environmental noise effects on the average fidelity for joint remotely preparing four types of specially chosen states of two qubits.

Case (I) We consider the case when the initial two-qubit state described in Eq. (1) is a general bipartite entangle state taking the form

$$|\phi\rangle_a=\cos\frac{\alpha}{2}|00\rangle+\sin\frac{\alpha}{2}\mathrm{e}^{\mathrm{i}\varphi_3}|11\rangle, \tag{19}$$

which indicates the real parameters $\lambda_1=\lambda_2=0$ for the input state $|\Psi\rangle$.

Case (I)-1 Firstly, we assume that the quantum channels consisting of the two GHZ states are affected by the dephasing noise. In this case, substituting $\lambda_1=\lambda_2=0$ into Eq. (11), the JRSP fidelity reduces to

$$F_a^{zz}=\frac{1}{4}\{3+\cos(2\alpha)+[1-\cos(2\alpha)]\mathrm{e}^{-12\gamma t}\}. \tag{20}$$

At this stage, the corresponding average fidelity is defined by

$$F_{{\rm av},a}=\frac{\int_0^{2\pi}\mathrm{d}\varphi_3\int_0^{\pi}\mathrm{d}\alpha F\sin\alpha}{4\pi}. \tag{21}$$

Substituting Eq. (20) into Eq. (21), the average fidelity can be derived as

$$F_{{\rm av},a}^{zz}=\frac{2}{3}+\frac{1}{3}\mathrm{e}^{-12\gamma t}. \tag{22}$$

Case (I)-2 Next, we assume that the two GHZ states are subject to the bit-flip noise. In this case, substituting $\lambda_1 = \lambda_2 = 0$ into Eq. (15), the fidelity reduces to

$$\begin{aligned} F_a^{xx} &= \frac{1}{8}[(1+3\mathrm{e}^{-8\gamma t})+(1-\mathrm{e}^{-8\gamma t})\cos\varphi_3 \\ &\quad +(1-2\mathrm{e}^{-4\gamma t}+\mathrm{e}^{-8\gamma t})\cos^2\varphi_3]\sin^2\alpha \\ &\quad +\frac{1}{4}(1+2\mathrm{e}^{-4\gamma t}+\mathrm{e}^{-8\gamma t})\left(1-\frac{1}{2}\sin^2\alpha\right). \end{aligned} \tag{23}$$

Then, by substituting Eq. (23) into Eq. (21), the average fidelity is obtained as

$$F_{\mathrm{av},a}^{xx} = \frac{7}{24}+\frac{1}{4}\mathrm{e}^{-4\gamma t}+\frac{11}{24}\mathrm{e}^{-8\gamma t}. \tag{24}$$

Case (I)-3 Thirdly, we assume that one GHZ state is subject to the dephasing noise and the other GHZ state to the bit-flip noise. Similarly, substituting $\lambda_1 = \lambda_2 = 0$ into Eq. (17), the fidelity reduces to

$$\begin{aligned} F_a^{zx} &= \frac{1}{16}[\mathrm{e}^{-6\gamma t}+3\mathrm{e}^{-10\gamma t}+(\mathrm{e}^{-6\gamma t}-\mathrm{e}^{-10\gamma t})\cos\varphi_3] \\ &\quad \times[1-\cos(2\alpha)]+\frac{1}{8}(1+\mathrm{e}^{-4\gamma t})[3+\cos(2\alpha)]. \end{aligned} \tag{25}$$

After substituting Eq. (25) into Eq. (21), we obtain the average fidelity

$$F_{\mathrm{av},a}^{zx} = \frac{1}{3}+\frac{1}{3}\mathrm{e}^{-4\gamma t}+\frac{1}{12}\mathrm{e}^{-6\gamma t}+\frac{1}{4}\mathrm{e}^{-10\gamma t}. \tag{26}$$

Case (II) We consider the case when the parameters of the initial two-qubit state in Eq. (1) are set to be

$$\lambda_0 = \lambda_2 = \frac{1}{\sqrt{2}}\cos\frac{\beta}{2},\ \ \lambda_1 = \lambda_3 = \frac{1}{\sqrt{2}}\sin\frac{\beta}{2},$$

$\varphi_1 = 0$, and $\varphi_2 = \varphi_3$,; that is, the initial state takes the form

$$|\phi\rangle_b = \frac{1}{\sqrt{2}}\left(|0\rangle+\mathrm{e}^{\mathrm{i}\varphi_3}|1\rangle\right)\left(\cos\frac{\beta}{2}|0\rangle+\sin\frac{\beta}{2}|1\rangle\right). \tag{27}$$

Clearly, $|\phi\rangle_b$ is a bipartite separable state consisting of one single-qubit equatorial state and one single-qubit state in real Hilbert space. In this case, the formula for calculating the average fidelity is the same as Eq. (21), only with β being replaced by α.

Case (II)-1 Firstly, we assume that every qubit of the two GHZ states is subject to the dephasing noise simultaneously. By using a similar method as Case (I), the fidelity and the average fidelity are calculated, respectively, as

$$F_b^{zz} = \frac{1}{8}[3+4\mathrm{e}^{-6\gamma t}+\mathrm{e}^{-12\gamma t}+(1-\mathrm{e}^{-12\gamma t})\cos(2\beta)], \tag{28}$$

$$F_{\mathrm{av},b}^{zz} = \frac{1}{3}+\frac{1}{2}\mathrm{e}^{-6\gamma t}+\frac{1}{6}\mathrm{e}^{-12\gamma t}. \tag{29}$$

Case (II)-2 Secondly, we assume that the two GHZ states are infected by the bit-flip noise. Similarly, the fidelity and the average fidelity can be derived, respectively, as

$$\begin{aligned} F_b^{xx} &= \frac{1}{32}(-1+2\mathrm{e}^{-4\gamma t}-\mathrm{e}^{-8\gamma t})[\cos(2\beta+2\varphi_3) \\ &\quad +\cos(2\beta-2\varphi_3)]+\frac{1}{512}[285+198\mathrm{e}^{-4\gamma t}+29\mathrm{e}^{-8\gamma t} \\ &\quad +(-97+66\mathrm{e}^{-4\gamma t}+31\mathrm{e}^{-8\gamma t})\cos(2\beta)] \\ &\quad +\frac{1}{16}(3-2\mathrm{e}^{-4\gamma t}-\mathrm{e}^{-8\gamma t})\cos(2\varphi_3), \end{aligned} \tag{30}$$

$$F_{\mathrm{av},b}^{xx} = \frac{119}{192}+\frac{11}{32}\mathrm{e}^{-4\gamma t}+\frac{7}{192}\mathrm{e}^{-8\gamma t}. \tag{31}$$

Case (II)-3 Thirdly, we assume that one GHZ state is infected by the dephasing noise and the other GHZ state by the bit-flip noise. Similarly, we obtain the fidelity and the corresponding average fidelity

$$\begin{aligned} F_b^{zx} &= \frac{1}{8}(1+\mathrm{e}^{-6\gamma t})[3-\cos(2\beta)] \\ &\quad +\frac{1}{8}(\mathrm{e}^{-4\gamma t}+\mathrm{e}^{-10\gamma t})[1+\cos(2\beta)], \end{aligned} \tag{32}$$

$$F_{\mathrm{av},b}^{zx} = \frac{5}{12}+\frac{1}{12}\mathrm{e}^{-4\gamma t}+\frac{5}{12}\mathrm{e}^{-6\gamma t}+\frac{1}{12}\mathrm{e}^{-10\gamma t}. \tag{33}$$

Case (III) We consider the case when the initial two-qubit state $|\Psi\rangle$ is in real Hilbert space with the form

$$|\phi\rangle_c = \lambda_0|00\rangle+\lambda_1|01\rangle+\lambda_2|10\rangle+\lambda_3|11\rangle, \tag{34}$$

which means the parameters of the state $\varphi_1 = \varphi_2 = \varphi_3 = 0$.

Case (III)-1 Firstly, the two GHZ states are assumed to be affected by the dephasing noise. In this situation, substituting $\varphi_1 = \varphi_2 = \varphi_3 = 0$ into Eq. (11), the fidelity reduces to

$$\begin{aligned} F_c^{zz} &= \lambda_0^4+\lambda_1^4+\lambda_2^4+\lambda_3^4 \\ &\quad +2(\lambda_0^2\lambda_1^2+\lambda_0^2\lambda_2^2+\lambda_3^2\lambda_1^2+\lambda_2^2\lambda_3^2)\mathrm{e}^{-6\gamma t} \\ &\quad +2(\lambda_0^2\lambda_3^2+\lambda_2^2\lambda_1^2)\mathrm{e}^{-12\gamma t}. \end{aligned} \tag{35}$$

At this stage, the corresponding average fidelity is defined by

$$F_{\mathrm{av},c} = \frac{\int_0^\pi\int_0^\pi\int_0^\pi \mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\delta F\sin\alpha\sin\beta\sin\delta}{8}. \tag{36}$$

After substituting Eq. (35) into Eq. (36), we get

$$F_{\mathrm{av},c}^{zz} = \frac{4}{9}+\frac{7}{18}\mathrm{e}^{-6\gamma t}+\frac{1}{6}\mathrm{e}^{-12\gamma t}. \tag{37}$$

Case (III)-2 Secondly, the two GHZ states are assumed to be infected by the bit-flip noise. In this case, substituting $\varphi_1 = \varphi_2 = \varphi_3 = 0$ into Eq. (15), the fidelity reduces to

$$\begin{aligned} F_c^{xx} &= \frac{1}{4}(\lambda_0^4+\lambda_1^4+\lambda_2^4+\lambda_3^4)(1+2\mathrm{e}^{-4\gamma t}+\mathrm{e}^{-8\gamma t}) \\ &\quad +\frac{1}{32}(\lambda_1^2\lambda_3^2+\lambda_0^2\lambda_2^2)(47+34\mathrm{e}^{-4\gamma t}-17\mathrm{e}^{-8\gamma t}) \\ &\quad +\frac{1}{2}(\lambda_0^2\lambda_1^2+\lambda_2^2\lambda_3^2)(3+2\mathrm{e}^{-4\gamma t}-\mathrm{e}^{-8\gamma t}) \\ &\quad +\frac{1}{2}(3-2\mathrm{e}^{-4\gamma t}+3\mathrm{e}^{-8\gamma t})(\lambda_0^2\lambda_3^2+\lambda_1^2\lambda_2^2). \end{aligned} \tag{38}$$

After substituting Eq. (38) into Eq. (36), the corresponding average fidelity can be derived as

$$F_{\mathrm{av},c}^{xx} = \frac{1}{1152}[605+18\pi^2+(390-12\pi^2)\mathrm{e}^{-4\gamma t} + (157-6\pi^2)\mathrm{e}^{-8\gamma t}]. \quad (39)$$

Case (III)-3 Thirdly, we assume that one GHZ state is infected by the dephasing noise and the other GHZ state by the bit-flip noise. In this situation, substituting $\varphi_1=\varphi_2=\varphi_3=0$ into Eq. (17), the fidelity is obtained as

$$F_c^{zx} = \frac{1}{2}(1+\mathrm{e}^{-4\gamma t})(\lambda_0^4+\lambda_1^4+\lambda_2^4+\lambda_3^4) + 4\lambda_0\lambda_1\lambda_2\lambda_3(\mathrm{e}^{-6\gamma t}-\mathrm{e}^{-10\gamma t}) + (3-\mathrm{e}^{-4\gamma t})(\lambda_0^2\lambda_1^2+\lambda_2^2\lambda_3^2) + (\mathrm{e}^{-6\gamma t}+\mathrm{e}^{-10\gamma t})(\lambda_0^2+\lambda_1^2)(\lambda_2^2+\lambda_3^2). \quad (40)$$

After substituting Eq. (40) into Eq. (36), the corresponding average fidelity is calculated as

$$F_{\mathrm{av},c}^{zx} = \frac{1}{288}[160+32\mathrm{e}^{-4\gamma t}+(48+3\pi^2)\mathrm{e}^{-6\gamma t} + (48-3\pi^2)\mathrm{e}^{-10\gamma t}]. \quad (41)$$

Case (IV) We consider the case when the initial state $|\Psi\rangle$ belongs to the equatorial-like ensemble[3,12,46] in the uniformly weighted superpositions of all the four computation basis states; namely, the state takes the form

$$|\phi\rangle_d = \frac{1}{2}\left(|00\rangle+\mathrm{e}^{\mathrm{i}\varphi_1}|01\rangle+\mathrm{e}^{\mathrm{i}\varphi_2}|10\rangle+\mathrm{e}^{\mathrm{i}\varphi_3}|11\rangle\right), \quad (42)$$

which implies the parameters of the initial state $\lambda_0=\lambda_1=\lambda_2=\lambda_3=1/2$.

Case (IV)-1 Firstly, the two GHZ states are assumed to be affected by the dephasing noise. At this moment, substituting $\lambda_0=\lambda_1=\lambda_2=\lambda_3=1/2$ into Eq. (11), the fidelity is simplified as

$$F_d^{zz} = \frac{1}{4}+\frac{1}{2}\mathrm{e}^{-6\gamma t}+\frac{1}{4}\mathrm{e}^{-12\gamma t}. \quad (43)$$

In this case, the average fidelity is defined by

$$F_{\mathrm{av},d} = \frac{\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi}\mathrm{d}\varphi_1\mathrm{d}\varphi_2\mathrm{d}\varphi_3 F}{8\pi^3}. \quad (44)$$

Then, substituting Eq. (43) into Eq. (44), we get the corresponding average fidelity

$$F_{\mathrm{av},d}^{zz} = \frac{1}{4}+\frac{1}{2}\mathrm{e}^{-6\gamma t}+\frac{1}{4}\mathrm{e}^{-12\gamma t}. \quad (45)$$

Case (IV)-2 Secondly, the two GHZ states are assumed to be infected by the bit-flip noise. In this case, substituting $\lambda_0=\lambda_1=\lambda_2=\lambda_3=1/2$ into Eq. (15), the fidelity reduces to

$$F_d^{xx} = \frac{1}{16}(7+6\mathrm{e}^{-4\gamma t}+3\mathrm{e}^{-8\gamma t}) + \frac{1}{256}[31+2\mathrm{e}^{-4\gamma t}-33\mathrm{e}^{-8\gamma t}] \times \cos(\varphi_1+\varphi_2-\varphi_3)+\frac{1}{32}\{(1-\mathrm{e}^{-8\gamma t})[\cos(2\varphi_1) + \cos(2\varphi_2)+\cos(2\varphi_1-2\varphi_3)+\cos(2\varphi_2-2\varphi_3)] + 4(1-\mathrm{e}^{-4\gamma t})[\cos(\varphi_1-\varphi_2-\varphi_3) + 4\cos(\varphi_1-\varphi_2+\varphi_3)]+(1-2\mathrm{e}^{-4\gamma t}+\mathrm{e}^{-8\gamma t}) \times [\cos(2\varphi_3)+\cos(2\varphi_1-2\varphi_2)]\}. \quad (46)$$

Then, by substituting Eq. (46) into Eq. (44), the corresponding average fidelity is calculated as

$$F_{\mathrm{av},d}^{xx} = \frac{7}{16}+\frac{3}{8}\mathrm{e}^{-4\gamma t}+\frac{3}{16}\mathrm{e}^{-8\gamma t}. \quad (47)$$

Case (IV)-3 Thirdly, we assume that one GHZ state is affected by the dephasing noise and the other GHZ state by the bit-flip noise. In this situation, by substituting $\lambda_0=\lambda_1=\lambda_2=\lambda_3=1/2$ into Eq. (17), we have

$$F_d^{zx} = \frac{1}{8}\{3+\mathrm{e}^{-4\gamma t}+2\mathrm{e}^{-6\gamma t}+3\mathrm{e}^{-10\gamma t} + 2(\mathrm{e}^{-6\gamma t}-\mathrm{e}^{-10\gamma t})[\cos(\varphi_1)\cos(\varphi_2-\varphi_3)]\} + \frac{1}{16}(1-\mathrm{e}^{-4\gamma t})[\cos(2\varphi_2-2\varphi_3)+\cos(2\varphi_1)]. \quad (48)$$

Then, by substituting Eq. (48) into Eq. (44), the corresponding average fidelity is calculated as

$$F_{\mathrm{av},d}^{zx} = \frac{1}{8}(3+\mathrm{e}^{-4\gamma t}+2\mathrm{e}^{-6\gamma t}+2\mathrm{e}^{-10\gamma t}). \quad (49)$$

Summing up the above four cases, we find that the average fidelities $F_{\mathrm{av},s}^{zz}$, $F_{\mathrm{av},s}^{xx}$, and $F_{\mathrm{av},s}^{zx}$ $(s=a,b,c,d)$ for JRSP of different initial two-qubit states also display different time evolution behaviors, even if the two GHZ states are subject to the same environmental noises simultaneously. Next, we analyze the average fidelity of the JRSP protocols under various Pauli noisy environments. The average fidelity versus γt is plotted for the joint remote preparation of four types of special input states in Fig. 3. From the figure, we can see that when $\gamma t=0$, all the average fidelities have the same value; that is, $F_{\mathrm{av},s}^{zz}=F_{\mathrm{av},s}^{xx}=F_{\mathrm{av},s}^{zx}=1$. Moreover, according to Fig. 3(a), it can be seen that in the long time limit, the dephasing noise acting on the two GHZ states has the weakest effect on the average fidelity, whereas the bit-flip noise acting on the two GHZ states has the strongest effect. However, we can observe that from Figs. 3(b), 3(c), 3(d) that in the limit of $\gamma t\to\infty$, the dephasing noise acting on the two quantum channels has the largest influence on the three average fidelities $F_{\mathrm{av},b}$, $F_{\mathrm{av},c}$, and $F_{\mathrm{av},d}$, while the bit-flip noise acting on the two channels has the smallest influence, which is clearly different from the result obtained in Fig. 3(a).

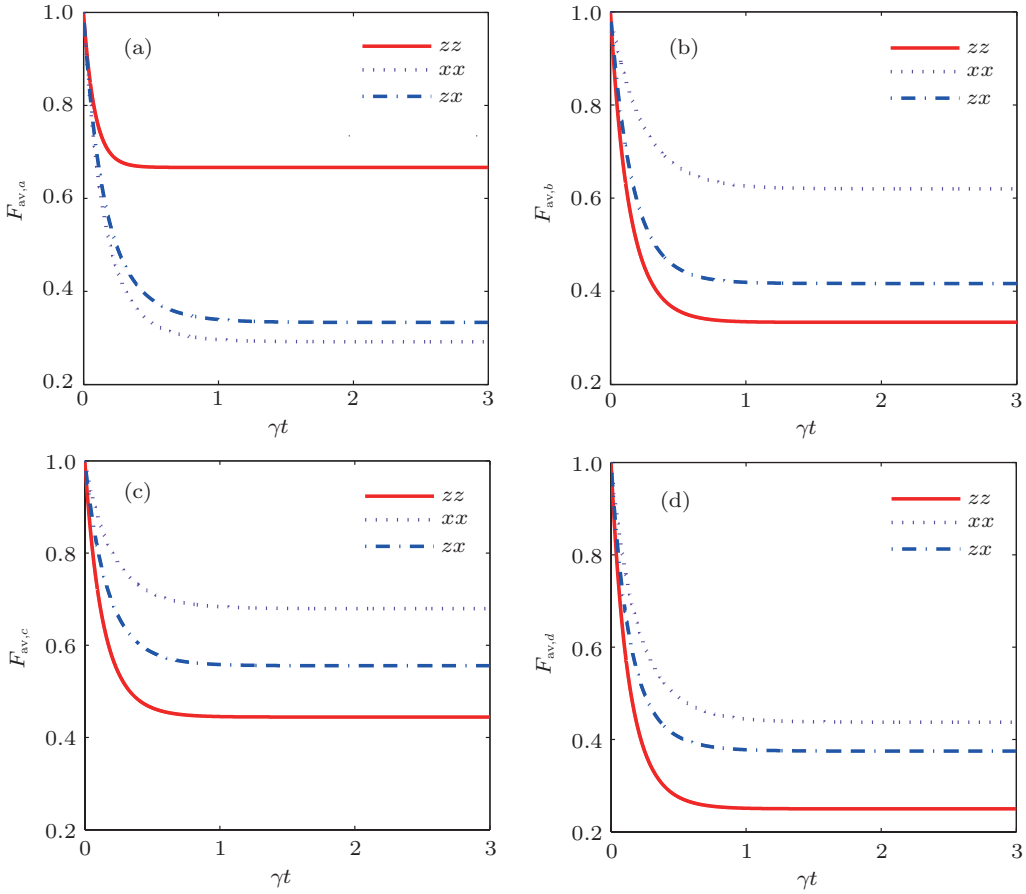


Fig. 3. (color online) Corresponding to JRSP of four types of specially chosen two-qubit input states, the average fidelity under the influence of Pauli noises is plotted as a function of γt. Here, the solid and dotted lines denote the two GHZ states infected by the dephasing noise and the bit-flip noise, respectively, while the dash–dotted line denotes the situation of one GHZ state infected by the dephasing noise and the other by the bit-flip noise.

4. Conclusions

In conclusion, by employing two tripartite GHZ states as the entangled channels, we have investigated the influence of Pauli noises on the deterministic JRSP of an arbitrary two-qubit pure state and of four types of special initial two-qubit states. Based on the decomposition method of arbitrarily unitary operations, we have first constructed the quantum circuit of the deterministic JRSP protocol[33] with a combination of single-qubit logic gates and two-qubit controlled-rotation gates. It deserves emphasizing that, when compared with the protocol that was proposed in Ref. [33], in our quantum circuit Charlie required less unitary operations due to Bob performing different local unitary operations on his two particles. After obtaining the time evolution of the tripartite GHZ states affected by the dephasing noise and by the bit-flip noise, we have then analytically derived the fidelity and the corresponding average fidelity for the JRSP protocol under various noisy environments. It is found that the fidelity depends on the types of noises acting on the quantum channels, the qubit-environment coupling strength, and the state to be remotely prepared. Moreover, for JRSP with different initial states of two qubits, there exhibit different time evolution actions for the average fidelities, even if the two GHZ states are infected by identical Pauli noises. Furthermore, for JRSP with the same initial two-qubit states, the average fidelities under the action of different noisy environments also display different evolution behaviors. It is also found that, in the long-time limit, when compared with the cases for two GHZ states affected by the dephasing noise as well as for one GHZ state affected by the dephasing noise and the other by the bit-flip noise, the bit-flip noise acting on two GHZ states has the weakest influence on the average fidelities $F_{\mathrm{av},b}$, $F_{\mathrm{av},c}$, and $F_{\mathrm{av},d}$ but it has the strongest influence on the average fidelities $F_{\mathrm{av},a}$ and F_{av}. Thus, we hope that our results may shed light on the design of quantum communication protocols in order to improve the fidelities of their initial state transmission.

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