Resonant interaction scheme for GHZ state preparation and quantum phase gate with superconducting qubits in a cavity*

Liu Xin(刘 欣)[a], Liao Qing-Hong(廖庆洪)[b], Fang Guang-Yu(方光宇)[a], Wang Yue-Yuan(王月媛)[c], and Liu Shu-Tian(刘树田)[a]†

[a] *Department of Physics, Harbin Institute of Technology, Harbin* 150001, *China*
[b] *Department of Electronic Information Engineering, Nanchang University, Nanchang* 330031, *China*
[c] *Key Laboratory for Advanced Functional Materials and Excited State Process of Heilongjiang Province, School of Physics and Electronic Engineering, Harbin Normal University, Harbin* 150025, *China*

(Received 26 April 2013; revised manuscript received 22 July 2013; published online 20 December 2013)

A scheme is proposed to generate GHZ state and realize quantum phase gate for superconducting qubits placed in a microwave cavity. This scheme uses resonant interaction between the qubits and the cavity mode, so that the interaction time is short, which is important in view of decoherence. In particular, the phase gate can be realized simply with a single interaction between the qubits and the cavity mode. With cavity decay being considered, the fidelity and success probability are both very close to unity.

Keywords: superconducting qubits, GHZ state, phase gate

PACS: 03.65.Ud, 03.67.Lx, 85.25.Cp **DOI:** 10.1088/1674-1056/23/2/020311

1. Introduction

Superconducting qubits are one of the promising candidates for the implementation of quantum information processing.[1–4] Over the past few years, various approaches for generating entangled states and realizing logical gates in superconducting qubits have been proposed in the literature. Several important experimental milestones have been achieved.[5–8] In some proposals, qubits are coupled directly via a capacitor or an insulator[9–12] or using additional elements.[13–15] Liu *et al.*[9] proposed a scheme to couple large detuning flux qubits with time-dependent magnetic flux in appropriate frequency. ISWAP gate could then be realized by XY coupling. Based on the so-called FLICFORQ (Fixed Linear Couplings between Fixed Off-Resonant Qubits) architecture,[10] Rigetti *et al.* developed a microwave scheme[11] to realize universal gates with tunable effective interaction, which has recently been experimentally demonstrated.[16] The flexibility of superconducting qubits has provided various coupling styles. In some other proposals, qubits are coupled through a 'quantum bus', such as a microwave cavity or a one-dimensional superconducting transmission line,[17–21] composing the cavity QED architecture, which has been well studied in atom cavity QED systems.[22–26] In Ref. [24], Zheng proposed a fast scheme to realize quantum phase gates for atoms by using an assistant atom state. Blais *et al.*[19] investigated five types of gates including schemes based on virtual photons, real excitation of the resonator and geometric phase for two charge qubits, which were coupled to a superconducting transmission line. Quantum states transferring between two superconducting qubits were then demonstrated experimentally.[8] In Ref. [20], Wang *et al.* proposed to generate GHZ states with an effective XX coupling. However, implementing entangled states and logical gates with high fidelity is still under pursuit.

In this paper, we propose an alternative scheme to realize GHZ state and quantum phase gate based on the interaction of flipping two superconducting qubits simultaneously with one photon. The three-qubit GHZ state is composed of two charge qubits and a flux qubit, which are based on a pair of conjugate degrees of freedom. In the generation of GHZ state, no auxiliary energy level is needed. Moreover, our scheme is deterministic in the absence of decoherence, instead of the scheme in a probabilistic way.[27,28] The phase gate is realized between two charge qubits in a cavity. In contrast to previous schemes working in the dispersive limit,[18,29,30] which leads to a long operation time, we construct the GHZ state and phase gate with resonant interaction. The situation with cavity decay is also considered.

This paper is organized as follows. In Section 2, we propose a scheme for realizing GHZ state and quantum phase gate. In Section 3, we discuss the proposed scheme with the cavity decay being considered. A conclusion appears in Section 4.

2. Generation of GHZ state and realization of quantum phase gate

Consider a system composed of three superconducting qubits coupled to a single-mode cavity field (assuming that

*Project supported by the National Basic Research Program of China (Grant No. 2013CBA01702).
†Corresponding author. E-mail: stliu@hit.edu.cn
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all other cavity modes are well decoupled to the two levels of the qubits), a schematic diagram of the system is shown in Fig. 1. Two superconducting charge qubits 1 and 2 share a large Josephson junction on chip, and are biased by a static magnetic flux Φ_{ec}.[15] Qubit 3 is a SQUID qubit (it also can be a persistent-current qubit working on its degeneracy point[2]) biased by a static magnetic flux Φ_{ef} and manipulated by classical microwave pulses. The Hamiltonian of the system can be written as

$$H = H_c + H_q + H_I, \tag{1}$$

where H_c, H_q, and H_I are the Hamiltonian of the cavity field, the qubits, and the interaction energy, respectively.

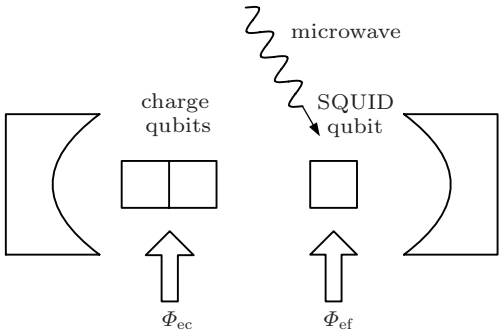


Fig. 1. Schematic setup of the scheme, where two coupled charge qubits and a SQUID qubit are placed in a microwave cavity and biased by static magnetic flux Φ_{ec} and Φ_{ef} respectively.

The Hamiltonian of the cavity field is

$$H_c = \hbar\omega\left(a^\dagger a + 1/2\right), \tag{2}$$

where $a^\dagger$ and a are creation and annihilation operators of the cavity mode.

The Hamiltonian of the qubits is expressed as[15,17]

$$\begin{aligned} H_q = & \sum_{i=1,2}\left[E_i(V_{Xi}) - 2E_{Ji}\cos\left(\frac{\pi\Phi_{ec}}{\Phi_0} - \frac{\gamma}{2}\right)\cos\varphi_i\right] \\ & - E_{J0}\cos\gamma + \frac{{Q_3}^2}{2C_3} + \frac{(\Phi_3 - \Phi_{ef})^2}{2L_3} \\ & - E_{J3}\cos\left(2\pi\frac{\Phi_3}{\Phi_0}\right), \end{aligned} \tag{3}$$

where

$$E_i(V_{Xi}) = E_{ci}(n_i - C_i V_{Xi}/2e)^2.$$

E_{ci} and E_{Ji} are the charge and Josephson energies of the i-th qubit. E_{J0} is the Josephson energy of the large Josephson junction.

Here, we illustrate our proposal for generating the GHZ state in three steps.

Step 1 First, after applying an ac voltage, adjust the level spacings of two charge qubits so that two charge qubits' frequencies ω_1 and ω_2 satisfy the condition $\omega = \omega_1 + \omega_2$. The level spacing of the flux qubit is also adjusted to be far-off resonant with the cavity field (in this case, there are only interactions between the two charge qubits and the cavity field). Then, in the interaction picture the interaction Hamiltonian can be written as[15] (setting $\hbar = 1$ below)

$$H_{I1} = \xi\left(a^\dagger\sigma_-^{(1)}\sigma_-^{(2)} + a\sigma_+^{(1)}\sigma_+^{(2)}\right), \tag{4}$$

in the spin-1/2 representation. Here $\sigma_+^{(i)} = |1\rangle_i\langle 0|$, $\sigma_-^{(i)} = |0\rangle_i\langle 1|$ with the charge states $|0\rangle$ and $|1\rangle$ correspond to zero and one excess Cooper pair in each Cooper-pair box. Notice that, under this Hamiltonian, the two qubits can be flipped simultaneously with one photon.

Assume that the cavity mode is initially in the vacuum state $|0\rangle_c$, and the two qubits 1 and 2 are initially in the state $|1\rangle_1|1\rangle_2$. The evolution of the two charge qubits and the cavity field under Hamiltonian (4) is expressed as

$$\begin{aligned} |\Psi(\tau_1)\rangle_{1,2,c} = & -\mathrm{i}\sin(\xi\tau_1)|0\rangle_1|0\rangle_2|1\rangle_c \\ & + \cos(\xi\tau_1)|1\rangle_1|1\rangle_2|0\rangle_c. \end{aligned} \tag{5}$$

With the choice of $\xi\tau_1 = \pi/4$, we obtain

$$|\Psi\rangle_{1,2,c} = -\mathrm{i}\frac{1}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2|1\rangle_c + \mathrm{i}|1\rangle_1|1\rangle_2|0\rangle_c\right). \tag{6}$$

Step 2 Adjust the level spacings of two charge qubits so that the cavity field is far-off resonant with ω_1, ω_2, $\omega_1 + \omega_2$, and $\omega_1 - \omega_2$.[15] The level spacing of the flux qubit is simultaneously brought to resonance with cavity mode. The interaction Hamiltonian in the interaction picture is[17]

$$H_{I2} = g\left(a\sigma_+^{(3)} + a^\dagger\sigma_-^{(3)}\right), \tag{7}$$

where $\sigma_+^{(3)} = |1\rangle_3\langle 0|$ and $\sigma_-^{(3)} = |0\rangle_3\langle 1|$ with $|0\rangle_3$ and $|1\rangle_3$ being the ground and second excited states of qubit 3.

If qubit 3 is initially in the ground state $|0\rangle_3$, then the three qubits and the cavity field are in the state

$$|\Psi\rangle_{1,2,3,c} = -\mathrm{i}\frac{1}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2|0\rangle_3|1\rangle_c + \mathrm{i}|1\rangle_1|1\rangle_2|0\rangle_3|0\rangle_c\right). \tag{8}$$

Under the Hamiltonian (7), the state $|0\rangle_3|0\rangle_c$ keeps unchanged while the state $|0\rangle_3|1\rangle_c$ evolves as

$$|\varphi(\tau_2)\rangle_{3,c} = \cos(g\tau_2)|0\rangle_3|1\rangle_c - \mathrm{i}\sin(g\tau_2)|1\rangle_3|0\rangle_c. \tag{9}$$

If the interaction time is chosen to fulfill $g\tau_2 = \pi/2$, then the state becomes $|\varphi\rangle_{3,c} = -\mathrm{i}|1\rangle_3|0\rangle_c$. The state of the three qubits and the cavity field is

$$|\Psi\rangle_{1,2,3,c} = -\frac{1}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2|1\rangle_3 - |1\rangle_1|1\rangle_2|0\rangle_3\right)|0\rangle_c. \tag{10}$$

Step 3 Decouple all three qubits from the cavity field, and apply a resonant classical microwave pulse ($\omega_m = \omega_3$) to

qubit 3. The interaction Hamiltonian (between qubit 3 and the cavity field) in the interaction picture is[17]

$$H_{\mathrm{I3}} = \Omega\left(|1\rangle_3\langle 0| + |0\rangle_3\langle 1|\right). \tag{11}$$

Based on Eq. (11), it is easy to get the following state rotation

$$\begin{aligned} |0\rangle_3 &\to \cos\Omega\tau_3|0\rangle_3 - \mathrm{i}\sin\Omega\tau_3|1\rangle_3, \\ |1\rangle_3 &\to -\mathrm{i}\sin\Omega\tau_3|0\rangle_3 + \cos\Omega\tau_3|1\rangle_3. \end{aligned} \tag{12}$$

If the interaction time is chosen to satisfy $\Omega\tau_3 = \pi/2$, then it turns out to be

$$\begin{aligned} |0\rangle_3 &\to -\mathrm{i}|1\rangle_3, \\ |1\rangle_3 &\to -\mathrm{i}|0\rangle_3. \end{aligned} \tag{13}$$

Now the three qubits and the cavity are in the state

$$|\Psi\rangle_{1,2,3,\mathrm{c}} = \frac{1}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2|0\rangle_3 - |1\rangle_1|1\rangle_2|1\rangle_3\right)|0\rangle_{\mathrm{c}}, \tag{14}$$

where the common phase factor i has been discarded. Thus, a GHZ state of three qubits

$$|\Psi\rangle_{1,2,3} = \frac{1}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2|0\rangle_3 - |1\rangle_1|1\rangle_2|1\rangle_3\right)$$

is obtained, leaving the cavity mode in the vacuum state.

Now, we illustrate the scheme for realizing quantum phase gate between two charge qubits. Based on Eq. (5), if the interaction time is chosen to satisfy $\xi\tau_{\mathrm{p}} = \pi$, then the initial state $|1\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}}$ evolves to $-|1\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}}$. On the other hand, the states $|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}}$, $|0\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}}$, and $|1\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}}$ do not undergo any transition since $H_{\mathrm{I1}}|1\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}} = H_{\mathrm{I1}}|0\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}} = H_{\mathrm{I1}}|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}} = 0$. Therefore, we have

$$\begin{aligned} |0\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}} &\to |0\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}}, \\ |0\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}} &\to |0\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}}, \\ |1\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}} &\to |1\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}}, \\ |1\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}} &\to -|1\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}}. \end{aligned} \tag{15}$$

In this way, a quantum phase gate is obtained for the two charge qubits with a cavity mode left in the vacuum state.

3. Scheme including cavity decay

We now take into account the cavity decay and show how the fidelity and success probability of the scheme are affected.

First, the preparation of the GHZ state with cavity decay is shown below. Suppose that, during the preparation process, there is no photon detected due to the leakage of a photon through the cavity mirrors. In the first step, the evolution of the system is governed by the conditional Hamiltonian

$$H_{\mathrm{con1}} = \xi\left(a^\dagger\sigma_-^{(1)}\sigma_-^{(2)} + a\sigma_+^{(1)}\sigma_+^{(2)}\right) - \mathrm{i}\frac{\kappa}{2}a^\dagger a, \tag{16}$$

where κ is the cavity decay rate. If the system is initially in the state $|1\rangle_1|1\rangle_2|0\rangle_3|0\rangle_{\mathrm{c}}$, then the evolution of the system can be written as

$$\begin{aligned} &\left|\Psi^1\left(\tau_1{}'\right)\right\rangle \\ &= \mathrm{e}^{-\kappa\tau_1{}'/4}\left[\cos\left(\alpha\tau_1{}'\right) + \frac{\kappa}{4\alpha}\sin\left(\alpha\tau_1{}'\right)\right]|1\rangle_1|1\rangle_2|0\rangle_3|0\rangle_{\mathrm{c}} \\ &\quad -\mathrm{i}\,\mathrm{e}^{-\kappa\tau_1{}'/4}\frac{\xi}{\alpha}\sin\left(\alpha\tau_1{}'\right)|0\rangle_1|0\rangle_2|0\rangle_3|1\rangle_{\mathrm{c}}, \end{aligned} \tag{17}$$

where

$$\alpha = \left(\xi^2 - \frac{\kappa^2}{16}\right)^{1/2}. \tag{18}$$

If the interaction time is chosen to satisfy $\tau_1{}' = \pi/4\alpha$, then the state of the system reads

$$\begin{aligned} \left|\Psi^1\right\rangle &= \frac{1}{\sqrt{2}}\mathrm{e}^{-\pi\kappa/(16\alpha)}\left[\left(1+\frac{\kappa}{4\alpha}\right)|1\rangle_1|1\rangle_2|0\rangle_3|0\rangle_{\mathrm{c}}\right. \\ &\quad \left. -\mathrm{i}\frac{\xi}{\alpha}|0\rangle_1|0\rangle_2|0\rangle_3|1\rangle_{\mathrm{c}}\right]. \end{aligned} \tag{19}$$

In the second step, the evolution of the system is dominated by the conditional Hamiltonian

$$H_{\mathrm{con}}^2 = g\left(a\sigma_+^{(3)} + a^\dagger\sigma_-^{(3)}\right) - \mathrm{i}\frac{\kappa}{2}a^\dagger a. \tag{20}$$

So, during the second period time $\tau_2{}'$, the evolution of the system can be written as

$$\begin{aligned} &\left|\Psi^2\left(\tau_2{}'\right)\right\rangle \\ &= \frac{1}{\sqrt{2}}\mathrm{e}^{-\pi\kappa/(16\alpha)}\left\{-\mathrm{i}\frac{\xi}{\alpha}\mathrm{e}^{-\kappa\tau_2{}'/4}\right. \\ &\quad \times\left[\cos\left(\beta\tau_2{}'\right) - \frac{\kappa}{4\beta}\sin\left(\beta\tau_2{}'\right)\right]|0\rangle_1|0\rangle_2|0\rangle_3|1\rangle_{\mathrm{c}} \\ &\quad -\mathrm{e}^{-\kappa\tau_2{}'/4}\frac{g\xi}{\alpha\beta}\sin\left(\beta\tau_2{}'\right)|0\rangle_1|0\rangle_2|1\rangle_3|0\rangle_{\mathrm{c}} \\ &\quad \left.+\left(1+\frac{\kappa}{4\alpha}\right)|1\rangle_1|1\rangle_2|0\rangle_3|0\rangle_{\mathrm{c}}\right\}, \end{aligned} \tag{21}$$

where

$$\beta = \left(g^2 - \frac{\kappa^2}{16}\right)^{1/2}. \tag{22}$$

If the interaction time is taken to be $\tau_2{}' = \left[\tan^{-1}\left(4\beta/\kappa\right)\right]/\beta$, then we get the state

$$\begin{aligned} \left|\Psi^2\right\rangle &= \frac{1}{\sqrt{2}}\mathrm{e}^{-\pi\kappa/(16\alpha)} \\ &\quad \times\left[-\mathrm{e}^{-\kappa\tau_2{}'/4}\frac{g\xi}{\alpha\beta}\sin\left(\beta\tau_2{}'\right)|0\rangle_1|0\rangle_2|1\rangle_3|0\rangle_{\mathrm{c}}\right. \\ &\quad \left.+\left(1+\frac{\kappa}{4\alpha}\right)|1\rangle_1|1\rangle_2|0\rangle_3|0\rangle_{\mathrm{c}}\right]. \end{aligned} \tag{23}$$

In the third step, the evolution of the cavity field state is dominated by the conditional Hamiltonian

$$H_{\mathrm{con}}^3 = -\mathrm{i}\frac{\kappa}{2}a^\dagger a. \tag{24}$$

This Hamiltonian produces no effect on the vacuum state, so the state evolves to

$$\begin{aligned}|\Psi^3\rangle &= -\mathrm{i}\frac{1}{\sqrt{2}}\mathrm{e}^{-\pi\kappa/(16\alpha)} \\ &\times\left[-\mathrm{e}^{-\kappa\tau_2'/4}\frac{g\xi}{\alpha\beta}\sin(\beta\tau_2')\,|0\rangle_1|0\rangle_2|0\rangle_3\right. \\ &\left.+\left(1+\frac{\kappa}{4\alpha}\right)|1\rangle_1|1\rangle_2|1\rangle_3\right]|0\rangle_{\mathrm{c}}. \end{aligned} \tag{25}$$

After normalization, the state of the three qubits is

$$\begin{aligned}|\Psi'\rangle_{1,2,3} &= \frac{1}{N}\left[\mathrm{e}^{-\kappa\tau_2'/4}\frac{g\xi}{\alpha\beta}\sin(\beta\tau_2')\,|0\rangle_1|0\rangle_2|0\rangle_3\right. \\ &\left.-\left(1+\frac{\kappa}{4\alpha}\right)|1\rangle_1|1\rangle_2|1\rangle_3\right], \end{aligned} \tag{26}$$

where N is a normalization factor given by

$$N=\left[\mathrm{e}^{-\kappa\tau_2'/2}\left(\frac{g\xi}{\alpha\beta}\right)^2\sin^2(\beta\tau_2')+\left(1+\frac{\kappa}{4\alpha}\right)^2\right]^{1/2}. \tag{27}$$

The fidelity is

$$\begin{aligned}F_{\mathrm{s}} &= \left|\langle\Psi'|\Psi\rangle_{1,2,3}\right|^2 \\ &= \frac{1}{2N^2}\left[\mathrm{e}^{-\kappa\tau_2'/4}\frac{g\xi}{\alpha\beta}\sin(\beta\tau_2')+\left(1+\frac{\kappa}{4\alpha}\right)\right]^2. \end{aligned} \tag{28}$$

The probability of success is

$$\begin{aligned}P_{\mathrm{s}} &= \frac{1}{2}\mathrm{e}^{-\pi\kappa/(8\alpha)}\left[\left(1+\frac{\kappa}{4\alpha}\right)^2\right. \\ &\left.+\frac{\mathrm{e}^{-\kappa\tau_2'/2}g^2\xi^2\sin^2(\beta\tau_2')}{\alpha^2\beta^2}\right]. \end{aligned} \tag{29}$$

We now take into account the cavity decay in the scheme of quantum phase gate. Based on Eq. (17), for $\alpha\tau_{\mathrm{p}}{}' = \pi$, the initial state evolves to $-\mathrm{e}^{-\kappa\pi/(4\alpha)}|1\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}}$. Then, we have

$$\begin{aligned}&|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}} \to |0\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}}, \\ &|0\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}} \to |0\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}}, \\ &|1\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}} \to |1\rangle_1|0\rangle_2|0\rangle_{\mathrm{c}}, \\ &|1\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}} \to -\mathrm{e}^{-\kappa\pi/(4\alpha)}|1\rangle_1|1\rangle_2|0\rangle_{\mathrm{c}}. \end{aligned} \tag{30}$$

Suppose that the two charge qubits are initially in the state

$$|\varphi\rangle=\frac{1}{2}(|0\rangle_1+|1\rangle_1)(|0\rangle_2+|1\rangle_2). \tag{31}$$

The ideal phase gate produces the maximally entangled state

$$|\varphi_i\rangle=\frac{1}{2}[|0\rangle_1(|0\rangle_2+|1\rangle_2)+|1\rangle_1(|0\rangle_2-|1\rangle_2)], \tag{32}$$

while the conditional Hamiltonian results in (after normalization)

$$\begin{aligned}|\varphi_{\mathrm{con}}\rangle &= \frac{1}{\sqrt{3+\mathrm{e}^{-\kappa\pi/(2\alpha)}}}\left[|0\rangle_1(|0\rangle_2+|1\rangle_2)\right. \\ &\left.+|1\rangle_1(|0\rangle_2-\mathrm{e}^{-\kappa\pi/(4\alpha)}|1\rangle_2)\right]. \end{aligned} \tag{33}$$

The fidelity is given by

$$F_{\mathrm{p}}=|\langle\varphi_{\mathrm{con}}\mid\varphi_i\rangle|^2=\frac{\left[3+\mathrm{e}^{-\kappa\pi/(4\alpha)}\right]^2}{4\left[3+\mathrm{e}^{-\kappa\pi/(2\alpha)}\right]}. \tag{34}$$

The probability of success is

$$P_{\mathrm{p}}=\frac{1}{4}\left(3+\mathrm{e}^{-\kappa\pi/(2\alpha)}\right). \tag{35}$$

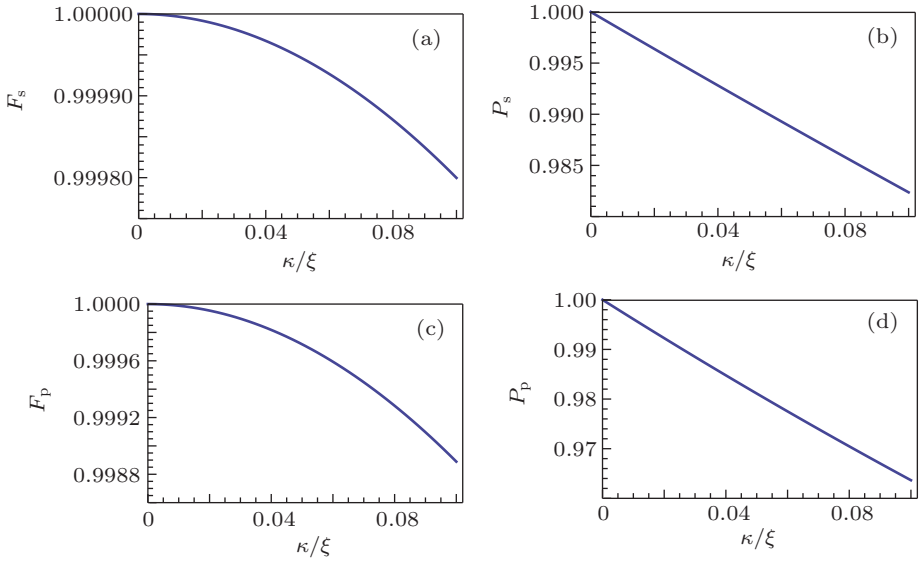


Fig. 2. (color online) Dependence of the fidelity and success probability on the cavity mode decay κ/ξ. The coupling strength ratio is $g=10\xi$. The scale of the independent variable is set to be $0\le\kappa/\xi\le 0.1$. (a) The fidelity (F_{s}) of GHZ state. (b) The probability (P_{s}) of GHZ state. (c) The fidelity (F_{p}) of the quantum phase gate. (d) The probability (P_{p}) of the quantum phase gate.

Figures 2(a) and 2(b) show the fidelity (F_s) and success probability (P_s) of the generated GHZ state with κ/ξ, respectively, after setting $g = 10\xi$. Figures 2(c) and 2(d) show the fidelity (F_p) and success probability (P_p) of the quantum phase gate with κ/ξ. The scale of the independent variable is set to be $0 \leq \kappa/\xi \leq 0.1$. For a SQUID, the typical transition frequency between the ground state and the second excited state is ~ 49.2 GHz.[30] Hence, we choose $\nu_c = 49.2$ GHz as the cavity mode frequency. The typical coupling constant ξ can reach about several hundred MHz[9] and $g \sim 1.2$ GHz[30] (here, we choose $\xi = g/10 = 120$ MHz for simplicity). For a cavity with $Q = 1 \times 10^5$, which can be achieved for the on-chip superconducting cavity under the recent experimental condition,[31] the decay rate should be $\kappa \simeq 0.0260\xi$, it can then be calculated that $F_s = 0.999986$, $P_s = 0.9953$, $F_p = 0.999923$ and $P_p = 0.9900$. Furthermore, we calculate the interaction time to be $\tau_1' = \pi/4\alpha \simeq 6.5$ ns, $\tau_2' = \left[\tan^{-1}(4\beta/\kappa)\right]/\beta \simeq 1.3$ ns, and we choose the time $\tau_3' = 5\tau_1' = 32.5$ ns.

In the above discussion, the qubit decay is neglected respecting the quite long lifetimes that are about a microsecond for a charge qubit[32] and 1.6×10^{-5} s for an SQUID qubit[30] in a recent experiment. Consequently, we just consider the effect of a bad cavity.

4. Conclusion

In summary, we have proposed a scheme to generate GHZ state and realize quantum phase gate for superconducting qubits in a cavity. The scheme uses resonant interaction and the GHZ state can be generated in a fast way. As the implementation of quantum phase gate only requires a single resonant interaction, the required time is very short. In principle, we can obtain the required state and gate with the fidelity and success probability of unity. Moreover, taking into account the cavity decay, the fidelity and success probability of the generated state and the phase gate are very close to unity.

References

[1] Makhlin Y, Schon G and Shnirman A 2001 *Rev. Mod. Phys.* **73** 357
[2] Orlando T P, Mooij J E, Tian L, Wal C H, Levitov L S, Lloyd S and Mazo J J 1999 *Phys. Rev. B* **60** 15398
[3] You J Q and Nori F 2005 *Phys. Today* **58** 42
[4] Clarke J and Wilhelm F K 2008 *Nature* **453** 1031
[5] Pashkin Y A, Yamamoto T, Astafiev O, Nakamura Y, Averin D V and Tsai J S 2003 *Nature* **421** 823
[6] Majer J B, Paauw F G, Haar A C G, Harmans C J P M and Mooij J E 2005 *Phys. Rev. Lett.* **94** 090501
[7] Niskanen A O, Harrabi K, Yoshihara F, Nakamura Y, Lloyd S and Tsai J S 2007 *Science* **316** 723
[8] Majer J, Chow J M, Gambetta J M, Koch J, Johnson B R, Schreier J A, Frunzio L, Schuster D I, Houck A A, Wallraff A, Blais A, Devoret M H, Girvin S M and Schoelkopf R J 2007 *Nature* **449** 443
[9] Liu Y X, Wei L F, Tsai J S and Nori F 2006 *Phys. Rev. Lett.* **96** 067003
[10] Rigetti C, Blais A and Devoret M 2005 *Phys. Rev. Lett.* **94** 240502
[11] Rigetti C and Devoret M 2010 *Phys. Rev. B* **81** 134507
[12] Tanamoto T, Liu Y X, Fujita S, Hu X D and Nori F 2006 *Phys. Rev. Lett.* **97** 230501
[13] Niskanen A O, Nakamura Y and Tsai J S 2006 *Phys. Rev. B* **73** 094506
[14] You J Q, Tsai J S and Nori F 2003 *Phys. Rev. B* **68** 024510
[15] He X L, Liu Y X, You J Q and Nori F 2007 *Phys. Rev. A* **76** 022317
[16] Chow J M, Corcoles A D, Gambetta J M, Rigetti C, Johnson B R, Smolin J A, Rozen J R, Keefe G A, Rothwell M B, Ketchen M B and Steffen M 2011 *Phys. Rev. Lett.* **107** 080502
[17] Yang C P and Chu S I 2003 *Phys. Rev. A* **67** 042311
[18] Blais A, Huang R S, Wallraff A, Girvin S M and Schoelkopf R J 2004 *Phys. Rev. A* **69** 062320
[19] Blais A, Gambetta J, Wallraff A, Schuster D I, Girvin S M, Devoret M H and Schoelkopf R J 2007 *Phys. Rev. A* **75** 032329
[20] Wang Y D, Chesi S, Loss D and Bruder C 2010 *Phys. Rev. B* **81** 104524
[21] Shi Z G, Chen X W, Zhu X X and Song K H 2009 *Chin. Phys. B* **18** 910
[22] Li Y L and Fang M F 2011 *Chin. Phys. B* **20** 050314
[23] Guo G C and Zhang Y S 2002 *Phys. Rev. A* **65** 054302
[24] Zheng S B 2005 *Phys. Rev. A* **71** 062335
[25] Li Y L, Fang M F and Zeng K 2010 *Chin. Phys. B* **19** 010307
[26] Liao C G, Chen Z H and Luo C L 2010 *Acta Phys. Sin.* **59** 8526 (in Chinese)
[27] Gong S Q, Wang Z Y, Feng X L and Xu Z Z 2000 *Chin. Phys.* **9** 94
[28] Yang Z B 2007 *Chin. Phys.* **16** 1963
[29] Song K H, Zhou Z W and Guo G C 2005 *Phys. Rev. A* **71** 052310
[30] Yang C P and Han S 2004 *Phys. Rev. A* **70** 062323
[31] Schoelkopf R J and Girvin S M 2008 *Nature* **451** 664
[32] Houck A A, Schreier J A, Johnson B R, Chow J M, Koch J, Gambetta J M, Schuster D I, Frunzio L, Devoret M H, Girvin S M and Schoelkopf R J 2008 *Phys. Rev. Lett.* **101** 080502