Correlation dynamics of a qubit–qutrit system in a spin-chain environment with Dzyaloshinsky–Moriya interaction*

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We study the dynamics of correlations for a hybrid qubit–qutrit system in an XY spin-chain environment with Dzyaloshinsky–Moriya interaction. Our discussion involves a comparative analysis of negativity, quantum discord, and measurement-induced disturbance. It is found that the quantum discord is optimal of the three quantum correlations to detect the critical point of quantum phase transition. Only when the qubit interacts with the environment, is the phenomenon of sudden transition between the classical correlation and the quantum discord observed. Moreover, the Dzyaloshinsky–Moriya interaction enhances the decay of quantum correlations.

Keywords: quantum correlation, quantum discord, measurement-induced disturbance, Dzyaloshinsky–Moriya interaction

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1. Introduction

Quantum correlation has attracted considerable attention from researchers and significant progress has been achieved. Quantum correlation without entanglement has been applied to implement some quantum tasks; for example, deterministic quantum computation with one pure qubit (DQC1),[1,2] quantum phase transitions (QPTs),[3] Grover's search,[4,5] and quantum remote state preparation.[6] This implies that entanglement is just one special kind of quantum correlation. To characterize the quantum correlation, quantum discord has been introduced by Olliver and Zurek.[7] However, the calculation of the quantum discord requires an optimization measurement procedure, and analytical results are only obtained in a few restricted cases.[8–10] In order to overcome this difficulty, Luo introduced measurement-induced disturbance (MID) to characterize correlations as classical or quantum case.[11]

In the real world, a quantum system will be unavoidably affected by its surrounding environment and lose coherence. The dynamics of quantum correlation under the influence of Markovian or non-Markovian environment have previously been investigated.[12–19] Entanglement may exhibit a sudden death, while quantum correlations vanish at asymptotic time.[20–23] In other words, quantum discord is more robust than entanglement against decoherence.

However, in the previous research, most of the works consider only the two-qubit system. We know that the bipartite system with higher dimension can improve the efficiency of quantum information processing.[24] So, in this paper, we will study the dynamics of correlations for a hybrid qubit–qutrit system in an XY spin-chain environment with Dzyaloshinsky–Moriya (DM) interaction. The DM interaction is an antisymmetric spin–orbit coupling and is often present in models of magnetic materials.[25–27] We will consider the dynamical behaviors of the quantum correlation as quantified by negativity, quantum discord, and MID. By comparing analyses of these quantum correlations, we find that the quantum discord is the optimal of these three measures, and it can always detect the existence of QPTs. We also find that the sudden transition between the quantum discord and the classical correlation only occurs when the qubit couples to the environment. DM interaction does not affect the critical point of QPTs and it only enhances the decay of quantum correlation.

This paper is organized as follows. In Section 2, we introduce the spin chain environment. In Section 3, we review the different measures of quantum correlation that are used in our discussion. In Section 4, we discuss the dynamics of these correlation measures for one class of initial states. In Section 5, we give a summary of our results.

2. Model

In this paper, we consider a composite system of noninteracting spin-1/2 and spin-1 transversely coupled with an XY spin-chain with DM interaction environment. The total Hamiltonian is $H = H_{\rm E} + H_{\rm I}$, where

$$H_{\rm E} = \sum_{i=1}^{N_t} \left[\frac{1+\gamma}{2}\sigma_i^x\sigma_{i+1}^x + \frac{1-\gamma}{2}\sigma_i^y\sigma_{i+1}^y + \lambda\sigma_i^z + D_z\cdot(\sigma_i^x\sigma_{i+1}^y - \sigma_i^y\sigma_{i+1}^x)\right], \quad (1)$$

$$H_{\rm I} = \sum_{i=1}^{N_t} \sigma_i^z g(\sigma_A^z + f s_B^z), \quad (2)$$

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$H_{\rm E}$ and $H_{\rm I}$ denote the Hamiltonian of the spin chain and the interaction between the system and the environment, respectively. σ_i^{α} $(\alpha = x,y,z)$ are the familiar Pauli matrices representing the i-th site and s_B^z is spin-1 operator along the z direction for B. The number of the spins in the chain is N_t. γ denotes the anisotropy. λ describes the strength of the transverse field and D_z denotes the intensity of the DM interaction imposed along the z direction. g characterizes the coupling strength of the central atoms with its spin chain. If $g \ll \lambda_{\rm c} = 1$, it belongs to the weak couplings. In this paper, we will only discuss the weak couplings case. The parameter f exhibits the asymmetry of strength between A and B coupled to the environment, respectively.

Based on the states $\{|00\rangle,|01\rangle,|02\rangle,|10\rangle,|11\rangle,|12\rangle\}$, $H = \sum_{\mu}|\chi_{\mu}\rangle\langle\chi_{\mu}| \otimes H_{\rm E}^{\lambda_{\mu}}$, $H_{\rm E}^{\lambda_{\mu}} = \sum_{i=1}^{N_t}((1+\gamma)/2\sigma_i^x\sigma_{i+1}^x + (1-\gamma)/2\sigma_i^y\sigma_{i+1}^y + \lambda_{\mu}\sigma_i^z)$, where $\lambda_1 = \lambda + g(1+f)$, $\lambda_2 = \lambda + g$, $\lambda_3 = \lambda + g(1-f)$, $\lambda_4 = \lambda - g(1-f)$, $\lambda_5 = \lambda - g, \lambda_6 = \lambda - g(1+f)$.

Using, successively, Jordan–Wigner, Frouier, and Bogoliubov transforms, $H_{\rm E}^{\lambda_{\mu}}$ can be diagonalized as[28,29]

$$H_{\rm E}^{\lambda_{\mu}} = \sum_k E_k^{\lambda_{\mu}}\left(b_{k,\lambda_{\mu}}^{\dagger}b_{k,\lambda_{\mu}} - \frac{1}{2}\right), \tag{3}$$

where the energy spectrum $E_k^{\lambda_{\mu}}$ reads

$$E_k^{\lambda_{\mu}} = 2\sqrt{\varepsilon_{\mu}^2 + \gamma^2\sin^2(2\pi k/N_t)} + 4D_z\sin(2\pi k/N_t), \tag{4}$$

with $\varepsilon_{\mu} = \lambda_{\mu} - \cos(2\pi k/N_t)$, $k = -M,\ldots,M, M = (N_t-1)/2$.

The initial state of the total system can be written as $\rho_s(0)\otimes|G_{\lambda}\rangle\langle G_{\lambda}|$, where $|G_{\lambda}\rangle$ is the ground state of $H_{\rm E}^{(\lambda)}$ and $\rho_s(0) = \sum_{\mu,\nu}c_{\mu\nu}|\chi_{\mu}\rangle\langle\chi_{\nu}|$ $(\mu,\nu = 1,\ldots,6)$ is the initial state of the qubit–qutrit system. The evolved reduced density matrix of the system can be obtained as

$$\begin{aligned}\rho_s(t) &= {\rm Tr}_{\rm E}[{\rm e}^{-{\rm i}Ht}\rho_s(0)\otimes|G_{\lambda}\rangle\langle G_{\lambda}|\,{\rm e}^{{\rm i}Ht}]\\ &= \sum_{\mu,\nu}c_{\mu\nu}F_{\mu\nu}(t)\,|\chi_{\mu}\rangle\langle\chi_{\nu}|,\end{aligned} \tag{5}$$

where $F_{\mu\nu}(t) = \langle G_{\lambda}|\,{\rm e}^{{\rm i}H^{\lambda_{\mu}}t}\,{\rm e}^{-{\rm i}H^{\lambda_{\nu}}t}\,|G_{\lambda}\rangle$, whose specific expression is

$$\begin{aligned}|F_{\mu\nu}(t)| &= \Pi_{k=0}^{M}[1 - 4\sin(2\Theta_k^{\lambda_{\mu}})\sin(2\Theta_k^{\lambda_{\nu}})\sin^2(\Theta_k^{\lambda_{\mu}} - \Theta_k^{\lambda_{\nu}})\\ &\times \sin^2(E_k^{\lambda_{\mu}}t)\sin^2(E_k^{\lambda_{\nu}}t) + 2\sin(2\Theta_k^{\lambda_{\mu}})\sin(2\Theta_k^{\lambda_{\nu}})\\ &\times \sin(E_k^{\lambda_{\mu}}t)\sin(E_k^{\lambda_{\nu}}t)\cos(E_k^{\lambda_{\mu}}t - E_k^{\lambda_{\nu}}t)\\ &- \sin^2(2\Theta_k^{\lambda_{\mu}})\sin^2(E_k^{\lambda_{\mu}}t) - \sin^2(2\Theta_k^{\lambda_{\nu}})\\ &\times \sin^2(E_k^{\lambda_{\nu}}t)]^{1/2}.\end{aligned} \tag{6}$$

Here,

$$\Theta_k^{\lambda_{\mu}} = (\theta_k^{\lambda_{\mu}} - \theta_k^{\lambda})/2,$$
$$\theta_k^{\lambda_{\mu}} = \arctan\frac{\gamma\sin(2\pi k/N_t)}{\lambda_{\mu} - \cos(2\pi k/N_t)}.$$[25]

When $\mu = \nu$, the decoherence factor $|F_{\mu\nu}(t)|$ is equal to 1.

3. Measures of correlations

Before discussing correlations dynamics, we introduce the correlation measures used in this paper, namely: negativity, quantum discord, and MID.

Negativity is an entanglement measurement that can be computed effectively for a bipartite system of any dimension. All positive partial transposed (PPT) states of qubit–qubit and qubit–qutrit systems are separable.[30,31] So the qubit–qubit and qubit–qutrit entanglement is fully characterized by the negativity. The negativity of an arbitrary state ρ_{AB} is defined as

$$N(\rho_{AB}) = \sum_i(|\eta_i| - \eta_i), \tag{7}$$

where η_i is the eigenvalues of $\rho_{AB}^{{\rm T}_A}$ and $\rho_{AB}^{{\rm T}_A}$ is the partial transpose matrix of ρ_{AB}.

Quantum discord is defined as the difference generated by two definitions of mutual information in quantum theory,[7]

$$D(\rho) = I(\rho_{AB}) - C(\rho_{AB}), \tag{8}$$

where $I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$ and $C(\rho_{AB}) = \max_{\{A_j\}}\{S(\rho_B) - \sum_j p_j S(\rho_j)\}$ represent the total correlation and classical correlation, respectively, $S(\rho) = -{\rm Tr}\rho\log\rho$ is the von Neumann entropy, ρ_A (ρ_B) is reduced density of ρ_{AB}. By performing the measurement locally on the party A, we get the state of the system $\rho_j = {\rm Tr}_A[(A_j\otimes I_3)\rho^{AB}]/p_j$, $p_j = {\rm Tr}_{AB}[(A_j\otimes I_3)\rho^{AB}]$. Here, $\{A_j\}$ represents the positive operator valued measurements. We note that the quantum discord may not be a symmetric quantity in general; that is, its value depends on whether the measurement is performed on subsystem A or B.

By performing the measurement on the subsystems A and B, Luo introduced MID to characterize the correlations.[11] The MID is defined as

$$D_{\rm M}(\rho_{AB}) = I(\rho_{AB}) - I(\Pi(\rho_{AB})), \tag{9}$$

where the measurement $\Pi = \{\Pi_i^A\otimes\Pi_j^B\}$ is induced by the spectral resolutions of the reduced states, $\rho_A = \sum_i p_i^A\Pi_i^A$ and $\rho_B = \sum_j p_j^B\Pi_j^B$. $I(\Pi(\rho_{AB}))$ quantifies the classical correlation in ρ_{AB}.

In the following, we will discuss the evolutions of quantum correlation by using negativity, quantum discord, and MID.

4. Dynamics of correlations

We consider the following initial state of the central qubit–qutrit[20]

$$\rho(p) = \frac{p}{2}(|00\rangle\langle 00| + |01\rangle\langle 01| + |00\rangle\langle 12| + |11\rangle\langle 11|$$

$$+ |12\rangle\langle 12| + |12\rangle\langle 00|) + \frac{1-2p}{2}(|02\rangle\langle 02| + |02\rangle\langle 10| + |10\rangle\langle 02| + |10\rangle\langle 10|), \quad (10)$$

where $0 \le p \le 1/2$, $\rho(p)$ is an entangled state except $p = 1/3$. According to Eqs. (5) and (6), we can obtain the evolved density operator of the cental qubit–qutrit system at time t. The analytical results of the three quantum correlations can be calculated.

The negativity has the following expression:

$$N(\rho) = \frac{1}{2}|(1-2p) - pF_{16}(t)| + \frac{1}{2}|p-(1-2p)F_{34}(t)| - \frac{1}{2}(1-3p) + pF_{16}(t) + (1-2p)F_{34}(t). \quad (11)$$

The classical correlation and the total correlation can be given by

$$C(\rho) = -(1-p)\log\left(\frac{1-p}{2}\right) + \frac{1-p-\theta}{2}\log\left(\frac{1-p-\theta}{2}\right) + \frac{1-p+\theta}{2}\log\left(\frac{1-p+\theta}{2}\right), \quad (12)$$

where $\theta = \max\{|1-3p|, pF_{16}(t) + (1-2p)F_{34}(t)\}$.

$$I(\rho) = \log 2 - p\log 2 - (1-p)\log\left(\frac{1-p}{2}\right) + \frac{p(1-F_{16}(t))}{2}\log\frac{p(1-F_{16}(t))}{2} + \frac{p(1+F_{16}(t))}{2}\log\frac{p(1+F_{16}(t))}{2} + \frac{(1-2p)(1-F_{34}(t))}{2}\log\frac{(1-2p)(1-F_{34}(t))}{2} + \frac{(1-2p)(1+F_{34}(t))}{2}\log\frac{(1-2p)(1+F_{34}(t))}{2}. \quad (13)$$

Then, the quantum discord $D(\rho)$ is interpreted as the difference between the total correlation $I(\rho)$ and the classical correlation $C(\rho)$.

The MID is obtained as

$$D_{\mathrm{M}}(\rho) = -p\log\frac{p}{2} - (1-2p)\log\frac{1-2p}{2} + \frac{p(1-F_{16}(t))}{2}\log\frac{p(1-F_{16}(t))}{2} + \frac{p(1+F_{16}(t))}{2}\log\frac{p(1+F_{16}(t))}{2} + \frac{(1-2p)(1-F_{34}(t))}{2}\log\frac{(1-2p)(1-F_{34}(t))}{2} + \frac{(1-2p)(1+F_{34}(t))}{2}\log\frac{(1-2p)(1+F_{34}(t))}{2}. \quad (14)$$

Case 1 $f = 0$, namely, only the qubit of the qubit–qutrit system couples to the spin chain. The coherence factors are $F_{16}(t) = F_{34}(t) = F_{61}^{\dagger}(t) = F_{43}^{\dagger}(t) = F(t)$.

In Figs. 1(a), 1(b), and 1(c), the correlations are plotted as functions of the magnetic intensity λ and time t. We find

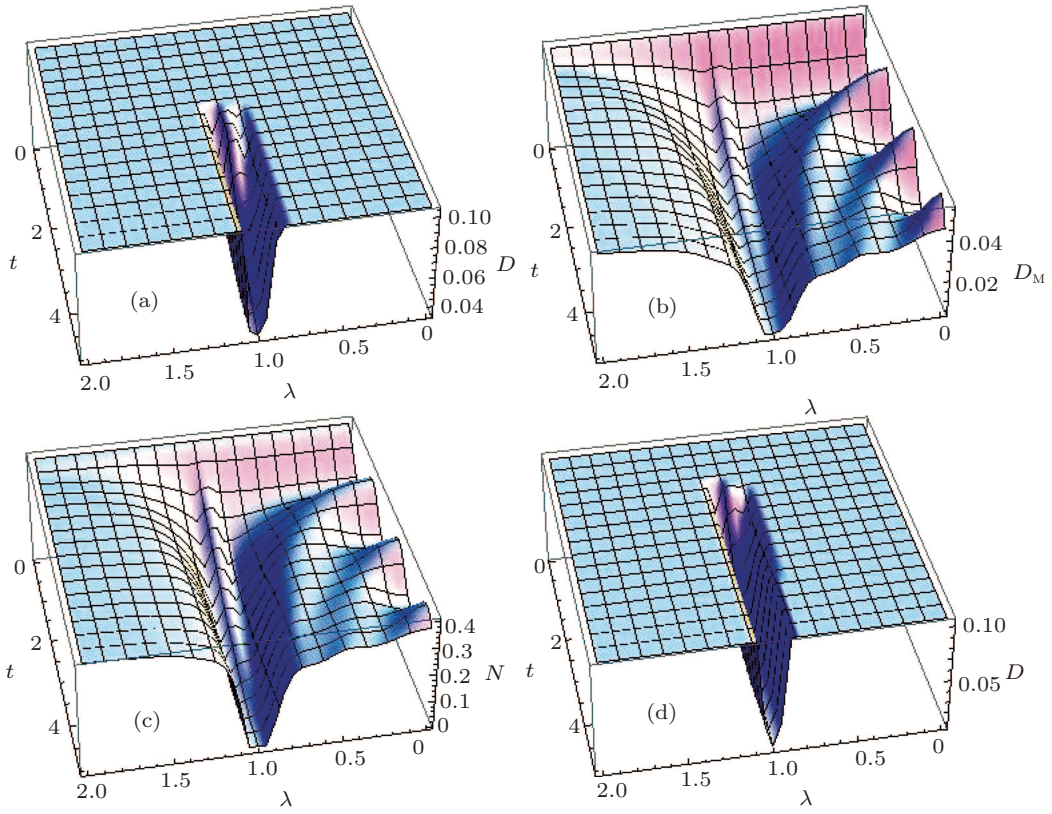


Fig. 1. (color online) The dynamics of correlations as functions of the magnetic intensity λ and time t. The correlations are: (a) quantum discord, (b) MID, and (c) negativity for $D_z = 0$; and, (d) quantum discord for $D_z = 0.5$. Other parameters are set as $p = 0.2, \gamma = 1, g = 0.05, N_t = 201$.

that quantum correlations are minimized at the critical point $\lambda = \lambda_c = 1$. However, in contrast to negativity and MID, the behavior of quantum discord is most obvious near the critical point. So the quantum discord characterizing QPTs is optimal. The DM interaction is considered in Fig. 1(d), the figure is similar to Fig. 1(a). Therefore, the DM interaction does not effect QPTs induced by the external magnetic field and does not induce the new critical phenomenon in the XY spin chain.

Figure 2 shows the decay of correlations at $\lambda = \lambda_c = 1$ for different values of p, with other parameters $\gamma = 1$, $g = 0.05$, $D_z = 0.5$, $N_t = 601$. From the behavior of negativity, we find that entanglement disappears in a finite time suffering from the sudden death. Interestingly, the quantum discord remains constant at an interval of time for the four different values of p. After the critical time t_c, the quantum discord and MID have the same variation behaviors. This implies that the optimal positive operator-valued measurements $\{A_j\}$ for the quantum discord are the eigenvectors $\{\Pi_i^A\}$ of the reduced density ρ_A.

The quantum discord remains invariant at an interval of time t_c in Fig. 2. This condition indicates that there exists a sudden transition between the quantum discord and classical correlation at t_c. In Fig. 3, we plot the total correlation, classical correlation, and quantum discord as a function of time t according to the condition presented in Fig. 2. We can clearly observe the sudden transition phenomenon from Fig. 3. The transition time t_c satisfies $|F(t_c)| = |1-3p|/(1-p)$. From Figs. 3(a), 3(b), and 3(c), we see that the time t_c for $p = 0.3$ is larger than that for $p = 0.2$ and $p = 0.4$, this is coincident with the expression of $|F(t_c)|$. Moreover, when $p = 0.5$, $|F(t_c)| = 1$ and $t_c = 0$. So, the sudden transition in Fig. 3(d) does not exist.

We turn to study the case of $\gamma \neq 1$, which means a general XY spin chain. Figure 4 shows the variation behaviors of correlations for different values of D_z at $\gamma = 0.5$, $g = 0.05$, $\lambda = 1.0$, $N_t = 601$. We observe that quantum discord still remains constant at an interval of time. This situation indicates there still exists a sudden transition between classical correlation and quantum discord. As D_z increases, the transition time decreases and the decay of quantum correlation increases in Fig. 4. Therefore, the DM interaction enhances the decay of quantum correlation and reduces the critical transition time.

Case 2 $f = 1$, both the qubit and qutrit coupled to the same spin chain. The coherence factors are $F_{16}(t) = F_{61}^{\dagger}(t) = F'(t)$, $F_{34}(t) = F_{43}^{\dagger}(t) = 1$.

The dynamics evolution of the quantum discord is similar to Fig. 1(a), which still exhibits a clear sudden change behavior near $\lambda_c = 1$. The analytical result of negativity for $p = 0.2$ is constant. Therefore, the negativity does not characterize the critical behavior. Figure 5 shows the decay of correlations at $\lambda = \lambda_c = 1$ for different values of p, with other parameters $\gamma = 1$, $g = 0.05$, $D_z = 0.5$, $N_t = 601$. The entanglement quantified by negativity is not influenced by spin chain environment in Figs. 5(a) and 5(b). We can prove that the dynamics evolution of the negativity remains unchanged as $p < 1/3$. The negativity for other two cases (Fig. 5(c) and Fig. 5(d)) suffers

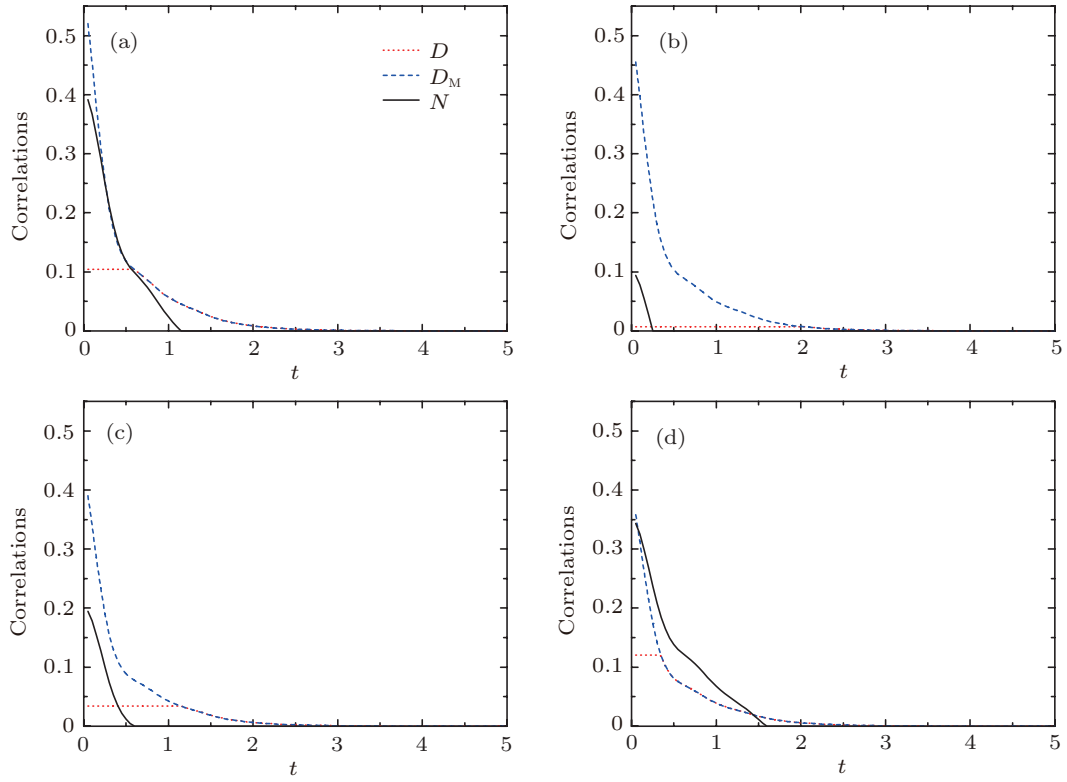


Fig. 2. (color online) Dynamics of quantum discord (red dot line), MID (blue dash line), and negativity (solid black line) versus time t for (a) $p = 0.2$, (b) $p = 0.3$, (c) $p = 0.4$, (d) $p = 0.45$. Other parameters are set as $\gamma = 1, g = 0.05, D_z = 0.5, N_t = 601$.

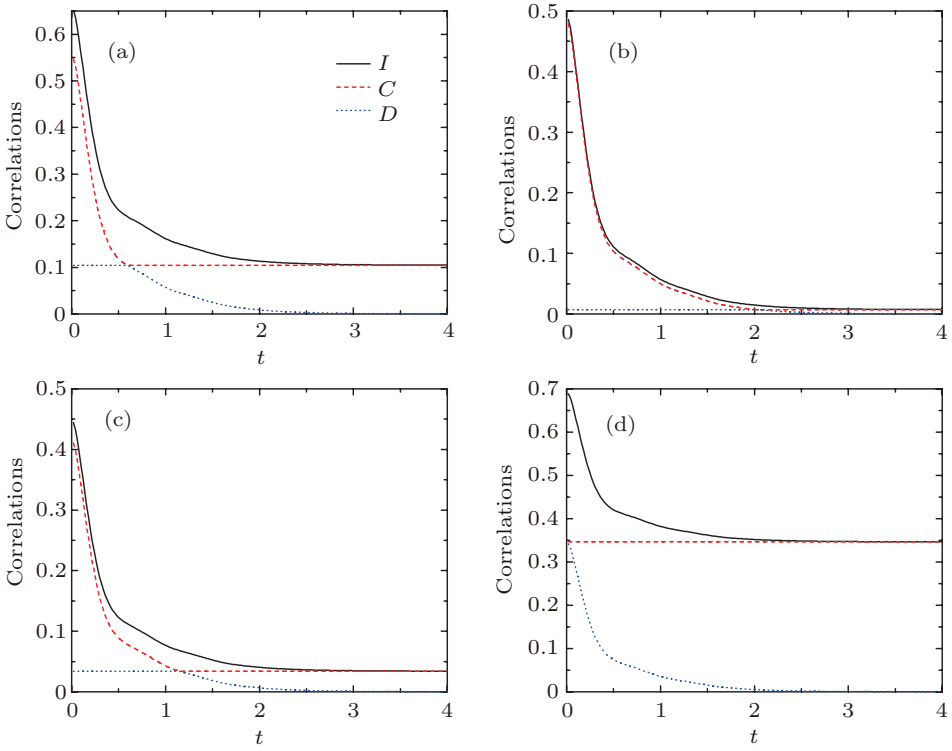


Fig. 3. (color online) Dynamics of the total correlation (solid black line), classical correlation (red dash line), and quantum discord (blue dot line) versus time t for (a) $p = 0.2$, (b) $p = 0.3$, (c) $p = 0.4$, (d) $p = 0.5$. Other parameters are set as $\gamma = 1, g = 0.05, D_z = 0.5, N_t = 601$.

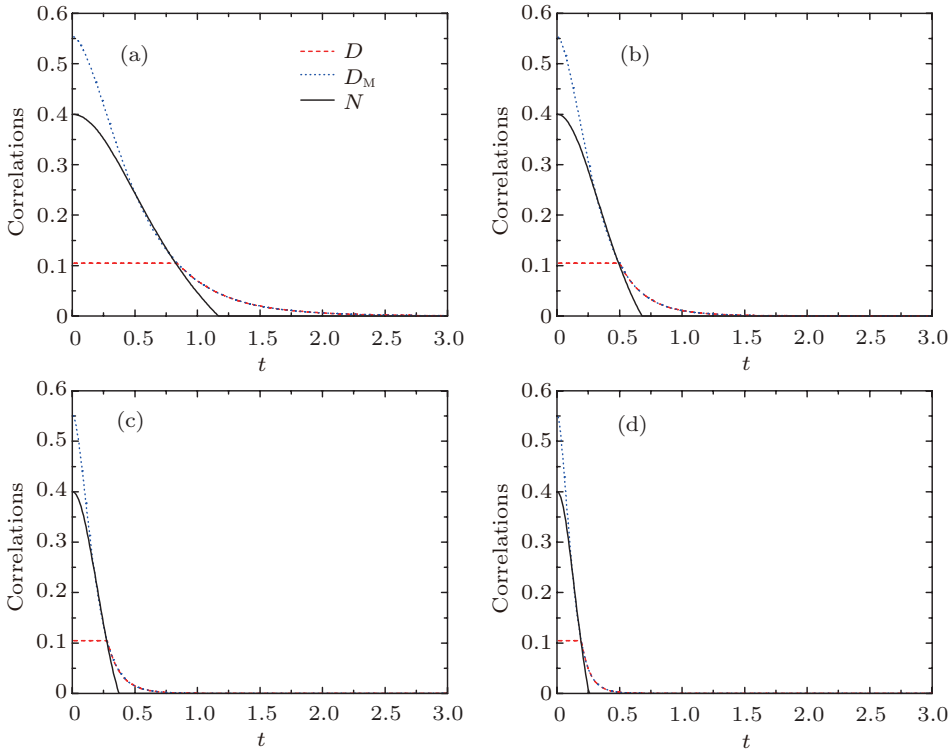


Fig. 4. (color online) Dynamics of the quantum discord (red dash line), MID (blue dot line) and negativity (solid black line) versus time t for (a) $D_z = 0$, (b) $D_z = 0.2$, (c) $D_z = 0.6$, (d) $D_z = 1.0$. Other parameters are set as $p = 0.2$, $\gamma = 0.5$, $g = 0.05$, $\lambda = 1.0$, $N_t = 601$.

from the sudden death. Moreover, we find that when $p \leq 0.4$, the variation tendency of the quantum discord and MID are different. The main reason for this is the difference between the optimal positive operator-valued measurements $\{A_j\}$ for the quantum discord and the eigenvectors $\{\Pi_i^A\}$ of the reduced density ρ_A. From Fig. 5, we observe that the dynamic evolution of MID is consistent with that of the total correlation. When time satisfies $|F'(t)| \leq (5p-2)/p$, the measurements $\{A_j\}$ of the quantum discord become eigenvectors $\{\Pi_i^A\}$ of the reduced density ρ_A. So the values of the quantum discord and MID are identical. For example, when $p = 0.4$, $|F'(t)| \to 0$, the change behaviors of quantum discord and MID are coincident.

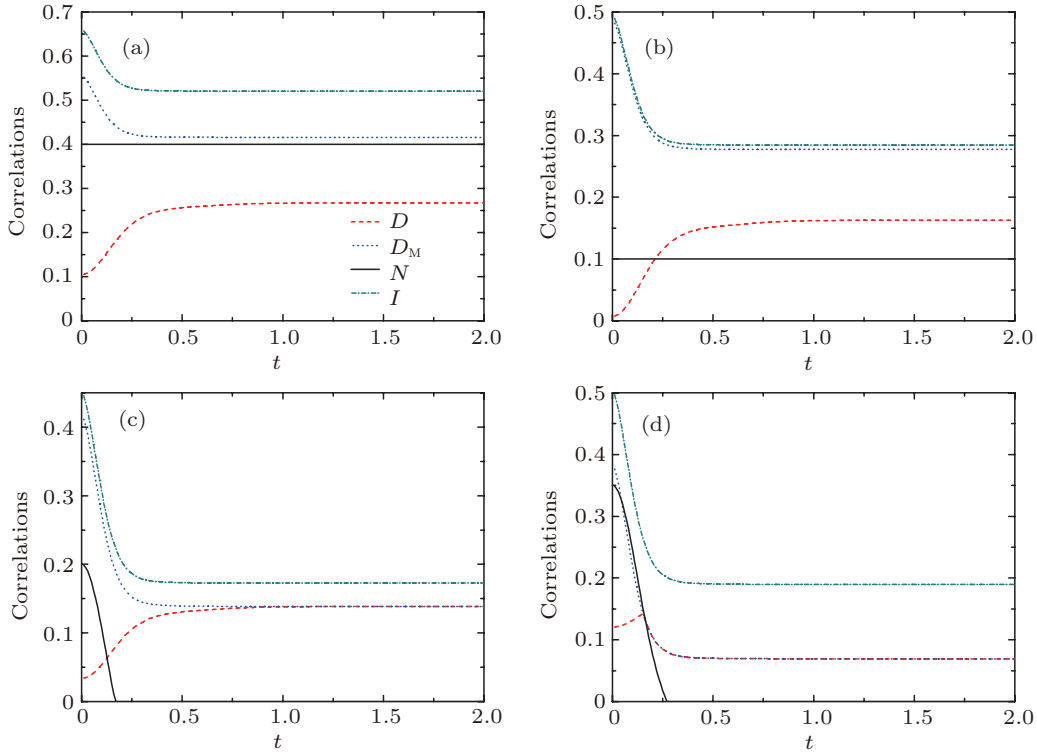


Fig. 5. (color online) Dynamics of the quantum discord (red dash line), MID (green dot line), negativity (solid black line), and total correlation (dark cyan dash–dot line) versus time t for (a) $p = 0.2$, (b) $p = 0.3$, (c) $p = 0.4$, (d) $p = 0.45$. Other parameters are set as $\gamma = 1$, $g = 0.05$, $D_z = 0.5$, $N_t = 601$.

When $0.4 < p < 0.5$, the behavior of the quantum discord is different from the property of the quantum discord in Case 1, where the quantum discord monotonically change before the sudden change point. The total correlation, classical correlation, and quantum discord as a function of time t are plotted in Fig. 6. We observe that the sudden transition between the quantum discord and classical correlation does not exist. Moreover, when $p = 0.5$, the classical correlation remains unchanged.

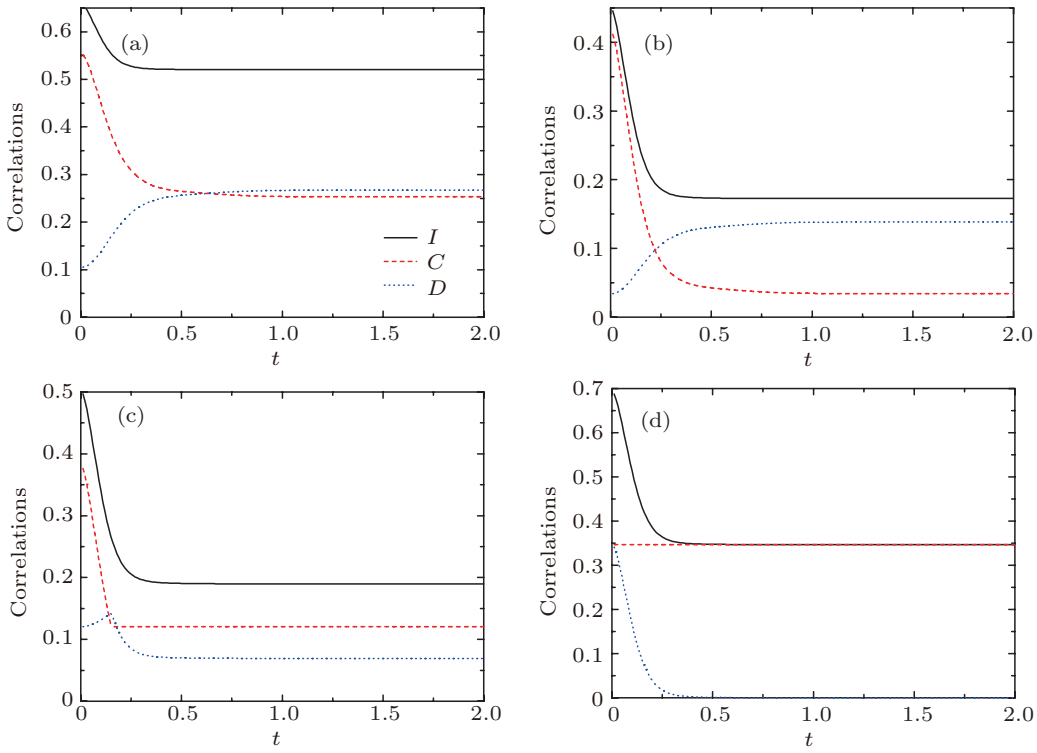


Fig. 6. (color online) Dynamics of the total correlation (solid black line), classical correlation (red dash line), and quantum discord (green dot line) versus time t for (a) $p = 0.2$, (b) $p = 0.4$, (c) $p = 0.45$, (d) $p = 0.5$. Other parameters are set as $\gamma = 1$, $g = 0.05$, $D_z = 0.5$, $N_t = 601$.

For the case $\gamma \neq 1$, we consider the effect of the DM interaction. The results is similar to Fig. 4. The DM interaction also enhances the decay of quantum correlation. Moreover, we find that the critical transition time between the quantum discord and MID decreases with the increase of the DM interaction D_z, these properties are similar to Case 1.

5. Conclusions

We have investigated the correlation dynamics of a hybrid qubit–qutrit system coupled in two different ways to an XY spin-chain environment with the DM interaction under the weak-coupling condition. We find that the quantum discord is the optimal correlation measurement among the three measures used in this paper, and it can always detect the existence of QPTs. The sudden transition between the quantum discord and classical correlation only occurs when the qubit couples to the environment. If both the qubit and the qutrit couple to the environment, then the phenomenon of the sudden transition disappears. The DM interaction does not affect the critical point of QPTs and only enhances the decay of quantum correlation. Furthermore, the quantum discord and measurement-induced disturbance is much more robust than entanglement. Entanglement suffers from a sudden death while the quantum discord and measurement-induced disturbance decay asymptotically.

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