Preserving entanglement and the fidelity of three-qubit quantum states undergoing decoherence using weak measurement*

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We demonstrate a method to preserve entanglement and improve fidelity of three-qubit quantum states undergoing amplitude-damping decoherence using weak measurement and quantum measurement reversal. It is shown that we are able to enhance entanglement to the greatest extent, and to circumvent entanglement sudden death by increasing the weak measurement strength both for the GHZ state and the W state. The weak measurement technique can also enhance the fidelity to the quantum region and even close to 1 for the whole range of the decoherence parameter in both of the two cases. In addition, the W state can maintain more fidelity than the GHZ state in the protection protocol. However, the GHZ state has a higher success probability than the W state.

Keywords: entanglement protection, quantum fidelity, weak measurement

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1. Introduction

Historically, quantum entanglement was regarded as a fundamental concept, nowadays it has been identified as a vital resource for quantum computing,[1] quantum cryptography,[2,3] quantum teleportation,[4–6] quantum metrology,[7] quantum secret sharing,[8–10] and so on. However, the unavoidable coupling of the quantum system with the environment will lead to quantum decoherence. For a multipartite quantum system, decoherence often results in degradation of entanglement and, in certain cases, entanglement sudden death (ESD). Therefore, understanding entanglement dynamics[11–21] and designing strategies[22–32] to manipulate entanglement become a long-term pursuit in the field of entanglement-based quantum information technology.

Thus, tackling decoherence and protecting entanglement in open systems is an important concern. Recently, the application of weak measurements has been suggested as a practically implementable method for protecting entanglement and quantum fidelity of quantum states undergoing decoherence through the amplitude damping channel.[33–38] In Ref. [37], Kim *et al.* experimentally demonstrated a scheme for protecting entanglement from amplitude-damping decoherence using weak measurement and quantum measurement reversal. However, their discussion is mainly limited to bipartite systems. For the tripartite systems, as we know, there are two distinct classes of multipartite entangled states (i.e., the Greenberger–Horne–Zeilinger (GHZ) class and the W class), which bear incompatible multipartite correlations in the sense that they cannot be transformed into each other under stochastic local operations and classical communication.[39] In particular, it has been demonstrated that besides the Bell states, the three-qubit GHZ and W class states in the form of $|\psi_{\rm GHZ}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ and $|\psi_{\rm W}\rangle = (|100\rangle + |010\rangle + \sqrt{2}|001\rangle)/2$ can also be adopted as quantum channels for perfect teleportation under ideal circumstance.[40,41] Since they are both unavoidably disturbed by the surrounding environments, it is natural to seek some methods to tame decoherence in terms of their teleportation capacity.

In this paper, we demonstrate a method to preserve entanglement and improve fidelity of three-qubit quantum states undergoing amplitude-damping decoherence using weak measurement and quantum measurement reversal. It is shown that we are able to enhance entanglement to the greatest extent and to circumvent entanglement sudden death by increasing the weak measurement strength, both for the GHZ state and the W state. The weak measurement technique can also enhance the fidelity to the quantum region and even close to 1 for the whole range of the decoherence parameter in both the two cases. In addition, the W state can maintain more fidelity than the GHZ state in the protection protocol. However, the GHZ state has a higher success probability than the W state.

2. Theoretical framework

The dynamics of three qubits interacting independently with individual environments is described by the solutions of the appropriate Born–Markov–Lindblad equations[42] that can

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be obtained conveniently by the Kraus operator approach.[1] Given an initial state for three qubits $\rho(0)$, its evolution can be written as

$$\rho(t) = \sum P_{ijk}\rho(0)P_{ijk}^{\dagger}. \tag{1}$$

The Kraus operator P_{ijk} describes the interaction of the qubits with the noise and satisfies the CPTP relation $\sum_{ijk} P_{ijk}^{\dagger}P_{ijk} = 1$. Since the noise operates locally on individual subsystems, P_{ijk} can be expressed in terms of the tensor products of E_i^A, E_j^B, and E_k^C as $P_{ijk} = E_i^A \otimes E_j^B \otimes E_k^C$. Here E_i^A, E_j^B, and E_k^C are the Kraus operators describing the time evolution of each qubit alone, and they individually satisfy the usual completeness condition for the operator-sum decomposition of CPTP maps.[43] The action of amplitude-damping noise can be described by the following Kraus operators[1]

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-D} \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & \sqrt{D} \\ 0 & 0 \end{bmatrix}, \tag{2}$$

where D is the magnitude of the decoherence. Amplitude-damping decoherence is applicable to many practical qubit systems, including vacuum-single-photon qubit with photon loss, atomic qubit with spontaneous decay, and superconducting qubit with zero-temperature energy relaxation.

The tripartite negativity N, introduced by Sabín and García-Alcaine[44] is particularly convenient for the case of current interest. It can be calculated explicitly from the density matrix $\rho(t)$ as

$$N = (N_{A-BC}N_{B-CA}N_{C-AB})^{1/3}, \tag{3}$$

where $N_{A-BC} = -\sum_i \mu_i^A$, $N_{B-CA} = -\sum_i \mu_i^B$, and $N_{C-AB} = -\sum_i \mu_i^C$ are the negativities introduced by Vidal and Werner,[45,46] and the sums are taken over all the negative eigenvalues μ_i^K of the partial transpose ρ^{T_K} of $\rho(t)$ with respect to the subsystem K. Note that for the mixed states, N is not able to quantify multipartite entanglement fully but that its positivity ensures that the state under consideration is not separable. N is used to study the time evolution of entanglement for a number of tripartite systems.

To characterize the quality of the teleported output state ρ_{out} under the influence of amplitude-damping noise, we calculate the average fidelity (the fidelity $F(\theta,\phi) = \langle\varphi_{\text{in}}|\rho_{\text{out}}|\varphi_{\text{in}}\rangle$ averaged over all pure input states on the Bloch sphere),[1] defined as

$$F_{\text{av}} = \frac{1}{4\pi}\int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \mathrm{d}\theta \sin\theta F(\theta,\phi), \tag{4}$$

where $|\varphi_{\text{in}}\rangle = \cos(\theta/2)|0\rangle + \mathrm{e}^{\mathrm{i}\phi}\sin(\theta/2)|1\rangle$ is the input state needs to be teleported, with $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$ being the polar and azimuthal angles, respectively. When the initial GHZ class state $|\psi_{\text{GHZ}}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ or the initial W class state $|\psi_{\text{W}}\rangle = (|100\rangle + |010\rangle + \sqrt{2}|001\rangle)/2$ is used as a quantum channel, by following the methodology of Refs. [47] and [48], the average teleportation fidelity can be readily derived.

3. Preserving entanglement and quantum fidelity for GHZ state

First, we consider that the initial state is GHZ state

$$|\psi_{\text{GHZ}}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle). \tag{5}$$

Although the initial GHZ state is a pure three-qubit entangled state, owing to decoherence, the three-qubit quantum state $\rho_{1\text{G}}$ is given as

$$\begin{aligned}
&\rho_{1\text{G}} \\
&= \sum_{i,j,k=1,2} P_{ijk}|\psi_{\text{GHZ}}\rangle\langle\psi_{\text{GHZ}}|P_{ijk}^{\dagger} \\
&= \frac{1}{2}\begin{bmatrix}
(D^3+1) & 0 & 0 & 0 & 0 & 0 & 0 & \bar{D}^{3/2} \\
0 & D^2\bar{D} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & D^2\bar{D} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & D\bar{D}^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & D^2\bar{D} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & D\bar{D}^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & D\bar{D}^2 & 0 \\
\bar{D}^{3/2} & 0 & 0 & 0 & 0 & 0 & 0 & \bar{D}^3
\end{bmatrix},
\end{aligned} \tag{6}$$

here

$$\bar{D} = 1 - D.$$

From Eqs. (3), (4), and (6), one can derive the tripartite negativity $N(\rho_{1\text{G}})$ and the average fidelity $F_{\text{av}}(\rho_{1\text{G}})$ analytically as

$$\begin{aligned}
N(\rho_{1\text{G}}) &= \frac{1}{4}[(1-D)^2(4D^4 - 4D^3 + D^2 - 4D + 4)^{1/2} \\
&\quad + D^2 - D], \\
F_{\text{av}}(\rho_{1\text{G}}) &= \frac{1}{3}[(1-D)^{3/2} + D^2 - D + 2].
\end{aligned} \tag{7}$$

Let us now consider the case that a weak measurement (M_{wk}) and the reversing measurement (M_{rev}) are performed on each qubit before and after the decoherence channels, respectively. The weak measurement and reversing measurement can be written, respectively, as a non-unitary quantum operation[37]

$$M_{\text{wk}}(p) = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, \quad M_{\text{rev}}(p_{\text{r}}) = \begin{bmatrix} \sqrt{1-p_{\text{r}}} & 0 \\ 0 & 1 \end{bmatrix}, \tag{8}$$

where p is the weak measurement strength and p_{r} is the reversing measurement strength. The optimal reversing measurement strength that aims at restoring the initial state of a single qubit is calculated to be $p_{\text{r}} = p + D\bar{p}$, where $\bar{p} = 1 - p$. And, we choose this optimal reversing measurement strength in this paper. For simplicity, we assume that the weak measurement strengths of the three qubits are the same. The three-qubit state

now (after the sequence of weak measurement, decoherence, and reversing measurement) is given as

$$\rho_{2G} = \frac{1}{2\lambda}\begin{bmatrix} \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma \\ 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 \\ \gamma & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_4 \end{bmatrix} \tag{9}$$

with

$$\begin{aligned} \lambda &= \frac{1}{2}[(D^3(p-1)^3-1)(p_r-1)^3+(D-1)^3(p-1)^3 \\ &\quad + 3D^2(D-1)(p-1)^3(p_r-1)^2 \\ &\quad + 3D(D-1)^2(p-1)^3(p_r-1)], \\ \alpha_1 &= [D^3(p-1)^3-1](p_r-1)^3, \\ \alpha_2 &= D^2(D-1)(p-1)^3(p_r-1)^2, \\ \alpha_3 &= D(D-1)^2(p-1)^3(p_r-1), \\ \alpha_4 &= (D-1)^3(p-1)^3, \\ \gamma &= (1-D)^{3/2}(1-p)^{3/2}(1-p_r)^{3/2}. \end{aligned} \tag{10}$$

The success probability is given by

$$\begin{aligned} S_G &= \frac{1}{2}[(D^3(p-1)^3-1)(p_r-1)^3+(D-1)^3(p-1)^3 \\ &\quad + 3D^2(D-1)(p-1)^3(p_r-1)^2 \\ &\quad + 3D(D-1)^2(p-1)^3(p_r-1)]. \end{aligned} \tag{11}$$

The tripartite negativity $N(\rho_{2G})$ and the average fidelity $F_{av}(\rho_{2G})$ are calculated as

$$\begin{aligned} N(\rho_{2G}) &= -\Big\{2\Big[\frac{1}{4}D\xi^3(D-1)^2(p_r-1) \\ &\quad - \frac{1}{4}(D^2\xi^6(D-1)^2(p_r-1)^2(Dp_r-2D+1)^2 \\ &\quad - 4\xi^3(D-1)^3(p_r-1)^3)^{1/2} \\ &\quad + \frac{1}{4}D^2\xi^3(D-1)(p_r-1)^2\Big]\Big\} \\ &\quad /[\xi^3(D-1)^3+(D^3\xi^3-1)(p_r-1)^3 \\ &\quad + 3D\xi^3(D-1)^2(p_r-1) \\ &\quad + 3D^2\xi^3(D-1)(p_r-1)^2], \\ F_{av}(\rho_{2G}) &= \frac{1}{3}[2D\xi^3-2(p_r-1)^3-2\xi^3-2D^2\xi^3 \\ &\quad + 2D^3\xi^3+2D^3\xi^3(p_r-1)^3-4D^3p_r\xi^3 \\ &\quad - 4D^2p_r^2\xi^3+4D^3p_r^2\xi^3+2(1-D)^{3/2} \\ &\quad \times (1-p)^{3/2}(1-p_r)^{3/2} \\ &\quad + 4Dp_r\xi^3]/[\xi^3(Dp_r-1)^3 \\ &\quad - 3p_r+3p_r^2-p_r^3+1], \end{aligned} \tag{12}$$

here

$$\xi = p-1. \tag{13}$$

The dependence of $N(\rho_{1G})$, $N(\rho_{2G})$, and the difference $\Delta N_G = N(\rho_{2G}) - N(\rho_{1G})$ on the strengths of decoherence and weak measurement is clearly shown in Fig. 1. From Fig. 1(a), we obtain that we are able to enhance entanglement and to circumvent entanglement sudden death (ESD) by increasing the weak measurement strength p, while entanglement decays monotonically and reaches zero under noise environments without the help of weak measurement. Moreover, it is interesting to observe that $\lim_{p\to 1} N(\rho_{2G}) = 0.5$. This implies that, in spite of sufficiently strong decoherence, the initial entangled state could be preserved to the most degree. From Fig. 1(b), it can be seen that the stronger the weak measurement strength p is, the larger the difference of entanglement ΔN_G is.

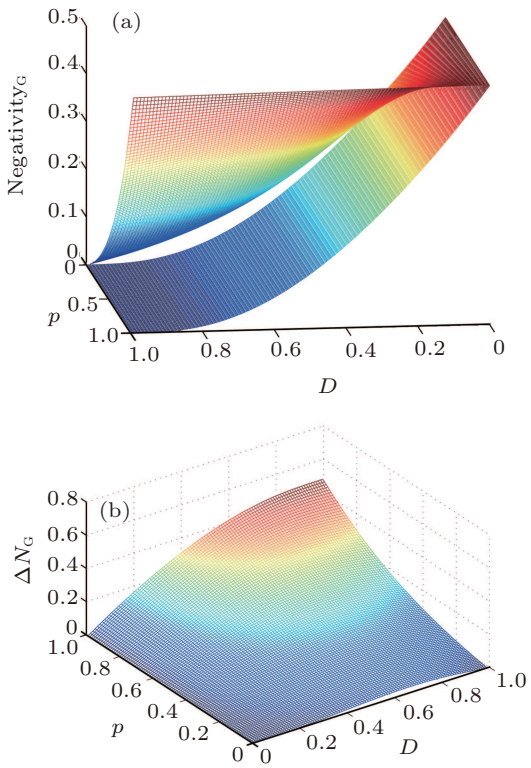


Fig. 1. (color online) (a) The dependence of $N(\rho_{1G})$ (the lower plot), $N(\rho_{2G})$ (the upper plot) and (b) the difference ΔN_G on the strengths of decoherence and weak measurement.

In Fig. 2, we plot the dependence of $F_{av}(\rho_{1G})$ and $F_{av}(\rho_{2G})$ on the strengths of decoherence and weak measurement. In this figure, we compare the $F_{av}(\rho_{1G})$ with the $F_{av}(\rho_{2G})$. One sees that, for strong noise environmental interaction, the fidelity could fall below the quantum region without the help of weak measurement. This finding corroborates our analysis presented in Eq. (7), which in the range $(\sqrt{5}-1)/2 \le D \le 1$, the teleportation fidelity $F_{av}(\rho_{1G})$ lies in the classical region, and for others values lies in the quantum region. However, as detailed in our protocol mentioned above, when one performs weak measurement and its subsequent reversal, the fidelity is improved above the classical value for all

strengths of decoherence by increasing the weak measurement strength p.

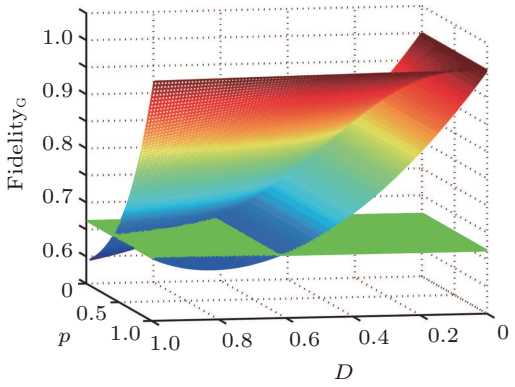


Fig. 2. (color online) The dependence of $F_{\rm av}(\rho_{1\rm G})$ (the lower plot) and $F_{\rm av}(\rho_{2\rm G})$ (the upper plot) on the strengths of decoherence and weak measurement. The flat plane represents the average classical fidelity 2/3.

Moreover, by comparing Fig. 1(a) with Fig. 2, one can note that when the tripartite negativity is small, the $F_{\rm av}(\rho_{1\rm G})$ and $F_{\rm av}(\rho_{2\rm G})$ lie in the classical region. This phenomenon indicates that not all of the three-qubit entangled states generated from the initial GHZ class state are useful for nonclassical teleportation.

4. Preserving entanglement and quantum fidelity for W state

When the initial entangled state is W state

$$|\psi_{\rm W}\rangle = \frac{1}{2}(|100\rangle + |010\rangle + \sqrt{2}|001\rangle). \tag{14}$$

After directly undergoing the decoherence, the final three-qubit state is given as

$$\rho_{1\rm W} = \sum_{i,j,k=1,2} P_{ijk}|\psi_{\rm W}\rangle\langle\psi_{\rm W}|P_{ijk}^{\dagger} = \begin{bmatrix} D & 0 & 0 & 0 & 0 & 0\,0\,0 \\ 0 & (1-D)/2 & \sqrt{2}(1-D)/4 & 0 & \sqrt{2}(1-D)/4 & 0\,0\,0 \\ 0 & \sqrt{2}(1-D)/4 & (1-D)/4 & 0 & (1-D)/4 & 0\,0\,0 \\ 0 & 0 & 0 & 0 & 0 & 0\,0\,0 \\ 0 & \sqrt{2}(1-D)/4 & (1-D)/4 & 0 & (1-D)/4 & 0\,0\,0 \\ 0 & 0 & 0 & 0 & 0 & 0\,0\,0 \\ 0 & 0 & 0 & 0 & 0 & 0\,0\,0 \\ 0 & 0 & 0 & 0 & 0 & 0\,0\,0 \end{bmatrix}. \tag{15}$$

The negativity $N(\rho_{1\rm W})$ and the average fidelity $F_{\rm av}(\rho_{1\rm W})$ are

$$\begin{aligned} &N(\rho_{1\rm W}) \\ &= \frac{1}{4}\Big\{-[D-3(1-D)-2\sqrt{7D^2-6D+3}+3]^2 \\ &\times\Big[\frac{D}{4}-\frac{1}{4}(1-D)-\frac{1}{2}\sqrt{2D^2-2D+1}+\frac{1}{4}\Big]\Big\}^{1/3}, \\ &F_{\rm av}(\rho_{1\rm W}) = 1-\frac{2}{3}D. \end{aligned} \tag{16}$$

As before, if a weak measurement and corresponding optimal reversing measurement are performed on each qubit before and after the decoherence channels, respectively, the three qubits will finally be in the state

$$\rho_{2\rm W} = \frac{1}{\delta}\begin{bmatrix} \beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & \beta_4 & 0 & \beta_4 & 0 & 0 & 0 \\ 0 & \beta_4 & \beta_3 & 0 & \beta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_4 & \beta_3 & 0 & \beta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{17}$$

where

$$\begin{aligned} \delta &= (Dp_{\rm r}-1)(p-1)(p_{\rm r}-1)^2, \\ \beta_1 &= D(p-1)(p_{\rm r}-1)^3, \\ \beta_2 &= \frac{1}{2}(D-1)(p-1)(p_{\rm r}-1)^2, \\ \beta_3 &= \frac{1}{4}(D-1)(p-1)(p_{\rm r}-1)^2, \\ \beta_4 &= \frac{\sqrt{2}}{4}(D-1)(p-1)(p_{\rm r}-1)^2. \end{aligned} \tag{18}$$

In this case (i.e., the initial entangled state is W state) the success probability is calculated to be

$$S_{\rm W} = (Dp_{\rm r}-1)(p-1)(p_{\rm r}-1)^2. \tag{19}$$

Then, the tripartite negativity $N(\rho_{2\rm W})$ and the average fidelity $F_{\rm av}(\rho_{2\rm W})$ are given by

$$\begin{aligned} &N(\rho_{2\rm W}) \\ &= \frac{1}{4}\Big\{\Big\{\Big[\frac{1}{2}(\zeta^4(p-1)^2(D^2p_{\rm r}^2-2D^2p_{\rm r} \\ &+2D^2-2D+1))^{1/2}+\frac{1}{4}(\zeta^4(D-1)^2(p-1)^2)^{1/2} \\ &-\frac{1}{4}\zeta^2(p-1)(D-1)-\frac{1}{2}D\zeta^3(p-1)\Big] \\ &\times[\zeta^2(2p-2)(D-1)-3(\zeta^4(D-1)^2(p-1)^2)^{1/2} \\ &-2(\zeta^4(p-1)^2(4D^2p_{\rm r}^2-8D^2p_{\rm r}+7D^2-6D+3))^{1/2} \\ &+4D\zeta^3(p-1)+\zeta^2(D-1)(p-1)]^2\Big\} \\ &/[\zeta^6(Dp_{\rm r}-1)^3(p-1)^3]\Big\}^{1/3}, \end{aligned}$$

$$F_{\rm av}(\rho_{\rm 2W}) = \frac{1}{3} + \frac{2}{3}\frac{(1-D)}{(1-Dp_{\rm r})}, \tag{20}$$

here

$$\zeta = p_{\rm r} - 1. \tag{21}$$

The dependence of $N(\rho_{\rm 1W})$, $N(\rho_{\rm 2W})$, and the difference $\Delta N_{\rm W} = N(\rho_{\rm 2W}) - N(\rho_{\rm 1W})$ on the strengths of decoherence and weak measurement, is shown in Fig. 3. From Fig. 3(a), we conclude that for W state we can improve entanglement and to circumvent ESD for all values of the decoherence parameter even the weak measurement strength p is very small. Moreover, it is interesting to observe that $\lim_{p\to 1} N(\rho_{\rm 2W}) = 0.4543$, meaning that this is in the maximum protection of the entanglement. From Fig. 3(b), it can be seen that the stronger the weak measurement strength is, the larger $\Delta N_{\rm W}$ will be. More interestingly, $\Delta N_{\rm W}$ increases with the increase of the decoherence D, meaning that the stronger D is, the more useful the weak measurement will be in protecting entanglement.

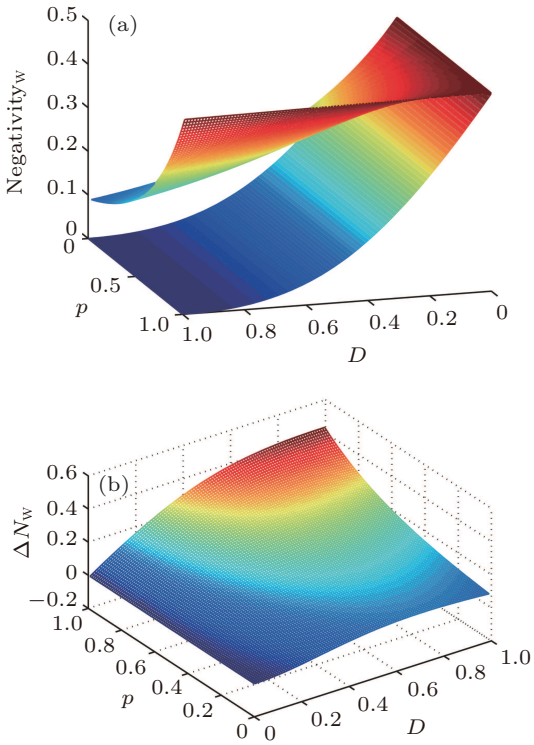


Fig. 3. (color online) (a) The dependence of $N(\rho_{\rm 1W})$ (the lower plot), $N(\rho_{\rm 2W})$ (the upper plot) and (b) the difference $\Delta N_{\rm W}$ on the strengths of decoherence and weak measurement.

In Fig. 4, we plot the dependence of $F_{\rm av}(\rho_{\rm 1W})$, $F_{\rm av}(\rho_{\rm 2W})$ on the strengths of decoherence and weak measurement. For the W state, the result shows that the weak measurement technique can enhance the fidelity $F_{\rm av}(\rho_{\rm 2W})$ above $F_{\rm av}(\rho_{\rm 1W})$ and the classical value for the whole range of the decoherence parameter and the weak measurement strength p. Note that, when D is chosen in the range $1/2 \le D \le 1$, though $F_{\rm av}(\rho_{\rm 1W})$ lies in the classical region, it was shown that $F_{\rm av}(\rho_{\rm 2W})$ is quantum fidelity.

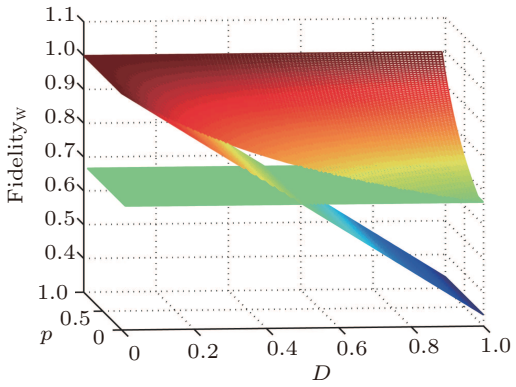


Fig. 4. (color online) The dependence of $F_{\rm av}(\rho_{\rm 1W})$ (the lower plot) and $F_{\rm av}(\rho_{\rm 2W})$ (the upper plot) on the strengths of decoherence and weak measurement. The flat plane represents the average classical fidelity $2/3$.

Moreover, by comparing Fig. 3(a) with Fig. 4, one can see that when D is chosen in the range $1/2 \le D \le 1$, that is, the tripartite negativity $N(\rho_{\rm 1W})$ is small, $F_{\rm av}(\rho_{\rm 1W})$ lies in the classical region. However, by using the weak measurement and its subsequent reversal, the tripartite negativity $N(\rho_{\rm 2W})$ is improved greatly and the $F_{\rm av}(\rho_{2w})$ is quantum fidelity for all values of the decoherence parameter. Thus, a certain amount of tripartite negativity $N(\rho_{\rm W})$ is always necessary for $F_{\rm av}(\rho_{\rm W}) > 2/3$.

5. Discussion

From Figs. 2 and 4, we observe that $\lim_{p\to 1} F_{\rm av}(\rho_{\rm 2G}) = 1$ and $\lim_{p\to 1} F_{\rm av}(\rho_{\rm 2W}) = 1$ no matter what the strengths of decoherence is. This implies that the initial quantum fidelity could be preserved to the most degree in spite of decoherence both for the GHZ state and the W state. On the other hand, as shown in Eqs. (12) and (20), under the same decoherence and fidelity protection protocol, the GHZ state and W state, which have equal fidelity initially, may evolve into inequivalent states containing unequal fidelity. For the two cases, when the optimal reversing measurement strength $p_{\rm r} = p + D(1-p)$ is chosen, the difference of the fidelity of the final states is given as

$$\begin{aligned}
&\Delta F_{\rm av} \\
&= F_{\rm av}(\rho_{\rm 2W}) - F_{\rm av}(\rho_{\rm 2G}) \\
&= \frac{2}{3D-3Dp+3} + \frac{1}{3} - [2\xi^3(D-1)^3 + 2D\xi^3 - 2\xi^3 \\
&\quad - 2D^2\xi^3 + 2D^3\xi^3 + 2(\xi(D-1))^{3/2}(1-D)^{3/2}(1-p)^{3/2} \\
&\quad - 4D^2\xi^3(D+p-Dp)^2 + 4D^3\xi^3(D+p-Dp)^2 \\
&\quad + 4D\xi^3(D+p-Dp) - 2D^3\xi^6(D-1)^3 \\
&\quad - 4D^3\xi^3(D+p-Dp)]/[3\xi^3(D-1)^3 \\
&\quad (-D^3p^3 + 3D^3p^2 - 3D^3p + D^3 + 3D^2p^2 \\
&\quad - 6D^2p + 3D^2 - 3Dp + 3D + 2)],
\end{aligned} \tag{22}$$

here

$$\xi = p - 1. \tag{23}$$

We obtain that $\Delta F_{\rm av} \geq 0$, meaning that $\rho_{2\rm W}$ (evolved from the W state) has more fidelity than $\rho_{2\rm G}$ (evolved from the GHZ state). In Fig. 5, we show how $\Delta F_{\rm av}$ behaves for different decoherence D and weak measurement p. It can be seen that the larger D is, the larger $\Delta F_{\rm av}$ becomes. Consequently, the W state can maintain more fidelity than the GHZ state in the aforementioned protection protocol, although they initially have the same amount of fidelity. The success probabilities for the two cases are also not the same, as shown in Eqs. (11) and (19). The difference of the success probabilities for the two cases is

$$\begin{aligned}\Delta S &= S_{\rm G} - S_{\rm W} \\ &= \frac{1}{2}D(1-D)^3(1-p)^4 \\ &\quad \times [D^2(1-p)^2 + 3D(1-p) + 1]. \end{aligned} \tag{24}$$

Obviously, $\Delta S \geq 0$, meaning that the GHZ state has a higher success probability. Figure 6 shows ΔS as a function of p and D. Clearly, ΔS has a different variation trend with ΔF, and tends to zero with the increase of p.

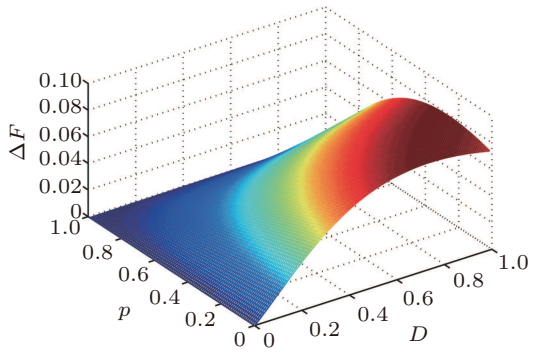


Fig. 5. (color online) The dependence of the difference of fidelity $\Delta F_{\rm av}$ on the strengths of decoherence and weak measurement.

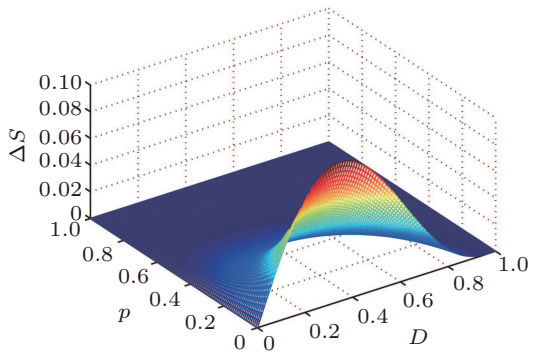


Fig. 6. (color online) The dependence of the difference of success probabilities ΔS on the strengths of decoherence and weak measurement.

According to the above discussion, one can find that there is a tradeoff between fidelity and success probability. When the initial entangled state is W, the three qubits can maintain a larger amount of fidelity than is the case when the initial entangled state is GHZ. However, the latter case has a higher success probability than the former case.

6. Conclusion

In this paper, we demonstrate a method to preserve entanglement and improve the fidelity of three-qubit quantum states undergoing decoherence using weak measurement and quantum measurement reversal. It is shown that we are able to enhance entanglement to the greatest extent, and to circumvent entanglement sudden death by increasing the weak measurement strength for both the GHZ state and the W state. The weak measurement technique can also enhance the fidelity to the quantum region and is even close to 1 for the whole range of the decoherence parameter in both the two cases. In addition, the W state can maintain more fidelity than the GHZ state in the protection protocol. However, the GHZ state has a higher success probability than the W state.

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