Photon number cumulant expansion and generating function for photon added- and subtracted-two-mode squeezed states*

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For the first time, we derive the photon number cumulant for two-mode squeezed state and show that its cumulant expansion leads to normalization of two-mode photon subtracted-squeezed states and photon added- squeezed states. We show that the normalization is related to Jacobi polynomial, so the cumulant expansion in turn represents the new generating function of Jacobi polynomial.

Keywords: Jacobi polynomial, photon number cumulant expansion, photon subtracted- and photon added-two-mode squeezed states

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1. Introduction

Einstein in 1951 wrote: "all the fifty years of conscious brooding have brought me no closer to the answer to the question, what are light quanta? Of course today every rascal thinks he knows the answer, but he is deluding himself." Today this problem still bothers physicists working on quantum optics since the nonclassical properties of light have not been completely revealed. In quantum optics theory, to examine optical field nonclassical properties, researchers often focus on photon detection and photon number distribution, so studying photon number cumulant expansion and moment will bring much convenience. For a pure quantum state $|\psi\rangle$, by defining the cumulant expansion as

$$\langle\psi|\,\mathrm{e}^{fa^\dagger a}\,|\psi\rangle = \sum_n \frac{f^n}{n!}\,\langle\psi|\,(a^\dagger a)^n\,|\psi\rangle\,, \tag{1}$$

we know that all moments are given by

$$\langle\psi|\,(a^\dagger a)^n\,|\psi\rangle = \frac{\mathrm{d}^n}{\mathrm{d}f^n}\,\langle\psi|\,\mathrm{e}^{fa^\dagger a}\,|\psi\rangle\,|_{f=0}. \tag{2}$$

In this work, we shall study the photon number cumulant expansion for two-mode squeezed state. As will be seen later, this cumulant is just new generating function of Jacobi polynomial and its expansion is just related to normalization of two-mode photon added- and subtracted-squeezed states. Since these states exhibit nonclassical properties and may be useful in optical engineering[1–3] and quantum state engineering, they are worth studying; therefore, their correct normalization is needed.

The rest of this paper is arranged as follows. In Section 2, we derive the cumulant. In Sections 3 and 4, we show that this cumulant expansion leads to normalization of two-mode photon subtracted-squeezed states (PSSS) and photon added-squeezed states (PASS). The application is presented in Section 5. Finally, our findings are summarized in Section 6. Our calculations show that the normalization is Jacobi polynomial, so the cumulant in turn represents the new generating function of Jacobi polynomial. To achieve our result, in the following we shall make full use of the technique of integral within an ordered product (IWOP) of operators.[4,5] This technique has been proven to be effective in theoretical calculation of quantum optics; for example, for the coherent state

$$|z\rangle = \exp(-|z|^2/2 + za^\dagger)\,|0\rangle\,, \quad a\,|z\rangle = z\,|z\rangle\,. \tag{3}$$

Its completeness relation can be directly demonstrated by virtue of the IWOP method

$$\begin{aligned}\int \frac{\mathrm{d}^2 z}{\pi}\,|z\rangle\,\langle z| &= \int \frac{\mathrm{d}^2 z}{\pi} : \exp(-|z|^2 + za^\dagger + z^* a - a^\dagger a) : \\ &= \int \frac{\mathrm{d}^2 z}{\pi} : \exp[-(z^* - a^\dagger)(z - a)] = 1,\end{aligned} \tag{4}$$

where : : denotes the normal ordering.

2. Cumulant for two-mode squeezed state

For the two-mode squeezed state we define the photon number cumulant as

$$\langle 00|\,S_2^\dagger(\lambda)\exp[fa_1^\dagger a_1 + ga_2^\dagger a_2]S_2(\lambda)\,|00\rangle = C, \tag{5}$$

where $S_2(\lambda) = \exp[\lambda(a_1^\dagger a_2^\dagger - a_1 a_2)]$ is the two-mode squeezing operator[6] with λ being a real squeezing parameter. The photon annihilation operator and the creation operators obey

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$[a_i, a_j^+] = \delta_{i,j},\ i,j = 1,2$. The two-mode squeezed vacuum state is given by

$$S_2(\lambda)\,|00\rangle = \operatorname{sech}\lambda \exp(a_1^\dagger a_2^\dagger \tanh\lambda)\,|00\rangle\,. \tag{6}$$

Using the completeness relation of the two-mode coherent state:

$$\int \frac{\mathrm{d}^2 z_1\,\mathrm{d}^2 z_2}{\pi^2}\,\mathrm{e}^{-|z_1|^2-|z_2|^2}\,\|z_1,z_2\rangle\,\langle z_1,z_2\| = 1, \tag{7}$$

where $\|z_1,z_2\rangle = \exp(z_1 a_1^\dagger + z_2 a_2^\dagger)\,|00\rangle$ is the un-normalized coherent state, and $\langle z_1,z_2\|\ 00\rangle = 1$,

$$\begin{aligned}&\exp(f a_1^\dagger a_1)\,a_1^\dagger \exp(-f a_1^\dagger a_1) = a_1^\dagger\,\mathrm{e}^f,\\ &\exp(f a_1^\dagger a_1)\,\|z_1\rangle = \left\|z_1\,\mathrm{e}^f\right\rangle,\end{aligned} \tag{8}$$

we derive the cumulant

$$\begin{aligned}C =\ & (\operatorname{sech}\lambda)^2\,\langle 00|\,\mathrm{e}^{a_1 a_2 \tanh\lambda}\,\mathrm{e}^{f a_1^\dagger a_1 + g a_2^\dagger a_2}\\ &\times\int\frac{\mathrm{d}^2 z_1\,\mathrm{d}^2 z_2}{\pi^2}\exp(-|z_1|^2-|z_2|^2)\\ &\times\|z_1,z_2\rangle\,\langle z_1,z_2\|\exp(a_1^\dagger a_2^\dagger\tanh\lambda)\,|00\rangle\\ =\ & (\operatorname{sech}\lambda)^2\,\langle 00|\int\frac{\mathrm{d}^2 z_1\,\mathrm{d}^2 z_2}{\pi^2}\,\mathrm{e}^{a_1 a_2\tanh\lambda}\,\mathrm{e}^{-|z_1|^2-|z_2|^2}\\ &\times\left\|z_1\,\mathrm{e}^f, z_2\,\mathrm{e}^g\right\rangle\langle z_1,z_2\|\exp(a_1^\dagger a_2^\dagger\tanh\lambda)\,|00\rangle\\ =\ & (\operatorname{sech}\lambda)^2\int\frac{\mathrm{d}^2 z_1\,\mathrm{d}^2 z_2}{\pi^2}\exp[-|z_1|^2-|z_2|^2\\ &+z_1 z_2\,\mathrm{e}^{f+g}\tanh\lambda + z_1^* z_2^*\tanh\lambda]\\ =\ & (\operatorname{sech}\lambda)^2\int\frac{\mathrm{d}^2 z_1}{\pi}\exp[-|z_1|^2(1-\mathrm{e}^{f+g}\tanh^2\lambda)]\\ =\ & \frac{1}{\cosh^2\lambda - \mathrm{e}^{f+g}\sinh^2\lambda},\end{aligned} \tag{9}$$

where in the last step we have used the mathematical formula

$$\begin{aligned}&\int\frac{\mathrm{d}^2 z}{\pi}\exp(\varsigma|z|^2+\xi z+\eta z^*) = -\frac{1}{\varsigma}\exp\left(-\frac{\xi\eta}{\varsigma}\right),\\ &\operatorname{Re}\varsigma<0.\end{aligned} \tag{10}$$

3. Cumulant expansion and the normalization of two-mode photon-subtracted squeezed state

In Eq. (9) letting $\ln(1+t) = f$, $\ln(1+t') = g$, and using the operator identity

$$\mathrm{e}^{\lambda a^\dagger a} =: \exp[(\mathrm{e}^\lambda - 1)a^\dagger a]:, \tag{11}$$

we have

$$\begin{aligned}C\to C_1 =\ & \frac{1}{\cosh^2\lambda - (1+t)(1+t')\sinh^2\lambda}\\ =\ & \langle 00|\,S_2^\dagger(\lambda)\exp[a_1^\dagger a_1\ln(1+t)\\ &+a_2^\dagger a_2\ln(1+t')]S_2(\lambda)\,|00\rangle\\ =\ & \langle 00|\,S_2^\dagger(\lambda):\exp[t a_1^\dagger a_1 + t' a_2^\dagger a_2]:S_2(\lambda)\,|00\rangle\\ =\ & \sum_{n,m=0}^{\infty}\langle 00|\frac{t^n t'^m}{n!m!}S_2^{-1}a_1^{\dagger n}a_2^{\dagger m}a_1^n a_2^m S_2\,|00\rangle\\ =\ & \sum_{n,m=0}^{\infty}\frac{t^n t'^m}{n!m!}{}_{m,n}\langle\lambda|\lambda\rangle_{m,n},\end{aligned} \tag{12}$$

where

$$|\lambda\rangle_{m,n} = a_1^n a_2^m S_2(\lambda)\,|00\rangle \tag{13}$$

is just the PSSS, obtained by repeatedly operating photon annihilation operators a_1 and a_2 on $S_2(\lambda)\,|00\rangle$. Thus, we realize that the cumulant expansion leads to the normalization ${}_{m,n}\langle\lambda|\lambda\rangle_{m,n}$ of the two-mode PSSS.

We now calculate ${}_{m,n}\langle\lambda|\lambda\rangle_{m,n}$. Noticing the transform relations,

$$\begin{aligned}&S_2^\dagger a_1 S_2 = a_1\cosh\lambda + a_2^\dagger\sinh\lambda,\\ &S_2^\dagger a_2 S_2 = a_2\cosh\lambda + a_1^\dagger\sinh\lambda,\end{aligned} \tag{14}$$

we have

$$\begin{aligned}|\lambda\rangle_{m,n} =\ & S_2 S_2^\dagger a_1^n a_2^m S_2\,|00\rangle\\ =\ & S_2(a_1\cosh\lambda + a_2^\dagger\sinh\lambda)^n\\ &\times(a_2\cosh\lambda + a_1^\dagger\sinh\lambda)^m\,|00\rangle\\ =\ & S_2\sum_{l=0}^{n}\frac{n!}{l!(n-l)!}(a_2^\dagger\sinh\lambda)^{n-l}\\ &\times(a_1\cosh\lambda)^l(a_1^\dagger\sinh\lambda)^m\,|00\rangle\\ =\ & S_2\sinh^{n+m}\lambda\sum_{l=0}^{n}\frac{n!\coth^l\lambda}{l!(n-l)!}a_2^{\dagger n-l}a_1^l a_1^{\dagger m}\,|00\rangle\,.\end{aligned} \tag{15}$$

Using the completeness relation of coherent state and Eq. (4) as well as the IWOP technique, we can directly put operator $a_1^l a_1^{\dagger m}$ into normal ordering,

$$\begin{aligned}a_1^l a_1^{\dagger m} =\ & \int\frac{\mathrm{d}^2 z}{\pi}z^l z^{*m}\,|z\rangle\,\langle z|\\ =\ & \int\frac{\mathrm{d}^2 z}{\pi}z^l z^{*m}:\exp(-|z|^2 + z a_1^\dagger + z^* a_1 - a_1^\dagger a_1):\\ =\ & \sum_{k=0}^{\min(m,l)}:\frac{l!m!}{k!(l-k)!(m-k)!}a_1^{\dagger m-k}a_1^{l-k}:,\end{aligned} \tag{16}$$

where we have used the mathematical formula

$$\begin{aligned}&\int\frac{\mathrm{d}^2 z}{\pi}z^n z^{*m}\exp(A|z|^2+Bz+Cz^*)\\ =\ & \exp\left(-\frac{BC}{A}\right)\\ &\times\sum_{l=0}^{\min(m,n)}\frac{n!m!}{l!(n-l)!(m-l)!(-A)^{n+m-l+1}}B^{m-l}C^{n-l},\\ &\operatorname{Re}A<0.\end{aligned} \tag{17}$$

The form in Eq. (17) reminds us of the form of the two-variable Hermite polynomial[7]

$$\mathrm{H}_{m,n}(\alpha,\beta) = \sum_{l=0}^{\min(m,n)}(-1)^l\frac{n!m!}{l!(n-l)!(m-l)!}\alpha^{m-l}\beta^{n-l}, \tag{18}$$

so we have the operator identity

$$a_1^l a_1^{\dagger m} = (-\mathrm{i})^{m+l} : H_{m,l}(\mathrm{i}a_1^\dagger, \mathrm{i}a_1) : . \tag{19}$$

It then follows that

$$\begin{aligned} & a_1^l a_1^{\dagger m} |00\rangle \\ = & (-\mathrm{i})^{m+l} H_{m,l}(\mathrm{i}a_1^\dagger, 0)\,|00\rangle \\ = & (-\mathrm{i})^{m+l} \sum_{k=0}^{\min(m,l)} (-1)^l \frac{l!m!}{k!(l-k)!(m-k)!} (\mathrm{i}a_1^\dagger)^{m-l} \delta_{l,k} |00\rangle \\ = & \frac{m!}{(m-l)!} a_1^{\dagger m-l} |00\rangle , \end{aligned} \tag{20}$$

and equation (15) becomes

$$\begin{aligned} |\lambda\rangle_{m,n} = & S_2 m!n! \sinh^{m+n}\lambda \\ & \times \sum_{l=0}^{\min(m,n)} \frac{\coth^l \lambda}{l!(m-l)!(n-l)!} a_2^{\dagger n-l} a_1^{\dagger n-l} |00\rangle . \end{aligned} \tag{21}$$

Thus, the overlap is

$$\begin{aligned} {}_{m,n}\langle\lambda|\lambda\rangle_{m,n} = & (m!n!)^2 \sinh^{2(m+n)}\lambda \\ & \times \sum_{l=0}^{\min(m,n)} \frac{\coth^{2l}\lambda}{(m-l)!(n-l)!(l!)^2} . \end{aligned} \tag{22}$$

Using the definition of Jacobi polynomial[8]

$$\mathrm{P}_n^{(\alpha,\beta)}(x) = \left(\frac{x-1}{2}\right)^n \sum_{s=0}^{n} \binom{n+\alpha}{s} \binom{n+\beta}{n-s} \left(\frac{x+1}{x-1}\right)^s , \tag{23}$$

we have

$$\begin{aligned} & \mathrm{P}_n^{(m-n,0)}(\cosh 2\lambda) \\ = & m!n! \sinh^{2n}\lambda \sum_{l=0}^{n} \frac{\coth^{2l}\lambda}{(l!)^2(m-l)!(n-l)!}, \quad m > n, \\ & \mathrm{P}_m^{(n-m,0)}(\cosh 2\lambda) \\ = & m!n! \sinh^{2m}\lambda \sum_{l=0}^{m} \frac{\coth^{2l}\lambda}{(l!)^2(m-l)!(n-l)!}, n > m. \end{aligned} \tag{24}$$

Therefore, equation (22) is recast into

$$\begin{aligned} & {}_{m,n}\langle\lambda|\lambda\rangle_{m,n} \\ = & \begin{cases} m!n! \sinh^{2m}\lambda \mathrm{P}_n^{(m-n,0)}(\cosh 2\lambda), & m > n, \\ m!n! \sinh^{2n}\lambda \mathrm{P}_m^{(n-m,0)}(\cosh 2\lambda), & m < n, \end{cases} \end{aligned} \tag{25}$$

which shows that the PSSS, expressed by $a_1^n a_2^m S_2(\lambda)\,|00\rangle$, is normalized as Jacobi polynomial of the squeezing parameter λ. Further, let $\cosh(2\lambda) = x$, from Eqs. (12) and (25) we derive the new generating function formula of Jacobi polynomial as

$$\begin{aligned} & \sum_{n,m=0,m\geq n}^{t^n t'^m} \left(\frac{x-1}{2}\right)^m \mathrm{P}_n^{(m-n,0)}(x) \\ & + \sum_{n,m=0,m<n}^{t^n t'^m} \left(\frac{x-1}{2}\right)^n \mathrm{P}_m^{(n-m,0)}(x) \\ = & \frac{2}{x+1-(1+t)(1+t')(x-1)} . \end{aligned} \tag{26}$$

So far as we know, this formula has not previously been reported in the literature.

4. Cumulant expansion and normalization of two-mode photon added-squeezed state

On the other hand, in Eq. (9) letting

$$\ln\frac{1}{1-s} = f, \quad \ln\frac{1}{1-s'} = g,$$

and using the antinormally ordered expansion of $\mathrm{e}^{\lambda a^\dagger a}$,[9]

$$\mathrm{e}^{\lambda a^\dagger a} = \mathrm{e}^{-\lambda} {:}[(1-\mathrm{e}^{-\lambda})aa^\dagger]{:}, \tag{27}$$

where $:\ :$ denotes the antinormal ordering, we have

$$\begin{aligned} & C \to C_2 \\ = & \frac{1}{\cosh^2\lambda - \dfrac{1}{(1-s)(1-s')}\sinh^2\lambda} \\ = & \langle 00| S_2^{-1}(\lambda) \exp[a_1^\dagger a_1 \ln\frac{1}{1-s} + a_2^\dagger a_2 \ln\frac{1}{1-s'}] S_2(\lambda)\,|00\rangle \\ = & (1-s)(1-s')\,\langle 00|\, S_2^{-1} {:}\exp[s a_1^\dagger a_1 + s' a_2^\dagger a_2]{:} S_2\,|00\rangle \\ = & (1-s)(1-s')\,\langle 00|\, S_2^{-1} \sum_{n,m=0}^{\infty} \frac{s^n s'^m}{n!m!} a_1^n a_2^m a_1^{\dagger n} a_2^{\dagger m} S_2\,|00\rangle \\ = & (1-s)(1-s') \sum_{n,m=0}^{\infty} \frac{s^n s'^m}{n!m!} \langle\lambda,n,m|\lambda,n,m\rangle, \end{aligned} \tag{28}$$

where

$$|\lambda,n,m\rangle = a_1^{\dagger n} a_2^{\dagger m} S_2(\lambda)\,|00\rangle \tag{29}$$

is just the two-mode PASS. From Eq. (14), we have

$$\begin{aligned} |\lambda,n,m\rangle = & S_2 S_2^\dagger a_1^{\dagger n} a_2^{\dagger m} S_2\,|00\rangle \\ = & S_2 \sum_{l=0}^{n} n! \frac{(a_1^\dagger \cosh\lambda)^{n-l} (a_2 \sinh\lambda)^l}{l!(n-l)!} \\ & \times (a_2^\dagger \cosh\lambda)^m\,|00\rangle . \end{aligned} \tag{30}$$

Using Eq. (16) we obtain

$$\begin{aligned} |\lambda,n,m\rangle = & S_2 m!n! \cosh^{m+n}\lambda \\ & \times \sum_{l=0}^{n} \frac{\tanh^l\lambda}{l!(n-l)!(m-l)!} a_1^{\dagger n-l} a_2^{\dagger m-l}\,|00\rangle . \end{aligned} \tag{31}$$

So the overlap of $|\lambda,n,m\rangle$ is

$$\begin{aligned} \langle\lambda,n,m|\lambda,n,m\rangle = & (m!n!)^2 \cosh^{2(m+n)}\lambda \\ & \times \sum_{l=0}^{n} \frac{\tanh^{2l}\lambda}{(m-l)!(n-l)!(l!)^2} . \end{aligned} \tag{32}$$

Using Eq. (23) we see

$$\mathrm{P}_n^{(m-n,0)}(-\cosh 2\lambda)$$

$$= (-\cosh^2\lambda)^n \sum_{s=0}^{n} \frac{m!n!\tanh^{2s}\lambda}{(s!)^2(m-s)!(n-s)!}, \tag{33}$$

so equation (32) can be re-expressed as

$$\langle\lambda,n,m|\lambda,n,m\rangle = \begin{cases} m!n!\cosh^{2m}\lambda \mathrm{P}_n^{(0,m-n)}(\cosh 2\lambda), & m \geq n, \\ m!n!\cosh^{2n}\lambda \mathrm{P}_m^{(0,n-m)}(\cosh 2\lambda), & m < n, \end{cases} \tag{34}$$

where we have used the property of the Jacobi polynomial

$$\mathrm{P}_n^{(\alpha,\beta)}(-x) = (-1)^n \mathrm{P}_n^{(\alpha,\beta)}(x). \tag{35}$$

Equation (34) indicates that the PASS, expressed by $a_1^\dagger a_2^{\dagger m} S_2(\lambda)|00\rangle$, is normalized as Jacobi polynomial of the squeezing parameter λ. Then, equation (28) leads to

$$\begin{aligned} &\sum_{n,m=0}^{\infty} \frac{s^n s'^m}{m!n!} \langle\lambda,n,m|\lambda,n,m\rangle \\ &= \sum_{n,m=0,m\geq n}^{\infty} s^n s'^m \cosh^{2m}\lambda \mathrm{P}_n^{(0,m-n)}(\cosh 2\lambda) \\ &\quad + \sum_{n,m=0,m<n}^{\infty} s^n s'^m \cosh^{2n}\lambda \mathrm{P}_m^{(0,n-m)}(\cosh 2\lambda) \\ &= \frac{1}{(1-s)(1-s')\cosh^2\lambda - \sinh^2\lambda}. \end{aligned} \tag{36}$$

Let $\cosh 2\lambda = x$, equation (36) becomes

$$\begin{aligned} &\sum_{n,m=0,m\geq n}^{\infty} s^n s'^m \left(\frac{x+1}{2}\right)^m \mathrm{P}_n^{(0,m-n)}(x) \\ &\quad + \sum_{n,m=0,m<n}^{\infty} s^n s'^m \left(\frac{x+1}{2}\right)^n \mathrm{P}_m^{(0,n-m)}(x) \\ &= \frac{2}{(1-s)(1-s')(x+1)-x+1}, \end{aligned} \tag{37}$$

this is another new generating function of Jacobi polynomials, which is different from Eq. (26).

5. Applications

Using Eqs. (5) and (9) we can directly calculate the single-mode photon number average in $S_2(\lambda)|00\rangle$

$$\begin{aligned} &\langle 00|S_2^\dagger(\lambda) a_1^\dagger a_1 S_2(\lambda)|00\rangle \\ &= \frac{\partial}{\partial f}(\langle 00|S_2^\dagger(\lambda)\exp[f a_1^\dagger a_1 + g a_2^\dagger a_2] S_2(\lambda)|00\rangle)|_{f=g=0} \\ &= \frac{\partial}{\partial f} \frac{1}{\cosh^2\lambda - \mathrm{e}^{f+g}\sinh^2\lambda}\bigg|_{f=g=0} = \sinh^2\lambda, \end{aligned} \tag{38}$$

and

$$\begin{aligned} &\langle 00|S_2^\dagger(\lambda)(a_1^\dagger a_1)^2 S_2(\lambda)|00\rangle \\ &= \frac{\partial^2}{\partial f^2}(\langle 00|S_2^\dagger(\lambda)\exp[f a_1^\dagger a_1 + g a_2^\dagger a_2] S_2(\lambda)|00\rangle)|_{f=g=0} \\ &= \frac{\partial}{\partial f} \frac{\mathrm{e}^{f+g}\sinh^2\lambda}{(\cosh^2\lambda - \mathrm{e}^{f+g}\sinh^2\lambda)^2}\bigg|_{f=g=0} \\ &= \sinh^2\lambda + 2\sinh^4\lambda. \end{aligned} \tag{39}$$

So the fluctuation is

$$\left\langle (a_1^\dagger a_1)^2 \right\rangle - \left\langle a_1^\dagger a_1 \right\rangle^2 = \frac{1}{4}\sinh^2 2\lambda. \tag{40}$$

6. Summary

In summary, we have derived the photon number cumulant for two-mode squeezed state and shown that its cumulant expansion leads to normalization of PSSS and PASS. We find that the normalization is related to Jacobi polynomial, so the cumulant expansion in turn indicates the new generating function of the Jacobi polynomial. For single-mode photon-subtracted squeezed state we refer to Refs. [10] and [11]. Both photon subtraction and addition in one-mode case are realized in experiments.[12–14] Through subtracting a photon from a two-mode squeezed state to prepare the non-Gaussian two-mode entangled state is also realized in experiment.[12,15] For a review of quantum-state engineering by photon subtraction and addition, we refer to Refs. [16] and [17].

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