Exponential synchronization of complex dynamical networks with Markovian jumping parameters using sampled-data and mode-dependent probabilistic time-varying delays

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In this paper, the problem of exponential synchronization of complex dynamical networks with Markovian jumping parameters using sampled-data and Mode-dependent probabilistic time-varying coupling delays is investigated. The sampling period is assumed to be time-varying and bounded. The information of probability distribution of the time-varying delay is considered and transformed into parameter matrices of the transferred complex dynamical network model. Based on the condition, the design method of the desired sampled data controller is proposed. By constructing a new Lyapunov functional with triple integral terms, delay-distribution-dependent exponential synchronization criteria are derived in the form of linear matrix inequalities. Finally, two numerical examples are given to illustrate the effectiveness of the proposed methods.

Keywords: complex networks, exponential synchronization, mode-dependent time-varying delays, linear matrix inequalities, sampled-data control

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1. Introduction

During the last few years, complex dynamical networks (CDNs) have received increasing research interest in various fields of science and engineering.[1–3] CDNs are the set of interconnected nodes unfold according to their respective dynamical equations. Some of these nodes are usually coupled according to the network topology. In the real world, a lot of practical systems can be described by models of complex networks such as the world wide web, social networks, electrical power grids, global economic markets, coupled biological and chemical engineering systems. Complex dynamical networks are one of the most popular topics in this area of scientific research. In particular, one of the interesting phenomena in CDNs is the synchronization of all dynamical nodes in a network. The synchronization problem for complex dynamical networks has been received increasing research interest in recent years and the results are made available in the literature.[4–12] In Ref. [5], the authors focused the synchronization stability of general CDNs with coupling delay. Chaos synchronization of a general CDNs with coupling delays was investigated in Ref. [6]. In Ref. [8], the authors studied the global exponential synchronization and synchronizability for general dynamical networks. Recently, the mean square exponential synchronization in Lagrange sense for uncertain CDNs have been analyzed in Ref. [11].

In actual systems, complex networks may be subject to networks mode switching, a network sometimes has finite modes that switch from one to another with certain transition rate, such switching can be governed by a Markovian chain and many relevant results were available in the literature.[13–19] In Ref. [14], the adaptive synchronization for a class of stochastic neural networks which involve both mixed delays and Markovian jumping parameters have been investigated. In Ref. [16], the authors pointed out the stability and synchronization problem for Markovian jump delayed neural networks with partially unknown transition probabilities. The synchronization of discrete-time Markovian jumping stochastic complex networks with mode-dependent mixed time-delays have been investigated in Ref. [15]. The synchronization of complex networks with nonhomogeneous Markovian jump topology has been discussed in Ref. [18]. In Ref. [13], the authors studied the exponential synchronization problem for complex networks with Markovian jump and mixed time delays. Recently, in Ref. [19], the authors analyzed the exponential synchronization criteria for CDNs with Markovian jumping parameters and stochastic delays.

The main purpose of the sampled-data controller is to validate digital control techniques in real time. In the digital controller, the output of the chaotic system is sampled. Also, the sampled data control is used to make the suitable control signal. The sampled data control technology is used to solve the control problems in various systems such as chaotic systems, fuzzy systems, neural networks, and so on. Synchronization is one of the important characteristics of complex networks.

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In Ref. [20], the authors have proposed master-slave (drive-response) concept for achieving the synchronization of chaotic systems where different schemes have been studied to ensure the control and synchronization of such systems. The problem of synchronization of chaotic systems has been widely investigated in Ref. [21] due to its potential applications in image processing, secure communication, information science, harmonic oscillation generation, chaos generator design, chemical reaction, and biological systems. The general class of nonlinear chaotic systems such as chaos circuit, n-scroll attractors, and hyperbolic attractors has been represented as Lure's systems. In Ref. [22], chaotic Lure's systems have been discussed in several useful ways for master-slave synchronization schemes, dependent on the controllers such as adaptive control, observer-based control, state-delayed feedback control, and sampled-data control. The above techniques have used continuous time controllers to synchronize the chaos. However, in real world applications, it is difficult to ensure that the state variables that are transmitted to controllers are continuous. Sampled-data control technology has large number of development results in computer hardware and it is most powerful when compared with other control approaches. In addition, in order to make full use of modern computer techniques, the sampled-data control method is applied to synchronize the chaotic systems. Moreover, in Refs. [23], [24], and [25], the authors have developed sampled-data techniques for synchronization of master-slave systems at discrete time instants, as well as the input delay approach. The operation of synchronization based on sampled-data feedback control can be described as follows. First, the system states that both the master and slave systems form an error signal, which is fed to the sampler with a sampling interval h_k. Then, the sampled system states information is processed by the sampled-data controller to produce an appropriate control signal. Finally, the control signal is kept constant during the sampling interval by the zero-order-hold (ZOH) and sent to the slave system to realize the synchronization. The synchronization scheme has been illustrated in Fig. 1.

Sampled-data control technologies have also been developed in control theory. In particular, the sampled-data synchronization problem for dynamical networks has received very little attention. In Refs. [26] and [27], authors have discussed the sampled-data control problems by converting the sampling period into a time-varying but bounded delay. The synchronization problem for complex dynamical networks using sampled-data and other control approaches has been discussed in Refs. [28], [29], [30], [31], and [32]. In Ref. [28], the authors studied the synchronization of general CDNs with sampled-data. The sampled-data synchronization control of dynamical networks with stochastic sampling have been studied in Ref. [29]. In Ref. [30], the authors pointed out the synchronization of CDNs with coupling time-varying delays via sampled-data control. Recently, in Ref. [31], the authors have proposed the exponential synchronization for CDNs by using the sampled-data control method.

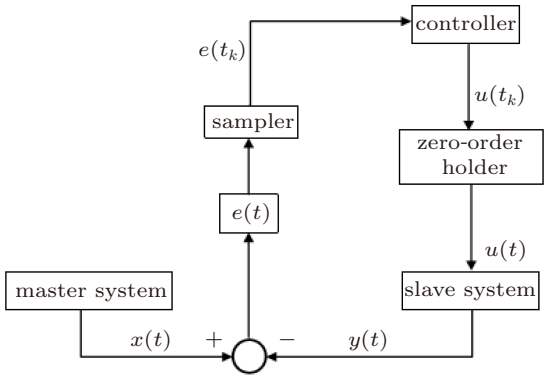


Fig. 1. Master-slave synchronization scheme under sampled-data feedback controller.

On the other hand, time delays often occur in complex networks due to the fact that the information transmission in complex networks is generally not instantaneous since the traveling speed of signals is limited and it is well known that time delays may cause undesirable dynamic network behaviors, such as oscillation and instability. In recent years, the synchronization problem for complex networks with time delays has gained increasing research attention.[33–48] In Ref. [38], authors studied the synchronization of complex dynamical networks with time delays. In Ref. [39], the authors analyzed the synchronization stability of general CDNs with time-varying delays. In Ref. [40], the authors introduced the global synchronization of linearly hybrid coupled networks with time-varying delay. The synchronization stability of general CDNs with time-varying delays have been investigated in Ref. [41]. Recently, in Ref. [42], authors discussed the novel criteria for exponential synchronization of inner time-varying complex networks with coupling delay. Moreover, in many practical systems, the probability distribution of time delay in the interval is an important characteristic for the network conditions. In Ref. [44], the authors proposed the synchronization stability of complex dynamical networks with probabilistic time-varying delays. In Ref. [45], the authors derived the synchronization criteria for Markovian jumping stochastic complex networks with distributed time delays and probabilistic interval discrete time-varying delays. The cluster synchronization stability for stochastic CDNs with probabilistic interval time-varying delays have been studied in Ref. [46]. The synchronization stability for discrete-time stochastic complex networks with probabilistic interval time-varying delays have been established in Ref. [47]. The global synchronization stability for stochastic complex dynamical networks with probabilistic interval time-varying delays have been discussed in Ref. [48]. To the best of the author's knowledge, exponential synchronization for Markovian jumping CDNs using sampled-data with mode-dependent probabilistic time-varying coupling delays has not yet been studied in the literature.

Motivated by the above discussion, in this paper the problem of exponential synchronization of CDNs with Markovian jumping parameters using sampled-data control and mode-dependent probabilistic time-varying coupling delays is considered. The sampling period is assumed to be time-varying and bounded. The information of the delay-probability-distribution is introduced in the CDNs model. The design method of the desired sampled-data synchronization controller is proposed. The delay probability distribution conditions depend not only on the size of the delay but they also depend on the delay intervals. By constructing a new Lyapunov functional with triple integral terms and employing Jensen's inequality, some sufficient conditions are derived for the considered CDNs in terms of linear matrix inequalities (LMIs). The obtained LMIs can be easily solved by MATLAB LMI control Toolbox. Numerical simulations are finally given to demonstrate the effectiveness of the theoretical results.

Notation: Some notations and symbols used in this paper are presented below. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ the set of all $n\times m$ real matrices.

For any matrix $A=[a_{ij}]_{n\times n}$, let A^{T} and A^{-1} denote the transpose and the inverse of A, respectively. For real symmetric matrices X and Y, the notation $X\le Y$ (respectively, $X\ge Y$) means that the matrix $X-Y$ is real negative-definite (respectively, positive-definite). The superscript "T" denotes the transposition of the matrix of appropriate dimension. "I" is the identity matrix. diag$\{\cdots\}$ stands for block diagonal matrix. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the largest and smallest eigenvalue of a given matrix, respectively. The Kronecker product of matrices $P\in\mathbb{R}^{n\times m}$ and $Q\in\mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{np\times mq}$ which is denoted as $P\otimes Q$. For an arbitrary matrix B and two symmetric matrices A and C,

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix}$$

denotes a symmetric matrix, where $*$ denotes the symmetric terms in a symmetric matrix. Let $(\Omega,\mathbb{F},\mathbb{P})$ be a complete probability space with filtration $\{\mathbb{F}_t\}_{t\ge 0}$ satisfying usual conditions; that is, the filtration contains all $\mathbb{P}$-null sets and is right continuous. Denote by $L^{\mathbb{P}}_{\mathbb{F}_0}([-\varsigma,0])$ the family of all $\mathbb{F}_0$-measurable $C([-\varsigma,0];\mathbb{R}^n)$-valued random variables $\varphi=\{\varphi(s):-\varsigma\le s\le 0\}$ such that $\sup_{-\varsigma\le s\le 0}\mathbb{E}\|\varphi(s)\|^{\mathbb{P}}<\infty$, where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$; $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator.

2. Problem description and preliminaries

Let $(\Omega,\mathbb{F},\mathbb{P})$ be a complete probability space, where Ω is the sample space, $\mathbb{F}$ is the σ-algebra of events and $\mathbb{P}$ is the probability measure defined on $\mathbb{F}$. Let $M=\{1,2,\ldots,m\}$ and the random form process $\{r(t),t\in[0,+\infty)\}$ be a homogeneous finite state Markov process with right continuous trajectories with generator $Q=(q_{ij})_{m\times m}$ $(i,j\in M)$ given by

$$Pr\{r(t+\Delta)=j|r(t)=i\}=\begin{cases} q_{ij}\Delta+O(\Delta), & i\ne j,\\ 1+q_{ii}\Delta+O(\Delta), & i=j,\end{cases}$$

where $\Delta>0$, $\lim_{\Delta\to 0}[O(\Delta)/\Delta]=0$, and $q_{ij}\ge 0$, for $j\ne i$, is the transition rate from mode i at time t to mode j at time $t+\Delta$ and $q_{ii}=-\sum_{j=1,\ j\ne i}^{m}q_{ij}$.

Consider the following CDN with Markovian jumping parameters consisting of N identical nodes with each node being n-dimensional dynamical system

$$\begin{aligned}\dot{x}_w(t) \;=\; & f(x_w(t))+\sum_{l=1}^{N}G_{wl}D(r(t))x_l(t)\\ & +\sum_{l=1}^{N}G_{wl}A(r(t))x_l(t-\tau(t))+u_w(t),\\ & w=1,2,\ldots,N,\end{aligned}\tag{1}$$

where $x_w(t)$ is the state variable and $u_w(t)$ is the control input of the node w, $D(r(t))=(d_{wl})_{n\times n}\in\mathbb{R}^{n\times n}$ and $A(r(t))=(a_{wl})_{n\times n}\in\mathbb{R}^{n\times n}$ are constant inner-coupling matrices of the nodes and $G=(G_{wl})_{N\times N}$ is the outer coupling matrix of the network. If there is a connection between node w and node l $(w\ne l)$, then $G_{wl}=1$, otherwise, $G_{wl}=0$ $(w\ne l)$. The diagonal elements of matrix G are defined by

$$G_{ww}=-\sum_{l=1,\ l\ne w}^{N}G_{wl},w=1,2,\ldots,N.$$

Assumption 1 Considering the information of probability distribution of the time-delay $\tau(t)$, two sets and functions are defined

$$\begin{aligned}&\Omega_1=\{t:\tau(t)\in[0,\tau_1)\},\qquad \Omega_2=\{t:\tau(t)\in[\tau_1,\tau_2]\},\\ &\tau_1(t)=\begin{cases}\tau(t), & \text{for } t\in\Omega_1,\\ \overline{\tau}_1, & \text{for } t\in\Omega_2,\end{cases}\qquad \tau_2(t)=\begin{cases}\tau(t), & \text{for } t\in\Omega_2,\\ \overline{\tau}_2, & \text{for } t\in\Omega_1,\end{cases}\\ &\dot{\tau}_1(t)\le\nu_1<\infty,\qquad \dot{\tau}_2(t)\le\nu_2<\infty,\end{aligned}$$

where $\tau_1\in[0,\tau_2]$, $\overline{\tau}_1\in[0,\tau_0)$, and $\overline{\tau}_2\in[\tau_1,\tau_2]$.

It is easy to know $t\in\Omega_1$ means the event $\tau(t)\in[0,\tau_1)$ occurs and $t\in\Omega_2$ means the event $\tau(t)\in[\tau_1,\tau_2]$ occurs. Therefore, a stochastic variable $\alpha(t)$ can be defined as follows:

$$\alpha(t)=\begin{cases}1, & t\in\Omega_1,\\ 0, & t\in\Omega_2.\end{cases}$$

Assumption 2 $\alpha(t)$ is the Bernoulli distributed sequence with

$$\begin{aligned}\mathrm{Prob}\{\alpha(t)=1\} \;&=\; \mathbb{E}\{\alpha(t)\}=\alpha_0,\\ \mathrm{Prob}\{\alpha(t)=0\} \;&=\; 1-\mathbb{E}\{\alpha(t)\}=1-\alpha_0,\end{aligned}$$

where $0\le\alpha_0\le 1$ is a constant and $\mathbb{E}\{\alpha(t)\}$ is the expectation of $\alpha(t)$.

From Assumption 2, it is easy to see that

$$\mathbb{E}\{\alpha(t)-\alpha_0\}=0,\ \ \mathbb{E}\{(\alpha(t)-\alpha_0)^2\}=\alpha_0(1-\alpha_0).$$

By Assumptions 1 and 2, the system (1) can be rewritten as

$$\begin{aligned}\dot{x}_w(t) &= f(x_w(t))+\sum_{l=1}^{N} G_{wl}D(r(t))x_l(t)\\ &\quad+\alpha_0\sum_{l=1}^{N} G_{wl}A(r(t))x_l(t-\tau_1(r(t),t))\\ &\quad+(1-\alpha_0)\sum_{l=1}^{N} G_{wl}A(r(t))x_l(t-\tau_2(r(t),t))\\ &\quad+(\alpha(t)-\alpha_0)\Big[\sum_{l=1}^{N} G_{wl}\times A(r(t))x_l(t-\tau_1(r(t),t))\\ &\quad-\sum_{l=1}^{N} G_{wl}A(r(t))x_l(t-\tau_2(r(t),t))\Big]+u_w(t), \qquad (2)\end{aligned}$$

where $\tau_1(r(t),t)$ and $\tau_2(r(t),t)$ are the mode-dependent time-varying delays. For each $r(t)=i\in M$, we denote $\tau_1(r(t),t)=\tau_{1i}(t)$, $\tau_2(r(t),t)=\tau_{2i}(t)$. For convenience, denote $D(r(t)=i)=D_i$, $A(r(t)=i)=A_i$. The system (2) can be written as

$$\begin{aligned}\dot{x}_w(t) &= f(x_w(t))+\sum_{l=1}^{N} G_{wl}D_ix_l(t)\\ &\quad+\alpha_0\sum_{l=1}^{N} G_{wl}A_ix_l(t-\tau_{1i}(t))\\ &\quad+(1-\alpha_0)\sum_{l=1}^{N} G_{wl}A_ix_l(t-\tau_{2i}(t))+(\alpha(t)-\alpha_0)\\ &\quad\times\Big[\sum_{l=1}^{N} G_{wl}A_ix_l(t-\tau_{1i}(t))\\ &\quad-\sum_{l=1}^{N} G_{wl}A_ix_l(t-\tau_{2i}(t))\Big]+u_w(t). \qquad (3)\end{aligned}$$

It is assumed that there exist scalars $\tau_{1i}>0,\tau_{2i}>0$, for each $i=1,2,\ldots,m$, such that

$$\begin{aligned}&0\le\tau_{1i}(t)<\tau_{1i},\quad \dot{\tau}_{1i}(t)\le\nu_{1i},\\ &\tau_{1i}\le\tau_{2i}(t)\le\tau_{2i},\quad \dot{\tau}_{2i}(t)\le\nu_{2i},\end{aligned}$$

where

$$\tau_1=\max_{i\in M}\{\tau_{1i}\},\quad \tau_2=\max_{i\in M}\{\tau_{2i}\}.$$

Assumption 3 Let $f:\mathbb{R}^n\to\mathbb{R}^n$ is a continuous vector valued function and satisfies the following sector-bounded condition:

$$[f(x)-f(y)-U(x-y)]^{\mathrm{T}}[f(x)-f(y)-V(x-y)]\le 0, \quad (4)$$

for all $x,y\in\mathbb{R}^n$, where U and V are constant matrices of appropriate dimensions.

Let $e_w(t)=x_w(t)-s(t)$ be the synchronization error, where $s(t)\in\mathbb{R}^n$ is the state trajectory of the unforced isolate node $\dot{s}(t)=f(s(t))$. Then the error dynamics of CDN (3) can be obtained as follows:

$$\begin{aligned}\dot{e}_w(t) &= g(e_w(t))+\sum_{l=1}^{N} G_{wl}D_ie_l(t)\\ &\quad+\alpha_0\sum_{l=1}^{N} G_{wl}A_ie_l(t-\tau_{1i}(t))\\ &\quad+(1-\alpha_0)\sum_{l=1}^{N} G_{wl}A_ie_l(t-\tau_{2i}(t))\\ &\quad+(\alpha(t)-\alpha_0)\Big[\sum_{l=1}^{N} G_{wl}A_ie_l(t-\tau_{1i}(t))\\ &\quad-\sum_{l=1}^{N} G_{wl}A_ie_l(t-\tau_{2i}(t))\Big]+u_w(t),\\ &\qquad w=1,2,\ldots,N,\ \ i=1,2,\ldots,m, \qquad (5)\end{aligned}$$

where $g(e_w(t))=f(x_w(t))-f(s(t))$.

The control signal is assumed to be generated by using a zero-order-hold (ZOH) function with a sequence of hold times $0=t_0<t_1<\cdots<t_k<\cdots$. Therefore, the state feedback controller takes the following form

$$\begin{aligned}&u_w(t)=K_ie_w(t_k),\ \ t_k\le t<t_{k+1},\\ &w=1,2,\ldots,N,\ \ i=1,2,\ldots,m, \qquad (6)\end{aligned}$$

where K_i is sampled-data feedback controller gain matrix to be determined, $e_w(t_k)$ is the discrete measurement of $e_w(t)$ at the sampling instant t_k, $\lim_{k\to+\infty}t_k=+\infty$. It is assumed that $t_{k+1}-t_k=h_k\le d$, for any integer $k\ge 0$, where d is a positive scalar and represents the largest sampling interval.

By substituting Eq. (6) into Eq. (5), we obtain

$$\begin{aligned}\dot{e}_w(t) &= g(e_w(t))+\sum_{l=1}^{N} G_{wl}D_ie_l(t)\\ &\quad+\alpha_0\sum_{l=1}^{N} G_{wl}A_ie_l(t-\tau_{1i}(t))\\ &\quad+(1-\alpha_0)\sum_{l=1}^{N} G_{wl}A_ie_l(t-\tau_{2i}(t))\\ &\quad+(\alpha(t)-\alpha_0)\Big[\sum_{l=1}^{N} G_{wl}A_ie_l(t-\tau_{1i}(t))\\ &\quad-\sum_{l=1}^{N} G_{wl}A_ie_l(t-\tau_{2i}(t))\Big]+K_ie_w(t-d(t)),\\ &\qquad w=1,2,\ldots,N,\ \ i=1,2,\ldots,m, \qquad (7)\end{aligned}$$

where $d(t)=t-t_k$. Thus, we have

$$0\le d(t)\le d. \qquad (8)$$

Equation (7) can be rewritten as

$$\begin{aligned}\dot{e}(t) &= \overline{g}(e(t))+(G\otimes D_i)e(t)+\alpha_0(G\otimes A_i)e(t-\tau_{1i}(t))\\ &\quad+(1-\alpha_0)(G\otimes A_i)e(t-\tau_{2i}(t))\\ &\quad+(\alpha(t)-\alpha_0)\big[(G\otimes A_i)e(t-\tau_{1i}(t))\\ &\quad-(G\otimes A_i)e(t-\tau_{2i}(t))\big]+K_ie(t-d(t)), \qquad (9)\end{aligned}$$

where $K_i=\mathrm{diag}\{K_{i1},K_{i2},\ldots,K_{iN}\}$ and

$$e(t)=\begin{bmatrix}e_1(t)\\ e_2(t)\\ \cdot\\ \cdot\\ \cdot\\ e_N(t)\end{bmatrix},\quad \overline{g}(e(t))=\begin{bmatrix}g(e_1(t))\\ g(e_2(t))\\ \cdot\\ \cdot\\ \cdot\\ g(e_N(t))\end{bmatrix}.$$

The following lemmas and definitions will be used to derive

our main results.

Lemma 1[31] For any matrix $W>0$, scalars a and b, satisfying $b>a$, a vector function $x:[a,b]\to\mathbb{R}^n$, such that the integrations concerned are well defined, then

$$\left(\int_a^b x(s)\mathrm{d}s\right)^{\mathrm{T}} W\left(\int_a^b x(s)\mathrm{d}s\right)\le(b-a)\int_a^b x^{\mathrm{T}}(s)Wx(s)\mathrm{d}s.$$

Lemma 2[19] For a given matrix

$$S=\begin{bmatrix}S_{11} & S_{12}\\ S_{12}^{\mathrm{T}} & S_{22}\end{bmatrix}$$

with $S_{11}=S_{11}^{\mathrm{T}}$, $S_{22}=S_{22}^{\mathrm{T}}$, then the following conditions are equivalent:

i) $S<0$;

ii) $S_{22}<0$, $S_{11}-S_{12}S_{22}^{-1}S_{12}^{\mathrm{T}}<0$.

Definion 1[31] The CDN (3) is said to be exponentially synchronized if the error system (9) is exponentially stable; that is, there exist two constants $\alpha>0$ and $\beta>0$ such that

$$\|e(t)\|^2\le\alpha\mathrm{e}^{-\beta t}\sup_{-\max\{\tau_1,\tau_2,p\}\le s\le 0}\|e(s)\|^2.$$

3. Main results

In this section, the exponential stability of the error system (9) is first investigated. Then, the sampled data feedback controllers are designed to achieve the exponential synchronization of the CDN (3). For representation convenience, the following notations are introduced:

$$\overline{U}=\frac{(I_N\otimes U)^{\mathrm{T}}(I_N\otimes V)}{2}+\frac{(I_N\otimes V)^{\mathrm{T}}(I_N\otimes U)}{2},$$
$$\overline{V}=-\frac{(I_N\otimes U)^{\mathrm{T}}+(I_N\otimes V)^{\mathrm{T}}}{2}.$$

Theorem 1 Let the controller gain K_i $(i=1,2,\ldots,m)$ be given. The error system (9) is exponentially stable in the mean square, if there exist matrices P_i $(i\in M)$, $H_v>0$ $(v=1,2,3,4)$, $N_y>0$ $(y=1,2,3,4)$, $T_1>0$, $T_2>0$, $Z_1>0$, $Z_2>0$, $Z_3>0$, and some positive scalar $\mu>0$ such that the following LMIs hold

$$N_1+(1-\nu_{1i})N_2>0,\quad N_3+(1-\nu_{2i})N_4>0, \tag{10}$$
$$\Pi_1=\begin{bmatrix}\Omega_1-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1-d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & L^{\mathrm{T}}\\ * & -R\end{bmatrix}<0, \tag{11}$$
$$\Pi_2=\begin{bmatrix}\Omega_1-\xi_2^{\mathrm{T}}\mathcal{X}\xi_2-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2-d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & L^{\mathrm{T}}\\ * & -R\end{bmatrix}<0, \tag{12}$$
$$\Pi_3=\begin{bmatrix}\Omega_1-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2-d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & L^{\mathrm{T}}\\ * & -R\end{bmatrix}<0, \tag{13}$$
$$\Pi_4=\begin{bmatrix}\Omega_1-\xi_2^{\mathrm{T}}\mathcal{X}\xi_2-\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1-d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & L^{\mathrm{T}}\\ * & -R\end{bmatrix}<0, \tag{14}$$
$$\Pi_5=\begin{bmatrix}\Omega_1-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2-d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & L^{\mathrm{T}}\\ * & -R\end{bmatrix}<0, \tag{15}$$
$$\Pi_6=\begin{bmatrix}\Omega_1-\xi_2^{\mathrm{T}}\mathcal{X}\xi_2-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2-d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & L^{\mathrm{T}}\\ * & -R\end{bmatrix}<0, \tag{16}$$

where

$$\Omega_1=\begin{bmatrix}\Omega_{1,1} & \Omega_{1,2} & 0 & \Omega_{1,4} & 0 & \Omega_{1,6} & \Omega_{1,7} & \Omega_{1,8} & 0 & \Omega_{1,10} & \Omega_{1,11}\\ * & \Omega_{2,2} & \Omega_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \Omega_{3,3} & \Omega_{3,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & \Omega_{4,4} & \Omega_{4,5} & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \Omega_{5,5} & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & \Omega_{6,6} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & \Omega_{7,7} & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & \Omega_{8,8} & \Omega_{8,9} & 0 & 0\\ * & * & * & * & * & * & * & * & \Omega_{9,9} & 0 & 0\\ * & * & * & * & * & * & * & * & * & \Omega_{10,10} & 0\\ * & * & * & * & * & * & * & * & * & * & \Omega_{11,11}\end{bmatrix}, \tag{17}$$

$$R=N_1\mathcal{M}_1+N_2\mathcal{M}_1+N_3\mathcal{M}_2+N_4\mathcal{M}_2+\overline{q}N_2\mathcal{M}_3+\overline{q}N_4\mathcal{M}_4+T_1\mathcal{M}_3+T_2\mathcal{M}_4+Z_2\mathcal{M}_5+Z_3\mathcal{M}_6,$$
$$L=[R(G\otimes D_i)\ \ R(G\otimes A_i)\alpha_0\ \ 0\ \ R(G\otimes A_i)(1-\alpha_0)\ \ 0\ \ 0\ \ 0\ \ RK_i\ \ 0\ \ 0\ \ R],$$
$$\mathcal{X}=\tau_1^{-1}(N_1+(1-\nu_{1i})N_2),\quad \mathcal{Y}=(\tau_2-\tau_1)^{-1}(N_3+(1-\nu_{2i})N_4),$$
$$\xi_1=[I\ \ -I\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],\quad \xi_2=[0\ I\ \ -I\ 0\ 0\ 0\ 0\ 0\ 0\ 0],\quad \phi_1=[0\ 0\ I\ \ -I\ 0\ 0\ 0\ 0\ 0\ 0],$$
$$\phi_2=[0\ 0\ 0\ I\ \ -I\ 0\ 0\ 0\ 0\ 0],\quad \rho_1=[I\ 0\ 0\ 0\ 0\ 0\ 0\ -I\ 0\ 0],\quad \rho_2=[0\ 0\ 0\ 0\ 0\ 0\ 0\ I\ \ -I\ 0],$$

$$\Omega_{1,1} = 2P_i(G\otimes D_i) + \sum_{j=1}^{m} q_{ij}P_j + H_1 + H_2 + \overline{q}H_2\mathcal{M}_1 + \overline{q}H_4\mathcal{M}_2 - \mathcal{X} - 2T_1 - \frac{2}{\tau_2^2-\tau_1^2} \times (\tau_2-\tau_1)^2T_2 - d^{-1}Z_2 + Z_1 - 2Z_3 - \mu\overline{U},$$

$$\Omega_{1,2} = P_i\alpha_0(G\otimes A_i) + \mathcal{X},\ \ \Omega_{1,4} = P_i(1-\alpha_0)(G\otimes A_i),$$

$$\Omega_{1,6} = \frac{2}{\tau_1}T_1,\ \ \Omega_{1,7} = \frac{2}{(\tau_2^2-\tau_1^2)}(\tau_2-\tau_1)T_2,$$

$$\Omega_{1,8} = P_iK_i + d^{-1}Z_2,\ \ \Omega_{1,10} = \frac{2}{d}Z_3,\ \ \Omega_{1,11} = P_i - \mu\overline{V},$$

$$\Omega_{2,2} = -\mathrm{e}^{-\alpha\tau_1}(1-\nu_{1i})H_2 - \mathcal{X} - \tau_1^{-1}N_1,\ \ \Omega_{2,3} = \tau_1^{-1}N_1,$$

$$\Omega_{3,3} = -\mathrm{e}^{-\alpha\tau_1}H_1 + \mathrm{e}^{-\alpha\tau_1}H_3 + \mathrm{e}^{-\alpha\tau_1}H_4 - \tau_1^{-1}N_1 - \mathcal{Y},$$

$$\Omega_{3,4} = \mathcal{Y},\ \ \Omega_{4,4} = -\mathrm{e}^{-\alpha\tau_2}(1-\nu_{2i})H_4 - \mathcal{Y} - (\tau_2-\tau_1)^{-1}N_3,$$

$$\Omega_{4,5} = (\tau_2-\tau_1)^{-1}N_3,\ \ \Omega_{5,5} = -\mathrm{e}^{-\alpha\tau_2}H_3 - (\tau_2-\tau_1)^{-1}N_3,$$

$$\Omega_{6,6} = \frac{-2}{\tau_1^2}T_1,\ \ \Omega_{7,7} = -\frac{2}{(\tau_2^2-\tau_1^2)}(\tau_2-\tau_1)T_2,$$

$$\Omega_{8,8} = -2d^{-1}Z_2,\ \ \Omega_{8,9} = d^{-1}Z_2,$$

$$\Omega_{9,9} = -\mathrm{e}^{-\alpha d}Z_1 - d^{-1}Z_2,\ \ \Omega_{10,10} = \frac{-2}{d^2}Z_3,$$

$$\Omega_{11,11} = -\mu I,$$

$$\mathcal{M}_1 = \frac{1}{\alpha}\left[\mathrm{e}^{\alpha\tau_1}-1\right],\ \ \mathcal{M}_2 = \frac{1}{\alpha}\left[\mathrm{e}^{\alpha\tau_2}-\mathrm{e}^{\alpha\tau_1}\right],$$

$$\mathcal{M}_3 = \left[\frac{\mathrm{e}^{\alpha\tau_1}}{\alpha^2} - \frac{\tau_1}{\alpha} - \frac{1}{\alpha^2}\right],\ \ \mathcal{M}_4 = \left[\frac{\mathrm{e}^{\alpha\tau_2}}{\alpha^2} + \frac{\tau_1}{\alpha} - \frac{\mathrm{e}^{\alpha\tau_1}}{\alpha^2} - \frac{\tau_2}{\alpha}\right],$$

$$\mathcal{M}_5 = \frac{1}{\alpha}\left[\mathrm{e}^{\alpha d}-1\right],\ \ \mathcal{M}_6 = \left[\frac{\mathrm{e}^{\alpha d}}{\alpha^2} - \frac{1}{\alpha^2} - \frac{d}{\alpha}\right],$$

$$\overline{q} = \max\{|q_{ii}|, i\in M\},\ \ \tau_{12} = \tau_2 - \tau_1,$$

and the other entries of Ω_{ij} are zero. Then, the error system (9) is exponentially stable.

Proof Consider the following Lyapunov–Krasovskii functional

$$V(e(t),t,i) = \sum_{i=1}^{5} V_i(e(t),t,i), \tag{18}$$

where

$$\begin{aligned}
V_1(e(t),t,i) &= \mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)P_ie(t),\\
V_2(e(t),t,i) &= \int_{t-\tau_1}^{t}\mathrm{e}^{\alpha s}e^{\mathrm{T}}(s)H_1e(s)\mathrm{d}s + \int_{t-\tau_{1i}(t)}^{t}\mathrm{e}^{\alpha s}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s + \int_{t-\tau_2}^{t-\tau_1}\mathrm{e}^{\alpha s}e^{\mathrm{T}}(s)H_3e(s)\mathrm{d}s\\
&\quad + \int_{t-\tau_{2i}(t)}^{t-\tau_1}\mathrm{e}^{\alpha s}e^{\mathrm{T}}(s)H_4e(s)\mathrm{d}s + \overline{q}\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s\mathrm{d}\theta\\
&\quad + \overline{q}\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}e^{\mathrm{T}}(s)H_4e(s)\mathrm{d}s\mathrm{d}\theta,\\
V_3(e(t),t,i) &= \int_{-\tau_1}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)N_1\dot{e}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_{1i}(t)}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)N_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\\
&\quad + \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)N_3\dot{e}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_{2i}(t)}^{-\tau_1}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)N_4\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\\
&\quad + \overline{q}\int_{-\tau_1}^{0}\int_{\lambda}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)N_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\lambda\\
&\quad + \overline{q}\int_{-\tau_2}^{-\tau_1}\int_{\lambda}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)N_4\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\lambda,\\
V_4(e(t),t,i) &= \int_{-\tau_1}^{0}\int_{\lambda}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)T_1\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\lambda\\
&\quad + \int_{-\tau_2}^{-\tau_1}\int_{\lambda}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)T_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\lambda,\\
V_5(e(t),t,i) &= \int_{t-d}^{t}\mathrm{e}^{\alpha s}e^{\mathrm{T}}(s)Z_1e(s)\mathrm{d}s + \int_{-d}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)Z_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\\
&\quad + \int_{-d}^{0}\int_{\lambda}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\alpha(s-\theta)}\dot{e}^{\mathrm{T}}(s)Z_3\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\lambda.
\end{aligned}$$

Let $\mathcal{L}$ be the weak infinitesimal generator of the random process $\{e(t), r(t),\ t\geq 0\}$. Then, for each $r(t)=i, \forall\ i\in M$, we get

$$\mathcal{L}V(e(t),t,i) = \sum_{i=1}^{5}\mathcal{L}V_i(e(t),t,i), \tag{19}$$

where

$$\begin{aligned}
\mathcal{L}V_1(e(t),t,i) &= 2\mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)P_i\big[\overline{g}(e(t)) + (G\otimes D_i)e(t)\\
&\quad + \alpha_0(G\otimes A_i)e(t-\tau_{1i}(t)) + (1-\alpha_0)\\
&\quad \times (G\otimes A_i)e(t-\tau_{2i}(t)) + (\alpha(t)-\alpha_0)\\
&\quad \times \big[(G\otimes A_i)e(t-\tau_{1i}(t)) - (G\otimes A_i)e(t-\tau_{2i}(t))\big]\\
&\quad + K_ie(t-d(t))\big] + \mathrm{e}^{\alpha t}\sum_{j=1}^{m}q_{ij}e^{\mathrm{T}}(t)P_je(t), \qquad (20)\\
\mathcal{L}V_2(e(t),t,i) &\leq \mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)H_1e(t)\\
&\quad - \mathrm{e}^{\alpha t}\mathrm{e}^{-\alpha\tau_1}e^{\mathrm{T}}(t-\tau_1)H_1e(t-\tau_1)\\
&\quad + \mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)H_2e(t) - \mathrm{e}^{\alpha t}\mathrm{e}^{-\alpha\tau_1}(1-\nu_{1i})\\
&\quad \times \big[e^{\mathrm{T}}(t-\tau_{1i}(t))H_2e(t-\tau_{1i}(t))\big]\\
&\quad + \mathrm{e}^{\alpha t}\mathrm{e}^{-\alpha\tau_1}e^{\mathrm{T}}(t-\tau_1)H_3e(t-\tau_1)\\
&\quad - \mathrm{e}^{\alpha t}\mathrm{e}^{-\alpha\tau_2}e^{\mathrm{T}}(t-\tau_2)H_3e(t-\tau_2)\\
&\quad + \mathrm{e}^{\alpha t}\mathrm{e}^{-\alpha\tau_1}e^{\mathrm{T}}(t-\tau_1)H_4e(t-\tau_1) - \mathrm{e}^{\alpha t}\mathrm{e}^{-\alpha\tau_2}\\
&\quad \times (1-\nu_{2i})[e^{\mathrm{T}}(t-\tau_{2i}(t))H_4e(t-\tau_{2i}(t))]\\
&\quad + \overline{q}\mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)H_2e(t)\mathcal{M}_1\\
&\quad - \mathrm{e}^{\alpha t}\overline{q}\int_{t-\tau_1}^{t}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s\\
&\quad + \overline{q}\mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)H_4e(t)\mathcal{M}_2\\
&\quad - \mathrm{e}^{\alpha t}\overline{q}\int_{t-\tau_2}^{t-\tau_1}e^{\mathrm{T}}(s)H_4e(s)\mathrm{d}s
\end{aligned}$$

$$+\mathrm{e}^{\alpha t}\sum_{j=1}^{m}q_{ij}\int_{t-\tau_{1j}(t)}^{t}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s$$
$$+\mathrm{e}^{\alpha t}\sum_{j=1}^{m}q_{ij}\int_{t-\tau_{2j}(t)}^{t-\tau_1}e^{\mathrm{T}}(s)H_4e(s)\mathrm{d}s, \tag{21}$$

$$\begin{aligned}\mathcal{L}V_3(e(t),t,i)\le{}&\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)N_1\dot e(t)\mathcal{M}_1-\mathrm{e}^{\alpha t}\int_{t-\tau_1}^{t}\dot e^{\mathrm{T}}(s)N_1\dot e(s)\mathrm{d}s\\&+\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)N_2\dot e(t)\mathcal{M}_1\\&-(1-\nu_{1i})\mathrm{e}^{\alpha t}\int_{t-\tau_{1i}(t)}^{t}\dot e^{\mathrm{T}}(s)N_2\dot e(s)\mathrm{d}s\\&+\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)N_3\dot e(t)\mathcal{M}_2\\&-\mathrm{e}^{\alpha t}\int_{t-\tau_2}^{t-\tau_1}\dot e^{\mathrm{T}}(s)N_3\dot e(s)\mathrm{d}s\\&+\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)N_4\dot e(t)\mathcal{M}_2\\&-(1-\nu_{2i})\mathrm{e}^{\alpha t}\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot e^{\mathrm{T}}(s)N_4\dot e(s)\mathrm{d}s\\&+\overline{q}\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)N_2\dot e(t)\mathcal{M}_3\\&-\mathrm{e}^{\alpha t}\overline{q}\int_{-\tau_1}^{0}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)N_2\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&+\overline{q}\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)N_4\dot e(t)\mathcal{M}_4\\&-\mathrm{e}^{\alpha_t}\overline{q}\int_{-\tau_2}^{-\tau_1}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)N_4\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&+\mathrm{e}^{\alpha t}\sum_{j=1}^{m}q_{ij}\int_{-\tau_{1j}(t)}^{0}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)N_2\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&+\mathrm{e}^{\alpha t}\sum_{j=1}^{m}q_{ij}\int_{-\tau_{2j}(t)}^{-\tau_1}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)N_4\dot e(s)\mathrm{d}s\mathrm{d}\lambda,\end{aligned} \tag{22}$$

$$\begin{aligned}\mathcal{L}V_4(e(t),t,i)\le{}&\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)T_1\dot e(t)\mathcal{M}_3\\&-\mathrm{e}^{\alpha t}\int_{-\tau_1}^{0}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)T_1\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&+\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)T_2\dot e(t)\mathcal{M}_4\\&-\mathrm{e}^{\alpha t}\int_{-\tau_2}^{-\tau_1}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)T_2\dot e(s)\mathrm{d}s\mathrm{d}\lambda,\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{L}V_5(e(t),t,i)\le{}&\mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)Z_1e(t)\\&-\mathrm{e}^{\alpha t}\mathrm{e}^{-\alpha d}e^{\mathrm{T}}(t-d)Z_1e(t-d)\\&+\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)Z_2\dot e(t)\mathcal{M}_5\\&-\mathrm{e}^{\alpha t}\int_{t-d}^{t}\dot e^{\mathrm{T}}(s)Z_2\dot e(s)\mathrm{d}s\\&+\mathrm{e}^{\alpha t}\dot e^{\mathrm{T}}(t)Z_3\dot e(t)\mathcal{M}_6\\&-\mathrm{e}^{\alpha t}\int_{-d}^{0}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)Z_3\dot e(s)\mathrm{d}s\mathrm{d}\lambda.\end{aligned} \tag{24}$$

Noting that $q_{ij}\ge 0$, for $j\ne i$ and $q_{ii}<0$, we have

$$\begin{aligned}&\mathrm{e}^{\alpha t}\sum_{j=1}^{m}q_{ij}\int_{t-\tau_{1j}(t)}^{t}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s\\&\le\mathrm{e}^{\alpha t}\sum_{j\ne i}q_{ij}\int_{t-\tau_{1j}(t)}^{t}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s\\&\quad+q_{ii}\mathrm{e}^{\alpha t}\int_{t-\tau_{1i}(t)}^{t}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s\\&\le\mathrm{e}^{\alpha t}\sum_{j\ne i}q_{ij}\int_{t-\tau_1(t)}^{t}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s\\&\le-q_{ii}\mathrm{e}^{\alpha t}\int_{t-\tau_1(t)}^{t}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s\\&\le\overline{q}\mathrm{e}^{\alpha t}\int_{t-\tau_1}^{t}e^{\mathrm{T}}(s)H_2e(s)\mathrm{d}s,\end{aligned} \tag{25}$$

$$\begin{aligned}&\mathrm{e}^{\alpha t}\sum_{j=1}^{m}q_{ij}\int_{t-\tau_{2j}(t)}^{t-\tau_1}e^{\mathrm{T}}(s)H_4e(s)\mathrm{d}s\\&\le\overline{q}\mathrm{e}^{\alpha t}\int_{t-\tau_2}^{t-\tau_1}e^{\mathrm{T}}(s)H_4e(s)\mathrm{d}s,\end{aligned} \tag{26}$$

$$\begin{aligned}&\mathrm{e}^{\alpha t}\sum_{j=1}^{m}q_{ij}\int_{-\tau_{1j}(t)}^{0}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)N_2\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&\le\mathrm{e}^{\alpha t}\sum_{j\ne i}q_{ij}\int_{-\tau_{1i}(t)}^{0}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)N_2\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&\le\overline{q}\mathrm{e}^{\alpha t}\int_{-\tau_1}^{0}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)N_2\dot e(s)\mathrm{d}s\mathrm{d}\lambda,\end{aligned} \tag{27}$$

$$\begin{aligned}&\mathrm{e}^{\alpha t}\sum_{j=1}^{m}q_{ij}\int_{-\tau_{2j}(t)}^{-\tau_1}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)N_4\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&\le\overline{q}\mathrm{e}^{\alpha t}\int_{-\tau_2}^{-\tau_1}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)N_4\dot e(s)\mathrm{d}s\mathrm{d}\lambda.\end{aligned} \tag{28}$$

Thus, we have

$$\begin{aligned}&-\mathrm{e}^{\alpha t}\int_{-\tau_1}^{0}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)T_1\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&\le-2\mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)T_1e(t)+\mathrm{e}^{\alpha t}\frac{2}{\tau_1}e^{\mathrm{T}}(t)T_1\int_{t-\tau_1}^{t}e(s)\mathrm{d}s\\&\quad+\frac{2}{\tau_1}\mathrm{e}^{\alpha t}\int_{t-\tau_1}^{t}e^{\mathrm{T}}(s)\mathrm{d}sT_1e(t)\\&\quad-\frac{2}{{\tau_1}^2}\mathrm{e}^{\alpha t}\int_{t-\tau_1}^{t}e^{\mathrm{T}}(s)\mathrm{d}sT_1\int_{t-\tau_1}^{t}e(s)\mathrm{d}s,\end{aligned} \tag{29}$$

$$\begin{aligned}&-\mathrm{e}^{\alpha t}\int_{-\tau_2}^{-\tau_1}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)T_2\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&\le-\frac{2}{(\tau_2^2-\tau_1^2)}(\tau_2-\tau_1)^2\mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)T_2e(t)\\&\quad+\frac{2}{(\tau_2^2-\tau_1^2)}(\tau_2-\tau_1)\mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)T_2\int_{t-\tau_2}^{t-\tau_1}e(s)\mathrm{d}s\\&\quad+\frac{2}{(\tau_2^2-\tau_1^2)}(\tau_2-\tau_1)\mathrm{e}^{\alpha t}\int_{t-\tau_2}^{t-\tau_1}e^{\mathrm{T}}(s)\mathrm{d}sT_2e(t)\\&\quad-\frac{2}{(\tau_2^2-\tau_1^2)}\mathrm{e}^{\alpha t}\int_{t-\tau_2}^{t-\tau_1}e^{\mathrm{T}}(s)\mathrm{d}sT_2\int_{t-\tau_2}^{t-\tau_1}e(s)\mathrm{d}s,\end{aligned} \tag{30}$$

$$\begin{aligned}&-\mathrm{e}^{\alpha t}\int_{-d}^{0}\int_{t+\lambda}^{t}\dot e^{\mathrm{T}}(s)Z_3\dot e(s)\mathrm{d}s\mathrm{d}\lambda\\&\le-2\mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)Z_3e(t)+\frac{2}{d}\mathrm{e}^{\alpha t}e^{\mathrm{T}}(t)Z_3\int_{t-d}^{t}e(s)\mathrm{d}s\\&\quad+\frac{2}{d}\mathrm{e}^{\alpha t}\int_{t-d}^{t}e^{\mathrm{T}}(s)\mathrm{d}sZ_3e(t)\\&\quad-\frac{2}{d^2}\mathrm{e}^{\alpha t}\int_{t-d}^{t}e^{\mathrm{T}}(s)\mathrm{d}sZ_3\int_{t-d}^{t}e(s)\mathrm{d}s.\end{aligned} \tag{31}$$

On the other hand, denoting $\beta(t)=[(d-d(t))]/d$, then we have that $0\le\beta(t)\le 1$ and $d(t)=(1-\beta(t))d$. By using

Lemma 1 we get

$$
\begin{aligned}
&-\int_{t-d}^{t}\dot{e}^{\mathrm{T}}(s)Z_2\dot{e}(s)\,\mathrm{d}s\\
&=-d\int_{t-d(t)}^{t}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&\quad-d\int_{t-d}^{t-d(t)}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&=-d(t)\int_{t-d(t)}^{t}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&\quad-(d-d(t))\int_{t-d(t)}^{t}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&\quad-(d-d(t))\int_{t-d}^{t-d(t)}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&\quad-d(t)\int_{t-d}^{t-d(t)}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&\le-d(t)\int_{t-d(t)}^{t}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&\quad-\beta(t)d(t)\int_{t-d(t)}^{t}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&\quad-(d-d(t))\int_{t-d}^{t-d(t)}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&\quad-(1-\beta(t))(d-d(t))\int_{t-d}^{t-d(t)}\dot{e}^{\mathrm{T}}(s)d^{-1}Z_2\dot{e}(s)\,\mathrm{d}s\\
&\le-\int_{t-d(t)}^{t}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s d^{-1}Z_2\int_{t-d(t)}^{t}\dot{e}(s)\,\mathrm{d}s\\
&\quad-\beta(t)\int_{t-d(t)}^{t}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s d^{-1}Z_2\int_{t-d(t)}^{t}\dot{e}(s)\,\mathrm{d}s\\
&\quad-\int_{t-d}^{t-d(t)}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s d^{-1}Z_2\int_{t-d}^{t-d(t)}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(1-\beta(t))\int_{t-d}^{t-d(t)}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s d^{-1}Z_2\int_{t-d}^{t-d(t)}\dot{e}(s)\,\mathrm{d}s\\
&=\begin{bmatrix}e(t)\\ e(t-d(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-d^{-1}Z_2 & d^{-1}Z_2\\ * & -d^{-1}Z_2\end{bmatrix}\begin{bmatrix}e(t)\\ e(t-d(t))\end{bmatrix}\\
&\quad-\beta(t)\eta^{\mathrm{T}}(t)\rho_1^{\mathrm{T}}d^{-1}Z_2\rho_1\eta(t)\\
&\quad+\begin{bmatrix}e(t-d(t))\\ e(t-d)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-d^{-1}Z_2 & d^{-1}Z_2\\ * & -d^{-1}Z_2\end{bmatrix}\begin{bmatrix}e(t-d(t))\\ e(t-d)\end{bmatrix}\\
&\quad-(1-\beta(t))\eta^{\mathrm{T}}(t)\rho_2^{\mathrm{T}}d^{-1}Z_2\rho_2\eta(t),
\end{aligned}\tag{32}
$$

where

$$
\begin{aligned}
\eta^{\mathrm{T}}(t)=\Big[&e^{\mathrm{T}}(t),\ e^{\mathrm{T}}(t-\tau_{1i}(t)),\ e^{\mathrm{T}}(t-\tau_1),\ e^{\mathrm{T}}(t-\tau_{2i}(t)),\\
&e^{\mathrm{T}}(t-\tau_2),\ \int_{t-\tau_1}^{t}e^{\mathrm{T}}(s)\,\mathrm{d}s,\ \int_{t-\tau_2}^{t-\tau_1}e^{\mathrm{T}}(s)\,\mathrm{d}s,\ e^{\mathrm{T}}(t-d(t)),\\
&e^{\mathrm{T}}(t-d),\ \int_{t-d}^{t}e^{\mathrm{T}}(s)\,\mathrm{d}s,\ \overline{g}^{\mathrm{T}}(e(t))\Big].
\end{aligned}
$$

Similarly denoting $\gamma(t)=[(\tau_1-\tau_{1i}(t))]/\tau_1$, then we have that $0\le\gamma(t)\le 1$ and $\tau_{1i}(t)=(1-\gamma(t))\tau_1$. By using Lemma 1 we get

$$
\begin{aligned}
&-\int_{t-\tau_1}^{t}\dot{e}^{\mathrm{T}}(s)N_1\dot{e}(s)\,\mathrm{d}s-(1-\nu_{1i})\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)N_2\dot{e}(s)\,\mathrm{d}s\\
&=-\tau_1\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)\tau_1^{-1}N_1\dot{e}(s)\,\mathrm{d}s\\
&\quad-\tau_1\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}^{\mathrm{T}}(s)\tau_1^{-1}N_1\dot{e}(s)\,\mathrm{d}s\\
&\quad-(1-\nu_{1i})\tau_1\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)\tau_1^{-1}N_2\dot{e}(s)\,\mathrm{d}s\\
&=-\tau_1\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)\mathcal{X}\dot{e}(s)\,\mathrm{d}s\\
&\quad-\tau_1\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}^{\mathrm{T}}(s)\tau_1^{-1}N_1\dot{e}(s)\,\mathrm{d}s\\
&=-\tau_{1i}(t)\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)\mathcal{X}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(\tau_1-\tau_{1i}(t))\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)\mathcal{X}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(\tau_1-\tau_{1i}(t))\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}^{\mathrm{T}}(s)\tau_1^{-1}N_1\dot{e}(s)\,\mathrm{d}s\\
&\quad-\tau_{1i}(t)\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}^{\mathrm{T}}(s)\tau_1^{-1}N_1\dot{e}(s)\,\mathrm{d}s\\
&\le-\tau_{1i}(t)\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)\mathcal{X}\dot{e}(s)\,\mathrm{d}s\\
&\quad-\gamma(t)\tau_{1i}(t)\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)\mathcal{X}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(\tau_1-\tau_{1i}(t))\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}^{\mathrm{T}}(s)\tau_1^{-1}N_1\dot{e}(s)\,\mathrm{d}s\\
&\quad-(1-\gamma(t))(\tau_1-\tau_{1i}(t))\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}^{\mathrm{T}}(s)\tau_1^{-1}N_1\dot{e}(s)\,\mathrm{d}s\\
&\le-\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s\mathcal{X}\int_{t-\tau_{1i}(t)}^{t}\dot{e}(s)\,\mathrm{d}s\\
&\quad-\gamma(t)\int_{t-\tau_{1i}(t)}^{t}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s\mathcal{X}\int_{t-\tau_{1i}(t)}^{t}\dot{e}(s)\,\mathrm{d}s\\
&\quad-\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s\tau_1^{-1}N_1\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(1-\gamma(t))\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s\tau_1^{-1}N_1\int_{t-\tau_1}^{t-\tau_{1i}(t)}\dot{e}(s)\,\mathrm{d}s\\
&=\begin{bmatrix}e(t)\\ e(t-\tau_{1i}(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-\mathcal{X} & \mathcal{X}\\ * & -\mathcal{X}\end{bmatrix}\begin{bmatrix}e(t)\\ e(t-\tau_{1i}(t))\end{bmatrix}\\
&\quad-\gamma(t)\eta^{\mathrm{T}}(t)\xi_1^{\mathrm{T}}\mathcal{X}\xi_1\eta(t)\\
&\quad+\begin{bmatrix}e(t-\tau_{1i}(t))\\ e(t-\tau_1)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-\tau_1^{-1}N_1 & \tau_1^{-1}N_1\\ * & -\tau_1^{-1}N_1\end{bmatrix}\begin{bmatrix}e(t-\tau_{1i}(t))\\ e(t-\tau_1)\end{bmatrix}\\
&\quad-(1-\gamma(t))\eta^{\mathrm{T}}(t)\xi_2^{\mathrm{T}}\tau_1^{-1}N_1\xi_2\eta(t).
\end{aligned}\tag{33}
$$

Similarly, denoting $\psi(t)=[(\tau_2-\tau_{2i}(t))]/(\tau_2-\tau_1)$, then we have that $0\le\psi(t)\le 1$ and $\tau_{2i}(t))=(1-\psi(t))\tau_2+\tau_1\psi(t)$. By using Lemma 1 we get

$$
\begin{aligned}
&-\int_{t-\tau_2}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)N_3\dot{e}(s)\,\mathrm{d}s-(1-\nu_{2i})\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)N_4\dot{e}(s)\,\mathrm{d}s\\
&=-(\tau_2-\tau_1)\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)(\tau_2-\tau_1)^{-1}N_3\dot{e}(s)\,\mathrm{d}s\\
&\quad-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}^{\mathrm{T}}(s)(\tau_2-\tau_1)^{-1}N_3\dot{e}(s)\,\mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
&-(1-\nu_{2i})(\tau_2-\tau_1)\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)(\tau_2-\tau_1)^{-1}N_4\dot{e}(s)\,\mathrm{d}s\\
&=-(\tau_2-\tau_1)\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)\mathcal{Y}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}^{\mathrm{T}}(s)(\tau_2-\tau_1)^{-1}N_3\dot{e}(s)\,\mathrm{d}s\\
&=-(\tau_{2i}(t)-\tau_1)\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)\mathcal{Y}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(\tau_2-\tau_{2i}(t))\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)\mathcal{Y}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(\tau_2-\tau_{2i}(t))\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}^{\mathrm{T}}(s)(\tau_2-\tau_1)^{-1}N_3\dot{e}(s)\,\mathrm{d}s\\
&\quad-(\tau_{2i}(t)-\tau_1)\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}^{\mathrm{T}}(s)(\tau_2-\tau_1)^{-1}N_3\dot{e}(s)\,\mathrm{d}s\\
&\le-(\tau_{2i}(t)-\tau_1)\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)\mathcal{Y}\dot{e}(s)\,\mathrm{d}s\\
&\quad-\psi(t)(\tau_{2i}(t)-\tau_1)\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)\mathcal{Y}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(\tau_2-\tau_{2i}(t))\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}^{\mathrm{T}}(s)(\tau_2-\tau_1)^{-1}N_3\dot{e}(s)\,\mathrm{d}s\\
&\quad-(1-\psi(t))(\tau_2-\tau_{2i}(t))\\
&\quad\times\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}^{\mathrm{T}}(s)(\tau_2-\tau_1)^{-1}N_3\dot{e}(s)\,\mathrm{d}s\\
&\le-\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s\mathcal{Y}\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}(s)\,\mathrm{d}s\\
&\quad-\psi(t)\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s\mathcal{Y}\int_{t-\tau_{2i}(t)}^{t-\tau_1}\dot{e}(s)\,\mathrm{d}s\\
&\quad-\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}^{\mathrm{T}}(s)(\tau_2-\tau_1)^{-1}N_3\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}(s)\,\mathrm{d}s\\
&\quad-(1-\psi(t))\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}^{\mathrm{T}}(s)\,\mathrm{d}s\\
&\quad\times(\tau_2-\tau_1)^{-1}N_3\int_{t-\tau_2}^{t-\tau_{2i}(t)}\dot{e}(s)\,\mathrm{d}s\\
&=\begin{bmatrix}e(t-\tau_1)\\ e(t-\tau_{2i}(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-\mathcal{Y} & \mathcal{Y}\\ * & -\mathcal{Y}\end{bmatrix}\begin{bmatrix}e(t-\tau_1)\\ e(t-\tau_{2i}(t))\end{bmatrix}\\
&\quad-\psi(t)\eta^{\mathrm{T}}(t)\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1\eta(t)\\
&\quad+\begin{bmatrix}e(t-\tau_{2i}(t))\\ e(t-\tau_2)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-(\tau_2-\tau_1)^{-1}N_3 & (\tau_2-\tau_1)^{-1}N_3\\ * & -(\tau_2-\tau_1)^{-1}N_3\end{bmatrix}\\
&\quad\times\begin{bmatrix}e(t-\tau_{2i}(t))\\ e(t-\tau_2)\end{bmatrix}\\
&\quad-(1-\psi(t))\eta^{\mathrm{T}}(t)\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2\eta(t). \qquad (34)
\end{aligned}
$$

Based on expression (4), we have that for any $\mu>0$,

$$
b(t)=\mu\begin{bmatrix}e(t)\\ \overline{g}(e(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\overline{U} & \overline{V}\\ * & I\end{bmatrix}\begin{bmatrix}e(t)\\ \overline{g}(e(t))\end{bmatrix}\le 0. \qquad (35)
$$

Substituting expressions (20)–(34) into Eq. (19) and subtracting expression (35) from Eq. (19), we get

$$
\begin{aligned}
\mathcal{L}V(e(t),t,i)\ \le\ &\mathcal{L}V_1(e(t),t,i)+\mathcal{L}V_2(e(t),t,i)+\mathcal{L}V_3(e(t),t,i)\\
&+\mathcal{L}V_4(e(t),t,i)+\mathcal{L}V_5(e(t),t,i)-b(t)\\
=\ &\eta^{\mathrm{T}}(t)\beta(t)\big(\Omega_1+L^{\mathrm{T}}RL-\rho_1^{\mathrm{T}}d^{-1}Z_2\rho_1\\
&-\gamma(t)\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-(1-\gamma(t))\xi_2^{\mathrm{T}}\tau_1^{-1}N_1\xi_2\\
&-\psi(t)\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1-(1-\psi(t))\phi_2^{\mathrm{T}}\\
&\times(\tau_2-\tau_1)^{-1}N_3\phi_2\big)\eta(t)\\
&+\eta^{\mathrm{T}}(t)(1-\beta(t))(\Omega_1+L^{\mathrm{T}}RL-\rho_2^{\mathrm{T}}d^{-1}Z_2\rho_2\\
&-\gamma(t)\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-(1-\gamma(t))\xi_2^{\mathrm{T}}\tau_1^{-1}N_1\xi_2\\
&-\psi(t)\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1-(1-\psi(t))\phi_2^{\mathrm{T}}\\
&\times(\tau_2-\tau_1)^{-1}N_3\phi_2)\eta(t)\\
=\ &\eta^{\mathrm{T}}(t)\beta(t)\gamma(t)\psi(t)(\Omega_1+L^{\mathrm{T}}RL-\rho_1^{\mathrm{T}}d^{-1}Z_2\rho_1\\
&-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1)\eta(t)\\
&+\eta^{\mathrm{T}}(t)\beta(t)(1-\gamma(t))(1-\psi(t))\\
&\times\big(\Omega_1+L^{\mathrm{T}}RL-\rho_1^{\mathrm{T}}d^{-1}Z_2\rho_1-\xi_2^{\mathrm{T}}\tau_1^{-1}N_1\xi_2\\
&-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2\big)\eta(t)\\
&+\eta^{\mathrm{T}}(t)\beta(t)\gamma(t)(1-\psi(t))\\
&\times\big(\Omega_1+L^{\mathrm{T}}RL-\rho_1^{\mathrm{T}}d^{-1}Z_2\rho_1-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1\\
&-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2\big)\eta(t)\\
&+\eta^{\mathrm{T}}(t)(1-\beta(t))(1-\gamma(t))\psi(t)\\
&\times\big(\Omega_1+L^{\mathrm{T}}RL-\rho_2^{\mathrm{T}}d^{-1}Z_2\rho_2-\xi_2^{\mathrm{T}}\tau_1^{-1}N_1\xi_2\\
&-\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1\big)\eta(t)\\
&+\eta^{\mathrm{T}}(t)(1-\beta(t))\gamma(t)(1-\psi(t))\\
&\times\big(\Omega_1+L^{\mathrm{T}}RL-\rho_2^{\mathrm{T}}d^{-1}Z_2\rho_2-\xi_1^{\mathrm{T}}\tau_1^{-1}N_1\xi_1\\
&-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2\big)\eta(t)\\
&+\eta^{\mathrm{T}}(t)(1-\beta(t))(1-\gamma(t))(1-\psi(t))\\
&\times\big(\Omega_1+L^{\mathrm{T}}RL-\rho_2^{\mathrm{T}}d^{-1}Z_2\rho_2-\xi_2^{\mathrm{T}}\tau_1^{-1}N_1\xi_2\\
&-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2\big)\eta(t). \qquad (36)
\end{aligned}
$$

By using the Schur complement formula, we can find from formulae (11)–(16) that there exists a scalar $\lambda>0$ such that

$$
\begin{aligned}
&\Omega_1+L^{\mathrm{T}}RL-\rho_1^{\mathrm{T}}d^{-1}Z_2\rho_1-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1<-\lambda I, \qquad (37)\\
&\Omega_1+L^{\mathrm{T}}RL-\rho_1^{\mathrm{T}}d^{-1}Z_2\rho_1-\xi_2^{\mathrm{T}}\tau_1^{-1}N_1\xi_2\\
&\quad-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2<-\lambda I, \qquad (38)\\
&\Omega_1+L^{\mathrm{T}}RL-\rho_1^{\mathrm{T}}d^{-1}Z_2\rho_1-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1\\
&\quad-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2<-\lambda I, \qquad (39)\\
&\Omega_1+L^{\mathrm{T}}RL-\rho_2^{\mathrm{T}}d^{-1}Z_2\rho_2-\xi_2^{\mathrm{T}}\tau_1^{-1}N_1\xi_2-\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1<-\lambda I, \qquad (40)\\
&\Omega_1+L^{\mathrm{T}}RL-\rho_2^{\mathrm{T}}d^{-1}Z_2\rho_2-\xi_1^{\mathrm{T}}\tau_1^{-1}N_1\xi_1\\
&\quad-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2<-\lambda I, \qquad (41)\\
&\Omega_1+L^{\mathrm{T}}RL-\rho_2^{\mathrm{T}}d^{-1}Z_2\rho_2-\xi_2^{\mathrm{T}}\tau_1^{-1}N_1\xi_2\\
&\quad-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2<-\lambda I. \qquad (42)
\end{aligned}
$$

From expression (36), we get

$$
E\{\mathcal{L}V(e(t),t,i)\}\le \mathrm{e}^{\alpha t}E\{\eta^{\mathrm{T}}(t)\Pi^*\eta(t)\}, \qquad (43)
$$

where $\Pi^*=\sum_{i=1}^{6}\Pi_i$.

From formulars (11)–(16),we have

$$
E\{\mathcal{L}V(e(t),t,i)\}\le 0, t\in[t_{k-1},t_k). \qquad (44)
$$

It follows from formular (44) and the generalized Ito's formula we have

$$E\{V(e(t),t,r(t))\}-E\{V(e(0),0,r(0))\} = \int_0^t E\{\mathcal{L}V(e(s),s,r(s))\,\mathrm{d}s\} \le 0,$$

which implies that $E\{V(e(t),t,i)\} \le E\{V(e(0),0,r(0))\} < \infty, t \ge 0$. Let

$$\begin{aligned}
\mathcal{N}_1 &= \left[\frac{1}{\alpha}-\frac{\mathrm{e}^{-\alpha\tau_1}}{\alpha}\right],\ \mathcal{N}_2=\left[\frac{1}{\alpha}-\frac{\mathrm{e}^{-\alpha\tau_2}}{\alpha}\right],\\
\mathcal{N}_3 &= \left[\frac{\mathrm{e}^{-\alpha\tau_1}}{\alpha}-\frac{\mathrm{e}^{-\alpha\tau_2}}{\alpha}\right],\ \mathcal{N}_4=\left[\frac{\mathrm{e}^{\alpha\tau_1}}{\alpha^2}-\frac{\tau_1}{\alpha}-\frac{1}{\alpha^2}\right],\\
\mathcal{N}_5 &= \left[\frac{\mathrm{e}^{\alpha\tau_2}}{\alpha^2}+\frac{\tau_1}{\alpha}-\frac{\tau_2}{\alpha}-\frac{\mathrm{e}^{\alpha\tau_1}}{\alpha^2}\right],\\
\mathcal{N}_6 &= \left[\frac{\mathrm{e}^{\alpha\tau_1}}{\alpha^3}-\frac{1}{\alpha^3}-\frac{\tau_1}{\alpha^2}-\frac{\tau_1^2}{2\alpha}\right],\\
\mathcal{N}_7 &= \left[\frac{\mathrm{e}^{\alpha\tau_2}}{\alpha^3}+\frac{\tau_1^2}{2\alpha}-\frac{\tau_1}{\alpha^2}-\frac{\mathrm{e}^{\alpha\tau_1}}{\alpha^3}-\frac{\tau_2^2}{2\alpha}-\frac{\tau_2}{\alpha^2}\right],\\
\mathcal{N}_8 &= \left[\frac{1}{\alpha}-\frac{\mathrm{e}^{-\alpha d}}{\alpha}\right],\ \mathcal{N}_9=\left[\frac{\mathrm{e}^{\alpha d}}{\alpha^2}-\frac{1}{\alpha^2}-\frac{d}{\alpha}\right],\\
\mathcal{N}_{10} &= \left[\frac{\mathrm{e}^{\alpha d}}{\alpha^3}-\frac{d^2}{2\alpha}-\frac{1}{\alpha^3}-\frac{d}{\alpha^2}\right].
\end{aligned}$$

From the definition of $V(e(t),t,i)$, we have

$$\begin{aligned}
&E\{V(e(0),0,r(0))\}\\
&\le \big[\lambda_{\max}(P_i)+\lambda_{\max}(H_1)\mathcal{N}_1+\lambda_{\max}(H_2)\mathcal{N}_2+\lambda_{\max}(H_3)\mathcal{N}_3\\
&\quad+\lambda_{\max}(H_4)\mathcal{N}_3+\lambda_{\max}(N_1)\mathcal{N}_4+\lambda_{\max}(N_2)\mathcal{N}_4\\
&\quad+\lambda_{\max}(N_3)\mathcal{N}_5+\lambda_{\max}(N_4)\mathcal{N}_5+\overline{q}\lambda_{\max}(N_2)\mathcal{N}_6\\
&\quad+\overline{q}\lambda_{\max}(N_4)\mathcal{N}_7+\lambda_{\max}(T_1)\mathcal{N}_6+\lambda_{\max}(T_2)\mathcal{N}_7\\
&\quad+\lambda_{\max}(Z_1)\mathcal{N}_8+\lambda_{\max}(Z_2)\mathcal{N}_9+\lambda_{\max}(Z_3)\mathcal{N}_{10}\big]\\
&\quad\times \sup_{-\max\{d,\tau_1,\tau_2\}\le s\le 0} E\|e(s)\|^2. \qquad (45)
\end{aligned}$$

By using the Definition 1, we get

$$E\{\|e(t)\|^2\} \le \mathrm{e}^{-\alpha t}\,\mathcal{U} \sup_{-\max\{d,\tau_1,\tau_2\}\le s\le 0} E\|e(s)\|^2,$$

where

$$\begin{aligned}
\mathcal{U} = {}& [\lambda_{\max}(P_i)+\lambda_{\max}(H_1)\mathcal{N}_1+\lambda_{\max}(H_2)\mathcal{N}_2+\lambda_{\max}(H_3)\mathcal{N}_3\\
&+\lambda_{\max}(H_4)\mathcal{N}_3+\lambda_{\max}(N_1)\mathcal{N}_4+\lambda_{\max}(N_2)\mathcal{N}_4\\
&+\lambda_{\max}(N_3)\mathcal{N}_5+\lambda_{\max}(N_4)\mathcal{N}_5+\overline{q}\lambda_{\max}(N_2)\mathcal{N}_6\\
&+\overline{q}\lambda_{\max}(N_4)\mathcal{N}_7+\lambda_{\max}(T_1)\mathcal{N}_6+\lambda_{\max}(T_2)\mathcal{N}_7\\
&+\lambda_{\max}(Z_1)\mathcal{N}_8+\lambda_{\max}(Z_2)\mathcal{N}_9+\lambda_{\max}(Z_3)\mathcal{N}_{10}].
\end{aligned}$$

Hence, the error system (9) is exponentially stable in the mean square.

Remark 1 In this paper, the problem of sampled-data synchronization is studied for CDNs with probabilistic time-varying coupling delays. Here, the sampling period is considered and is assumed to be time-varying and bounded, which means that the sampled-data feedback controller (6) can be used to effectively deal with the case when the sampling periods are time-varying. In this work, we assume that control inputs from all nodes are sampled by one sampler. It is noted that, in the case of multiple samplers, the proposed control approach is feasible.

Remark 2 In Theorem 1, some sufficient conditions that depend on the probability distribution of the time-varying coupling delays are derived by constructing an appropriate Lyapunov functional and using Jensen's inequality. It should be mentioned that some newly developed approaches to deal with Markovian jumping parameters and mode-dependent probabilistic time-varying delays can be employed to further reduce the conservatism of the stability condition.

In the following theorem we will discuss the exponential synchronization for CDN (3) and derive the sufficient condition for the existence of the desired sampled-data controllers based on Theorem 1.

Theorem 2 The CDN (3) is exponentially synchronized in the mean square, by the sampled-data feedback controllers of the form (6), if there exist matrices $P_i = \mathrm{diag}\{P_{i1},P_{i2},\ldots,P_{iN}\} > 0$ $(i\in M)$, $H_v > 0$ $(v=1,2,3,4)$, $N_y > 0$ $(y=1,2,3,4)$, $T_1>0$, $T_2>0$, $Z_1>0$, $Z_2>0$, $Z_3>0$, $X_i = \mathrm{diag}\{X_{i1},X_{i2},\ldots,X_{iN}\}$ and a scalar $\mu>0$ such that the following LMIs hold

$$N_1+(1-\nu_{1i})N_2>0,\quad N_3+(1-\nu_{2i})N_4>0, \qquad (46)$$

$$\tilde{\Pi}_1=\begin{bmatrix}\tilde{\Omega}_1-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1-d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & \tilde{L}^{\mathrm{T}}\\ * & -2P_i+R\end{bmatrix}<0, \qquad (47)$$

$$\tilde{\Pi}_2=\begin{bmatrix}\tilde{\Omega}_1-\xi_2^{\mathrm{T}}\mathcal{X}\xi_2-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2-d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & \tilde{L}^{\mathrm{T}}\\ * & -2P_i+R\end{bmatrix}<0, \qquad (48)$$

$$\tilde{\Pi}_3=\begin{bmatrix}\tilde{\Omega}_1-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2-d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & \tilde{L}^{\mathrm{T}}\\ * & -2P_i+R\end{bmatrix}<0, \qquad (49)$$

$$\tilde{\Pi}_4=\begin{bmatrix}\tilde{\Omega}_1-\xi_2^{\mathrm{T}}\mathcal{X}\xi_2-\phi_1^{\mathrm{T}}\mathcal{Y}\phi_1-d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & \tilde{L}^{\mathrm{T}}\\ * & -2P_i+R\end{bmatrix}<0, \qquad (50)$$

$$\tilde{\Pi}_5=\begin{bmatrix}\tilde{\Omega}_1-\xi_1^{\mathrm{T}}\mathcal{X}\xi_1-\phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2-d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & \tilde{L}^{\mathrm{T}}\\ * & -2P_i+R\end{bmatrix}<0, \qquad (51)$$

$$\tilde{\Pi}_6 = \begin{bmatrix} \tilde{\Omega}_1 - \xi_2^{\mathrm{T}}\mathcal{X}\xi_2 - \phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2 - d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & \tilde{L}^{\mathrm{T}} \\ * & -2P_i+R \end{bmatrix} < 0, \tag{52}$$

where

$$\tilde{\Omega}_1 = \begin{bmatrix} \Omega_{1,1} & \Omega_{1,2} & 0 & \Omega_{1,4} & 0 & \Omega_{1,6} & \Omega_{1,7} & \Omega_{1,8} & 0 & \Omega_{1,10} & \Omega_{1,11} \\ * & \Omega_{2,2} & \Omega_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Omega_{3,3} & \Omega_{3,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{4,4} & \Omega_{4,5} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{5,5} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{6,6} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{7,7} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{8,8} & \Omega_{8,9} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Omega_{9,9} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Omega_{10,10} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Omega_{11,11} \end{bmatrix}, \tag{53}$$

$$\tilde{L} = [P_i(G\otimes D_i) \;\; P_i(G\otimes A_i)\alpha_0 \;\; 0 \;\; P_i(G\otimes A_i)(1-\alpha_0) \;\; 0 \;\; 0 \;\; 0 \;\; X_i \;\; 0 \;\; 0 \;\; P_i], \quad \Omega_{1,8} = X_i + d^{-1}Z_2,$$

and the other parameters follow the same definitions as in Theorem 1. Moreover the controller gain matrices are given by $K_i = P_i^{-1}X_i$, $i = 1,2,\ldots,m$.

Proof Define matrix $U = \mathrm{diag}\{I,I,I,I,I,I,I,I,I,I,P_iR^{-1}\}$ and $X_i = P_iK_i$. Then pre- and post-multiplying equations (11)–(16) with U and U^{T}, respectively, we obtain the following expressions

$$\tilde{\Pi}_1 = \begin{bmatrix} \tilde{\Omega}_1 - \xi_1^{\mathrm{T}}\mathcal{X}\xi_1 - \phi_1^{\mathrm{T}}\mathcal{Y}\phi_1 - d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & \tilde{L}^{\mathrm{T}} \\ * & -P_iR^{-1}P_i \end{bmatrix} < 0, \tag{54}$$

$$\tilde{\Pi}_2 = \begin{bmatrix} \tilde{\Omega}_1 - \xi_2^{\mathrm{T}}\mathcal{X}\xi_2 - \phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2 - d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & \tilde{L}^{\mathrm{T}} \\ * & -P_iR^{-1}P_i \end{bmatrix} < 0, \tag{55}$$

$$\tilde{\Pi}_3 = \begin{bmatrix} \tilde{\Omega}_1 - \xi_1^{\mathrm{T}}\mathcal{X}\xi_1 - \phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2 - d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & \tilde{L}^{\mathrm{T}} \\ * & -P_iR^{-1}P_i \end{bmatrix} < 0, \tag{56}$$

$$\tilde{\Pi}_4 = \begin{bmatrix} \tilde{\Omega}_1 - \xi_2^{\mathrm{T}}\mathcal{X}\xi_2 - \phi_1^{\mathrm{T}}\mathcal{Y}\phi_1 - d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & \tilde{L}^{\mathrm{T}} \\ * & -P_iR^{-1}P_i \end{bmatrix} < 0, \tag{57}$$

$$\tilde{\Pi}_5 = \begin{bmatrix} \tilde{\Omega}_1 - \xi_1^{\mathrm{T}}\mathcal{X}\xi_1 - \phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2 - d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & \tilde{L}^{\mathrm{T}} \\ * & -P_iR^{-1}P_i \end{bmatrix} < 0, \tag{58}$$

$$\tilde{\Pi}_6 = \begin{bmatrix} \tilde{\Omega}_1 - \xi_2^{\mathrm{T}}\mathcal{X}\xi_2 - \phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2 - d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & \tilde{L}^{\mathrm{T}} \\ * & -P_iR^{-1}P_i \end{bmatrix} < 0. \tag{59}$$

Noting that $R > 0$, we have $-P_iR^{-1}P_i \le -2P_i + R$. It is clear that if equations (47)–(52) hold, then equations (54)–(59) hold, which implies that equations (11)–(16) hold. The remaining proof follows from Theorem 1.

Remark 3 In the case when $M = \{1\}$, that is, there is only one mode in operation, the error system (9) for the synchronization of Markovian jumping complex dynamical networks (CDNs) using sampled data with probabilistic time-varying delays can be obtained as follows:

$$\begin{aligned} \dot{e}(t) = {} & \overline{g}(e(t)) + (G\otimes D_i)e(t) + \alpha_0(G\otimes A_i)e(t-\tau_1(t)) + (1-\alpha_0)(G\otimes A_i)e(t-\tau_2(t)) \\ & + (\alpha(t)-\alpha_0)[(G\otimes A_i)e(t-\tau_1(t)) - (G\otimes A_i)e(t-\tau_2(t))] + K_ie(t-d(t)). \end{aligned} \tag{60}$$

In the following corollary, we will discuss the exponential stability in the mean square for the error dynamical system (60).

Corollary 1 The error system (60) is exponentially stable in the mean square, if there exist matrices P_i $(i \in M), H_v > 0$ $(v = 1,2,3,4), N_y > 0$ $(y = 1,2,3,4)$, $T_1 > 0$, $T_2 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, and some positive scalar $\mu > 0$ such that the following LMIs hold

$$N_1 + (1-\nu_1)N_2 > 0, N_3 + (1-\nu_2)N_4 > 0, \tag{61}$$

$$\Pi_1 = \begin{bmatrix} \Omega_1 - \xi_1^{\mathrm{T}}\mathcal{X}\xi_1 - \phi_1^{\mathrm{T}}\mathcal{Y}\phi_1 - d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & L^{\mathrm{T}} \\ * & -R \end{bmatrix} < 0, \tag{62}$$

$$\Pi_2 = \begin{bmatrix} \Omega_1 - \xi_2^{\mathrm{T}}\mathcal{X}\xi_2 - \phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2 - d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & L^{\mathrm{T}} \\ * & -R \end{bmatrix} < 0, \tag{63}$$

$$\Pi_3 = \begin{bmatrix} \Omega_1 - \xi_1^{\mathrm{T}}\mathcal{X}\xi_1 - \phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2 - d^{-1}\rho_1^{\mathrm{T}}Z_2\rho_1 & L^{\mathrm{T}} \\ * & -R \end{bmatrix} < 0, \tag{64}$$

$$\Pi_4 = \begin{bmatrix} \Omega_1 - \xi_2^{\mathrm{T}}\mathcal{X}\xi_2 - \phi_1^{\mathrm{T}}\mathcal{Y}\phi_1 - d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & L^{\mathrm{T}} \\ * & -R \end{bmatrix} < 0, \tag{65}$$

$$\Pi_5 = \begin{bmatrix} \Omega_1 - \xi_1^{\mathrm{T}}\mathcal{X}\xi_1 - \phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2 - d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & L^{\mathrm{T}} \\ * & -R \end{bmatrix} < 0, \tag{66}$$

$$\Pi_6 = \begin{bmatrix} \Omega_1 - \xi_2^{\mathrm{T}}\mathcal{X}\xi_2 - \phi_2^{\mathrm{T}}(\tau_2-\tau_1)^{-1}N_3\phi_2 - d^{-1}\rho_2^{\mathrm{T}}Z_2\rho_2 & L^{\mathrm{T}} \\ * & -R \end{bmatrix} < 0, \tag{67}$$

where

$$\Omega_1 = \begin{bmatrix} \Omega_{1,1} & \Omega_{1,2} & 0 & \Omega_{1,4} & 0 & \Omega_{1,6} & \Omega_{1,7} & \Omega_{1,8} & 0 & \Omega_{1,10} & \Omega_{1,11} \\ * & \Omega_{2,2} & \Omega_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Omega_{3,3} & \Omega_{3,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{4,4} & \Omega_{4,5} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{5,5} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{6,6} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{7,7} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{8,8} & \Omega_{8,9} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Omega_{9,9} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Omega_{10,10} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Omega_{11,11} \end{bmatrix}, \tag{68}$$

$$\begin{aligned}
R &= N_1\mathcal{M}_1 + N_2\mathcal{M}_1 + N_3\mathcal{M}_2 + N_4\mathcal{M}_2 + T_1\mathcal{M}_3 + T_2\mathcal{M}_4 + Z_2\mathcal{M}_5 + Z_3\mathcal{M}_6, \\
L &= [R(G\otimes D_i)\ \ R(G\otimes A_i)\alpha_0\ \ 0\ \ R(G\otimes A_i)(1-\alpha_0)\ \ 0\ \ 0\ \ 0\ \ RK_i\ \ 0\ \ 0\ \ R], \\
\mathcal{X} &= \tau_1^{-1}(N_1 + (1-\nu_1)N_2),\ \ \mathcal{Y} = (\tau_2-\tau_1)^{-1}(N_3 + (1-\nu_2)N_4), \\
\xi_1 &= [I\ \ -I\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],\ \ \xi_2 = [0\ I\ \ -I\ 0\ 0\ 0\ 0\ 0\ 0\ 0], \\
\phi_1 &= [0\ 0\ I\ \ -I\ 0\ 0\ 0\ 0\ 0\ 0\ 0],\ \ \phi_2 = [0\ 0\ 0\ I\ \ -I\ 0\ 0\ 0\ 0\ 0], \\
\rho_1 &= [I\ 0\ 0\ 0\ 0\ 0\ 0\ \ -I\ 0\ 0\ 0],\ \ \rho_2 = [0\ 0\ 0\ 0\ 0\ 0\ 0\ I\ \ -I\ 0\ 0], \\
\Omega_{1,1} &= 2P_i(G\otimes D_i) + \sum_{j=1}^{N} q_{ij}P_j + H_1 + H_2 - \mathcal{X} - 2T_1 - \frac{2}{\tau_2^2-\tau_1^2}(\tau_2-\tau_1)^2T_2 - d^{-1}Z_2 + Z_1 \\
&\quad - 2Z_3 - \mu\overline{U},\ \ \Omega_{1,2} = P_i\alpha_0(G\otimes A_i) + \mathcal{X},\ \ \Omega_{1,4} = P_i(1-\alpha_0)(G\otimes A_i), \\
\Omega_{1,6} &= \frac{2}{\tau_1}T_1,\ \ \Omega_{1,7} = \frac{2}{(\tau_2^2-\tau_1^2)}(\tau_2-\tau_1)T_2,\ \ \Omega_{1,8} = P_iK_i + d^{-1}Z_2,\ \ \Omega_{1,10} = \frac{2}{d}Z_3, \\
\Omega_{1,11} &= P_i - \mu\overline{V},\ \ \Omega_{2,2} = -\mathrm{e}^{-\alpha\tau_1}(1-\nu_1)H_2 - \mathcal{X} - \tau_1^{-1}N_1,\ \ \Omega_{2,3} = \tau_1^{-1}N_1, \\
\Omega_{3,3} &= -\mathrm{e}^{-\alpha\tau_1}H_1 + \mathrm{e}^{-\alpha\tau_1}H_3 + \mathrm{e}^{-\alpha\tau_1}H_4 - \tau_1^{-1}N_1 - \mathcal{Y},\ \ \Omega_{3,4} = \mathcal{Y}, \\
\Omega_{4,4} &= -\mathrm{e}^{-\alpha\tau_2}(1-\nu_2)H_4 - \mathcal{Y} - (\tau_2-\tau_1)^{-1}N_3,\ \ \Omega_{4,5} = (\tau_2-\tau_1)^{-1}N_3, \\
\Omega_{5,5} &= -\mathrm{e}^{-\alpha\tau_2}H_3 - (\tau_2-\tau_1)^{-1}N_3,\ \ \Omega_{6,6} = \frac{-2}{\tau_1^2}T_1,\ \ \Omega_{7,7} = -\frac{2}{(\tau_2^2-\tau_1^2)}(\tau_2-\tau_1)T_2, \\
\Omega_{8,8} &= -2d^{-1}Z_2,\ \ \Omega_{8,9} = d^{-1}Z_2,\ \ \Omega_{9,9} = -\mathrm{e}^{-\alpha d}Z_1 - d^{-1}Z_2,\ \ \Omega_{10,10} = \frac{-2}{d^2}Z_3, \\
\Omega_{11,11} &= -\mu I,\ \ \tau_{12} = \tau_2 - \tau_1, \\
\mathcal{M}_1 &= \frac{1}{\alpha}[\mathrm{e}^{\alpha\tau_1} - 1],\ \ \mathcal{M}_2 = \frac{1}{\alpha}[\mathrm{e}^{\alpha\tau_2} - \mathrm{e}^{\alpha\tau_1}],\ \ \mathcal{M}_3 = \left[\frac{\mathrm{e}^{\alpha\tau_1}}{\alpha^2} - \frac{\tau_1}{\alpha} - \frac{1}{\alpha^2}\right], \\
\mathcal{M}_4 &= \left[\frac{\mathrm{e}^{\alpha\tau_2}}{\alpha^2} + \frac{\tau_1}{\alpha} - \frac{\mathrm{e}^{\alpha\tau_1}}{\alpha^2} - \frac{\tau_2}{\alpha}\right],\ \ \mathcal{M}_5 = \frac{1}{\alpha}\left[\mathrm{e}^{\alpha d} - 1\right],\ \ \mathcal{M}_6 = \left[\frac{\mathrm{e}^{\alpha d}}{\alpha^2} - \frac{1}{\alpha^2} - \frac{d}{\alpha}\right].
\end{aligned}$$

and other entries of $\Omega_{i,j}$ are zero.

The proof of this corollary follows from Theorem 1.

Remark 4 Recently, the authors in Refs. [44]–[48], studied the synchronization problem for complex dynamical networks with probabilistic time-varying delays. In Ref. [44], the authors studied the synchronization stability of complex

dynamical networks with probabilistic time-varying delays. The synchronization of Markovian jumping stochastic complex networks with distributed time delays and probabilistic interval discrete interval time-varying delays is investigated in Ref. [45]. In Ref. [46], the authors discussed the cluster synchronization stability for stochastic complex dynamical networks with probabilistic interval time-varying delays. In this paper, we have introduced CDNs with Markovian jumping parameters and mode-dependent probabilistic time-varying coupling delays. Moreover, by introducing triple integral terms in the Lyapunov functional and using Jensen's inequality, some sufficient conditions for the exponential synchronization criteria are derived in the form of LMIs. The derived results are theoretically and numerically proved to be less conservative than the existing results.

4. Numerical examples

Example 1 Consider the CDN (3) with three nodes. The outer-coupling matrix is assumed to be $G = (G_{wl})_{N\times N}$ with

$$G = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

The nonlinear function f is taken as

$$f(x_i(t)) = \begin{bmatrix} -0.5x_{i1} + \tanh(0.2x_{i1}) + 0.2x_{i2} \\ 0.95x_{i2} - \tanh(0.75x_{i2}) \end{bmatrix}.$$

It can be found from formular (4)

$$U = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, \quad V = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}.$$

The inner coupling matrices are given as

$$A_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

and

$$Q = \begin{bmatrix} -5 & 5 \\ 7 & -7 \end{bmatrix}.$$

The time-varying delays are chosen as $\tau_{11}(t) = 0.1 + 0.05\sin(10t)$, $\tau_{12}(t) = 0.1 + 0.1\sin(10t)$, $\tau_{21}(t) = 0.5 + 0.2\sin(10t)$, $\tau_{22}(t) = 0.6 + 0.2\sin(10t)$, $d(t) = 0.025 + 0.025\cos(10t)$. Then, the derivative of the time-varying delays are $\nu_{11} = 0.1$, $\nu_{12} = 0.2$, $\nu_{21} = 0.3$, $\nu_{22} = 0.4$, and $\alpha = 0.5$, $\alpha_0 = 0.25$. Using the MATLAB LMI control Toolbox to solve the LMIs in Theorem 2, we obtain the following gain matrices

$$K_{11} = P_{11}^{-1}X_{11} = \begin{bmatrix} -0.7546 & -0.0660 \\ -0.0645 & -1.2199 \end{bmatrix},$$

$$K_{12} = P_{12}^{-1}X_{12} = \begin{bmatrix} -0.7546 & -0.0660 \\ -0.0645 & -1.2199 \end{bmatrix},$$

$$K_{13} = P_{13}^{-1}X_{13} = \begin{bmatrix} -0.8007 & -0.0589 \\ -0.0574 & -1.2121 \end{bmatrix},$$

$$K_{21} = P_{21}^{-1}X_{21} = \begin{bmatrix} -2.1891 & -0.1343 \\ -0.1323 & -3.3574 \end{bmatrix},$$

$$K_{22} = P_{22}^{-1}X_{22} = \begin{bmatrix} -2.1891 & -0.1343 \\ -0.1323 & -3.3574 \end{bmatrix},$$

$$K_{23} = P_{23}^{-1}X_{23} = \begin{bmatrix} -2.3516 & -0.1201 \\ -0.1188 & -3.3957 \end{bmatrix}.$$

Using the above parameters, the state trajectories of the error system (9) are given in Fig. 2, and the control inputs $u_w(t)$ are shown in Fig. 3, where $x_1(0) = [3 \quad -2]^{\mathrm{T}}$, $x_2(0) = [-5 \quad 6]^{\mathrm{T}}$, $x_3(0) = [1 \quad 8]^{\mathrm{T}}$, $s(0) = [-2 \quad 4]^{\mathrm{T}}$.

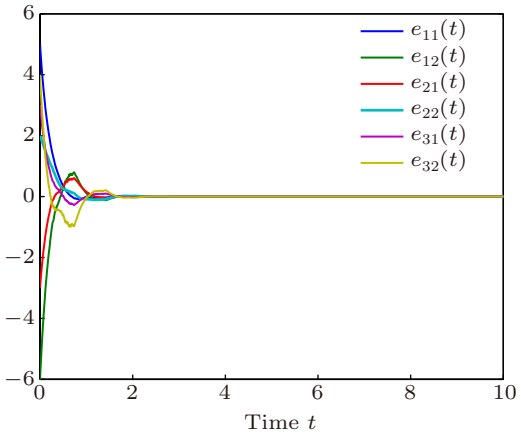


Fig. 2. (color online) The state trajectories of the error system (9).

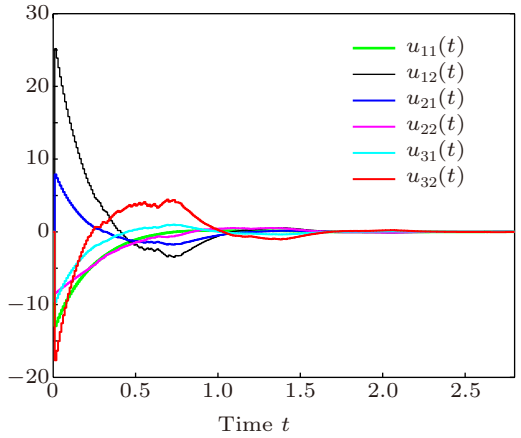


Fig. 3. (color online) Responses of the control inputs $u_w(t)$.

Example 2 Consider the CDN (3) with three nodes. The outer-coupling matrix is assumed to be $G = (G_{wl})_{N\times N}$ with

$$G = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

The nonlinear function f is taken as

$$f(x_i(t)) = \begin{bmatrix} -0.5x_{i1} + \tanh(0.2x_{i1}) + 0.2x_{i2} \\ 0.95x_{i2} - \tanh(0.75x_{i2}) \end{bmatrix}.$$

It can be found from formular (4)

$$U = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, \quad V = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}.$$

The inner coupling matrices are given as

$$A_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

and

$$Q = \begin{bmatrix} -6 & 6 \\ 7 & -7 \end{bmatrix}.$$

The time-varying delays is chosen as $\tau_{11}(t) = 0.2 + 0.04\sin(10t)$, $\tau_{12}(t) = 0.15 + 0.15\sin(10t)$, $\tau_{21}(t) = 0.5 + 0.2\sin(10t)$, $\tau_{22}(t) = 0.6 + 0.2\sin(10t)$, $d(t) = 0.025 + 0.025\cos(10t)$. Then, the derivative of the time-varying delays satisfies $\nu_{11} = 0.1$, $\nu_{12} = 0.2$, $\nu_{21} = 0.3$, $\nu_{22} = 0.4$, and $\alpha = 0.4$, $\alpha_0 = 0.25$. Using the MATLAB LMI control Toolbox to solve the LMIs in Theorem 2, we obtain the following gain matrices

$$K_{11} = P_{11}^{-1}X_{11} = \begin{bmatrix} -0.9431 & -0.0812 \\ -0.0809 & -1.5968 \end{bmatrix},$$
$$K_{12} = P_{12}^{-1}X_{12} = \begin{bmatrix} -0.9431 & -0.0812 \\ -0.0809 & -1.5968 \end{bmatrix},$$
$$K_{13} = P_{13}^{-1}X_{13} = \begin{bmatrix} -1.0101 & -0.0794 \\ -0.0791 & -1.6539 \end{bmatrix},$$
$$K_{21} = P_{21}^{-1}X_{21} = \begin{bmatrix} -1.3361 & -0.1206 \\ -0.1201 & -2.3992 \end{bmatrix},$$
$$K_{22} = P_{22}^{-1}X_{22} = \begin{bmatrix} -1.3361 & -0.1206 \\ -0.1201 & -2.3992 \end{bmatrix},$$
$$K_{23} = P_{23}^{-1}X_{23} = \begin{bmatrix} -1.4138 & -0.1167 \\ -0.1163 & -2.4482 \end{bmatrix}.$$

Using the above parameters, the state trajectories of the error system (9) are given in Fig. 4, and the control inputs $u_w(t)$ are shown in Fig. 5, where $x_1(0) = [-7 \quad 7]^{\mathrm{T}}$, $x_2(0) = [6 \quad 10]^{\mathrm{T}}$, $x_3(0) = [-5 \quad 6]^{\mathrm{T}}$, $s(0) = [-3 \quad 5]^{\mathrm{T}}$.

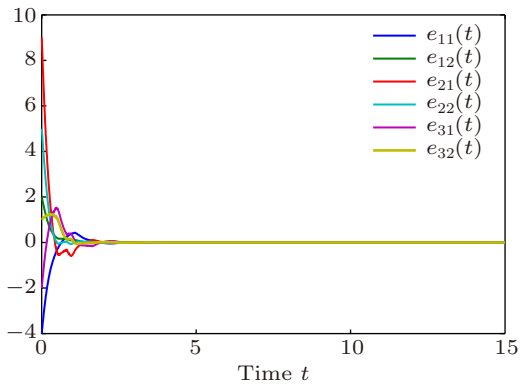


Fig. 4. (color online) The state trajectories of the error system (9).

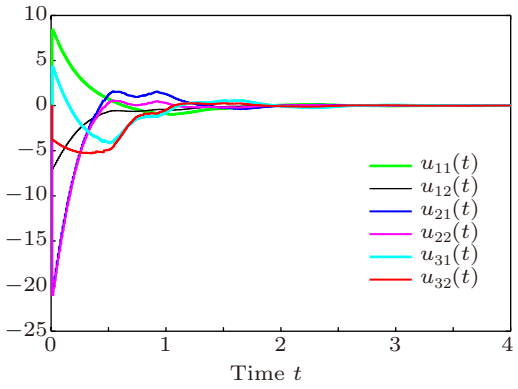


Fig. 5. (color online) Responses of the control inputs $u_w(t)$.

5. Conclusion

In this paper, the sampled-data exponential synchronization problem has been investigated for CDNs with Markovian jumping parameters and mode-dependent probabilistic time-varying coupling delays. The sampling period was assumed to be time-varying and bounded. By considering the information of probability distribution of time-varying delay, a new model of the complex network was introduced. A set of sampled-data controllers have been designed. Based on the new model, some sufficient conditions for delay-distribution-dependent exponential synchronization criteria were obtained in the form of LMIs. Finally, two numerical examples were given to illustrate the usefulness of the derived results.

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