Chaos control in the nonlinear Schrödinger equation with Kerr law nonlinearity*

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The nonlinear Schrödinger equation with Kerr law nonlinearity in the two-frequency interference is studied by the numerical method. Chaos occurs easily due to the absence of damping. This phenomenon will cause the distortion in the process of information transmission. We find that fiber-optic transmit signals still present chaotic phenomena if the control intensity is smaller. With the increase of intensity, the fiber-optic signal can stay in a stable state in some regions. When the strength is suppressed to a certain value, an unstable phenomenon of the fiber-optic signal occurs. Moreover we discuss the sensitivities of the parameters to be controlled. The results show that the linear term coefficient and the environment of two quite different frequences have less effects on the fiber-optic transmission. Meanwhile the phenomena of vibration, attenuation and escape occur in some regions.

Keywords: chaos control, fiber-optic signal, nonlinear Schrödinger equation

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1. Introduction

As is well known, fiber-optic signal transmission has a very wide application in real life, which makes our lives more convenient and rapid. One of the famous fiber-optic models is the nonlinear Schrödinger (NLS) equation with Kerr law nonlinearity, which is given as follows:[1]

$$\begin{aligned} &\mathrm{i}u_x + u_{xx} + \alpha|u|^2 u \\ &\quad + \mathrm{i}[\gamma_1 u_{xxx} + \gamma_2|u|^2 u_x + \gamma_3(|u|^2)_x u] = 0, \end{aligned} \tag{1}$$

where γ_1 is the third-order dispersion, γ_2 and γ_3 are the parameters of the nonlinear dispersion, and α is a positive constant. Equation (1) describes the propagation of an optical soliton in a nonlinear optical fiber that exhibits a Kerr law nonlinearity. Equation (1) has important applications in various fields, such as semiconductor materials, optical fiber communications, plasma physics, fluid, and solid mechanics.[1–8] Equation (1) has received more attention.[1–5,9,10] For example, by the first integral method, Zhang *et al.*[1] obtained new exact solutions of the perturbed nonlinear Schrödinger equation (NLSE) with Kerr law nonlinearity. Özgüla *et al.*[4] obtained new solutions of nonlinear partial differential equation by a direct algebraic method. Taghizadeh and Mirzazadeh[7] constructed the exact solutions of perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity by the simplest equation method. Miao and Zhang[8] used the (G'/G)-expansion method to obtain the traveling wave solutions for NLSEs. By the trigonometric function series method, Zhang[11] obtained new exact traveling wave solutions for the nonlinear Klein–Gordon equation. By the modified mapping method and the extended mapping method, Zhang *et al.*[1] acquired new exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity. Some other optical waves have been found in many equations.[12–21]

As is well known, the fiber-optic signal plays an important role in real life. It seems that the signal propagation cannot exist in a pure environment. It is always influenced by external environmental perturbations. It seems that chaos may be unavoidable for external perturbations and has been observed in many practical applications such as engineering, biology, industry, and production. In addition, many other systems with external perturbations have been widely investigated by using analytic methods and numerical simulations.[22–25] However, there are few studies of external perturbation for fiber-optic signal models. In fact, the fiber-optic signal propagation will be affected by lots of interference in the process of real propagation. Moreover, the dynamic characteristics of nonlinear system in two-frequency interference will be more rich and complicated. Consequently, the two-frequency disturbance has more practical meaning.

In this paper, we will study the chaotic behavior and control of Eq. (1) under two-frequency perturbations. Our interest is in the following two points.

i) Will our system present chaos under the two-frequency interference? If chaos appears, how is a controller designed to suppress chaos due to the complex nonlinear item of the system? Compared with the Duffing system,[6,24,25] our system has no damping. Once being perturbed with external forcing, the system may easily tend to the chaotic state. Therefore, we will select the controller which has the same function as the damping.

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ii) The other point is to analyze the parameter regions for fiber-optic signals to stably propagate in the controlled system. The research on this point is very important in practice. Through analyzing the sensitivities of parameters for the controlled system, we can obtain a group of reasonable parameters and guarantee the smooth propagation of fiber-optic signals. For example, due to the transmission medium of fiber-optic communication system, the parameters of the optical fiber signal directly affect the nature of the transmission system. The influence on the optical fiber communication system will cause signal attenuation and dispersion. By studying sensitivities of parameters to be controlled, we can obtain a preferable medium to reduce the influence of perturbation in the process of fiber-optic signal propagation.

The rest of this paper is organized as follows. In Section 2, we study the chaotic behavior of system (5) and eliminate the chaotic behavior by adding an appropriate controller. In Section 3, we consider the sensitivities of parameters to be controlled. Finally, in Section 4 we draw some conclusions from the present study.

2. Chaotic behavior and control

Assume that equation (1) has traveling wave solutions in the form of[1]

$$u(x,t) = \phi(\xi)\exp(-\mathrm{i}\Omega t), \quad \xi = x - ct, \tag{2}$$

where c is the propagation speed of a wave.

Substituting Eq. (2) into Eq. (1), we obtain

$$\begin{aligned} \mathrm{i}(\gamma_1\phi''' &+ \gamma_2\phi^2\phi' + 2\gamma_2\phi^2\phi' - c\phi') \\ &+ (\Omega\phi + \phi'' + \alpha\phi^3) = 0, \end{aligned} \tag{3}$$

where γ_i $(i = 1,2,3)$, α and k are the positive constants, and prime denotes the derivation with respect to ξ.

From Ref. [1], we have

$$A\phi'' + B\phi + C\phi^3 = 0 (A \neq 0), \tag{4}$$

where $A = \gamma_i$, $B = -c$, and $C = \gamma_2/3 + 2\gamma_3/3$ with γ_2 and γ_3 denoting the parameters of linear and nonlinear terms. Equation (4) describes the fiber-optic signal transmission system in an ideal environment. But it seems that the signal propagation cannot exist in a pure environment, it is always influenced by external environmental perturbations.

Consequently, we will consider the fiber-optic signal transmission system with two-frequency perturbation in the following form:

$$A\phi'' + B\phi + C\phi^3 = d_1\cos(w_1\xi) + d_2\cos(w_2\xi), \tag{5}$$

where the new variables d_i, w_i $(i = 1,2)$ denote the amplitude and the frequency of the parametric excitation, respectively. Here d_i and w_i $(i = 1,2)$ are real parameters. For this reason, C, d_i, and $w_i \geq 0$ $(i = 1,2)$.

Without loss of generality, in Eq. (5) we set $A = 1$. By the transformation $\phi = x_1$ and $\phi' = x_2$, equation (5) can be transformed into first-order non-autonomous equation

$$\begin{cases} x_1' = x_2, \\ x_2' = -Bx_1 - Cx_1^3 + d_1\cos(w_1\xi) + d_2\cos(w_2\xi). \end{cases} \tag{6}$$

Next, equation (6) is investigated in detail. We will discuss the behaviors of the fiber-optic signal transmission under two-frequency perturbation by the four-order Runge–Kutte method. We select parameters to be $B = 0.002$, $C = 0.4$, $d_2 = 1.5$, $w_1 = 1.2$, and $w_2 = 1$ with the initial conditions $x_1 = -10^{-4}$ and $x_2 = 10^{-4}$. The results are given by bifurcation diagram and maximum Lyapunov exponents.

According to the bifurcation diagram and Lyapunov exponents (see Fig. 1), we can observe that the values of Lyapunov exponents are all positive, so the system is easily converted into chaos even if there exists a small perturbation. So, an appropriate controller is needed to satisfy the practical applications of fiber-optic propagation.

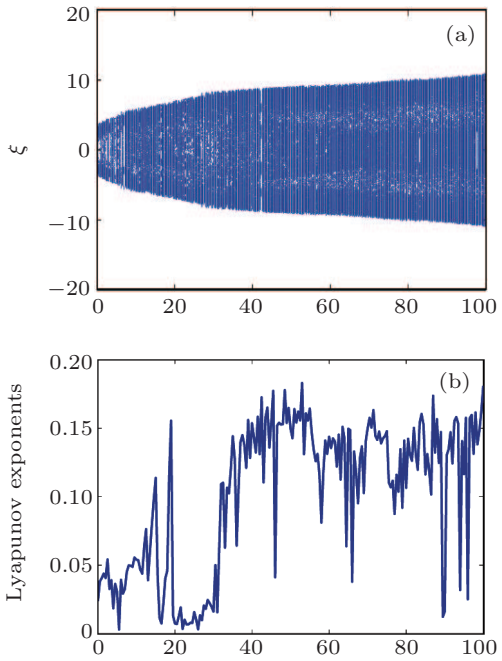


Fig. 1. (color online) (a) Bifurcation diagram of system (5) in (d_1, ξ) plane with $0 \leq d_1 \leq 100$, and (b) its corresponding maximum Lyapunov exponents.

Because the fiber-optic transmission system in the chaotic state is very sensitive to its initial condition and chaos often causes irregular behavior, chaos is undesirable. It is not hard to see that this system (6) is similar to a Duffing system, except for the absence of damping in the former. Therefore, we will select the controller that has the same function as that with the damping. To suppress chaos, we will add a controller

$e(k_1x_1+k_2x_2)$ to the system; that is,

$$\begin{cases} x_1' = x_2, \\ x_2' = -Bx_1 - Cx_1^3 + d_1\cos(w_1\xi) + d_2\cos(w_2\xi) \\ \quad + e(k_1x_1+k_2x_2), \end{cases} \tag{7}$$

where the parameters k_1 and k_2 are both real constants, and e denotes the controller strength.

Now, we will study the chaos control of system (7) with $k_1 = -0.005$, $k_1 = -0.005$, $d_1 = 1$, $B = 0.002$, $C = 0.4$, $d_2 = 1.5$, $w_1 = 1.2$, and $w_2 = 1$. According to the bifurcation diagram and Lyaponove exponents in Fig. 2, we can obtain that when $e < 3.186$, the parameter of controller is so small that the chaotic state cannot be controlled. With the parameter e continuously increasing, when $e > 3.186$, the chaotic state of system (7) can be controlled. With e increasing, the signal of system (7) is attenuated as shown in Fig. 2.

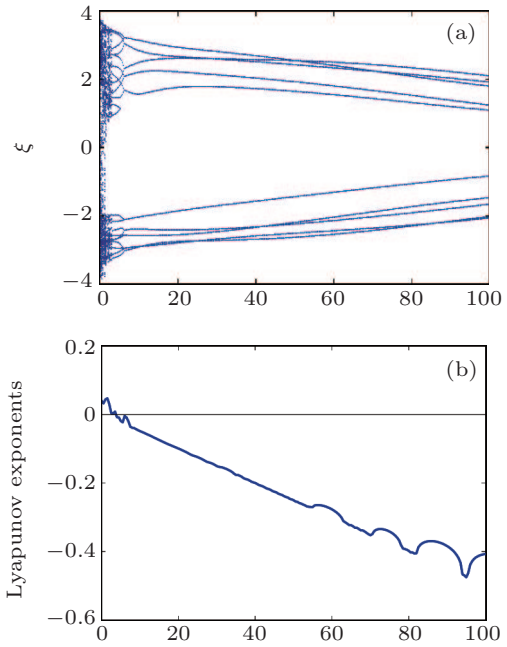


Fig. 2. (color online) (a) Bifurcation diagram of system (5) in (e, ξ) plane with $0 \le e \le 100$, (b) its corresponding maximum Lyapunov exponents.

Remark 1 According to the above analysis, the increase of parameter of controller will be beneficial to the behaviors of the system. It is not difficult to find that the attenuation may occur if the parameters of the controller are strong. Therefore, we recommend that the parameters of the controller should be selected appropriately.

3. Sensitivities of parameters to be controlled

In this section, the controller of fiber-optic transmission system has several parameters, each of which plays a different role in, and has an important influence on, the signal propagation.

Next, we will study the sensitivities of parameters for fixing controller.

3.1. Analyses of parameters of perturbations (Case A)

Varying d_1 in a range $0 \le C \le 100$ and notice the following.

Case A i) Setting $w_1 = 1.2$, $w_2 = 1$, $d_2 = 1.5$, $C = 0.4$, $B = 0.002$, $e = 7$, $k_1 = -0.005$, and $k_1 = -0.005$;

Case A ii) Setting $w_1 = 1.2$, $w_2 = 10$, $d_2 = 1.5$, $C = 0.4$, $B = 0.002$, $e = 7$, $k_1 = -0.005$, and $k_1 = -0.005$;

Case A iii) Setting $d_2 = 15$, $w_1 = 1.2$, $w_2 = 1$, $C = 0.4$, $B = 0.002$, $e = 7$, $k_1 = -0.005$, and $k_1 = -0.005$;

Case A iv) Setting $d_2 = 15$, $w_1 = 1.2$, $w_2 = 10$, $C = 0.4$, $B = 0.002$, $e = 7$, $k_1 = -0.005$, and $k_1 = -0.005$;

The bifurcation diagrams and Lyapunov exponents of system (7) are given in four groups as follows.

For Case A i), the bifurcation diagram of system (7) in (d_1, ξ) plane and its corresponding maximum Lyapunov exponents are given in Figs. 3(a) and 3(b), respectively. The local amplitudes of d_1 in Fig. 3(a) are in a range of 0–100; that is, $0 \le d_1 \le 100$. From them we observe that when the two interference frequencies w_i ($i = 1, 2$) are very small parameters, the system (6) separately with $d_1 \in$ (2.5, 5), (35, 48), (78.5, 83) presents a periodic solution form with a positive maximum Lyapunov exponent and is basically in a quasi-periodic or chaotic state in other regions.

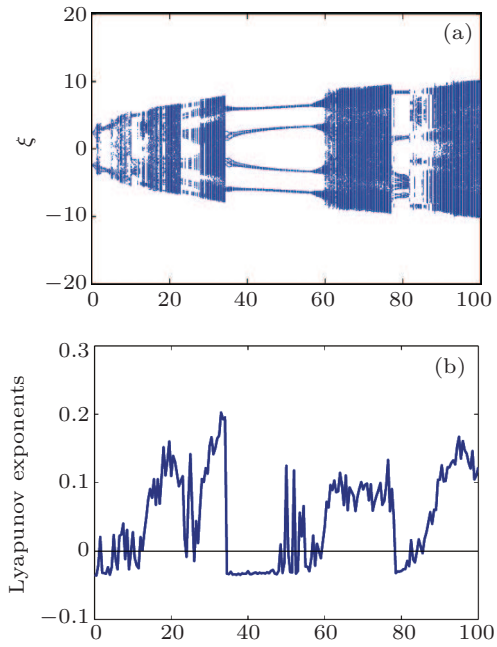


Fig. 3. (color online) (a) Bifurcation diagram of system (7) in (d_1, ξ) plane with $0 \le d_1 \le 100$, and (b) its corresponding maximum Lyapunov exponents.

For Case A ii), the bifurcation diagram of system (7) in (d_1, ξ) plane and its corresponding maximum Lyapunov exponents are given in Figs. 4(a) and 4(b) respectively. The local amplitudes of d_1 in Fig. 4(a) are in a range of 0–100; that is, $0 \le d_1 \le 100$. From them we observe that, when w_2 increases to 10, as shown in Fig. 4(b), the system (7) separately with

$d_1 \in (0, 14), (32, 43), (44, 49), (56, 61), (75.5, 81)$ presents a periodic solution but is almost in a quasi-periodic or chaotic state in other regions.

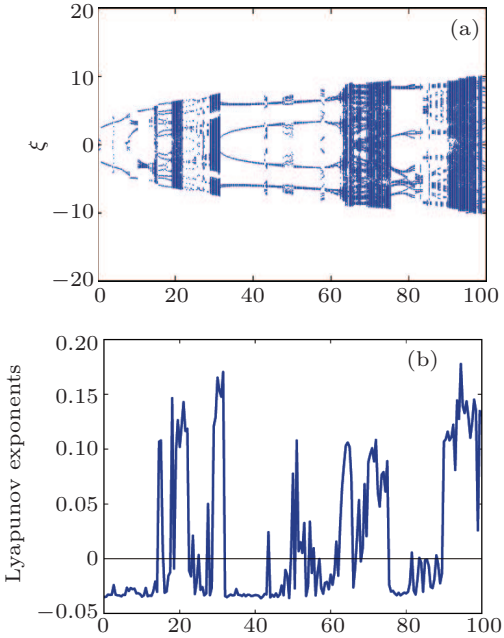


Fig. 4. (color online) (a) Bifurcation diagram of system (7) in (d_1, ξ) plane with $0 \le d_1 \le 100$, and (b) its corresponding maximum Lyapunov exponents.

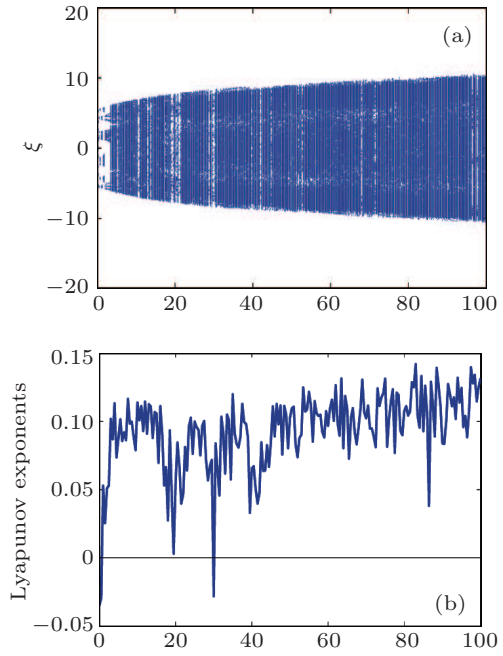


Fig. 5. (color online) (a) Bifurcation diagram of system (7) in (d_1, ξ) plane with $0 \le d_1 \le 100$, and (b) its corresponding maximum Lyapunov exponents.

For Case A iii), the bifurcation diagram of system (7) in (d_1, ξ) plane and its corresponding maximum Lyapunov exponents are given in Figs. 5(a) and 5(b), respectively. The local amplifications of Fig. 5(a) are for $0 \le d_1 \le 100$. From them we observe that, when $d_1 \in (0,0.68)$, the system (7) presents a periodic solution with a positive maximum Lyapunov exponent. Oscillation occurs when $d_1 > 0.68$, which is distinct with the increasing of amplitude.

For Case A iv), the bifurcation diagram of system (7) in (d_1, ξ) plane and its corresponding maximum Lyapunov exponents are given in Figs. 6(a) and 6(b), respectively. The local amplifications of Fig. 6(a) are for $0 \le d_1 \le 100$. From them we observe this kind of phenomenon, when $w_2 = 10$ is embodied. The system (7) separately in intervals of (0.8, 5), (9, 12.3), (32, 48.3), (52,57.4), etc. presents a periodic solution form.

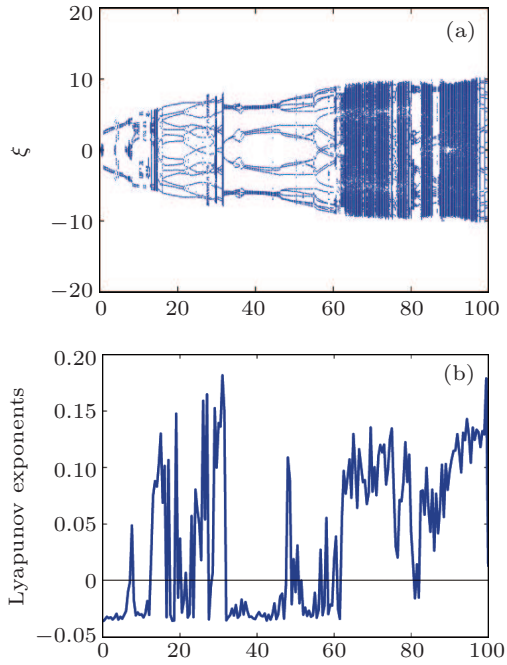


Fig. 6. (color online) (a) Bifurcation diagram of system (7) in (d_1, ξ) plane with $0 \le d_1 \le 100$, and (b) its corresponding maximum Lyapunov exponents.

Remark 2 From Figs. 3 and 5, it can be observed that if one of the frequencies of the perturbations is very close to another, then the system will more easily tend to resonance. On the contrary, the influence for fiber-optic signal is smaller.

3.2. Analysis of linear and nonlinear parameters (Case B)

Case B i) Varying B in a range $-50 \le B \le 50$ and fixing $C = 0.4$, $w_1 = 1.2$, $w_2 = 1$, $d_2 = 1.5$, and $d_1 = 1$;

Case B ii) Varying C in a range $0 \le C \le 40$ and fixing $B = 0.002$, $w_1 = 1.2$, $w_2 = 1$, $d_2 = 1.5$, and $d_1 = 1$.

For case B i), the bifurcation diagram of system (6) in (B, ξ) plane and its corresponding maximum Lyapunov exponents are given in Figs. 7(a) and 7(b), respectively. The local amplifications of Fig. 7(a) are for $-50 \le B \le 50$. From them we can obtain the following results: there are obvious oscillations with B being $(-4.15, -0.2)$, the cycle hopping behavior appears with B being $(-17.5, -3.32)$. When $B < -17.5$, the attenuation of the signal is intensified and escape appears. The system is relatively stable as $B > -3.32$, the signals strengthens attenuation and finally approaches to zero.

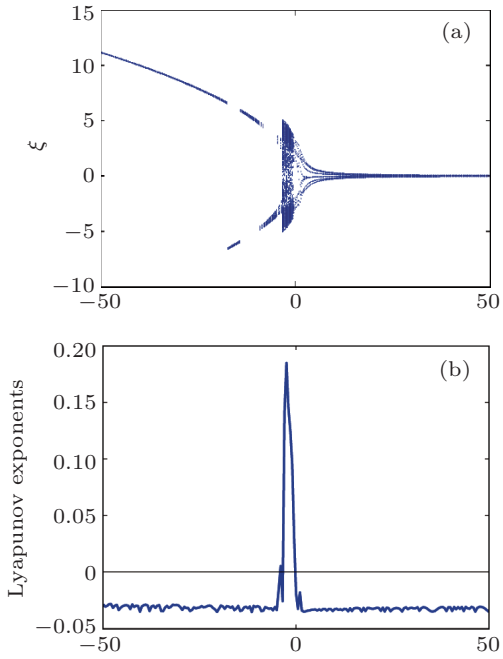


Fig. 7. (color online) (a) Bifurcation diagram of system (7) in (B, ξ) plane with $-50 \leq B \leq 50$, and (b) its corresponding maximum Lyapunov exponents.

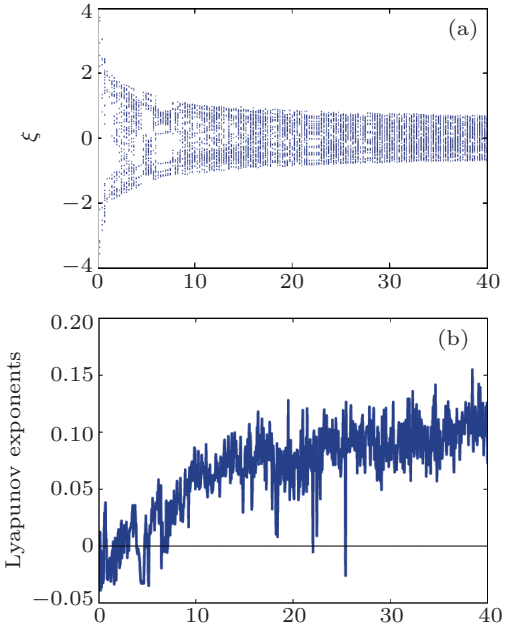


Fig. 8. (color online) (a) Bifurcation diagram of system (6) in (C, ξ) plane with $0 \leq C \leq 40$, and (b) its corresponding maximum Lyapunov exponents.

For Case B ii), the bifurcation diagram of system (6) in (C, ξ) plane and its corresponding maximum Lyapunov exponents are given in Figs. 8(a) and 8(b) respectively. The local amplitudes of C in Fig. 8(a) are in a range of 0–40; that is, $0 \leq C \leq 40$. From them we can obtain the following results. With the increase of C, the chaotic parameter region of the system can be expanded and has more easy access to a chaotic state in same external disturbance conditions. The behavior of the system is good and chaos is inhibited in the intervals of (0.15, 0.53), (0.81, 1.455), (3.9, 4.716). With C continuously increasing, the signals intensify attenuation and the system is sustained in a chaotic state.

Remark 3 The escape and attenuation of the fiber-optic signal appear very easily, the linear term parameter of system (7) is changed, and the change of the system nonlinear parameters leads to a fiber-optic signal attenuation and chaotic phenomenon is more pronounced. So the change of nonlinear term of the system leads to the change of chaotic region obviously, which can also be considered as a control strategy of chaos.

4. Conclusions

We conclude that chaos occurs easily due to the absence of damping in the system. This phenomenon will cause the distortion in the process of information transmission. One can add a controller to suppress the chaos. The efficiency of this controller is demonstrated. The complex fiber-optic transmission system of the perturbed NlSE with Kerr law nonlinearity is controlled. What is more, we discuss the sensitivities of parameters to be controlled and find the practical parameter regions. Some of the results obtained for other NLSE can be extended correspondingly.

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