# Calculation and analysis of the number of return photons from sodium laser beacon excited by the long pulse laser with circular polarization*

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The number of return photons from sodium laser beacon (SLB) greatly suffers down-pumping, recoil, and geomagnetic field when the long pulse laser with circular polarization interacts with sodium atoms in the mesosphere. Considering recoil and down-pumping effects on the number of return photons from SLB, the spontaneous radiation rates are obtained by numerical computations and fittings. Furthermore, combining with the geomagnetic field effects, a new expression is achieved for calculating the number of return photons. By using this expression and considering the stochastic distribution of laser intensity in the mesosphere under different turbulence models for atmosphere, the number of return photons excited by the narrow-band single mode laser and that by the narrow-band three-mode laser are respectively calculated. The results show that the narrow-band three-mode laser with a specific spectrum structure has a higher spontaneous radiation rate and more return photons than a narrow-band single mode laser. Of note, the effect of the atmospheric turbulence on the number of return photons is remarkable. Calculation results indicate that the number of return photons under the HV5/7 model for atmospheric turbulence is much higher than that under the Greenwood and ModHV models.

**Keywords:** sodium laser beacon, spontaneous radiation rates, the number of return photons, down-pumping, recoil, geomagnetic field

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## 1. Introduction

Using the long pulse laser to excite SLB is very useful for obtaining high laser launch power, effectively avoiding the Rayleigh scattering from the lower atmosphere and reducing the elongating of sodium laser beacon (SLB) by a laser with a duration of a few microseconds. However, in the process of the long pulse laser interacting with sodium atoms, there appear three adverse factors, i.e., down-pumping, recoil, and geomagnetic field. Besides these factors, the pulse formats and spectrum structures of laser should also be considered. Rochester *et al.*[1] investigated the interaction of a kind of narrow-band multi-mode laser produced by TIPC (Technical Institute of Physics and Chemistry of the Chinese Academy of Sciences) with sodium atoms in the mesosphere and the effects of down-pumping, recoil, and geomagnetic field on the number of return photons. The results indicated that the number of return photons is enhanced by adding repumping power. Holzlöhner *et al.*[2] worked on the efficiency of return photons excited by laser with multiform pulse formats and suggested that the laser with power 10 W, bandwidth 50 MHz, pulse width 3 μs, and adding repumping power should be introduced because its efficiency corresponds to that of the 20-W continuous-wave laser. Rampy *et al.*[3] calculated the number of return photons from SLB for Shane 3-meter telescope by numerical simulations and put forward the optimal scheme for the long pulse laser with pulse duration 30 μs. Céline d'Orgeville *et al.*[4] predicted the required power of SLB at Gemini Mauna Kea according to Milonni *et al.*'s[5] calculation expression about SLB excited by the long pulse laser. But ignored in all the above researches were the stochastic distributions of laser intensity in the mesosphere and the influences of atmosphere turbulence strength on the number of return photons. On the other hand, in the previous researches the above influencing factors were not taken into consideration simultaneously.

The purpose of this paper is to calculate the number of return photons from SLB for the long pulse laser interacting with sodium atoms in the mesosphere under different atmospheric turbulence and laser spectrum structures. The rest of this paper is organized as follows. In Section 2, down-pumping, recoil, and geomagnetic field effects are studied. An expression is achieved to calculate the number of return photons from SLB. In Section 3, the spontaneous radiation rates of sodium atoms in the excited states are computed by numerical method and fittings based on the theory models. Two expressions of the

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spontaneous radiation rates are obtained. In Section 4, the laser intensity distribution in the mesosphere and the number of return photons are simulated under different atmospheric turbulence models and different zenith angles. Finally, some significant conclusions are drawn in Section 5.

## 2. Theory models

According to the correlative theories[5,6] and experimental researches,[7,8] the excited sodium atoms attain steady states when the single-frequency circularly-polarized laser interacts with sodium atoms in an extremely short time, and the excited probability is given by

$$p_2 = \frac{I/2I_{\rm sat}}{1+16(\pi\tau(\nu-\nu_0-\nu_{\rm D}))^2+I/I_{\rm sat}}, \tag{1}$$

where $I$ is the incident intensity, $I_{\rm sat}$ is the saturated intensity, $\tau$ is the lifetime of sodium atom in an excited state, $\nu$ is the frequency of the incident laser, $\nu_0$ is the transition center frequency from an excited state to ground state, $\nu_{\rm D}$ is the Doppler shift, $\nu_{\rm D} = \boldsymbol{v}\cdot\boldsymbol{k}/2\pi$, $v$ is the component velocity of sodium atoms along the direction of laser transmission, and $k$ is the laser wave vector. Equation (1) shows that for a narrow-band laser, the laser launch frequency is required $\nu = \nu_0$ in order to realize the fluorescence resonance of laser and sodium atoms at $\nu_{\rm D} = 0$, where the excitation probability is maximum. But the sodium atoms at $\nu_{\rm D} \neq 0$ only show the near resonance[9] and the probability decreases as the deviation of Doppler shift from $\nu_{\rm D} = 0$ increases. Moreover, the spontaneous radiation rate of sodium atoms is defined as

$$R = p_2/\tau = \frac{I/2I_{\rm sat}\tau}{1+16(\pi\tau(\nu-\nu_0-\nu_{\rm D}))^2+I/I_{\rm sat}}. \tag{2}$$

Whereas the actual cases are complex to a certain extent, the further researches indicate that they are the near resonance for narrow-band laser (full width at half maximum (FWHM) $\leq$ 1 MHz) and the resonance excitation for wide-band laser (FWHM $\geq$ 10 MHz) in the process of the laser interacting with sodium atoms. As a result, those sodium atoms in the $F = 2$ ground state are excited to $F' = 1$ and $F' = 2$ excited states, then the sodium atoms in $F' = 1$ and $F' = 2$ decay to $F = 1$ ground states with 50% probability. In a period of time, a great many sodium atoms will reside in $F = 1$ ground states. This phenomenon is the so-called down-pumping. The down-pumping effect weakens the transition between $3^2S_{1/2}(2,\ 2)$ states and $3^2P_{3/2}(3,\ 3)$ states and makes sodium atoms more quickly reach a transition saturation state with a lower intensity. With no optical field exciting $F = 1$ ground states, the sodium atomic collisions can partially make these atoms return to $F = 2$ ground states and remain at the certain level.

On condition that there exists no external magnetic field in the mesosphere, the sodium atoms at the sodium layer are in the thermo-equilibrium and the rate distribution of sodium atoms meets the Maxwellian velocity distribution law. When laser irradiates the sodium layer, this thermo-equilibrium is broken because laser interacts with sodium atoms. While the sodium atoms are absorbing photons, the known $\boldsymbol{k}$ is still the wave vector, providing that $\boldsymbol{v}_1$ is the velocity before a sodium atom absorbs a photon, $\boldsymbol{v}_2$ is the velocity after a sodium atom has absorbed a photon, $h\nu$ is the photon energy of the incident laser, the decay energy of excited state is $h\nu_0' = E_2 - E_1$, where $\nu_0'$ is the transition frequency, $E_1$ denotes the ground state energy, and $E_2$ represents the excited state energy. According to the momentum conservation, the change in momentum can be obtained as

$$\Delta\boldsymbol{p} = m_{\rm Na}\boldsymbol{v}_2 - m_{\rm Na}\boldsymbol{v}_1 = \hbar\boldsymbol{k}, \tag{3}$$

where $\hbar = h/2\pi$. One can see from Eq. (3) that a sodium atom absorbing a photon leads to the momentum changes along the direction of laser transmission. As a result, the sodium atom, whose mass is $m_{\rm Na}$, obtains a recoil velocity $\boldsymbol{v}_{\rm r} = \hbar\boldsymbol{k}/m_{\rm Na}$ and enters into the next velocity groups when velocity groups are adequately small. At this time, the Doppler shift is $\nu_{\rm D}' = (\boldsymbol{v}+\boldsymbol{v}_{\rm r})\cdot\boldsymbol{k}/2\pi$ and the resonant frequency of laser and soudim atoms is $\nu = \nu_0' + \nu_{\rm D}'$. So, we obtain

$$\begin{aligned}\nu - \nu_0' &= \pm\nu_{\rm D} + h/(\lambda^2 m_{\rm Na}) \\ &= \pm\nu_{\rm D} + 50\ \text{kHz}.\end{aligned} \tag{4}$$

Equation (4) shows that the Doppler shifts will increase 50 kHz due to recoil effects after a sodium atom has absorbed a photon, which results in more sodium atoms drifting toward the higher Doppler shift range.[10]

On the basis of the Bohr model of sodium atom hyperfine structures[10] and on the assumption that the near resonance meets Lorentz line profile, the excitation probability of the level transition, brought by the down-pumping for narrow-band laser, can be phenomenologically expressed as follows:

$$\begin{cases} p_{F=2\to F'=2} = \dfrac{I/2I_{\rm sat}}{1+16(\pi\tau(58.3\times10^6))^2+I/I_{\rm sat}} \\ \qquad\qquad \otimes \dfrac{I/2I_{\rm sat}}{1+16(\pi\tau(58.3\times10^6-\nu_{\rm D}))^2+I/I_{\rm sat}}, \\ p_{F=2\to F'=1} = \dfrac{I/2I_{\rm sat}}{1+16(\pi\tau(92.7\times10^6))^2+I/I_{\rm sat}} \\ \qquad\qquad \otimes \dfrac{I/2I_{\rm sat}}{1+16(\pi\tau(92.7\times10^6-\nu_{\rm D}))^2+I/I_{\rm sat}}, \end{cases} \tag{5}$$

where $p_{F=2\to F'=2}$ and $p_{F=2\to F'=1}$ are respectively the probabilities with which the sodium atoms transit from $F = 2$ ground state to $F' = 2$ and $F' = 1$ excited states, the symbol $\otimes$ represents the second term multiplying the first one after the second term has been normalized by maximum.

Recoil and down-pumping effects reduce the excitation probability and the spontaneous radiation rate of sodium atom. In order to explore the effects of recoil and down-pumping

on the spontaneous radiation rate, numerical calculation is adopted. Firstly, we calculate the summation of the excited sodium atoms under the different intensities, where the percentage is adopted. Then, the summation is divided by the average collision time of sodium atoms with others. Consequently, the spontaneous radiation rate per unit sodium atom is expressed as

$$R = 5/8 \sum_{j=1}^{n} \left( \sum_{i=1}^{m} \mathrm{N}_{v_{\mathrm{D}i}}(\Delta v_{\mathrm{D}}) \times [p_{2i} - 0.5(p_{iF=2\to F'=2} + p_{iF=2\to F'=1})] \right)_j \Big/ (n\tau), \quad (6)$$

where $m$ is the number of velocity groups, $n$ is the times of the spontaneous radiation, $N_{v_{\mathrm{D}}i}(\Delta v_{\mathrm{D}})$ and $p_{2i}$ are, respectively, the percentage and the excitation probability of sodium atoms in the $i$-th velocity group, $\Delta v_{\mathrm{D}}$ represents a certain velocity interval, $p_{iF=2\to F'=2}$ and $p_{iF=2\to F'=1}$ are, respectively, the transition probabilities of the $F = 2$ ground state to $F' = 2$ and $F' = 1$ excited states in the $i$-th velocity group. According to the Maxwellian velocity distribution of the thermal motion, we have

$$N_{v_{\mathrm{D}}}(\Delta v_{\mathrm{D}}) = \frac{(4\ln 2/\pi)^{1/2}}{\delta v_{\mathrm{D}}} \mathrm{e}^{-4\ln 2 v_{\mathrm{D}}^2/\delta v_{\mathrm{D}}^2} \Delta v_{\mathrm{D}}. \quad (7)$$

Besides the recoil and down-pumping effects, the geomagnetic field is also an important factor that can influence the interaction between laser and sodium atoms due to Lamor precession. Moussaoui *et al.*[11] have investigated the probability of sodium atoms staying in the $3\mathrm{S}_{1/2}(2, 2)$ state during a precession period. Their results show that the geomagnetic field weakens the increase of excitation probability of sodium atoms and the influencing degree relates to the angle $\theta$ between the laser transmission direction and geomagnetic field direction. When $\theta = 90^\circ$, the excitation probability of sodium atoms is much less than that when $\theta = 30^\circ$. Drummond *et al.*[12] have studied experimentally the number of return photons from SLB at the Starfire Optical Range and summarized the influencing factor $f_{\mathrm{mag}}$ when the circularly-polarized laser excites SLB. The $f_{\mathrm{mag}}$ is expressed as

$$f_{\mathrm{mag}} = 1 - 0.6552 \sin\theta. \quad (8)$$

When laser transmits upward into the sodium layer and irradiates a certain area, the intensity distribution of laser takes on the random characteristics due to atmosphere absorption, scattering and turbulence.[13] Here, the backscatter radiative photons per unit solid angle and time is expressed as

$$\varphi = \beta' C_{\mathrm{Na}} R_{\mathrm{p}} \sum_i \Delta S_i (\tau_{\mathrm{p}} + \tau)(1 - 0.6552 \sin\theta) R/(4\pi), \quad (9)$$

where $\beta'$ is the backscattering coefficient of return photons, $\Delta S_i$ is the very small area irradiated by laser, $C_{\mathrm{Na}}$ is the column density in the sodium layer, $\tau_{\mathrm{p}}$ is the pulse duration, $R_{\mathrm{p}}$ is the repetition frequency of long pulse laser, and $R$ is the spontaneous radiation rate. On the assumption that laser is launched vertically, the number of return photons per unit time and area is

$$\Phi = T_0 \varphi / L^2, \quad (10)$$

where $T_0$ is the atmosphere transmittance, $L$ is the altitude from the telescope to the sodium layer center. In the case where the laser transmits along the slant path and the zenith is $\zeta$, the column density in the sodium layer is $C_{\mathrm{Na}} \sec(\zeta)$, the very small area irradiated by laser is $\Delta S_i'$, the transmission distance is $L\sec(\zeta)$. Combining with Eqs. (9) and (10), the number of return photons per unit time and area becomes

$$\Phi = T_0^{\sec(\zeta)} \beta' C_{\mathrm{Na}} R_{\mathrm{p}} \sum_i \Delta S_i' (\tau_{\mathrm{p}} + \tau) \times (1 - 0.6552 \sin\theta) R/(4\pi L^2 \sec(\zeta)). \quad (11)$$

If the area of receiving telescope $A_{\mathrm{a}} = \pi r^2$, $r$ is the radius of telescope, then the single pulse laser excites the number of return photons,

$$\Phi = T_0^{\sec(\zeta)} \beta' C_{\mathrm{Na}} A_{\mathrm{a}} \sum_i \Delta S_i' (\tau_{\mathrm{p}} + \tau) \times (1 - 0.6552 \sin\theta) R/(4\pi L^2 \sec(\zeta)). \quad (12)$$

From Eqs. (11) and (12), it is very clear that the atmosphere transmittance and the column density change when the laser transmits along the slant path, $C_{\mathrm{Na}}$ becomes the column density at the indecent point, and $\theta$ also changes as the zenith $\zeta$ changs. The $R$, in fact, relates to intensity and is influenced by characteristics of the sodium layer in the mesosphere. Here, equations (11) and (12) consider only the number of return photons from $3^2\mathrm{P}_{3/2}(3, 3)$ states to $3^2\mathrm{S}_{1/2}(2, 2)$ states. The number of return photons from down-pumping effects is ignored because the transition probability is only equal to or even lower than 1% of the excitation probability $p_2$ by computation.

## 3. Spontaneous radiation rate

### 3.1. For the narrow-band single mode laser

The bandwith of narrow-band laser is commonly less than or equal to 1 MHz. When calculating the number of return photons from SLB according to Eq. (11), the spontaneous radiation rate $R$ must be known. But the numerical computation of the spontaneous radiation rate relates to the long pulse duration. According to the average collision time of sodium atoms in the sodium layer, we take $n\tau = 100$ μs[14] for the long pulse with $\tau_{\mathrm{p}} \geq 100$ μs. Using Eqs. (4)–(7), the spontaneous radiation rate per unit sodium atom is found through numerical computations and fittings. Given the Doppler shift range is from −1 GHz to 1 GHz and $\Delta v_{\mathrm{D}} = 10^4$ Hz, the total number of velocity groups is 200001. Figure 1 shows the simulation results of the spontaneous radiation rates denoted as '○', with average collision time $n\tau$ of sodium atoms being 100 μs, and $I$ varying from 0 to 150 W/m$^2$: the points of these data are fitted as indicated by a solid curve.

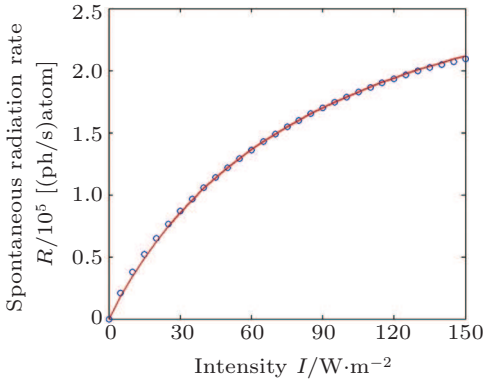



**Fig. 1.** (color online) Spontaneous radiation rates and numerical fitting curves with average collision time $n\tau$ of sodium atoms being 100 μs, and $I$ varying from 0 to 150 W/m$^2$.

It thus follows from this fitted curve that the spontaneous radiation rate can be achieved as follows:

$$R = 3.82 \times 10^3 \frac{I}{1+I/88}. \tag{13}$$

### 3.2. For the narrow-band three-mode laser

Compared with multimode laser, the single mode laser has a certain width of spectrum and has only a central frequency as power changes with frequency. But the power distribution of the multimode laser has several frequency centers and each center corresponds to a certain bandwith. Here, take the laser developed by TIPC for instance, the known modes are 3 and the interval between modes is 150 MHz. On the assumption that the laser spectrum presents a Gaussian distribution, because the pulse duration $\tau_p = 120$ μs, the full width at half maximum (FWHM) value of each mode is

$$\delta\nu_0 = 2\ln 2/(\pi\tau_p) = 3.7 \text{ kHz}.$$

Again, assuming that the envelope of the laser spectrum is Gaussian line profile whose FWHM value is $\delta\nu_b = 1$ GHz, the power distribution with respect to frequency[15] is

$$P(\nu) = P(0)\exp[-4\ln 2(\nu/\delta\nu_b)^2] \times \sum_{j=-k}^{+k} \exp[-4\ln 2(\nu - j\nu_1)^2/\delta\nu_0^2], \tag{14}$$

where $j$ denotes the mode, $k$ is an integer, $\nu_1$ is the interval between modes, and $\nu_1 = 150$ MHz. Taking $k = 1$ and $\nu = \nu_D$, the power distribution of the multimode laser is the same as that given by Rochester *et al.*[1] (Fig. 2).

In Fig. 2, the dashed curve is the envelope line. Each mode belongs to narrow-band, which looks like a line. Based on Eq. (14), the spectrum distribution of intensity is achieved when laser irradiates a certain area.

$$I(\nu) = I(0)\exp[-4\ln 2(\nu/\delta\nu_b)^2] \times \sum_{j=-k}^{+k} \exp[-4\ln 2(\nu - j\nu_1)^2/\delta\nu_0^2]. \tag{15}$$

From Eq. (15), the envelope line profile is

$$I(\nu) = I(0)\exp[-4\ln 2(\nu/\delta\nu_b)^2].$$

Furthermore, the peak intensity ratio among the three modes in Fig. 2 is achieved by computing $I_f(-\nu_1) : I_f(0) : I_f(\nu_1) = 1 : 1.0638 : 1$. Finally, the total intensity ratio among the three modes is $I_{sum}(-\nu_1) : I_{sum}(0) : I_{sum}(\nu_1) = 32.6\% : 34.8\% : 32.6\%$.

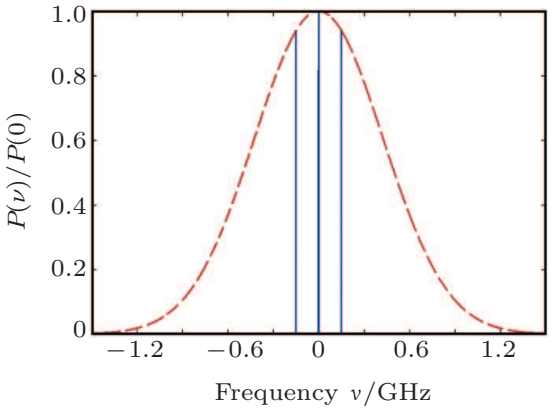



**Fig. 2.** (color online) Distribution of laser power with respect to frequency when $\delta\nu_0 = 3.7$ kHz.

In the case of knowing the intensity distribution with respect to frequency, regarding each spectral line as the narrow-band laser and applying Eq. (4)–Eq. (7), one can analyze recoil and down-pumping effects in the process of the long pulse laser interacting with sodium atoms. The normalized distributions of sodium atoms as Doppler shifts are simulated under the different incident intensities after the different times in Fig. 3.

Figure 3 shows that recoil effects increase as time and intensity increase under given conditions. As a result, more and more sodium atoms accumulate in the higher and higher Doppler shift ranges as time goes by.

According to Eqs. (4)–(7), taking the recoil effects and the Doppler distribution in the heat equilibrium into account, the sodium atom distribution is simulated in the case of losing $F = 2$ ground states with average collision time $n\tau$ of sodium atoms being 100 μs.

In Fig. 4, the range of Doppler shift is from −1 GHz to 1 GHz on the upper panel (a) and it is partially enlarged on the bottom panel (b). The dotted curves show the normalized distributions and the dash-dotted curves are the distributions of the $F = 2$ ground states in the thermo-equilibrium. The other curves indicate that the number of sodium atoms changes in the case of losing $F = 2$ ground states. It can be seen from Fig. 4 that recoil and down-pumping effects cause the loss of sodium atoms in the $F = 2$ ground staes as intensity increases and the middle mode is influenced most by down-pumping effects.

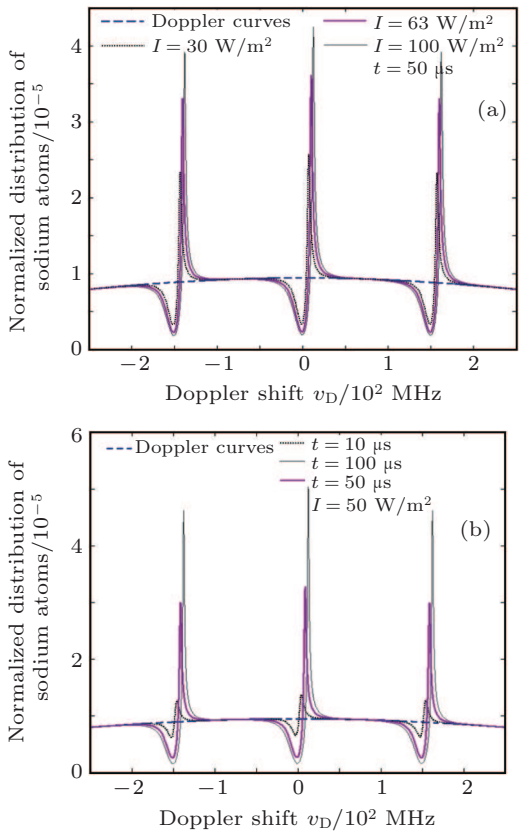



**Fig. 3.** (color online) Normalized distributions of sodium atoms versus Doppler shift (a) for $t = 50$ μs under different intensity and (b) after different time under $I = 50$ W/m$^2$.

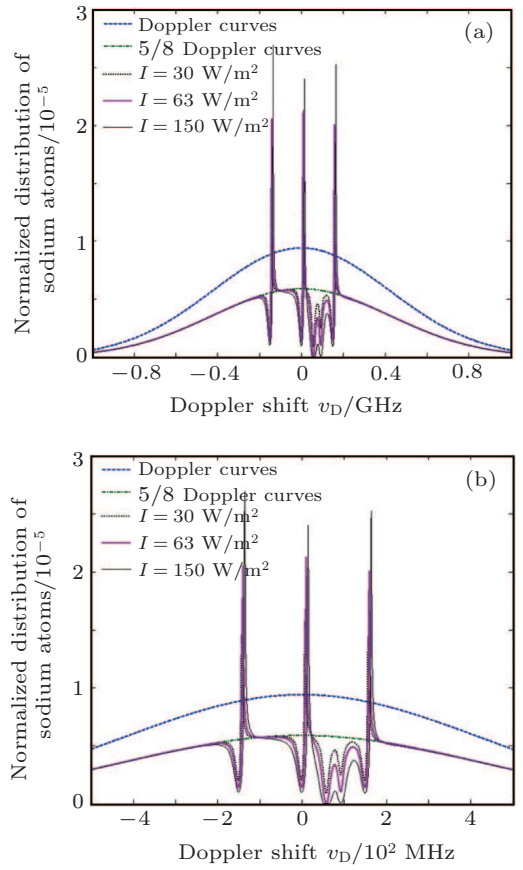



**Fig. 4.** (color online) Distributions of sodium atoms versus Doppler shift by $t = 100$ μs under different intensities.

Figure 5 shows the simulated spontaneous radiation rates of sodium atoms excited by long pulse laser with 3 modes and the rates are fitted to the solid curve.

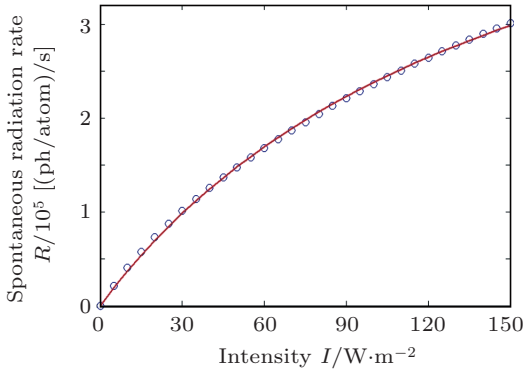



**Fig. 5.** (color online) Spontaneous radiation rates and numerical fitting curve with average collision time $n\tau$ of sodium atoms being 100 μs and $I$ varying from 0 to 150 W/m$^2$.

From the results of numerical computation and fitting curve in Fig. 5, a new expression of the spontaneous radiation rate is able to be achieved

$$R = 3.8994 \times 10^3 \frac{I}{1 + I/176} \tag{16}$$

It is worthwhile to note that the average collision time $n\tau$ of sodium atoms is chosen to be 100 μs in the above stimulations and computations. In fact, this choice is a reasonable substitute. The reason for this is that the collision time depends on the change of sodium layer in the mesosphere, including the abundance, centroid altitude, and specific structure of sodium layer. Holzlöhner *et al.*[16] have estimated the collision time of $v$-damping to be 35 μs and that of $s$-damping (spin damping) to be 490 μs according as those parameters that the centroid altitude of sodium atoms 92 km, the exponential distribution of the gas pressure, Gaussian distribution of sodium atoms, etc. However, Milonni and Fearn's[14] computation result shows that the average collision time is roughly 100 μs by using the so-called '$s$-damping (spin damping)'. Here, without accurate calculation, we directly take the Milonni and Fearn's[14] computation result as the average collision time.

## 4. Simulation and analysis

### 4.1. Simulation method

It can be seen from Eqs. (11) and (12) that the number of return photons has a quantitative relation with the intensity distribution in the mesosphere. We use the Maxwell equation to simulate the propagation of sodium laser in the atmospheric turbulence. Under the parabolic approximation and avoiding the backscattering, the Maxwell equation can be expressed as the following equation:

$$2ik\frac{\partial E}{\partial z} + \nabla_\perp^2 E + 2k^2 n_1 E = 0, \tag{17}$$

where $k$ is the wavenumber which is $2\pi/\lambda$, and $n_1$ is the variation of atmosphere refractive index. Firstly, the multi-phase screen method[17,18] is used to solve Eq. (17) and to obtain the intensity distribution $I$ of the exciting laser. Then, the number of return photons can be obtained according to Eqs. (11) and (12). Considering the stochastic process of atmospheric turbulene,[19] 100 realizations are simulated to obtain the average number of return photons.

### 4.2. Simulation parameters

Simulations of the number of return photons excited by the long pulse laser with circular polarization involve many parameters, mostly including characteristics of laser, laser launch parameters, atmosphere absorption, column density of sodium layers, receiving telescope diameter, etc. The specific parameters are listed in Table 1.

**Table 1.** Simulation parameters for the number of return photons from sodium laser beacon.[1] The unit 1 Gs = $10^{-4}$ T.

| Variable name | Symbol | Value | Variable name | Symbol | Value |
|---|---|---|---|---|---|
| Launched power | $P$ | 20 W | Pulse duration | $\tau_p$ | 120 μs |
| Laser center wavelength | $\lambda$ | 589.159 nm | Sodium layer center altitude | $L$ | 93 km |
| Repetition rate | $R_p$ | 800 Hz | Zenith of laser beam | $\zeta$ | 0° |
| Zenith of geomagnetic field | $\theta$ | 126°18′ | Beam quality factor | $\beta$ | 1.2 |
| Laser polarization | circular | +1 | Launch diameter | $D$ | 40 cm |
| Atmospheric transmittance | $T_0$ | 0.84 | Sodium column density | $C_{Na}$ | $4\times10^9$atoms/cm$^2$ |
| B-field | $B$ | 0.334 Gs | Backscattering coefficient | $\beta'$ | 1.5 |
| Temperature in the sodium layer | $T_K$ | 200 K | FWHM of Doppler curves | $\delta\nu_D$ | 1 GHz |

### 4.3. Results and discussion

Besides the above data in Table 1, the laser waveform is equivalent to a standard square shape. One can calculate the average power of single laser pulse $\bar{P} = P/(R_p\tau_p) = 208$ (W), which is the incident power in air. Assuming that the sodium laser is a narrow-band single mode laser which enters vertically into the mesosphere with a zero zenith angle and is launched in a collimated manner, the distributions of laser intensity and radiative photons in backscattering are simulated under HV5/7, Greenwood and ModHV turbulence models respectively. The results are shown in Figs. 6–8 and these figures show the results of random samples.

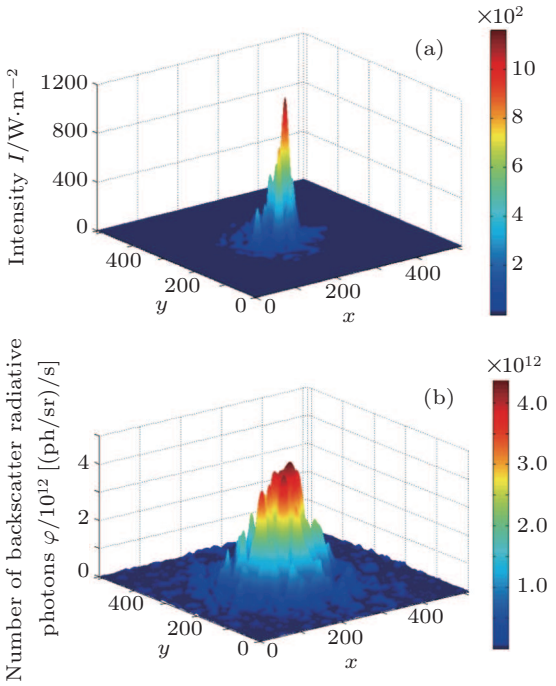



**Fig. 6.** (color online) (a) Distribution of laser intensity for one long pulse in the mesosphere and (b) distribution of radiation photons in one second in the sodium layer under the HV5/7 model for atmospheric turbulence.

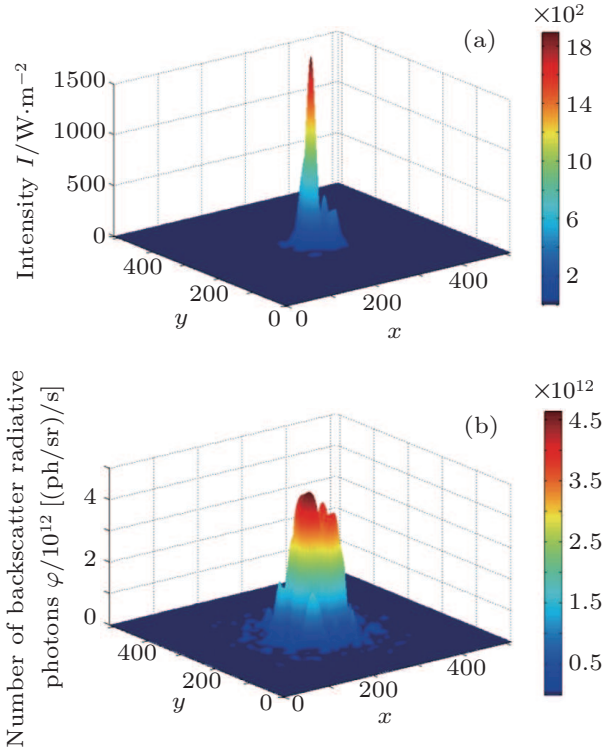



**Fig. 7.** (color online) (a) Distribution of laser intensity for one long pulse in the mesosphere and (b) distribution of radiation photons in one second in the sodium layer under the Greenwood model for atmospheric turbulence.

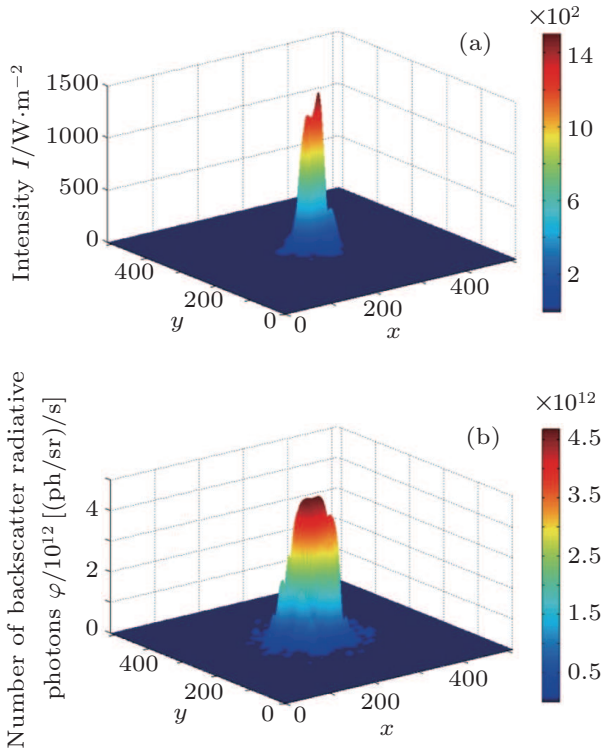



**Fig. 8.** (color online) (a) Distribution of laser intensity for one long pulse in the mesosphere and (b) distribution of radiation photons in one second in the sodium layer under the ModHV model for atmospheric turbulence.

From the above, one can see that the intensity distributions of one long pulse in the mesosphere are subjected to atmospheric turbulence. When the atmospheric turbulence is stronger, the dispersiveness of the intensity distribution becomes more severe. At the same time, the distribution of radiative photons per unit solid angle becomes more dispersive. According to Eqs. (9) and (13), because the relation between the number of radiative photons from SLB and intensity is nonlinear, the photon number has a remarkable increase under the lower intensity, but the very high intensity leads to a saturation effect that is the flat-topped phenomenon near the center of intensity distribution. Under the three atmospheric turbulence modes, the average values and standard deviations of the return photons in the receiving plane as well as the Fried's parameters are listed in Table 2 where zeniths of laser beam include two cases of 0° and 30°.

In Table 2, $r_0$ is the Fried's parameter along the vertical direction. When zenith of laser beam $\zeta$ is 30°, the value of the angle between laser beam and direction of geomagnetic field is taken to be $\theta = 180° - 126.3° - 30°$. When the laser transmits along the slant path, in spite of the atmospheric transmittance $T_0$ becoming worse, the angle $\theta$ becomes small and the effect of the geomagnetic field decreases. On the whole, the average values of return photons at $\zeta = 30°$ goes up. From the data in Table 2, the number of return photons increases as the Fried's parameter decreases. The photon number under the HV5/7 model for atmospheric turbulence is nearly twice that under the Greenwood model. Moreover, the standard deviations embody the fluctuation of the photon number due to atmospheric turbulence and have a maximum value at $r_0 = 6$ cm. The standard deviations, corresponding to the HV5/7 model, Greenwood model and the ModHV model for atmosphere turbulence are, respectively, the average values 7.8%, 12%, and 16.8% at $\zeta = 0°$.

**Table 2.** Average values and standard deviations of the return photons for sodium laser beacons as well as the Fried's parameters.

| Atmospheric turbulence models | Fried's parameters $r_0$/cm | Average return photons $\bar{\Phi}/10^6$ [(ph/s)/m$^2$] | | Standard deviations $\sigma_\Phi/10^5$ [(ph/s)/m$^2$] | |
|---|---|---|---|---|---|
| | | 0° | 30° | 0° | 30° |
| HV5/7 | 6.0 | 6.1425 | 9.2898 | 4.7645 | 7.5929 |
| Greenwood | 15.5 | 3.2720 | 5.2702 | 3.9300 | 5.3835 |
| ModHV | 21.8 | 2.6825 | 4.3396 | 4.5076 | 6.0281 |

Using the data in Table 1, for the narrow-band three-mode laser, at $\zeta = 0°$, the distributions of the intensity and radiative photons in backscattering are simulated as shown in Figs. 9–11 (results of random samples).

Figure 9–11 show that the distributions of the laser intensity and the radiative photons in the mesosphere are very similar to what the figures 6–8 display. Nevertheless, the flat-topped phenomenon is weaker due to saturated effects. Under the three atmospheric turbulence modes, the average values and standard deviations of the return photons on the receiving plane as well as the Fried's parameters are listed in Table 3 at $\zeta = 0°, 30°$.

The return photons excited by three-mode laser in Table 3 have more than one single mode laser under the three atmospheric turbulence modes and bigger standard deviations than the counterparts in Table 2. The reasons for this are that their spectrum structures are different and the interval between modes is 150 MHz, which effectively reduces the down-pumping damage and enhances the excited probability of sodium atoms. The standard deviations, corresponding to the HV5/7 model, Greenwood model, and the ModHV model for atmospheric turbulence, are respectively the average values 7.4%, 12.4%, and 14.6% at $\zeta = 0°$. It can be seen from Tables 2 and 3 that the geomagnetic field has a great influence on the number of return photons. When angle $\theta$ becomes very small, the return photon number is markedly enhanced.

A notable question is that the positive effects of sodium atom collisions are not taken into account.[20] For example, spin exchange and velocity exchange collision of sodium atoms with others will be beneficial to the enhancement of the return photon number. But these effects do not play a dominant role in the number of return photons.[14] In addition, the dwell velocity of laser beam can also influence the return photon number. But this influence may be ignored because the dwell time is a few milliseconds.[16] We predict that this simplification would not have an essential influence on the above results.

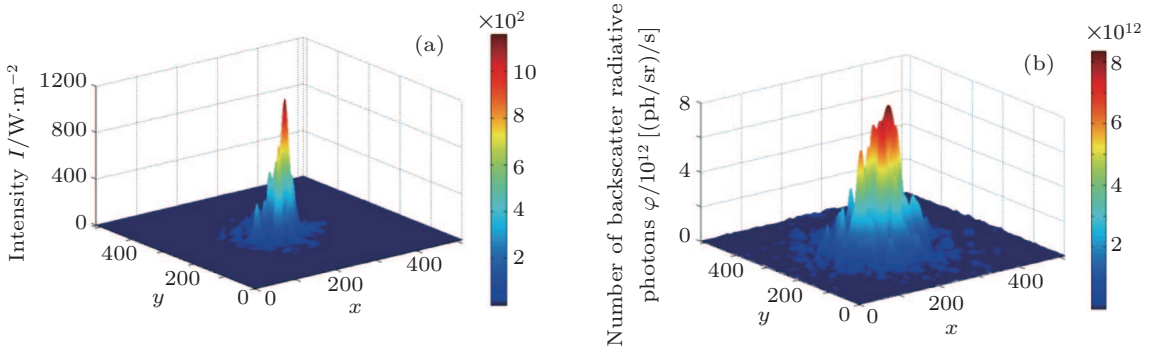



**Fig. 9.** (color online) (a) Distribution of laser intensity for one long pulse in the mesosphere and (b) distribution of radiation photons in one second in the sodium layer under the HV5/7 model for atmospheric turbulence.

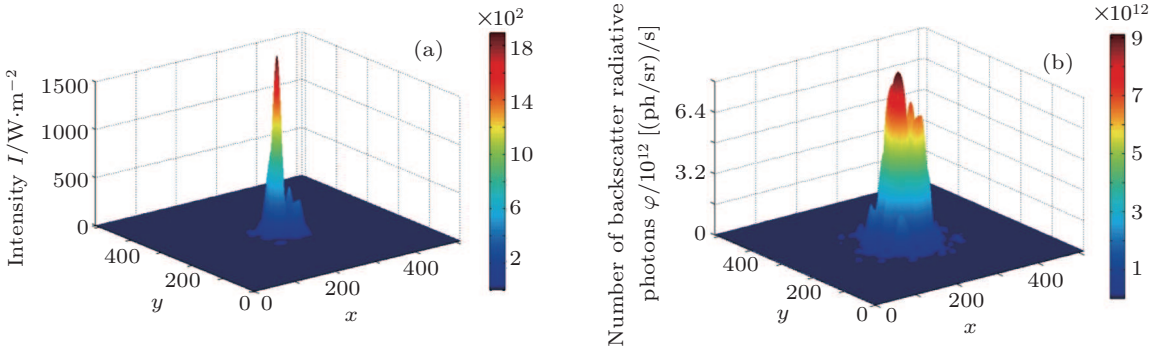



**Fig. 10.** (color online) (a) Distribution of laser intensity for one long pulse in the mesosphere and (b) distribution of radiation photons in one second in the sodium layer under the Greenwood model for atmospheric turbulence.

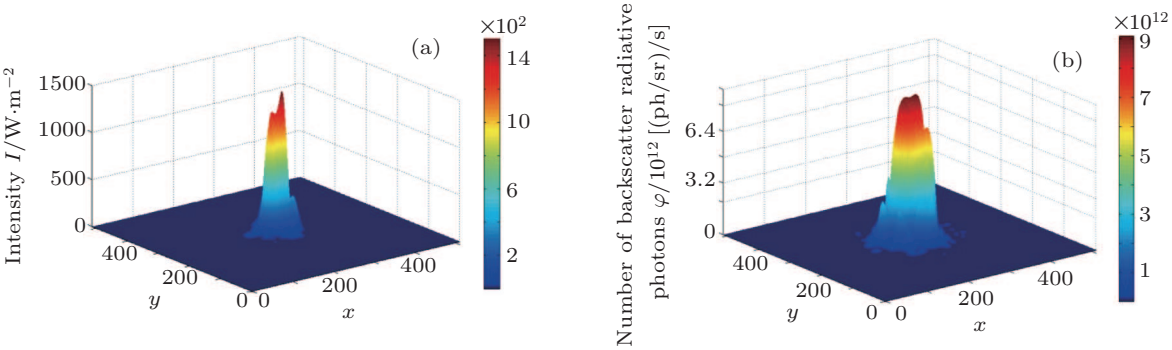



**Fig. 11.** (color online) (a) Distribution of laser intensity for one long pulse in the mesosphere and (b) distribution of radiation photons in one second in the sodium layer under the ModHV model for atmospheric turbulence.

**Table 3.** Average values and standard deviations of the return photons for laser sodium beacons as well as the Fried's parameters.

| Atmospheric turbulence models | Fried's parameters $r_0$/cm | Average return photons $\bar{\Phi}/10^6$ [(ph/s)/m$^2$] | | Standard deviations $\sigma_\Phi/10^5$ [(ph/s)/m$^2$] | |
|---|---|---|---|---|---|
| | | 0° | 30° | 0° | 30° |
| HV5/7 | 6.0 | 7.9803 | 11.895 | 5.9079 | 8.6457 |
| Greenwood | 15.5 | 4.6205 | 6.9991 | 5.7445 | 8.5794 |
| ModHV | 21.8 | 3.8064 | 6.1515 | 5.5654 | 8.8271 |

## 5. Conclusions

In this paper, we analyze the down-pumping, recoil, and geomagnetic field effects in the process of the long circularly-polarized pulse laser interacting with sodium atoms. Based on the phenomenological physics models, the spontaneous radiation rates are computed for narrow-band, three-mode, and single mode lasers. According to the intensity distributions in the mesosphere, we calculate the number of return photons from SLB. Some conclusions are drawn from the present study as follows.

(i) Atmospheric turbulence sufficiently influences the number of return photons from SLB. Under the HV5/7 model for atmospheric turbulence, the number of return photons is much higher than that under the Greenwood and ModHV models.

(ii) The down-pumping, recoil, and geomagnetic field effects reduce the excited probability of sodium atoms and weaken their spontaneous radiation rates. As a result, these effects reduce the number of return photons from SLB.

(iii) The spectrum structure of laser influences the spontaneous radiation rate of sodium atoms. The calculation results indicate that the spontaneous radiation rate excited by the three-mode laser with the interval between modes 150 MHz is more than that by the single mode laser.

(iv) The flat-topped phenomenon due to saturation effect caused by the three-mode laser is weaker than that by the single mode laser because of the different spectrum structures.

(v) Reducing the angle between laser beam direction and geomagnetic field direction is beneficial to achieving the number of more return photons for the long pulse (120 μs) laser.

In summary, the spectrum structure, down-pumping, recoil, and geomagnetic field effects should be considered when calculating the number of return photons from SLB. Particularly, the three-mode laser can excite more return photons, about 30%–40% increase. In fact, the pulse formats of the laser are diversiform. What has been discussed here is only one example. We should make a specific analysis for the case of a particular laser of SLB.

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