Research and application of regular phenomenon between periodic signals*

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(Received 6 May 2014; revised manuscript received 23 June 2014; published online 10 October 2014)

The phase change between periodic signals is regular. Research on the regular phenomenon between periodic signals is helpful to improve the precision of some measurements and develop some new measurement methods. So it is necessary to analyze the characteristics of the greatest common factor frequency and the least common multiple period universally existing in periodic signals. The regulation of the quantitative phase shift resolution between periodic signals is presented. The cause of difference in phase characteristics between periodic signals is explained well. In this paper we propose different application prospects based on the regular phenomenon between periodic signals, with focusing on the methods for high precision frequency measurement and transient stability measurement. The experimental results are satisfactory.

Keywords: periodic signal, phase coincidence, frequency, transient stability

PACS: 06.30.Ft, 06.20.Dk, 07.05.Fb, 07.50.Qx **DOI:** 10.1088/1674-1056/23/12/120601

1. Introduction

In recent years, the precision of frequency standard has been greatly improved, so the measurement technology needs to be developed accordingly.[1,2] Much high-precision equipment is achieved, however a new theory is more significant in the frequency measurement. New concepts and methods of time and frequency measurement technologies will provide more effective theoretical bases for high precision frequency measurement and transient frequency stability measurement.

2. Regular phenomenon between periodic signals

Phase difference between periodic signals changes periodically, which is closely related to a series of phase coincidence points, thus we can obtain the concepts of the greatest common factor frequency, the least common multiple period, the quantitative phase shift resolution, etc.

The greatest common factor frequency is defined as follows:

$$f_1 = A \cdot f_{\mathrm{maxc}}, \quad f_2 = B \cdot f_{\mathrm{maxc}}, \tag{1}$$

where f_1 and f_2 are the frequencies of two periodic signals, A and B are two relatively prime positive integers, f_{maxc} is the greatest common factor of f_1 and f_2,[3] T_{minc} is the least common multiple period which can be calculated as follows:

$$T_{\mathrm{minc}} = 1/f_{\mathrm{maxc}} = A/f_1 = AT_1, \tag{2}$$

$$T_{\mathrm{minc}} = 1/f_{\mathrm{maxc}} = B/f_2 = BT_2. \tag{3}$$

The relationship between periodic signals is shown in Fig. 1, where $f_1 : f_2$ equals $A : B$ which is 21:13 in Fig. 1.

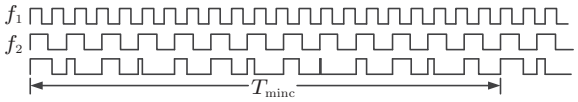


Fig. 1. Waveforms of the phase relationship between periodic signals.

Figure 1 shows that the phase differences between periodic signals have only B kinds of cases in a least common multiple period, and the phase differences can range only from 0 to T_1 with the rising edge of the lower frequency signal f_2 as reference. Narrow pulses which represent the phase differences are formed by the time difference between each rising edge of f_2 and the corresponding next rising edge of f_1. So the phase differences between signals can be calculated as follows:

$$T'_n = \left\{\frac{A}{B} \times n\right\} \times T_1 - nT_2, \tag{4}$$

where T_1 and T_2 are the periods of two signals, the parenthesis $\{\ \}$ means rounding up to an integer. Because it is the same in the cases of $n = 0$ and $n = B$, here the value of n is taken to be from 0 to $(B-1)$. There are B kinds of characteristic phase points of f_2, so there are B kinds of phase differences between periodic signals which can be obtained from Eq. (4). The sequence of B kinds of phase differences is represented by $T'_0, T'_1, \ldots, T'_{B-1}$, which is the same as the pulse width of the third line in Fig. 1. The analysis of these B kinds of continuous phase differences shows that none of the phase differences are the same.[4] So there are only B kinds of conditions for T'_n, that is,

$$0(T_1), \quad \frac{B-1}{B}T_1, \quad \frac{B-2}{B}T_1, \quad \ldots, \quad \frac{1}{B}T_1.$$

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10978017 and 61201288).

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For the actual condition, different A and B will cause the different permutations of phase differences. The permutations of these B kinds of phase differences may be ordered or unordered. But no matter what permutation the phase differences between periodic signals have, if we disorganize and arrange the phase differences in order, the difference between adjacent phase differences after rearrangement can be calculated as follows:

$$\Delta T = \frac{1}{B}T_1 = \frac{f_{\text{maxc}}}{f_1 f_2} = \frac{1}{ABf_{\text{maxc}}}, \tag{5}$$

where ΔT is called the quantitative step value of phase difference.[5,6] It is a fixed number for two determined periodic signals. So the frequencies of two signals should be as high as possible and the greatest common factor frequency of two signals should be relatively low so as to realize high resolution of phase change between periodic signals.

The analysis and calculation about the actual permutation of T'_n above are not obvious. The further analysis is shown as follows:

$$\begin{aligned} T'_n &= \left\{\frac{A}{B}\times n\right\}\times T_1 - nT_2 \\ &= \left(\left\{\frac{A}{B}\times n\right\} - \frac{A}{B}\times n\right)\times T_1 \\ &= \left(1 - \frac{(A\times n)_{\bmod B}}{B}\right)\times T_1 \\ &\Leftrightarrow (B - ((A-B)\times n)_{\bmod B})\times \Delta T \\ &\Leftrightarrow (B - ((A-B)_{\bmod B}\times n)_{\bmod B})\times \Delta T, \end{aligned} \tag{6}$$

where n ranges from zero to $(B-1)$. There is a great relationship between the permutation of T'_n and the value of $(A-B)_{\bmod B}$ which is assumed to be m. Then

$$T'_n = (B - (m\times n)_{\bmod B})\times \Delta T. \tag{7}$$

When n is equal to zero, the first phase difference T'_0 is equal to T_1. T'_1 is equal to the difference between T'_0 and $m\times\Delta T$, etc. Then the permutation of B kinds of phase differences in a least common multiple period is obtained. The following conclusions can be obtained by analysis. When $m<2$, the permutation of phase differences is monotonic. When $2\le m\le(B-1)$, the permutation of phase differences is non-monotonic. When $m>B/2$, the difference between adjacent phase differences is $(B-m)\times\Delta T$ instead of $m\times\Delta T$.

3. High precision frequency measurement

A new method of measuring frequency with high precision is designed by studying the regular phenomenon between periodic signals. The measurement precision is higher when the frequencies of measured signal and reference signal are nearly the same or have a multiple relationship with a small difference in actual measurement. By the above analysis it is shown that the permutation of phase differences is monotonic at this time. Then the rising edges of the two signals arrange from non-coincidently to coincidently and then arrange from coincidently to non-coincidently as shown in Fig. 2. Considering random factors, the actual trigger signal is near the first phase coincidence pulse which can trigger the circuit in the cluster of phase coincidence pulses. Then the phase differences between the start and the stop trigger signals are almost the same. Because of the high stability of circuit, high precision frequency measurement can be achieved. When the relationship between frequencies of two signals is complex, the permutation of phase differences is non-monotonic. Then the trigger signals are chaotic and some random factors are inevitable, so the phase differences between the start and the stop trigger signals are very different, which leads to the large error.

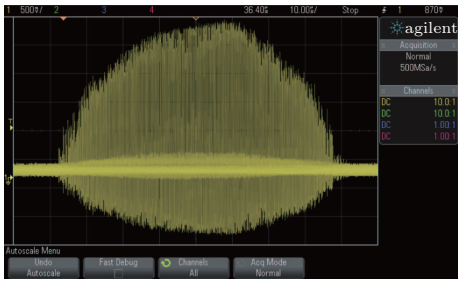


Fig. 2. (color online) Phase coincidence detection waveform of two signals with a small difference.

A signal of which the frequency is nearly the same as that of the measured signal (with a small difference in frequency) is produced as reference signal, so that high precision frequency measurement can be achieved. Some high stability frequency standards are available, such as 10 MHz, 5 MHz, 2.5 MHz, and 1 MHz. So the reference signal can be obtained by frequency synthesis technology and high stability frequency standards. Then the stability of the reference signal can be very high such as 1×10^{-12}/s or even higher, which is almost one order of magnitude higher than the stability of the measured signal in general conditions. The block diagram of high precision frequency measurement based on the phase coincidence detection technology is shown in Fig. 3.

The direct digital synthesizer (DDS) is used to synthesize a reference signal f_0 after measuring the measured signal f_x roughly. The measured signal and the reference signal pass through the AND gate after shaping. The phase coincidence pulses are output when the rising edges of the two signals have been aligned. The actual count gate is modified by the phase coincidence pulses according to the reference gate. So there is no ±1 count error caused by counting in the actual count gate and the measurement precision is greatly improved.[7,8] The frequency can be calculated from the following expression:

$$f_x = \frac{N_x}{N_0}\times f_0, \tag{8}$$

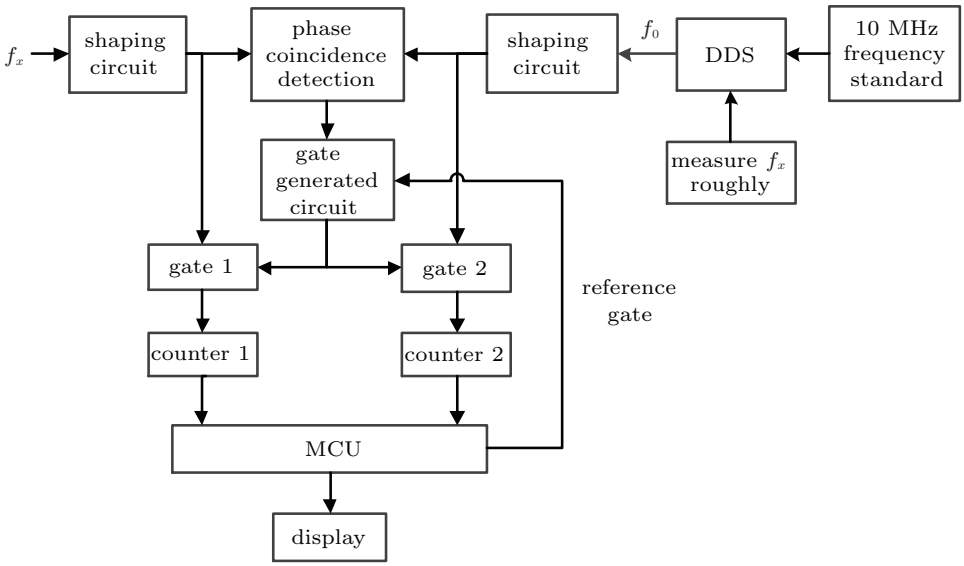


Fig. 3. Block diagram of high precision frequency measurement.

where f_0 is the frequency of the reference signal, f_x is the frequency of the measured signal, N_0 is the count number of the reference signal in the measurement gate, and N_x is the count number of the measured signal in the measurement gate.

Some experiments have been completed for showing the high precision by the method. A super-high-stability cesium atomic clock made by OSA Company has been used as the frequency standard in the experiments. Its stability is 1×10^{-12}/s. The frequency synthesizer HP8662A was used to output the measured frequency signal f_x. Its stability is 5×10^{-12}/s.

In Experiment 1, the standard frequency is 10 MHz, and the measured frequency is 16.384 MHz. The test data are shown in Table 1. The Allan variance calculated by the test data is 6.46×10^{-12}/s. When the frequency is measured without using DDS, that is to say, the relationship between the reference signal frequency and the measured signal frequency is complex, the Allan variance can reach only 2.53×10^{-10}/s.

Table 1. Data of frequency measurement.

The test data/Hz			
16384000.05520	16384000.05534	16384000.05557	16384000.05552
16384000.05552	16384000.05547	16384000.05523	16384000.05519
16384000.05519	16384000.05532	16384000.05532	16384000.05547
16384000.05520	16384000.05557	16384000.05562	16384000.05552
16384000.05562	16384000.05520	16384000.05557	16384000.05547

In Experiment 2, the standard frequency is 10 MHz, and the measured frequency is 8.192 MHz. The test data are shown in Table 2. The Allan variance calculated by the test data is 7.16×10^{-12}/s. When the frequency is measured without using DDS, Allan variance can reach only 3.75×10^{-10}/s.

From the test data we can see that the precision of the measured frequency can arrive at a picosecond level which is higher than that obtained by other general methods.

Table 2. Data of frequency measurement.

The test data/Hz			
8191999.89532	8191999.89538	8191999.89545	8191999.89558
8191999.89528	8191999.89534	8191999.89552	8191999.89555
8191999.89532	8191999.89545	8191999.89538	8191999.89566
8191999.89540	8191999.89538	8191999.89532	8191999.89558
8191999.89538	8191999.89549	8191999.89553	8191999.89562
8191999.89537	8191999.89555	8191999.89548	8191999.89556

4. Transient stability measurement

Transient stability measurement can be achieved by adjusting the phase difference and analyzing the coincidence information. The phase difference between periodic signals are measured and adjusted so that the phase difference can be stabilized at a small value. The resolution and the resolution stability of the phase coincidence detection circuit are fixed. When the phase difference between periodic signals is adjusted to the edge of coincidence detection resolution, continuous coincidence information can be obtained if there is no noise. When there is noise, the phase differences vary between inside and outside of the edge. Thus the information about the phase coincidence is not always existent. So the phase instability of signals can be obtained accurately from the continuity and discontinuity of coincidence information. The shortest response time can reach the nominal period of the signal (the nominal frequencies are the same), or the period of the lower frequency signal (the nominal frequencies are different).

Figure 4 shows the schematic diagram of the transient stability measurement between the same nominal frequency signals. The result of the phase coincidence detection is a se-

ries of pulse clusters. Due to the influence of phase noise, the pulse cluster is discontinuity. As a result, the transient stability as well as time and frequency domain phase noise index can be calculated based on the position of losing pulses and the discontinuity of coincidence information. The two signal frequencies can be equal or multiple. Then several frequency signals can be measured with a nominal frequency signal.

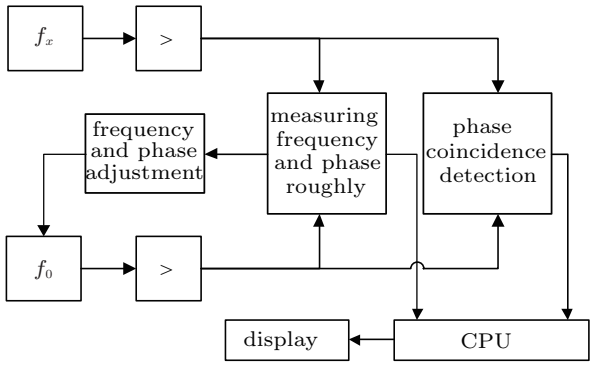


Fig. 4. Block diagram of transient stability measurement.

The transient stability can be calculated from the following expression, which is deduced from the common Allan variance calculation formula:[9]

$$\sigma_y(\tau) = \frac{1}{\tau}\sqrt{\sum_{i=1}^{m}\frac{(\Delta T_{i+1} - \Delta T_i)^2}{2m}}, \tag{9}$$

where ΔT_i is the phase difference variation, m is the number of sampling, and τ is the duration of the corresponding variation. If we consider the response time corresponding to the period of the lower frequency signal, the coincidence detection period τ is precisely the period of the lower frequency signal. Because a small frequency difference exists between two signals, the permutation of phase differences is monotonic. According to the positions of losing pulses and the discontinuity of coincidence information, the corresponding variation of phase difference ΔT_i can be obtained.

A super-high stability 10 MHz oven controlled crystal oscillator 8607 made by the OSA Company is used to output the reference frequency signal. The frequency synthesizer HP8662A is used to output the measured frequency signal. Seven groups of data are measured, and the Allan variance calculation results of 100-ns frequency stability are shown in Table 3.

Table 3. 100-ns frequency stability calculated by Allan variance.

Number	Frequency stability/100 ns
1	3.38×10^{-4}
2	3.45×10^{-4}
3	3.29×10^{-4}
4	3.28×10^{-4}
5	2.69×10^{-4}
6	2.73×10^{-4}
7	2.57×10^{-4}

Transient stability measurement plays an important role in the real-time short-term stability and the characterization of distal phase noise. The period of phase coincidence detection is the period of the measured signal, so both short response time and high resolution can be obtained.

5. Conclusion

In this paper, we show the regular change of phase differences between periodic signals and the permutation of phase differences between periodic signals in detail. The methods of precisely measuring frequency and transient stability are proposed by the regular phenomenon between periodic signals. The methods have the advantages of simple structure, high precision, short response time, etc. The regular phenomenon between periodic signals has universal significance, so it can be used in extensive measurement methods.

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