Impulsive stabilization of a class of nonlinear system with bounded gain error*

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(Received 2 May 2014; revised manuscript received 23 June 2014; published online 10 October 2014)

Considering mechanical limitation or device restriction in practical application, this paper investigates impulsive stabilization of nonlinear systems with impulsive gain error. Compared with the existing impulsive analytical approaches, the proposed impulsive control method is more practically applicable, which includes control gain error with an acceptable boundary. A sufficient criterion for global exponential stability of an impulsive control system is derived, which relaxes the condition for precise impulsive gain efficiently. The effectiveness of the proposed method is confirmed by theoretical analysis and numerical simulation based on Chua's circuit.

Keywords: bounded gain error, impulsive control, exponential stability, Chua's circuit

PACS: 05.45.Gg, 05.45.Pq **DOI:** 10.1088/1674-1056/23/12/120504

1. Introduction

In the last decade, impulsive control has attracted considerable attention because impulsive phenomenon can be found in the fields of physics, artificial intelligence, robotics, telecommunications, electronics, information science, and automatic control systems, etc. Many sudden and sharp changes, in the form of impulses which cannot be well described by a pure continuous-time or discrete-time model, occur instantaneously in these systems. Mathematically, these impulsive systems are generally described by the corresponding impulsive differential equations which is also the prototype of impulsive control systems.[1] The main idea of the impulsive control approach is to change the states of a system whenever certain conditions are satisfied. Compared with the other control schemes, impulsive control is an efficient method to deal with the dynamical systems which cannot be controlled by continuous control methods. In particular, it has been proved that the impulsive control method is relatively easily realized because it only uses small control impulses and can endure continuous disturbance.

Impulsive control theory can be applied in many kinds of system models, such as chaotic systems,[2–9] neural works,[10–16] and master–slave system synchronization models.[17–24] Generally, impulsive control systems can be classified into three types based on the characteristics of plants and control laws. In the first type, the plant is continuous whereas the control input is implemented by the "sudden jumps" of some state variables. In the second type, there are two kinds of control inputs: one is the continuous control input and the other is the impulsive control input. In the third type, the plant itself is impulsive and the control is continuous. In order to derive the control law for any type of impulsive control system, one general approach is to study the stability of the corresponding impulsive differential equations.

Based on impulsive differential equations, many approaches have been proposed to make the impulsive differential system stable. Researchers introduce some factors that can affect system stabilization into impulsive differential equations, such as system nonlinearity,[25,26] time-delay,[22,27] and parameters mismatch.[20,28] In particular, some nonlinear systems under impulsive effects also attract increasing attention[29–31] and derive some sufficient criteria for the stability of the impulsive differential system (such as impulsive neural networks).[32–34] In general, for the convenience of applications, impulsive control gain is often chosen as a constant value at certain impulsive instants. However, there tends to exist impulsive control gain error caused by mechanical limitation and device restriction inevitably when impulsive control is applied in a practical system. It is difficult to alter state variables to certain values precisely at the impulsive moment. Thus, it is more important to consider control gain error with an acceptable boundary in an impulsive control system.

Motivated by the aforementioned discussion, the main objective of this paper is to further investigate stability of the impulsive control system with bounded control gain error. Firstly, an impulsive differential equation of a class of nonlinear system is established. Secondly, the theorem is given to guarantee the global exponential stability of the impulsive

*Project supported by the Major State Basic Research Development Program of China (Grant No. 2012CB215202), the National Natural Science Foundation of China (Grant Nos. 61104080 and 61134001), and the Fundamental Research Funds for the Central Universities (Grant No. CDJZR13 175501).

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control system by using the theory of impulsive differential inequalities.[35] Moreover, the parametric uncertainty is also considered in this paper, which is more coincident with the real systems. As the extension of the main theorems, a special case for choosing equivalent impulsive control gains and impulsive distances is also studied. Finally, the proposed impulsive control scheme is illustrated by nonlinear Chua's oscillator. Stable regions with different impulsive control precisions are shown in the simulation section. By studying comparatively, we qualitatively summarize how the impulsive control gain error influences stable regions. The simulation results demonstrate the effectiveness of the method.

2. Problem description and preliminaries

Consider the general nonlinear system described by

$$\dot{x}(t) = Ax + f(x), \tag{1}$$

where $x \in R^n$ is the state vector, $f(x) \in R^n$ is a continuous function.

Suppose that a discrete time set $\{t_k\}$ satisfies $0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$, $\lim_{k\to\infty} t_k = \infty$, $0 \le t_0 < t_1$, $k \in \mathbb{N}$. Let

$$U(k,x) = \Delta x|_{t=t_k} \equiv x(t_k^+) - x(t_k^-) = x(t_k^+) - x(t_k)$$

be the "jump" of the state vector at the time instant t_k, where $x(t_k^+) = \lim_{t\to t_k^+} x(t)$ and $x(t_k^-) = \lim_{t\to t_k^-} x(t)$. Let $x(t_k^-) = x(t_k)$, i.e., $x(t)$ is left-continuous at $t = t_k$. Then, an impulsively controlled system with the initial condition at time t_0 can be described by

$$\begin{cases} \dot{x} = Ax + f(x), & t \ne t_k, \\ \Delta x = U(k,x), & t = t_k, \\ x(t_0) = x_0, & t_0 \ge 0, \end{cases} \tag{2}$$

which is called an impulsive differential equation.[1] From Eq. (2), we consider a class of impulsive nonlinear differential system with impulsive control gain error as follows:

$$\begin{cases} \dot{x} = Ax + f(x), & t \ne t_k, \\ x(t_k^+) = \Delta x + x(t_k^-) = (B_k + \Delta B_k)x, & t = t_k, \\ x(t_0) = x_0, & t_0 \ge 0, \end{cases} \tag{3}$$

where $x \in R^n$ is the state vector, $f : R^n \to R^n$ is a continuous nonlinear function, $A \in R^{n\times n}$ is a known constant matrix, $B_k \in R^{n\times n}$ is the impulsive control gain, and $\Delta B_k \in R^{n\times n}$ implies impulsive gain error caused by technology limitation or device restriction. With regard to the function $f(x)$ and the gain error ΔB_k, the following assumptions are given.

Assumption 1 Continuous nonlinear function $f : R^n \to R^n$ satisfies the Lipschitz condition with a Lipschitz constant $l > 0$, i.e.,

$$\|f(x)\| \le l\|x\|, \tag{4}$$

where $\|\cdot\|$ refers to Euclidean vector norm or the induced matrix 2-norm.

Remark 1 A lot of nonlinear systems can be expressed by Eq. (3) and meet Assumption 1, especially some well-known chaotic systems, such as Lorenz system, Chen system, Lü system, unified system, Chua's circuit, and so on.

Owing to the mechanical limitation and device restriction, the impulsive control gain may contain an uncertainty, which is often time-varying and unknown. Now, we assume that the gain error satisfies the following assumption:

Assumption 2 The parametric uncertainty $\Delta B_k \in R^{n\times n}$ has the form:

$$\Delta B_k = mF(t)B_k, \tag{5}$$

where $m > 0$ is a known constant, and the uncertain matrix $F(t) \in R^{n\times n}$ satisfies

$$F^{\mathrm{T}}(t)F(t) \le I. \tag{6}$$

In order to estimate the dynamic behavior of the impulsive differential system (3), in what follows, some necessary definitions and lemma are given.

Definition 1[1] The function $V : R_+ \times R^n \to R_+$ is said to belong to class v_0, if V is continuous in $(t_{k-1}, t_k] \times R^n$ and for any $x \in R^n$, $k \in \mathbb{N}$, there exists

$$\lim_{(t,y)\to(t_k^+,x)} V(t,y) = V(t_k^+,x).$$

Definition 2[1] For $(t,x) \in (t_{k-1}, t_k] \times R^n$, the right and upper Dini's derivative of $V \in v_0$ with respect to time variable is defined as

$$D^+V(t,x) \equiv \lim_{h\to 0^+} \sup \frac{1}{h}\{V[t+h, x+hf(t,x)] - V(t,x)\}. \tag{7}$$

Lemma 1[36] Given any real matrices $X_1 \in R^{p\times q}$, $X_2 \in R^{p\times q}$, $Q = Q^{\mathrm{T}} > 0$, and positive constant $\epsilon > 0$, the following inequality holds,

$$X_1^{\mathrm{T}}X_2 + X_2^{\mathrm{T}}X_1 \le \epsilon X_1^{\mathrm{T}}QX_1 + \epsilon^{-1}X_2^{\mathrm{T}}Q^{-1}X_2.$$

3. Main result

In this section, we will study the stabilization of the aforementioned system under the effects of bounded impulsive control gain error, and further derive the sufficient conditions such that system (3) is exponentially stable at the origin.

Theorem 1 Suppose that $\lambda_k > 0$ is the largest eigenvalue of $B_k^{\mathrm{T}}B_k$, the impulse intervals satisfy $t_k - t_{k-1} \le \theta$ $(k \in \mathbb{N})$, where $\theta > 0$ denotes the upper bound of the whole impulse intervals. If Assumptions 1 and 2 hold, and there exist positive constant scalars μ, β, ϵ such that

$$A + A^{\mathrm{T}} + (\mu + \mu^{-1}l^2 - \beta)I \le 0, \tag{8}$$

$$\frac{1}{\theta}\ln\gamma+\beta<0, \tag{9}$$

then the origin of system (3) is globally exponentially stable, where $\gamma=\max\{\gamma_k\}=\max\{\lambda_k(1+\epsilon)+\lambda_k m^2(1+\epsilon^{-1})\}$.

Proof We construct a Lyapunov-like function in the form of

$$V(t)=x^{\mathrm{T}}x. \tag{10}$$

For $t\in(t_{k-1},t_k]$, $k\in\mathbb{N}$, and from Assumption 1, Lemma 1, and inequality (8), the Dini's derivative of $V(t)$ along the trajectory of system (3) is obtained as follows:

$$\begin{aligned}D^+V(t) &= 2x^{\mathrm{T}}(Ax+f(x))\\ &\le 2x^{\mathrm{T}}Ax+\mu x^{\mathrm{T}}x+\mu^{-1}f^T(x)f(x)\\ &\le x^{\mathrm{T}}(A+A^{\mathrm{T}})x+\mu x^{\mathrm{T}}x+\mu^{-1}l^2x^{\mathrm{T}}x\\ &\le x^{\mathrm{T}}(A+A^{\mathrm{T}}+\mu I+\mu^{-1}l^2I-\beta I)x+\beta x^{\mathrm{T}}x\\ &\le \beta x^{\mathrm{T}}x.\end{aligned} \tag{11}$$

For $t=t_k$, from Assumption 2 and Lemma 1, we have

$$\begin{aligned}&V(t_k^+)\\ &= x^{\mathrm{T}}(B_k+\Delta B_k)^{\mathrm{T}}(B_k+\Delta B_k)x\\ &= x^{\mathrm{T}}(B_k^{\mathrm{T}}B_k+B_k^{\mathrm{T}}\Delta B_k+\Delta B_k^{\mathrm{T}}B_k+\Delta B_k^{\mathrm{T}}\Delta B_k)x\\ &\le x^{\mathrm{T}}(B_k^{\mathrm{T}}B_k+\epsilon B_k^{\mathrm{T}}B_k+\epsilon^{-1}\Delta B_k^{\mathrm{T}}\Delta B_k+\Delta B_k^{\mathrm{T}}\Delta B_k)x\\ &= x^{\mathrm{T}}((1+\epsilon)B_k^{\mathrm{T}}B_k+(1+\epsilon^{-1})\Delta B_k^{\mathrm{T}}\Delta B_k)x\\ &\le \lambda_k(1+\epsilon)x^{\mathrm{T}}x+x^{\mathrm{T}}(1+\epsilon^{-1})(mF(t)B_k)^{\mathrm{T}}(mF(t)B_k)x\\ &\le (\lambda_k(1+\epsilon)+\lambda_k m^2(1+\epsilon^{-1}))x^{\mathrm{T}}x\\ &= \gamma_k V(t_k),\end{aligned} \tag{12}$$

where $\gamma_k=\lambda_k(1+\epsilon)+\lambda_k m^2(1+\epsilon^{-1})$.

Let $t\in(t_0,t_1]$. Then, multiplying two sides of the inequality (11) by $\mathrm{e}^{-\beta(t-t_0)}$ and it yields

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t}\mathrm{e}^{-\beta(t-t_0)}-\beta V(t)\mathrm{e}^{-\beta(t-t_0)}\le 0,$$

namely,

$$\frac{\mathrm{d}}{\mathrm{d}t}(V(t)\mathrm{e}^{-\beta(t-t_0)})\le 0,$$

which yields after integrating from t_0 to t,

$$V(t)\mathrm{e}^{-\beta(t-t_0)}|_{t_0}^{t}=V(t)\mathrm{e}^{-\beta(t-t_0)}-V(t_0)\le 0,$$

namely,

$$V(t)\le V(t_0)\mathrm{e}^{\beta(t-t_0)}. \tag{13}$$

Let $t\in(t_1,t_2]$, similarly, it yields

$$V(t)\le V(t_1^+)\mathrm{e}^{\beta(t-t_1)}.$$

From expressions (12) and (13), we can obtain

$$\begin{aligned}V(t) &\le \gamma_1 V(t_1)\mathrm{e}^{\beta(t-t_1)}\\ &\le \gamma_1 V(t_0)\mathrm{e}^{\beta(t-t_0)}.\end{aligned}$$

In general, let $t\in(t_k,t_{k+1}]$, $k\in\mathbb{N}$, we also obtain

$$\begin{aligned}V(t) &\le V(t_k^+)\mathrm{e}^{\beta(t-t_k)}\\ &\le \gamma_k V(t_k)\mathrm{e}^{\beta(t-t_k)}\\ &\le \gamma_k V(t_{k-1}^+)\mathrm{e}^{\beta(t-t_{k-1})}\\ &\le \prod_{t_0<t_k<t}\gamma_k V(t_0)\mathrm{e}^{\beta(t-t_0)}.\end{aligned}$$

From inequality (9), we can further have

$$\begin{aligned}V(t) &\le \prod_{t_0<t_k<t}\gamma_k V(t_0)\mathrm{e}^{\beta(t-t_0)}\\ &\le \gamma^{(t-t_0)/\theta-1}V(t_0)\mathrm{e}^{\beta(t-t_0)}\\ &= \gamma^{-1}V(t_0)\exp\left[\left(\frac{1}{\theta}\ln\gamma+\beta\right)(t-t_0)\right].\end{aligned} \tag{14}$$

From expression (14), it is easy to conclude that system (3) is globally exponentially stable. This completes the proof.

In practice, for the sake of convenience, impulsive gain matrix B_k is always selected as a constant matrix and impulsive distance t_k-t_{k-1} is set to be a positive constant, i.e., $B_k=B$, $t_k-t_{k-1}=\theta$, then we have the following special case of Theorem 1.

Corollary 1 Suppose that $\lambda>0$ is the largest eigenvalue of $B^{\mathrm{T}}B$, the impulse intervals satisfy $t_k-t_{k-1}=\theta>0$ ($k\in\mathbb{N}$). If Assumptions 1 and 2 hold, and there exist positive constant scalars μ,β,ϵ such that

$$A+A^{\mathrm{T}}+(\mu+\mu^{-1}l^2-\beta)I\le 0, \tag{15}$$

$$\frac{1}{\theta}\ln\gamma+\beta<0, \tag{16}$$

then the origin of system (3) is globally exponentially stable, where $\gamma=\lambda(1+\epsilon)+\lambda m^2(1+\epsilon^{-1})$.

In what follows, we will consider the impulsive control with parameter uncertainty. The corresponding system dynamics can be described as

$$\begin{cases}\dot{x}=(A+\Delta A(t))x+f(x), & t\ne t_k,\\ x(t_k^+)=\Delta x+x(t_k^-)=(B_k+\Delta B_k)x, & t=t_k,\\ x(t_0)=x_0, & t_0\ge 0,\end{cases} \tag{17}$$

where $\Delta A(t)$ is the parametric uncertainty and has the following form

$$\Delta A(t)=EG(t)H,$$

where E and H are known real constant matrices with appropriate dimensions and the uncertain matrix $G(t)$ satisfies

$$\|G(t)\|\le 1.$$

From Theorem 1, it is easy to get the following theorem by a similar analysis.

Theorem 2 Suppose that $\lambda_k > 0$ is the largest eigenvalue of $B_k^{\mathrm{T}} B_k$, the impulse intervals satisfy $t_k - t_{k-1} \leq \theta$ $(k \in \mathbb{N})$, where $\theta > 0$ denotes upper bound of the whole impulse intervals. If Assumptions 1 and 2 hold, and there exist positive constant scalars μ, β, ϵ such that

$$A + A^{\mathrm{T}} + (2\|EH\| + \mu + \mu^{-1} l^2 - \beta) I \leq 0, \tag{18}$$

$$\frac{1}{\theta} \ln \gamma + \beta < 0, \tag{19}$$

then the origin of system (17) is globally exponentially stable, where $\gamma = \max\{\gamma_k\} = \max\{\lambda_k(1+\epsilon) + \lambda_k m^2 (1+\epsilon^{-1})\}$.

Proof We construct a Lyapunov-like function in the form of

$$V(t) = x^{\mathrm{T}} x. \tag{20}$$

For $t \in (t_{k-1}, t_k]$, $k \in \mathbb{N}$, and from Assumption 1, Lemma 1, and inequality (18), the Dini's derivative of $V(t)$ along the trajectory of system (17) is obtained as follows:

$$\begin{aligned} & D^+ V(t) \\ &= 2x^{\mathrm{T}}((A + \Delta A(t))x + f(x)) \\ &\leq 2x^{\mathrm{T}} A x + 2x^{\mathrm{T}} \Delta A(t) x + \mu x^{\mathrm{T}} x + \mu^{-1} f^T(x) f(x) \\ &\leq x^{\mathrm{T}}(A + A^{\mathrm{T}}) x + 2\|EH\| x^{\mathrm{T}} x + \mu x^{\mathrm{T}} x + \mu^{-1} l^2 x^{\mathrm{T}} x \\ &= x^{\mathrm{T}}(A + A^{\mathrm{T}} + 2\|EH\| + \mu I + \mu^{-1} l^2 I - \beta I) x + \beta x^{\mathrm{T}} x \\ &\leq \beta x^{\mathrm{T}} x. \end{aligned} \tag{21}$$

The rest of the proof is similar to that of Theorem 1, which is omitted here for simplicity. This completes the proof.

When impulsive distance $t_k - t_{k-1}$ and impulsive gain B_k are selected as constants, we can get the following corollary straightforward.

Corollary 2 Suppose that $\lambda > 0$ is the largest eigenvalue of $B^{\mathrm{T}} B$, the impulse intervals satisfy $t_k - t_{k-1} = \theta > 0$ $(k \in \mathbb{N})$. If Assumptions 1 and 2 hold, and there exist positive constant scalars μ, β, ϵ such that

$$A + A^{\mathrm{T}} + (2\|EH\| + \mu + \mu^{-1} l^2 - \beta) I \leq 0, \tag{22}$$

$$\frac{1}{\theta} \ln \gamma + \beta < 0, \tag{23}$$

then the origin of system (17) is globally exponentially stable, where $\gamma = \lambda(1+\epsilon) + \lambda m^2 (1+\epsilon^{-1})$.

4. Simulation

In this section, we investigate the control of Chua's oscillator by applying the theory presented in the previous section. The Chua's oscillator is described by the following state equation:[37]

$$\begin{cases} \dot{x_1} = p_1(x_2 - x_1 - g(x_1)), \\ \dot{x_2} = x_1 - x_2 + x_3, \\ \dot{x_3} = -p_2 x_2, \end{cases} \tag{24}$$

where p_1, p_2 are system parameters and $g(x_1)$ is the piecewise linear characteristic function, which is described by

$$g(x_1) = m_1 x_1 + 0.5(m_0 - m_1)(|x_1 + 1| - |x_1 - 1|), \tag{25}$$

where $m_0 < m_1 < 0$ are two given constants.

To employ the results given above, we rewrite system (24) into the following form,

$$\dot{x} = Ax + f(x), \tag{26}$$

where

$$A = \begin{pmatrix} -p_1(1+m_1) & p_1 & 0 \\ 1 & -1 & 1 \\ 0 & -p_2 & 0 \end{pmatrix}$$

and

$$f(x) = \begin{pmatrix} -0.5 p_1 (m_0 - m_1)(|x_1 + 1| - |x_1 - 1|) \\ 0 \\ 0 \end{pmatrix}.$$

This system exhibits chaotic behavior when $p_1 = 9.21$, $p_2 = 15.995$, $m_0 = -1.25$, and $m_1 = -0.758$. In this case, it is easy to estimate that $l = 4.53132$.

For notational and computational convenience, we choose the impulsive control gain $B_k = B$, and impulsive instant $t_k - t_{k-1} = \theta$. From sufficient condition (16), the stable region is given by

$$\theta < -\frac{\ln(\lambda((1+\epsilon) + m^2(1+\epsilon^{-1})))}{\beta}. \tag{27}$$

We choose $\mu = l = 4.53132$, and calculate $\beta = 25.61462$ via linear matrix inequality (LMI) toolbox.

For the sake of simplicity, the impulsive control gain error is specified as

$$\Delta B_k = \Delta B = 0.05 \sin t B,$$

i.e.,

$$m = 0.05, \quad F(t) = \begin{pmatrix} \sin t & 0 \\ 0 & \sin t \end{pmatrix},$$

let $\epsilon = m = 0.05$, then the stable region (27) is

$$\theta < -\frac{\ln(1.1025\lambda)}{25.61462}. \tag{28}$$

If the impulsive control gain error magnifies,

$$\Delta B_k = \Delta B = 0.2 \sin t B,$$

i.e.,

$$m = 0.2, F(t) = \begin{pmatrix} \sin t & 0 \\ 0 & \sin t \end{pmatrix},$$

let $\epsilon = m = 0.2$, then the stable region (27) is given as

$$\theta < -\frac{\ln(1.44\lambda)}{25.61462}. \tag{29}$$

Stable regions (28) and (29) are shown in Fig. 1 (with solid curve and dotted curve respectively). The entire region below the curve is the predicted stable region. With the strengthening of impulsive control gain error, the stable region becomes smaller, and thus the impulsive controller precision will decrease accordingly.

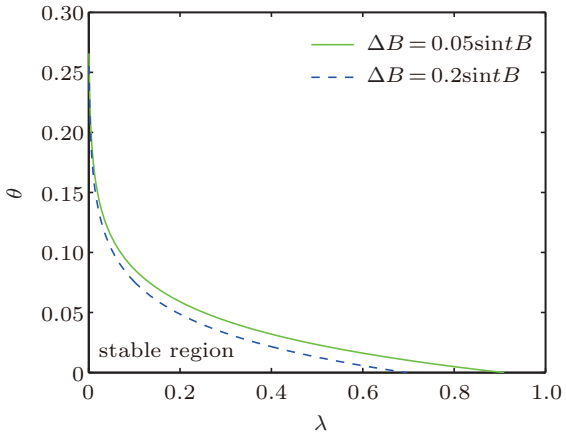


Fig. 1. (color online) Stable region with different impulsive control precisions.

The impulsive control gain is chosen as $B = -0.5I$, hence $\lambda = 0.25$. From the stable region (28), it yields $\theta < 0.0503$, hence the impulsive interval can be given as $\theta = 0.05$, the time response curves of Chua's oscillators with bounded gain error are shown in Fig. 2. Similarly, from the stable region (29), it yields $\theta < 0.0399$, hence the impulsive interval can be given as $\theta = 0.039$, the corresponding time response curves are shown in Fig. 3. Throughout the simulation, the initial condition of system (24) is chosen as $x(t_0) = (0.5, 0.3, -0.5)$. The simulation results display that the control system is globally exponentially stable.

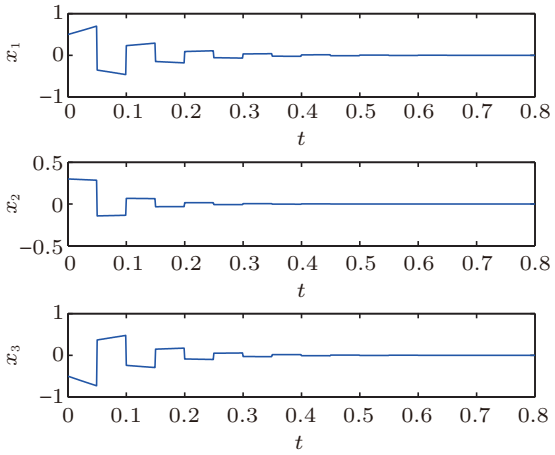


Fig. 2. (color online) Time response curves of Chua's oscillator for $\Delta B_k = 0.05\sin tB_k, \lambda = 0.25, \theta = 0.05$.

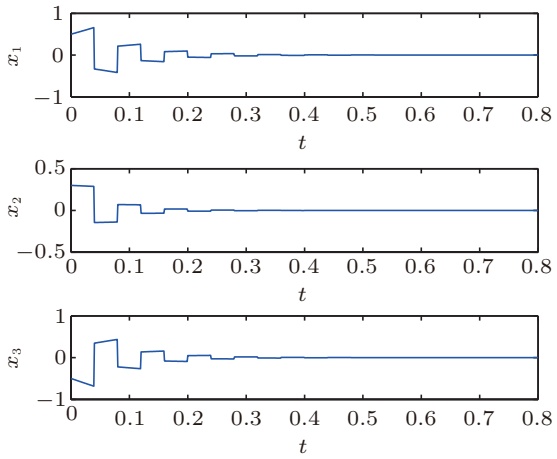


Fig. 3. (color online) Time response curves of Chua's oscillator for $\Delta B_k = 0.2\sin tB_k, \lambda = 0.25, \theta = 0.039$.

5. Conclusions

In this paper, we consider the impulsive gain error in practical applications. Based on the efficient impulsive differential inequality and the Lyapunov stability theory, a sufficient condition is derived without the need for precise impulsive gain. Finally, numerical simulation for typical Chua's oscillator is given to demonstrate the effectiveness of the method. The proposed issue is very practical and important, and continued research is desirable.

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