Geometric quantum discord and Berry phase between two charge qubits coupled by a quantum transmission line*

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Geometric quantum discord (GQD) and Berry phase between two charge qubits coupled by a quantum transmission line are investigated. We show how GQDs evolve and investigate their dependencies on the parameters of the system. We also calculate the energy and the Berry phase and compare them with GQD, finding that there are close connections between them.

Keywords: geometric quantum discord (GQD)，Berry phase, charge qubits

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1. Introduction

Quantum correlations have been a focus in quantum computation since their discovery. Quantum correlations differ from entanglement and may offer some advantages in quantum computation. Quantum discord,[1] a measure of non-classical correlations that indicates the discrepancy between two equivalent classical expressions for mutual information after being quantified, is an important type of quantum correlation. Owing to its significance and ubiquity, much attention has been paid to its characterization and quantification, and the properties of quantum discord have been widely investigated.[2–4] However, because its evaluation involves an optimization procedure, analytical results are known only in a few cases.[5,6] To avoid this difficulty, geometric quantum discord (GQD)[7] is introduced, which measures the quantum correlations through the minimum Hilbert–Schmidt distance between the given state and the zero discord state. This measure is significant in capturing quantum correlations from a geometric perspective. Recently, the deficiency of the proposal in Ref. [7] of GQD as a quantum correlation was pointed out,[8–10] showing that GQD may increase under local operations on the unmeasured subsystem. Despite this shortcoming, GQD is still an effective and convenient measure for quantum correlations.

Berry phase[11] has attracted much interest in quantum theory since Berry showed that the wave function of a system acquires a purely geometric feature in addition to the usual dynamical phase when it is varied slowly and eventually brought back to its initial form. It has nontrivial connections with the character of the system,[12] especially with entanglement.[13,14] It was shown that the Berry phase is able to detect the entanglement in a system.

Recently in solid state-based quantum computation, a one-dimensional transmission line resonator was used to reach the strong coupling limit of cavity quantum electrodynamics, and it has shown advantages over other data buses.[15] For this reason, the integration of the Josephson junction qubit and the quantum transmission line has become a focus in quantum computation. Finding ways to generate and control the quantum correlations of the system is important in realizing quantum computation. Much work has been devoted to producing, characterizing, and controlling quantum correlations in various systems.[16–20] Therefore, it is of great significance to explore the quantum correlations in this composite system.

In this paper, we investigate the behavior of GQD and Berry phase for two charge qubits coupled by a quantum transmission line. We calculate the eigenstates of the system and evaluate the time evolution of GQD, analyzing the effect of system parameters on it. The energy eigenvalue, Berry phase, and GQD for the ground state and the first excited state are given, and the relations among them are investigated.

2. The model and its solution

Our model describes two charge qubits coupled by a quantum transmission line in the presence of an external magnetic field. The Hamiltonian can be written as:[21]

$$H = H_0 + H', \tag{1}$$

with

$$\begin{aligned} H_0 &= 4E_{\rm C}\left(n_{\rm g}-\frac{1}{2}\right)\left(\sigma_z^1+\sigma_z^2\right) \\ &\quad -E_{\rm J}\cos\phi_{\rm c}\left(\sigma_x^1+\sigma_x^2\right)+\hbar\omega a^{\dagger}a, \\ H' &= \hbar\eta\left(a+a^{\dagger}\right)\left(\sigma_x^1-\sigma_x^2\right), \end{aligned}$$

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where σ_z^i, σ_x^i are the usual Pauli operators of the i-th qubit ($i = 1,2$), $a^\dagger$ (a) denote the creation (annihilation) operator of the photon (frequency ω) of the quantum magnetic field, E_C is the charge energy of a single electron, n_g is the gate charge, $E_J \cos\phi_c$ is the Josephson coupling energy of the qubits, where $\phi_c = \pi\Phi_c/\Phi_0$, Φ_c is the classic flux of the qubits, $\eta = \phi_q E_J \sin\phi_c$ represents the interaction strength between the qubits and a quantum magnetic field, and $\phi_q = \pi\Phi_q/\Phi_0$, where Φ_q is the quantum magnetic flux of qubits. In the following discussion, we assume that $\eta = 1$. From Ref. [15] we know that the following inequality exists:

$$\hbar\omega \gg E_J \gg \eta.$$

For simplicity, we set $\hbar\omega = 100$ and $E_J = 10$.

The Hamiltonian can be divided into two parts, where H' denotes the coupling between the qubits and the quantum magnetic field. In our model we have $\phi_q \ll \phi_c$, and hence $H' \ll H_0$. Obviously $N = a^\dagger a$ is a conserved quantity of H_0, and the basis of the subspace K_n is:

$$|00n\rangle,\ |01n\rangle,\ |10n\rangle,\ |11n\rangle,$$

where $|01n\rangle$ denotes the first qubit in the ground state and the second qubit in the excited state while $|n\rangle$ is the state of the quantum magnetic field.

3. Time evolution of GQD

We now calculate the wave function of the system when the quantum magnetic field is initially in the vacuum state. Since $H' \ll H_0$, we assume that the maximum number of photons is 1, which means that the state space is restricted to the direct sum of space K_0, K_1. We choose the basis as follows:

$$|\,00n\rangle\,,\ |\,01n\rangle\,,\ |\,10n\rangle\,,\ |\,11n\rangle\,,$$

with $n = 0,1$. Then the Hamiltonian can be written out in this basis. By solving the equation $H\psi = E\psi$, we acquire the energy eigenvalues and eigenstates, and we can obtain the wave function of the system for a given initial state.

In order to investigate the GQD between the two qubits, we need to obtain the reduced density matrix by tracing the combined system density matrix over the field variable. Here $\rho_{AB} = \mathrm{Tr}_c|\,\Psi\rangle\langle\Psi\,|$, where $|\,\Psi\rangle$ denotes the state of the system and Tr_c is the trace of the quantum magnetic field. Then the GQD of two qubits can be computed as follows:[7]

$$D_A(\rho) = \frac{1}{4}\left(\|\boldsymbol{a}\|^2 + \|T\|^2 - k_{\max}\right), \tag{2}$$

where $a_i = \mathrm{Tr}\,\rho\,(II\otimes\sigma_i)$, $b_i = \mathrm{Tr}\,\rho\,(II\otimes\sigma_i)$, $T_{ij} = \mathrm{Tr}\,\rho\,(\sigma_i\otimes\sigma_j)$, $k_{\max}$ is the largest eigenvalue of matrix $K = \boldsymbol{a}\boldsymbol{a}^{\mathrm{T}} + TT^{\mathrm{T}}$.

The GQD as a function of time and ϕ_c is shown in Fig. 1. We find in the figure that the GQD varies periodically with the interaction between the qubits and the quantum magnetic field. With $\phi_c \to 0$, the period of GQD during time evolution goes to infinity. This feature corresponds to the case when the H' can be ignored. Physically, the coupling between the qubits and the field generates the quantum correlation in the system of two qubits. Without the coupling, the system, which was prepared in $|000\rangle$, will remain separated. Unlike the period, the maximum value of GQD remains nearly unchanged with variation of ϕ_c, which indicates that the coupling cannot modify the maximum value of GQD during time evolution.

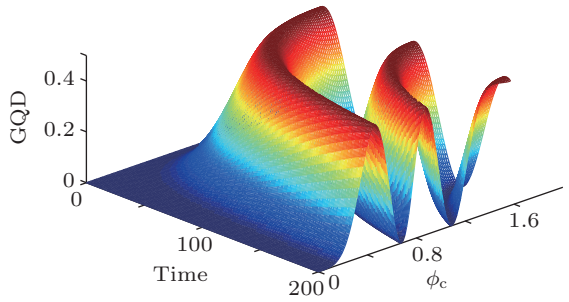


Fig. 1. (color online) GQD corresponds to the state of two qubits versus time and ϕ_c. The initial state is $|000\rangle$, and the gate charge $n_g = 0.5$.

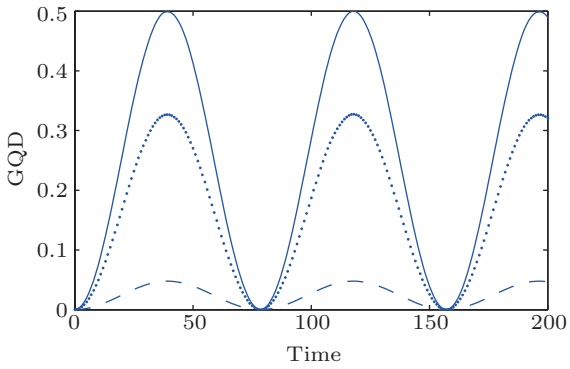


Fig. 2. (color online) GQD versus time for different initial states. The initial state is $\cos\theta|\,000\rangle + \sin\theta|\,100\rangle$. Solid line, dotted line, and dashed line represent $\theta = 0.5\pi, 0.4\pi$, and 0.3π, respectively. The gate charge $n_g = 0.5$, while $\phi_c = 0.3\pi$.

Figures 2 and 3 show how the initial state influences the maximum of GQD. The system was initially prepared in a separated state. The time evolution of GQD for different initial states of the first qubit is shown in Fig. 2. Clearly, without changing period, the maximum of GQD varies while the initial state changes. Considering a more general separated initial state, figure 3 presents the relation between the maximum of GQD and the initial state. Clearly, for different initial states of the first qubit, the GQD reaches different maximum values. This provides a method for us to alter the maximum value of GQD. The GQD can reach a relatively high value when two qubits both approach the eigenstate of σ_z initially. When the initial state of one qubit is close to the eigenstate of σ_x, the maximum of GQD goes to zero. The reason for this is that the operators σ_x^1, σ_x^2 commute with the Hamiltonian when

$n_g = 0.5$. This leads to the fact that the quantum correlations between two qubits remain zero when the system is prepared in a separated state and one of the qubits is initially in the eigenstate of σ_x.

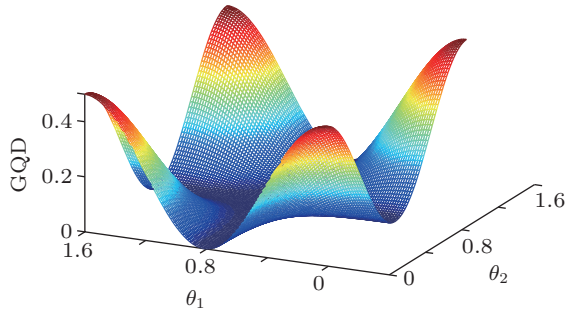


Fig. 3. (color online) Maximum value of GQD versus θ_1 and θ_2. The initial state is $(\cos\theta_1|0\rangle + \sin\theta_1|1\rangle) \otimes (\cos\theta_2|0\rangle + \sin\theta_2|1\rangle) \otimes |n\rangle$. The gate charge $n_g = 0.5$, while $\phi_c = 0.3\pi$.

4. State energy, Berry phase, and GQD

The eigenstate ψ_n^0 of H_0 on K_n has the following shape:

$$\psi_n^0 = (c_1^0|00\rangle + c_2^0|01\rangle + c_3^0|10\rangle + c_4^0|11\rangle) \otimes |n\rangle. \quad (3)$$

Since $H' \ll H_0$ and H' contains only the first order of $a^\dagger$ (a), we can treat H' as the disturbance. On the first approximation of $\hbar\eta$, the energy eigenstate ψ_n of H corresponding to ψ_n^0 can be assumed to have the following form:

$$\psi_n = \sum_{i=n-1}^{n+1} (c_1^i|00\rangle + c_2^i|01\rangle + c_3^i|10\rangle + c_4^i|11\rangle) \otimes |i\rangle. \quad (4)$$

Setting K as the direct sum of K_{n-1}, K_n, K_{n+1}, we confine the Hamiltonian in space K and set it as H_0'. Solving the equation $H_0'\psi_n = E\psi_n$ in the space K we can obtain ψ_n and the corresponding energy.

The dependence of the energy of the ground state and the first excited state (the next-to-lowest eigenvalue of the Hamiltonian) on the detuning $\Delta = \omega_0 - \omega (\hbar\omega_0 = 8E_C(n_g - 1/2))$ as well as ϕ_c are illustrated in Fig. 4, respectively. For the ground state, we find that the energy is a monotonic function of the detuning, while ϕ_c has little influence on the energy. For the first excited state, we can see that with $\phi_c = 0$, a non-differentiable point exists. This point originates from the level crossing when the coupling between qubits and field is zero.

In order to acquire the geometric properties of the system we need a process that is able to produce a phase change in the state of the field. Therefore we resort to the method of introducing the phase shift operator:[22]

$$U(\phi) = \exp(-\phi a^\dagger a), \quad (5)$$

where ϕ slowly changes from 0 to 2π. Then the Berry phase can be calculated as follows:

$$\gamma = \mathrm{i}\int_0^{2\pi} \mathrm{d}\phi \langle U(\phi)^\dagger \frac{\mathrm{d}}{\mathrm{d}\phi} U(\phi)|\psi\rangle, \quad (6)$$

where $|\psi\rangle$ is the eigenstate of H. Substituting Eq. (4) leads to

$$\gamma = 2\pi \sum_{j=n-1}^{n+1} j(|c_1^j|^2 + |c_2^j|^2 + |c_3^j|^2 + |c_4^j|^2). \quad (7)$$

After substituting c_k^j we get the Berry phase for the corresponding eigenstate.

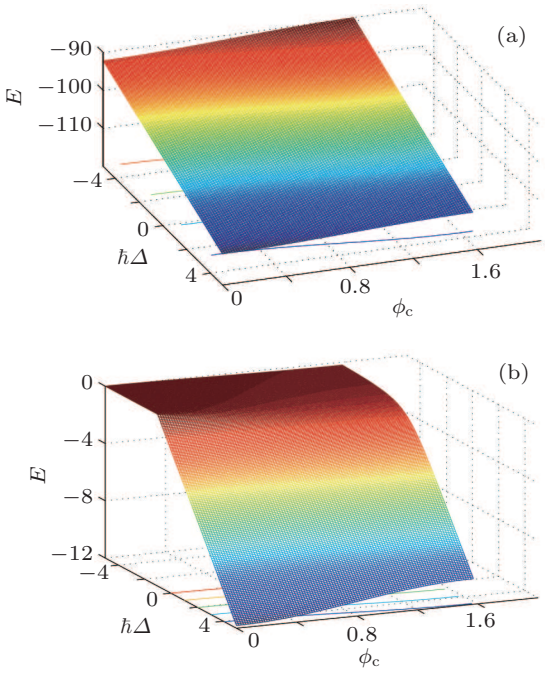


Fig. 4. (color online) The state energy versus detuning $\hbar\Delta$ and ϕ_c, where panels (a) and (b) correspond to the ground state and the first excited state, respectively. The gate charge $n_g = 0.5$.

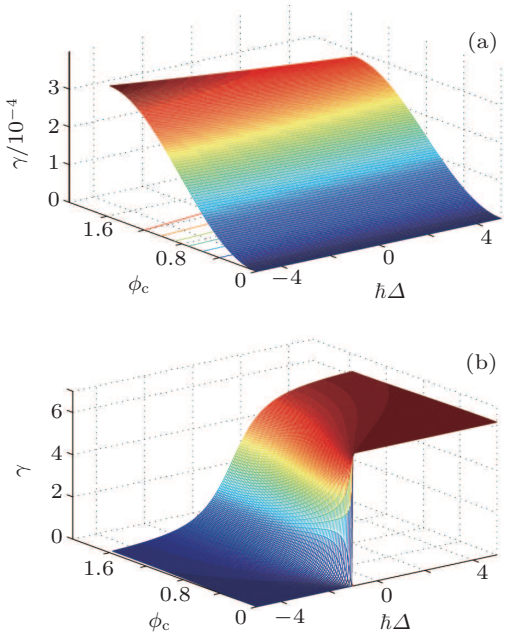


Fig. 5. (color online) The Berry phase for the energy eigenstates versus detuning $\hbar\Delta$ and ϕ_c. Panels (a) and (b) correspond to the ground state and the first excited state respectively.

The Berry phases for the ground state and the first excited state are shown in Fig. 5. In panel (a), we find that the Berry phase for the ground state is so small it can be considered as

zero. In panel (b), which corresponds to the first excited state, we see that the Berry phase is a monotonic function of the detuning, and it goes to 0 and 2π in the limits $\Delta \to \pm\infty$ respectively. It is evident that there is one discontinuity point when $\phi_c = 0$, which corresponds to the non-differentiable point in Fig. 4. This finding is similar to that of Ref. [23] which reveals that there is correspondence between the singularities of the Berry phase and the energy.

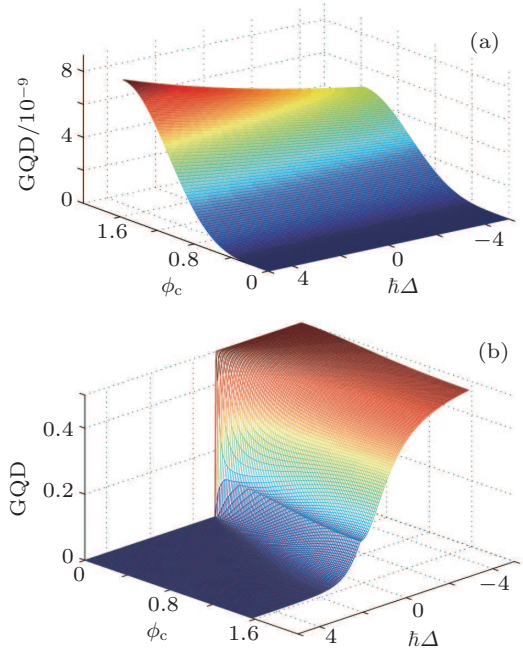


Fig. 6. (color online) The GQD for the energy eigenstates versus detuning $\hbar\Delta$ and ϕ_c. Panels (a) and (b) correspond to the ground state and the first excited state respectively.

Now we are in a position to study the GQD and to show what the relation between GQD and Berry phase is. Figure 6 presents GQD as a function of detuning Δ and ϕ_c for the ground state and the first excited state. As the figure shows, for the ground state, the GQD is very small for the ground state, similar to the Berry phase. For the first excited state, we find in the graph that when $\phi_c \to 0$, there exists a singularity for the GQD. Also, the image is centrosymmetric to some extent against the intersection curve of the GQD image and the plane where GQD is equal to 0.25. There is apparently the same singularity, and the symmetry corresponds to that of Berry phase. This correspondence shows that there are close connections between the Berry phase and GQD, and some properties of a system may be revealed by both GQD and the Berry phase. Some details in the figure of GQD, however, are obviously different from those of the Berry phase, such as the non-differentiable points of the GQD, which do not exist in the Berry phase. These results reflect the fact that the GQD may contain information beyond the energy and Berry phase.

5. Conclusion

To sum up, we calculated the eigenstates and eigenvalues for our chosen model. Then we investigated the time evolution of GQD and analyzed the effect of system parameters on it, finding that the external magnetic flux is able to control the period of GQD, and the initial state influences the maximum value of GQD. The relationship between energy, the Berry phase, and GQD is also presented and discussed. The results show that there are strong connections between them.

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