Improved delay-dependent robust H_∞ control of an uncertain stochastic system with interval time-varying and distributed delays*

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In this paper, the robust H_∞ control problem for a class of stochastic systems with interval time-varying and distributed delays is discussed. The system under study involves parameter uncertainty, stochastic disturbance, interval time-varying, and distributed delay. The aim is to design a delay-dependent robust H_∞ control which ensures the robust asymptotic stability of the given system and to express it in the form of linear matrix inequalities (LMIs). Numerical examples are given to demonstrate the effectiveness of the proposed method. The results are also compared with the existing results to show its conservativeness.

Keywords: distributed delay, interval time-varying delay, H_∞ control, linear matrix inequality (LMI), stochastic systems

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1. Introduction

Over the years, much attention has been paid to time-delay systems because they have been successfully and widely used in various fields, such as aircraft stabilization, nuclear reactors, models of lasers, ship stabilization, systems with lossless transmission lines, and neural networks. Thus for the time-delay systems, the stabilities,[1–3] especially the stabilities of delay-differential systems[4–6] have been analyzed in recent decades. Recently, a new type of time delay, that is, interval time-varying delay, was frequently confronted from various practical and theoretical systems and attracted much attention in the area of time-delay systems.[7–9] In recent years, the stabilities of uncertain systems with distributed delays have been extensively investigated in Refs. [10]–[12].

Since the existence of time-delays causes poor performance, oscillation, and even instability, it is very important to investigate the stabilities for systems with time delays before designing control systems.[13] The attention has been increasingly focused on the delay-dependent stability analysis of deterministic and stochastic time-delay systems.[13–20] Distributed delay systems constitute a particular case for time-delay systems, modeling the cumulative effect of the past values of the dynamics. There are many practical issues of such models in the literature. Indeed, distributed delay systems are often used to model the time lag phenomenon in thermodynamics,[21] ecology as well as epidemiology like predator-prey systems.[19]

Control design is an effective robust stability criterion for uncertain systems, which has been successfully applied to a wide variety of complex systems and the engineering field, including uncertain systems, time-delay systems. It is worth pointing out that there have been few results considering delay-dependent robust H_∞ control for uncertain stochastic nonlinear systems with time-varying delays and parameter uncertainties, which remains open but challenging. H_∞ control problem has attracted great attention due to its theoretical and practical significance in system control. The objective of H_∞ control is to design controllers such that the closed-loop system is internally stable and its H_∞ norm from the external input to the controlled output is less than a prescribed level. Since the theory of H_∞ control was proposed by Zames,[22] much effort has been devoted to H_∞ controller design in order to guarantee the desired stability.[23–28]

The problems of robust stochastic stabilization and robust H_∞ control for uncertain neutral stochastic time-delay systems were studied in Ref. [26]. For another research front of nonlinear system theory, the H_∞ control for the stochastic system has been an attractive subject for many years. For some recent studies in the stochastic case, we refer the reader to Refs. [29]–[32], and many methods have been proposed to deal with the H_∞ control for Markovian jump systems.[33,34] An effective method of designing the H_∞ filter of the neutral system was proposed in Ref. [35]. Over the past several years, much attention has been received by stochastic systems since stochastic modeling plays an important role in many branches of science.[36–42]

Motivated by the above discussion in this paper, we focus on the stability analyses of uncertain stochastic systems with interval time-varying and distributed delays. By Lyapunov–Krasovskii functional theory, new delay-dependent robust stability criteria are formulated in terms of linear matrix inequal-

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ities (LMIs) and the usual restriction on derivation of the time-varying delay where $\dot{\tau}(t) < 1$ is removed. The conditions obtained are simple and efficient, because neither model transformations, nor estimations for cross terms nor additional free-weighting matrices are involved. By introducing free-weighting matrices and using the convex combination technique, sufficient conditions (robust H_∞ control) are derived for the considered systems in terms of LMIs,[43] which can be easily calculated by using the Matlab LMI control toolbox.[44] Numerical examples are given to demonstrate the effectiveness and less conservatism of the proposed method.

Notation Throughout this paper, $\Re^n$ and $\Re^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times n$ real matrices. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ denote the symmetric matrix, where the notation $*$ represents the entries implied by symmetry. A^{T} and A^{-1} denote the transpose and inverse of matrix A respectively. We say $X > 0$ means that the matrix X is a real-symmetric positive-definite matrix with an appropriate dimension. I denotes the identity matrix with an appropriate dimension. $\mathcal{E}$ represents the expectation operator with respect to some probability measure $\mathcal{P}$. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space which relates to an increasing family $(\mathcal{F}_t)_{t>0}$ of σ algebras $(\mathcal{F}_t)_{t>0} \subset \mathcal{F}$, where Ω is the sample space, $\mathcal{F}$ is the σ algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. Let $\|f\|_2 = \sqrt{\int_0^\infty \|f\|^2 \mathrm{d}t}, f(t) \in L_2[0,\infty)\|f\|$ refers to the Euclidean norm of the function $f(t)$ at time t. $L_2[0,\infty)$ is the space of square integrable vector on $[0,\infty)$.

2. System description and preliminaries

Consider a delay-differential uncertain stochastic system with interval time-varying and distributed delays in the following form:

$$\begin{aligned}
\mathrm{d}x(t) &= \Big[A(t)x(t) + A_d(t)x(t-\tau(t)) + A_f(t)\int_{t-h}^{t} x(s)\mathrm{d}s \\
&\quad + B_1u(t) + B_v v(t)\Big]\mathrm{d}t + \Big[E(t)x(t) + E_d(t)x(t-\tau(t)) \\
&\quad + E_f(t)\int_{t-h}^{t} x(s)\mathrm{d}s\Big]\mathrm{d}w(t), \\
z(t) &= Cx(t) + Du(t), \\
x(s) &= \varphi(s), \quad s \in [-\max(\overline{\tau}, h), 0],
\end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the control input, $v(t) \in \Re^p$ is the disturbance input which belongs to $L_2[0,\infty)$, $z(t) \in \Re^q$ is the controlled output of the system, initial vector $\phi \in C_0$, where C_0 is the set of continuous functions from $[-\overline{\tau}, 0]$ to $\Re^n$, h denotes the distributed delay, $\tau(t)$ denotes the time-varying delay, and $\tau(t)$ is assumed to satisfy

$$0 \le \underline{\tau} \le \tau(t) \le \overline{\tau}, \ \dot{\tau}(t) \le \mu, \tag{2}$$

where $\underline{\tau}$, $\overline{\tau}$, h, and μ are constants. The matrices $A(t) = A + \triangle A(t)$, $A_d(t) = A_d + \triangle A_d(t)$, $A_f(t) = A_f + \triangle A_f(t)$, $E(t) = E + \triangle E(t)$, $E_d(t) = E_d + \triangle E_d(t)$, $E_f(t) = E_f + \triangle E_f(t)$, and B_1, B_v, C, D are known real constant matrices with appropriate dimensions.

$\triangle A(t)$, $\triangle A_d(t)$, $\triangle A_f(t)$, $\triangle E(t)$, $\triangle E_d(t)$, and $\triangle E_f(t)$ are real-valued unknown matrices representing time-varying parameter uncertainties, and are assumed to be in the following form:

$$\begin{aligned}
&\begin{bmatrix} \triangle A(t) & \triangle A_d(t) & \triangle A_f(t) & \triangle E(t) & \triangle E_d(t) & \triangle E_f(t) \end{bmatrix} \\
&= LF(t)\begin{bmatrix} G_1 & G_2 & G_3 & G_4 & G_5 & G_6 \end{bmatrix},
\end{aligned} \tag{3}$$

where L and E_i, $(i = 1,2,3,4,5,6)$ are known real constant matrices, and $F(t)$ is unknown time-varying matrix function satisfying

$$F^{\mathrm{T}}(t)F(t) \le I, \forall t. \tag{4}$$

Lemma 1[43] Let M, P, Q be given matrices such that $Q > 0$, then

$$\begin{bmatrix} P & M^{\mathrm{T}} \\ M & -Q \end{bmatrix} < 0 \Leftrightarrow P + M^{\mathrm{T}}Q^{-1}M < 0.$$

Lemma 2 Given any matrices X, Y, and S with appropriate dimensions such that $0 < S = S^{\mathrm{T}}$, the following inequality holds

$$2X^{\mathrm{T}}Y \le X^{\mathrm{T}}SX + Y^{\mathrm{T}}S^{-1}Y.$$

Lemma 3[18] For any constant matrix $M \in R^{n\times n}$, $M = M^{\mathrm{T}} > 0$, scalar $\eta > 0$, and vector function $w : [0,\eta] \to R^n$ such that the integrations concerned are well-defined, then

$$\left[\int_0^\eta w(s)\mathrm{d}s\right]^{\mathrm{T}} M \left[\int_0^\eta w(s)\mathrm{d}s\right] \le \eta \int_0^\eta w^{\mathrm{T}}(s)Mw(s)\mathrm{d}s.$$

Lemma 4[45] Let D, E, and $F(t)$ be real matrices of appropriate dimensions, and $F(t)$ satisfy $F^{\mathrm{T}}(t)F(t) \le I$, then the following inequality will hold for any constant $\varepsilon > 0$:

$$DF(t)E + E^{\mathrm{T}}F^{\mathrm{T}}(t)D^{\mathrm{T}} \le \epsilon DD^{\mathrm{T}} + \epsilon^{-1}E^{\mathrm{T}}E.$$

3. Problem formulation and main results

In this section, we derive a new delay-dependent criterion for the system (1) using the Lyapunov functional method combining with the linear matrix method. We can rewrite the system (1) as follows:

$$\begin{aligned}
\mathrm{d}x(t) &= \Big[Ax(t) + A_d x(t-\tau(t)) + A_f\int_{t-h}^{t} x(s)\mathrm{d}s + B_v v(t)\Big]\mathrm{d}t \\
&\quad + \Big[Ex(t) + E_d x(t-\tau(t)) + E_f(t)\int_{t-h}^{t} x(s)\mathrm{d}s\Big]\mathrm{d}w(t), \\
z(t) &= Cx(t), \\
x(s) &= \varphi(s), s \in [-\max(\overline{\tau}, h), 0].
\end{aligned} \tag{5}$$

Firstly, we establish a new delay-dependent robust stability criterion for the system (5) with $u(t) = 0$.

Theorem 1 For given scalars $0 \le \underline{\tau} < \overline{\tau}$, $h > 0$, and μ, the uncertain stochastic system (5) is robustly asymptotically stable, and under zero condition for nonzero $w(t) \in L_2[0,\infty)$ the inequality $\|z(t)\|_2 \le \gamma\|w(t)\|_2$ holds, if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $V > 0$, $Z_j > 0$, $(j = 1,2,3,4)$, T, and appropriately dimensioned matrices M_l, N_l, S_l, $(l = 1,2,3)$, such that the following LMI holds:

$$\widehat{\Psi} = \begin{bmatrix} \Psi_0 & \Psi_1 & \Psi_2 & PB_v & C^{\mathrm{T}} \\ * & -\Psi_{11} & 0 & 0 & 0 \\ * & * & -\Psi_{22} & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{6}$$

where

$$\Psi_0 = \begin{bmatrix} \Phi_1 & \Phi_2^{\mathrm{T}} U_1 & \Phi_3^{\mathrm{T}} U_2 \\ * & -U_1 & 0 \\ * & * & U_2 \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & \Phi_{12} & PA_f + N_3^{\mathrm{T}} & M_1 & -S_1 \\ * & \Phi_{22} & \Phi_{23} & M_2 & -S_2 \\ * & * & -V & M_3 & -S_3 \\ * & * & * & -Q & 0 \\ * & * & * & * & -R \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} \overline{\tau}N_1 & \tau M_1 & \tau S_1 \\ \overline{\tau}N_2 & \tau M_2 & \tau S_2 \\ \overline{\tau}N_3 & \tau M_3 & \tau S_3 \\ 0 & 0 & 0 \\ . & . & . \\ . & . & . \\ . & . & . \\ 0 & 0 & 0 \end{bmatrix}, \quad \Psi_2 = \begin{bmatrix} N_1 & M_1 & S_1 \\ N_2 & M_2 & S_2 \\ N_3 & M_3 & S_3 \\ 0 & 0 & 0 \\ . & . & . \\ . & . & . \\ . & . & . \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} A & A_d & A_f & 0 & 0 \end{bmatrix},$$

$$\Phi_3 = \begin{bmatrix} E & E_d & E_f & 0 & 0 \end{bmatrix},$$

$$U_1 = \overline{\tau}Z_1 + \tau Z_2, U_2 = P + \overline{\tau}Z_3 + \tau Z_4,$$

$$\Psi_{11} = \mathrm{diag}\{\overline{\tau}Z_1, \tau Z_2, \tau(Z_1 + Z_2)\},$$

$$\Psi_{22} = \mathrm{diag}\{\overline{\tau}Z_3, \tau Z_4, \tau(Z_3 + Z_4)\}$$

with

$$\Phi_{11} = PA + A^{\mathrm{T}}P + T + T^{\mathrm{T}} + Q + R + h^2V + N_1 + N_1^{\mathrm{T}},$$

$$\Phi_{12} = PA_d - N_1 + N_2^{\mathrm{T}} - M_1 + S_1 - (1-\mu)T,$$

$$\Phi_{22} = -N_2 - N_2^{\mathrm{T}} - M_2 - M_2^{\mathrm{T}} + S_2 + S_2^{\mathrm{T}},$$

$$\Phi_{23} = S_3^{\mathrm{T}} - N_3^{\mathrm{T}} - M_3^{\mathrm{T}}, \quad \tau = \overline{\tau} - \underline{\tau}.$$

Proof Define the Lyapunov functional candidate as

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t), \tag{7}$$

where

$$V_1(x_t) = x^{\mathrm{T}}(t)Px(t),$$

$$V_2(x_t) = 2x^{\mathrm{T}}(t)T\int_{t-\tau(t)}^{t} x(s)\,\mathrm{d}s + \int_{t-\underline{\tau}}^{t} x^{\mathrm{T}}(s)Qx(s)\,\mathrm{d}s + \int_{t-\overline{\tau}}^{t} x^{\mathrm{T}}(s)Rx(s)\,\mathrm{d}s,$$

$$V_3(x_t) = h\int_{-h}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(s)Vx(s)\,\mathrm{d}s\,\mathrm{d}\theta,$$

$$V_4(x_t) = \int_{-\overline{\tau}}^{0}\int_{t+\theta}^{t} f^{\mathrm{T}}(s)Z_1 f(s)\,\mathrm{d}s\,\mathrm{d}\theta + \int_{-\overline{\tau}}^{-\underline{\tau}}\int_{t+\theta}^{t} f^{\mathrm{T}}(s)Z_2 f(s)\,\mathrm{d}s\,\mathrm{d}\theta + \int_{-\overline{\tau}}^{0}\int_{t+\theta}^{t} g^{\mathrm{T}}(s)Z_3 g(s)\,\mathrm{d}s\,\mathrm{d}\theta + \int_{-\overline{\tau}}^{-\underline{\tau}}\int_{t+\theta}^{t} g^{\mathrm{T}}(s)Z_4 g(s)\,\mathrm{d}s\,\mathrm{d}\theta,$$

with

$$f(t) = Ax(t) + A_d x(t-\tau(t)) + A_f\int_{t-h}^{t} x(s)\,\mathrm{d}s,$$

$$g(t) = Ex(t) + E_d x(t-\tau(t)) + E_f\int_{t-h}^{t} x(s)\,\mathrm{d}s.$$

From the Leibniz–Newtons formula, the following equalities hold true for any matrices N_j, M_j, S_j, $(j = 1,2,3)$ with appropriate dimensions:

$$0 = 2\Big[x^{\mathrm{T}}(t)N_1 + x^{\mathrm{T}}(t-\tau(t))N_2 + \Big(\int_{t-h}^{t} x(s)\,\mathrm{d}s\Big)^{\mathrm{T}}N_3\Big] \times \Big[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} f(s)\,\mathrm{d}s - \int_{t-\tau(t)}^{t} g(s)\,\mathrm{d}w(s)\Big], \tag{8}$$

$$0 = 2\Big[x^{\mathrm{T}}(t)M_1 + x^{\mathrm{T}}(t-\tau(t))M_2 + \Big(\int_{t-h}^{t} x(s)\,\mathrm{d}s\Big)^{\mathrm{T}}M_3\Big] \times \Big[x(t-\underline{\tau}) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\underline{\tau}} f(s)\,\mathrm{d}s - \int_{t-\tau(t)}^{t-\underline{\tau}} g(s)\,\mathrm{d}w(s)\Big], \tag{9}$$

$$0 = 2\Big[x^{\mathrm{T}}(t)S_1 + x^{\mathrm{T}}(t-\tau(t))S_2 + \Big(\int_{t-h}^{t} x(s)\,\mathrm{d}s\Big)^{\mathrm{T}}S_3\Big] \times \Big[x(t-\tau(t)) - x(t-\overline{\tau}) - \int_{t-\overline{\tau}}^{t-\tau(t)} f(s)\,\mathrm{d}s - \int_{t-\overline{\tau}}^{t-\tau(t)} g(s)\,\mathrm{d}w(s)\Big]. \tag{10}$$

By Lemma 2,[40] for any matrices $Z_i, (i = 1,2,3,4)$, the following equalities hold:

$$-2\xi^{\mathrm{T}}(t)N\int_{t-\tau(t)}^{t} f(s)\,\mathrm{d}s \le \overline{\tau}\xi^{\mathrm{T}}(t)NZ_1^{-1}N^{\mathrm{T}}\xi(t) + \int_{t-\tau(t)}^{t} f^{\mathrm{T}}(s)Z_1 f(s)\,\mathrm{d}s, \tag{11}$$

$$-2\xi^{\mathrm{T}}(t)M\int_{t-\tau(t)}^{t-\underline{\tau}} f(s)\,\mathrm{d}s \le \tau\xi^{\mathrm{T}}(t)MZ_2^{-1}M^{\mathrm{T}}\xi(t) + \int_{t-\tau(t)}^{t-\underline{\tau}} f^{\mathrm{T}}(s)Z_2 f(s)\,\mathrm{d}s, \tag{12}$$

$$-2\xi^{\mathrm{T}}(t)S\int_{t-\overline{\tau}}^{t-\tau(t)} f(s)\,\mathrm{d}s \le \tau\xi^{\mathrm{T}}(t)S(Z_1+Z_2)^{-1}S^{\mathrm{T}}\xi(t)+\int_{t-\overline{\tau}}^{t-\tau(t)} f^{\mathrm{T}}(s)(Z_1+Z_2)f(s)\,\mathrm{d}s, \tag{13}$$

$$-2\xi^{\mathrm{T}}(t)N\int_{t-\tau(t)}^{t} g(s)\,\mathrm{d}w(s) \le \xi^{\mathrm{T}}(t)NZ_3^{-1}N^{\mathrm{T}}\xi(t)+\Big(\int_{t-\tau(t)}^{t} g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}}Z_3\Big(\int_{t-\tau(t)}^{t} g(s)\,\mathrm{d}w(s)\Big), \tag{14}$$

$$-2\xi^{\mathrm{T}}(t)M\int_{t-\tau(t)}^{t-\underline{\tau}} g(s)\,\mathrm{d}w(s) \le \xi^{\mathrm{T}}(t)MZ_4^{-1}M^{\mathrm{T}}\xi(t)+\Big(\int_{t-\tau(t)}^{t-\underline{\tau}} g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}}Z_4\Big(\int_{t-\tau(t)}^{t-\underline{\tau}} g(s)\,\mathrm{d}w(s)\Big), \tag{15}$$

$$-2\xi^{\mathrm{T}}(t)S\int_{t-\overline{\tau}}^{t-\tau(t)} g(s)\,\mathrm{d}w(s) \le \xi^{\mathrm{T}}(t)S(Z_3+Z_4)^{-1}S^{\mathrm{T}}\xi(t)+\Big(\int_{t-\overline{\tau}}^{t-\tau(t)} g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}}(Z_3+Z_4)\Big(\int_{t-\overline{\tau}}^{t-\tau(t)} g(s)\,\mathrm{d}w(s)\Big), \tag{16}$$

where

$$N=\begin{bmatrix} N_1^{\mathrm{T}} & N_2^{\mathrm{T}} & N_3^{\mathrm{T}} & 0 & 0\end{bmatrix}^{\mathrm{T}},\quad M=\begin{bmatrix} M_1^{\mathrm{T}} & M_2^{\mathrm{T}} & M_3^{\mathrm{T}} & 0 & 0\end{bmatrix}^{\mathrm{T}},\quad \text{and}\quad S=\begin{bmatrix} S_1^{\mathrm{T}} & S_2^{\mathrm{T}} & S_3^{\mathrm{T}} & 0 & 0\end{bmatrix}^{\mathrm{T}}. \tag{17}$$

Along the trajectories of Eq. (5) and making use of the Ito's formula (Ref. [36]), the generator LV for the evolution of $V(x_t)$ is therefore given by

$$LV(x_t)=LV_1(x_t)+LV_2(x_t)+LV_3(x_t), \tag{18}$$

where

$$\begin{aligned} LV_1(x_t) &= 2x^{\mathrm{T}}(t)P\Big[Ax(t)+A_dx(t-\tau(t))+A_f\int_{t-h}^{t}x(s)\,\mathrm{d}s+B_vv(t)\Big]+\Big[Ex(t)+E_dx(t-\tau(t))+E_f\int_{t-d}^{t}x(s)\,\mathrm{d}s\Big]^{\mathrm{T}}P\\ &\quad\times\Big[E(t)x(t)+E_d(t)x(t-\tau(t))+E_f(t)\int_{t-d}^{t}x(s)\,\mathrm{d}s\Big], \end{aligned} \tag{19}$$

$$\begin{aligned} LV_2(x_t) &= 2x^{\mathrm{T}}(t)T[x(t)-(1-\dot{d}(t))x(t-d(t))]+x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\underline{\tau})Qx(t-\underline{\tau})+x^{\mathrm{T}}(t)Rx(t)-x^{\mathrm{T}}(t-\overline{\tau})Rx(t-\overline{\tau})\\ &\le 2x^{\mathrm{T}}(t)T[x(t)-(1-\mu)x(t-d(t))]+x^{\mathrm{T}}(t)[Q+R]x(t)-x^{\mathrm{T}}(t-\underline{\tau})Qx(t-\underline{\tau})-x^{\mathrm{T}}(t-\overline{\tau})Rx(t-\overline{\tau}), \end{aligned} \tag{20}$$

$$LV_3(x_t)=h^2x^{\mathrm{T}}(t)Vx(t)-h\int_{t-h}^{t}x^{\mathrm{T}}(s)Vx(s)\,\mathrm{d}s \le h^2x^{\mathrm{T}}(t)Vx(t)-\Big(\int_{t-h}^{t}x(s)\,\mathrm{d}s\Big)^{\mathrm{T}}V\Big(\int_{t-h}^{t}x(s)\,\mathrm{d}s\Big), \tag{21}$$

$$\begin{aligned} LV_4(x_t) &= f^{\mathrm{T}}(t)[\overline{\tau}Z_1+\tau Z_2]f(t)-\int_{t-\overline{\tau}}^{t}f^{\mathrm{T}}(s)Z_1f(s)\,\mathrm{d}s-\int_{t-\overline{\tau}}^{t-\underline{\tau}}f^{\mathrm{T}}(s)Z_2f(s)\,\mathrm{d}s+g^{\mathrm{T}}(t)[\overline{\tau}Z_3+\tau Z_4]g(t)\\ &\quad-\int_{t-\overline{\tau}}^{t}g^{\mathrm{T}}(s)Z_3g(s)\,\mathrm{d}s-\int_{t-\overline{\tau}}^{t-\underline{\tau}}g^{\mathrm{T}}(s)Z_4g(s)\,\mathrm{d}s\\ &= f^{\mathrm{T}}(t)[\overline{\tau}Z_1+\tau Z_2]f(t)+g^{\mathrm{T}}(t)[\overline{\tau}Z_3+\tau Z_4]g(t)-\int_{t-\tau(t)}^{t}f^{\mathrm{T}}(s)Z_1f(s)\,\mathrm{d}s-\int_{t-\tau(t)}^{t-\underline{\tau}}f^{\mathrm{T}}(s)Z_2f(s)\,\mathrm{d}s\\ &\quad-\int_{t-\overline{\tau}}^{t-\tau(t)}f^{\mathrm{T}}(s)(Z_1+Z_2)f(s)\,\mathrm{d}s-\int_{t-\tau(t)}^{t}g^{\mathrm{T}}(s)Z_3g(s)\,\mathrm{d}s-\int_{t-\tau(t)}^{t-\underline{\tau}}g^{\mathrm{T}}(s)Z_4g(s)\,\mathrm{d}s-\int_{t-\overline{\tau}}^{t-\tau(t)}g^{\mathrm{T}}(s)(Z_3+Z_4)g(s)\,\mathrm{d}s. \end{aligned} \tag{22}$$

Combining Eqs. (8)–(22), we obtain

$$\begin{aligned} LV(x_t) &\le \xi^{\mathrm{T}}(t)\Big[\Phi_1+\Phi_2^{\mathrm{T}}U_1\Phi_2+\Phi_3^{\mathrm{T}}U_2\Phi_3+\overline{\tau}NZ_1^{-1}N^{\mathrm{T}}+\tau MZ_2^{-1}M^{\mathrm{T}}+\tau S(Z_1+Z_2)^{-1}S^{\mathrm{T}}+NZ_1^{-1}N^{\mathrm{T}}+MZ_2^{-1}M^{\mathrm{T}}+S(Z_1+Z_2)^{-1}S^{\mathrm{T}}\Big]\xi(t)\\ &\quad+2x^{\mathrm{T}}(t)PB_vv(t)+\Big(\int_{t-\tau(t)}^{t}g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}}Z_3\Big(\int_{t-\tau(t)}^{t}g(s)\,\mathrm{d}w(s)\Big)+\Big(\int_{t-\tau(t)}^{t-\underline{\tau}}g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}}Z_4\Big(\int_{t-\tau(t)}^{t-\underline{\tau}}g(s)\,\mathrm{d}w(s)\Big)\\ &\quad+\Big(\int_{t-\overline{\tau}}^{t-\tau(t)}g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}}(Z_3+Z_4)\Big(\int_{t-\overline{\tau}}^{t-\tau(t)}g(s)\,\mathrm{d}w(s)\Big)-\int_{t-\tau(t)}^{t}g^{\mathrm{T}}(s)Z_3g(s)\,\mathrm{d}s-\int_{t-\tau(t)}^{t-\underline{\tau}}g^{\mathrm{T}}(s)Z_4g(s)\,\mathrm{d}s\\ &\quad-\int_{t-\overline{\tau}}^{t-\tau(t)}g^{\mathrm{T}}(s)(Z_3+Z_4)g(s)\,\mathrm{d}s, \end{aligned} \tag{23}$$

where

$$\xi^{\mathrm{T}}(t)=\Big[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-\tau(t))\ \ \Big(\int_{t-h}^{t}x(s)\,\mathrm{d}s\Big)^{\mathrm{T}}\ \ x(t-\underline{\tau})\ \ x(t-\overline{\tau})\ \Big],\quad U_1=\overline{\tau}Z_1+\tau Z_2,\quad U_2=\overline{\tau}Z_3+\tau Z_4.$$

By using

$$\begin{aligned}
&\mathcal{E}\Big\{\Big(\int_{t-\tau(t)}^{t} g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}} Z_3 \Big(\int_{t-\tau(t)}^{t} g(s)\,\mathrm{d}w(s)\Big)\Big\}\\
&=\mathcal{E}\Big\{\int_{t-\tau(t)}^{t} g^{\mathrm{T}}(s) Z_3 g(s)\,\mathrm{d}s\Big\},\\
&\mathcal{E}\Big\{\Big(\int_{t-\tau(t)}^{t-\underline{\tau}} g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}} Z_4 \Big(\int_{t-\tau(t)}^{t-\underline{\tau}} g(s)\,\mathrm{d}w(s)\Big)\Big\}\\
&=\mathcal{E}\Big\{\int_{t-\tau(t)}^{t-\underline{\tau}} g^{\mathrm{T}}(s) Z_4 g(s)\,\mathrm{d}s\Big\},\\
&\mathcal{E}\Big\{\Big(\int_{t-\overline{\tau}}^{t-\tau(t)} g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}} (Z_3+Z_4) \Big(\int_{t-\overline{\tau}}^{t-\tau(t)} g(s)\,\mathrm{d}w(s)\Big)\Big\}\\
&=\mathcal{E}\Big\{\int_{t-\overline{\tau}}^{t-\tau(t)} g^{\mathrm{T}}(s)(Z_3+Z_4) g(s)\,\mathrm{d}s\Big\}, \qquad (24)
\end{aligned}$$

and by Lemma 2, we have

$$LV(x_t) \le \xi^{\mathrm{T}}(t)\Psi\xi(t) + \gamma^{-2}x^{\mathrm{T}}(t)PB_vB_v^{\mathrm{T}}Px(t) + \gamma^2 v^{\mathrm{T}}(t)v(t), \qquad (25)$$

where

$$\begin{aligned}
\Psi = \Big[&\Phi_1 + \Phi_2^{\mathrm{T}}U_1\Phi_2 + \Phi_3^{\mathrm{T}}U_2\Phi_3 + \overline{\tau}NZ_1^{-1}N^{\mathrm{T}}\\
&+ \tau MZ_2^{-1}M^{\mathrm{T}} + \tau S(Z_1+Z_2)^{-1}S^{\mathrm{T}} + NZ_1^{-1}N^{\mathrm{T}}\\
&+ MZ_2^{-1}M^{\mathrm{T}} + S(Z_1+Z_2)^{-1}S^{\mathrm{T}}\Big]
\end{aligned}$$

and

$$E[LV(x_t)\,\mathrm{d}t] = E[\xi^{\mathrm{T}}(t)\Psi\xi(t)] < 0. \qquad (26)$$

Now, we set

$$J(t) = E\Big\{\int_0^t [z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t)]\,\mathrm{d}t\Big\},$$

where $t > 0$. Since $V(x(t),0) = 0$ under the zero initial condition, that is, $x(t) = 0$ for $t \in [-\overline{\tau}, 0]$, then by Ito's formula, it follows that

$$\begin{aligned}
J(t) &= E\Big\{\int_0^t [z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) + LV(x_t)]\,\mathrm{d}t\Big\}\\
&\quad - E\Big\{V(x(t),t)\Big\},\\
&\le E\Big\{\int_0^t [z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) + LV(x_t)]\,\mathrm{d}t\Big\},\\
J(t) &\le E\Big\{\int_0^t \xi^{\mathrm{T}}(t)\widehat{\Psi}\xi(t)\,\mathrm{d}t\Big\},
\end{aligned}$$

where

$$\begin{aligned}
\widehat{\Psi} = \Big[&\widehat{\Phi}_1 + \Phi_2^{\mathrm{T}}U_1\Phi_2 + \Phi_3^{\mathrm{T}}U_2\Phi_3 + \overline{\tau}NZ_1^{-1}N^{\mathrm{T}} + \tau MZ_2^{-1}M^{\mathrm{T}}\\
&+ \tau S(Z_1+Z_2)^{-1}S^{\mathrm{T}} + NZ_1^{-1}N^{\mathrm{T}} + MZ_2^{-1}M^{\mathrm{T}}\\
&+ S(Z_1+Z_2)^{-1}S^{\mathrm{T}} + \gamma^{-2}PB_vB_v^{\mathrm{T}}P\Big], \qquad (27)
\end{aligned}$$

with

$$\widehat{\Phi}_1 = \Phi_{11} + \gamma^{-2}PB_vB_v^{\mathrm{T}}P + \widehat{C}^{\mathrm{T}}\widehat{C}$$

and

$$\widehat{C} = \begin{bmatrix} C & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We obtain that expression (27) is equivalent to inequality (6) by using Lemma 1, which means that $\|z\|_2 \le \gamma\|w\|_2$, for any nonzero $w(t) \in L_2[0,\infty)$. When $w(t) \equiv 0$, it follows from expression (27) that $\dot{V} < 0$, which ensures the robust asymptotic stability of the system (1).

Remark 1 Theorem 1 provides a new version of delay-dependent stability criteria for stochastic systems with interval time-varying and distributed delays. In the proof of Theorem 1, it can be seen that neither model transformation method (Ref. [46]) nor Moons inequality (Ref. [47]) is used, which reduces the conservatism in existing stability criteria derived by either model transformation methods or Moons inequality approach. It is worth pointing out that for a given $\tau > 0, h > 0$, the minimum γ, which satisfies the LMI in expression (31) can also be obtained by solving a quasi-convex optimization problem.

Remark 2 In Theorem 1 we established delay-dependent robust stability criterion for the system (5) with $u(t) = 0$. In the following Theorem 2 the uncertain terms are added to extend the Theorem 1 for the system (1) with $u(t) = 0$.

Theorem 2 For given scalars $0 \le \underline{\tau} < \overline{\tau}$, $h > 0$ and μ, the uncertain stochastic system (1) is robustly asymptotically stable, and under zero condition for nonzero $w(t) \in L_2[0,\infty)$ the inequality $\|z(t)\|_2 \le \gamma\|w(t)\|_2$ holds, if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $V > 0$, $Z_j > 0$, $(j = 1,2,3,4)$, T, and appropriately dimensioned matrices M_l, N_l, S_l, $(l = 1,2,3)$, and a scalar $\epsilon > 0$ such that the following LMI holds:

$$\widehat{\Psi} = \begin{bmatrix}
\Psi_0 & \Psi_1 & \Psi_2 & \Gamma_i & \Upsilon_i & PB_v & C^{\mathrm{T}}\\
* & -\Psi_{11} & 0 & 0 & 0 & 0 & 0\\
* & * & -\Psi_{22} & 0 & 0 & 0 & 0\\
* & * & * & -\epsilon_i I & 0 & 0 & 0\\
* & * & * & * & -\epsilon_i^{-1} I & 0 & 0\\
* & * & * & * & * & -\gamma^2 I & 0\\
* & * & * & * & * & * & -I
\end{bmatrix} < 0, \qquad (28)$$

where

$$\Psi_0 = \begin{bmatrix} \Phi_1 & \Phi_2^{\mathrm{T}}U_1 & \Phi_3^{\mathrm{T}}U_2\\ * & -U_1 & 0\\ * & * & U_2 \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix}
\Phi_{11} & \Phi_{12} & PA_f + N_3^{\mathrm{T}} & M_1 & -S_1\\
* & \Phi_{22} & \Phi_{23} & M_2 & -S_2\\
* & * & -V & M_3 & -S_3\\
* & * & * & -Q & 0\\
* & * & * & * & -R
\end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} \overline{\tau}N_1 & \tau M_1 & \tau S_1 \\ \overline{\tau}N_2 & \tau M_2 & \tau S_2 \\ \overline{\tau}N_3 & \tau M_3 & \tau S_3 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \quad \Psi_2 = \begin{bmatrix} N_1 & M_1 & S_1 \\ N_2 & M_2 & S_2 \\ N_3 & M_3 & S_3 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} A & A_d & A_f & 0 & 0 \end{bmatrix},$$

$$\Phi_3 = \begin{bmatrix} E & E_d & E_f & 0 & 0 \end{bmatrix},$$

$$\Gamma_1 = \begin{bmatrix} L^{\mathrm{T}}P & 0 & 0 & 0 & 0 & \overline{\tau}L^{\mathrm{T}}Z_1 + \tau L^{\mathrm{T}}Z_2 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\Gamma_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & L^{\mathrm{T}}P + \overline{\tau}L^{\mathrm{T}}Z_3 + \tau L^{\mathrm{T}}Z_4 \end{bmatrix}^{\mathrm{T}},$$

$$\Upsilon_1 = \begin{bmatrix} G_1 & G_2 & G_3 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} G_4 & G_5 & G_6 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$U_1 = \overline{\tau}Z_1 + \tau Z_2, U_2 = P + \overline{\tau}Z_3 + \tau Z_4,$$

$$\Psi_{11} = \mathrm{diag}\{\overline{\tau}Z_1, \tau Z_2, \tau(Z_1 + Z_2)\},$$

$$\Psi_{22} = \mathrm{diag}\{\overline{\tau}Z_3, \tau Z_4, \tau(Z_3 + Z_4)\}.$$

Proof Replace A, A_d, A_f, E, E_d, and E_f in LMI (6) by $A(t) = A + \triangle A(t)$, $A(t) = A + \triangle A(t)$, $A_d(t) = A_d + \triangle A_d(t)$, $A_f(t) = A_f + \triangle A_f(t)$, $E(t) = E + \triangle E(t)$, $E(t) = E + \triangle E(t)$, $E_d(t) = E_d + \triangle E_d(t)$, and $E_f(t) = E_f + \triangle E_f(t)$ respectively, then we will obtain

$$\widehat{\Psi} + \Gamma_1 F(t) \Upsilon_1 + \Upsilon_1^{\mathrm{T}} F^{\mathrm{T}}(t) \Gamma_1^{\mathrm{T}} + \Gamma_2 F(t) \Upsilon_2 + \Upsilon_2^{\mathrm{T}} F^{\mathrm{T}}(t) \Gamma_2^{\mathrm{T}}. \quad (29)$$

By Lemma 4, we know that expression (29) is equivalent to

$$\Psi + \epsilon_1^{-1} \Gamma_1 \Gamma_1^{\mathrm{T}} + \epsilon_1 \Upsilon_1^{\mathrm{T}} \Upsilon_1 + \epsilon_2^{-1} \Gamma_2 \Gamma_2^{\mathrm{T}} + \epsilon_2 \Upsilon_2^{\mathrm{T}} \Upsilon_2. \quad (30)$$

Obviously, by applying Schur complement, expression (30) is equivalent to Eq. (28).

Remark 3 Theorem 3 provides a delay-dependent method of analyzing the robust H_∞ performance and H_∞ synthesis, respectively, for a class of uncertain system with time-varying delay in a range. Note that using the method in Theorem 3, the problem of finding the maximum allowed delay $\overline{\tau}$ for a given γ and $\underline{\tau}$, or the smallest γ for giving $\underline{\tau}$, $\overline{\tau}$ can be easily solved without the need for explicitly tuning any parameter. For instance, the smallest γ for a given $\underline{\tau}$, $\overline{\tau}$ obtainable from Theorem 3 can be determined by solving the following convex optimization problem

$$\begin{cases} \min \gamma^2 \\ \text{s.t. } \overline{Q} > 0, \ \overline{R} > 0, \ \overline{V} > 0, \ \overline{Z}_j > 0, \ (j = 1,2,3,4), \\ X > 0 \ \text{ and } \ \text{LMI (31)}. \end{cases}$$

4. H_∞ control problem

In this section we present a delay-dependent robust H_∞ controller design for a delayed uncertain stochastic system. In order to solve this problem, we first consider the problem of robust H_∞ performance analysis for the system (1). Using state feedback controller $u(t) = Kx(t)$, the following theorem is derived by extending Theorem 2. That is, $u(t) \neq 0$.

Theorem 3 For given scalars $0 \leq \underline{\tau} < \overline{\tau}$, $h > 0$, and μ, the uncertain stochastic system (5) is robustly asymptotically stable by $u(t) = Kx(t)$, and under zero condition for nonzero $w(t) \in L_2[0,\infty)$ the inequality $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$ holds, if there exist matrices $\overline{Q} > 0, \overline{R} > 0, \overline{V} > 0, \overline{Z}_j > 0$, $(j = 1,2,3,4)$, $\overline{T}$, $X > 0$, Y, and appropriately dimensioned matrices $\overline{M}_l$, $\overline{N}_l$, $\overline{S}_l$, $(l = 1,2,3)$, and a scalar $\epsilon > 0$ such that the following LMI holds:

$$\Omega = \begin{bmatrix} \Omega_0 & \Omega_1 & \Omega_2 & \Gamma_i & \Upsilon_i & PB_v & (C+DK)^{\mathrm{T}} \\ * & -\Omega_{11} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\Omega_{22} & 0 & 0 & 0 & 0 \\ * & * & * & \epsilon_i(-2X+I) & 0 & 0 & 0 \\ * & * & * & * & \epsilon_i^{-1}(-2X+I) & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \ (i = 1,2), \quad (31)$$

where

$$\Omega_0 = \begin{bmatrix} \Xi_1 & \Xi_2^{\mathrm{T}}\overline{U}_1 & \Xi_3^{\mathrm{T}}\overline{U}_2 \\ * & -\overline{U}_1 & 0 \\ * & * & \overline{U}_2 \end{bmatrix}, \quad \Xi_1 = \begin{bmatrix} \Xi_{11} & \Xi_{12} & A_f X^{\mathrm{T}} + \overline{N}_3^{\mathrm{T}} & \overline{M}_1 & -\overline{S}_1 \\ * & \Xi_{22} & \Xi_{23} & \overline{M}_2 & -\overline{S}_2 \\ * & * & -\overline{V} & \overline{M}_3 & -\overline{S}_3 \\ * & * & * & -\overline{Q} & 0 \\ * & * & * & * & -\overline{R} \end{bmatrix}, \quad \Omega_1 = \begin{bmatrix} \overline{\tau}\overline{N}_1 & \tau\overline{M}_1 & \tau\overline{S}_1 \\ \overline{\tau}\overline{N}_2 & \tau\overline{M}_2 & \tau\overline{S}_2 \\ \overline{\tau}\overline{N}_3 & \tau\overline{M}_3 & \tau\overline{S}_3 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} \overline{N}_1 & \overline{M}_1 & \overline{S}_1 \\ \overline{N}_2 & \overline{M}_2 & \overline{S}_2 \\ \overline{N}_3 & \overline{M}_3 & \overline{S}_3 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_2 = \begin{bmatrix} A & A_d & A_f & 0 & 0 \end{bmatrix}, \ \Xi_3 = \begin{bmatrix} E & E_d & E_f & 0 & 0 \end{bmatrix}, \ \Gamma_1 = \begin{bmatrix} XL^{\mathrm{T}} & 0 & 0 & 0 & 0 & \overline{\tau}L^{\mathrm{T}}\overline{Z}_1 + \tau L^{\mathrm{T}}\overline{Z}_2 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\Gamma_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & XL^{\mathrm{T}} + \overline{\tau}L^{\mathrm{T}}\overline{Z}_3 + \tau L^{\mathrm{T}}\overline{Z}_4 \end{bmatrix}^{\mathrm{T}}, \ \Upsilon_1 = \begin{bmatrix} G_1X^{\mathrm{T}} & G_2X^{\mathrm{T}} & G_3X^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}, \ \Upsilon_2 = \begin{bmatrix} G_4X^{\mathrm{T}} & G_5X^{\mathrm{T}} & G_6X^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\overline{U}_1 = \overline{\tau}\overline{Z}_1 + \tau\overline{Z}_2, \ \overline{U}_2 = X^{\mathrm{T}} + \overline{\tau}\overline{Z}_3 + \tau\overline{Z}_4, \ \Omega_{11} = \mathrm{diag}\{\overline{\tau}\overline{Z}_1, \tau\overline{Z}_2, \tau(\overline{Z}_1 + \overline{Z}_2)\}, \ \Omega_{22} = \mathrm{diag}\{\overline{\tau}\overline{Z}_3, \tau\overline{Z}_4, \tau(\overline{Z}_3 + \overline{Z}_4)\}$$

with

$$\Xi_{11} = AX^{\mathrm{T}} + B_1 Y + XA^{\mathrm{T}} + Y^{\mathrm{T}} B_1^{\mathrm{T}} + \overline{T} + \overline{T}^{\mathrm{T}} + \overline{Q} + \overline{R} + h^2 \overline{V} + \overline{N}_1 + \overline{N}_1^{\mathrm{T}},$$

$$\Xi_{12} = A_d X^{\mathrm{T}} - \overline{N}_1 + \overline{N}_2^{\mathrm{T}} - \overline{M}_1 + \overline{S}_1 - (1-\mu)\overline{T},$$

$$\Xi_{22} = -\overline{N}_2 - \overline{N}_2^{\mathrm{T}} - \overline{M}_2 - \overline{M}_2^{\mathrm{T}} + \overline{S}_2 + \overline{S}_2^{\mathrm{T}},$$

$$\Xi_{23} = \overline{S}_3^{\mathrm{T}} - \overline{N}_3^{\mathrm{T}} - \overline{M}_3^{\mathrm{T}}, \quad \tau = \overline{\tau} - \underline{\tau},$$

and the feedback gain matrix is given by $K = YX^{-1}$.

Proof Now we make the following definitions:

$$P = X^{-1}, \quad \overline{Q} = XQX^{\mathrm{T}}, \quad \overline{R} = XRX^{\mathrm{T}}, \quad \overline{V} = XVX^{\mathrm{T}},$$

$$\overline{Z}_j = XZ_jX^{\mathrm{T}} \ (i = 1,2,3,4), \quad \overline{N}_l = X^{\mathrm{T}} N_l X, \quad \overline{M}_l = X^{\mathrm{T}} M_l X,$$

$$\overline{S}_l = X^{\mathrm{T}} S_l X, \ (l = 1,2,3). \tag{32}$$

Then pre and post multiplying $\widehat{\Psi}$ in Eq. (6) by $(\mathrm{diag}\overbrace{X,\ldots,X}^{15 \text{ times}},I,I)$ and its transpose respectively, we can obtain Eq. (31).

Remark 4 In order to show the reduced conservatism of our stability criteria, we consider the following system as a special case of the system (1):

$$\begin{aligned} \dot{x}(t) &= \Big[A(t)x(t) + A_d(t)x(t-\tau(t))\Big]\mathrm{d}t \\ &\quad + \Big[E(t)x(t) + E_d(t)x(t-\tau(t))\Big]\mathrm{d}w(t), \\ x(s) &= \varphi(s), \ \ s \in [-\overline{\tau}, 0], \end{aligned} \tag{33}$$

where $x(t) \in \Re^n$ is state, the initial vector $\phi \in C_0$, where C_0 is the set of continuous functions from $[-\overline{\tau}, 0]$ to $\Re^n$. $\tau(t)$ denotes the time-varying delay, and is assumed to satisfy Eq. (2).

The matrices $A(t) = A + \triangle A(t)$, $A_d(t) = A_d + \triangle A_d(t)$, $E(t) = E + \triangle E(t)$, and $E_d(t) = E_d + \triangle E_d(t)$ are known real constant matrices with appropriate dimensions. $\triangle A(t)$, $\triangle A_d(t)$, $\triangle E(t)$, and $\triangle E_d(t)$ are real-valued unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form:

$$\begin{bmatrix} \triangle A(t) & \triangle A_d(t) & \triangle E(t) & \triangle E_d(t) \end{bmatrix} = LF(t) \begin{bmatrix} G_1 & G_2 & G_5 & G_6 \end{bmatrix}. \tag{34}$$

Using the same method in Theorem 1, we can obtain the following results.

Corollary 1 For given scalars $0 \le \underline{\tau} < \overline{\tau}$, and μ, the uncertain stochastic system (33) is robustly asymptotically stable, and under zero condition for nonzero $w(t) \in L_2[0,\infty)$ the inequality $\|z(t)\|_2 \le \gamma \|w(t)\|_2$ holds, if there exist matrices $P > 0, Q > 0, R > 0, V > 0$, $Z_j > 0$, $(j = 1,2,3,4)$, T, and appropriately dimensioned matrices M_l, N_l, S_l, $(l = 1,2,3)$, and a scalar $\epsilon > 0$ such that the following LMI holds:

$$\widehat{\Psi} = \begin{bmatrix} \Psi_0 & \Psi_1 & \Psi_2 & \Gamma_i & \Upsilon_i \\ * & -\Psi_{11} & 0 & 0 & 0 \\ * & * & -\Psi_{22} & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & -\epsilon_i^{-1} I \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix} < 0, \tag{35}$$

where

$$\Psi_0 = \begin{bmatrix} \Phi_1 & \Phi_2^{\mathrm{T}} U_1 & \Phi_3^{\mathrm{T}} U_2 \\ * & -U_1 & 0 \\ * & * & U_2 \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & \Phi_{12} & N_3^{\mathrm{T}} & M_1 & -S_1 \\ * & \Phi_{22} & \Phi_{23} & M_2 & -S_2 \\ * & * & -V & M_3 & -S_3 \\ * & * & * & -Q & 0 \\ * & * & * & * & -R \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} \overline{\tau}N_1 & \tau M_1 & \tau S_1 \\ \overline{\tau}N_2 & \tau M_2 & \tau S_2 \\ \overline{\tau}N_3 & \tau M_3 & \tau S_3 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \quad \Psi_2 = \begin{bmatrix} N_1 & M_1 & S_1 \\ N_2 & M_2 & S_2 \\ N_3 & M_3 & S_3 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} A & A_d & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_3 = \begin{bmatrix} E & E_d & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_1 = \begin{bmatrix} L^{\mathrm{T}}P & 0 & 0 & 0 & 0 & \overline{\tau}L^{\mathrm{T}}Z_1 + \tau L^{\mathrm{T}}Z_2 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\Gamma_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & L^{\mathrm{T}}P + \overline{\tau}L^{\mathrm{T}}Z_3 + \tau L^{\mathrm{T}}Z_4 \end{bmatrix}^{\mathrm{T}},$$

$$\Upsilon_1 = \begin{bmatrix} G_1 & G_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} G_5 & G_6 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$U_1 = \overline{\tau}Z_1 + \tau Z_2, U_2 = P + \overline{\tau}Z_3 + \tau Z_4,$$

$$\Psi_{11} = \mathrm{diag}\{\overline{\tau}Z_1, \tau Z_2, \tau(Z_1 + Z_2)\},$$

$$\Psi_{22} = \mathrm{diag}\{\overline{\tau}Z_3, \tau Z_4, \tau(Z_3 + Z_4)\}$$

with

$$\Phi_{11} = PA + A^{\mathrm{T}}P + T + T^{\mathrm{T}} + Q + R + h^2 V + N_1 + N_1^{\mathrm{T}},$$

$$\Phi_{12} = PA_d - N_1 + N_2^{\mathrm{T}} - M_1 + S_1 - (1-\mu)T,$$

$$\Phi_{22} = -N_2 - N_2^{\mathrm{T}} - M_2 - M_2^{\mathrm{T}} + S_2 + S_2^{\mathrm{T}},$$

$$\Phi_{23} = S_3^{\mathrm{T}} - N_3^{\mathrm{T}} - M_3^{\mathrm{T}}, \quad \tau = \overline{\tau} - \underline{\tau}.$$

Proof Like the proof of Theorem 1, inequality (35) can be obtained.

Remark 5 Since the LMIs (6), (28), (31), and (35) in Theorem 1, Theorem 2, Theorem 3, and Corollary 1 respectively, can be easily solved by various efficient convex algorithms, in this paper, we utilize Matlabs LMI Control Toolbox which implements the interior-point algorithm. This algorithm

is significantly faster than classical convex optimization algorithms.

Remark 6 By iteratively solving the LMI (35) given in Corollary 1 with respect to $\overline{\tau}$ for different values of $\underline{\tau}$ and μ, one can find the maximum upper bound of time delay $\overline{\tau}$ for guaranteeing asymptotic stability of the system (33).

5. Numerical examples

In this section, we provide numerical examples and compare our results with the previous ones to demonstrate the effectiveness of the method proposed in this paper.

Example 1 Consider the system (33) with the following parameters:

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\quad \text{and } A_d=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},$$

and the uncertainties are described by inequality (4) with

$$\|\triangle A(t)\|\le 0.2,\ \ \|\triangle A_d(t)\|\le 0.2,$$
$$\|\triangle E(t)\|\le 0.2,\ \ \|\triangle E_d(t)\|\le 0.2,$$
$$L=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},\ G_1=G_2=G_5=G_6=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \tag{36}$$

By taking the parameter $\underline{\tau}=0$, and solving this convex optimization problem by Corollary 1, we obtain maximum admissible upper bound (MAUB) $\overline{\tau}$ for different values of μ as shown in Table 1, which clearly shows that the proposed method presented in this paper provides less conservative results than the previous results. The results obtained in this paper are significantly better than those in Ref. [50]. In general, the more free-weighting matrices are considered, and the less conservative performance can be achieved. However, the aforementioned method fails to obtain the results for the case where $\underline{\tau}=0$ is nonzero, which shows the merits of the results obtained in this paper.

Table 1. Comparisons among maximum allowable upper bounds of $\overline{\tau}$ for given μ, in Example 1.

Methods	$\underline{\tau}=0$			$\underline{\tau}=0.5$		$\underline{\tau}=1$
	$\mu=0.3$	$\mu=0.5$	$\mu=0.9$	$\mu=0.5$	$\mu=0.9$	$\mu=0.9$
$\overline{\tau}$ in Ref. [50]	0.7288	0.5252	0.1489	–	–	–
$\overline{\tau}$ by Corollary 1	0.7481	0.7571	0.7736	1.1028	1.2536	1.7536

Example 2 Consider the following uncertain system with time-varying delay satisfying condition (3):

$$\dot{x}(t)=A(t)x(t)+A_d(t)x(t-\tau(t)),$$
$$x(s)=\varphi(s),\ \ s\in[-\overline{\tau},0],$$
$$A=\begin{bmatrix}-2 & 0\\ 1 & -3\end{bmatrix},\ A_d=\begin{bmatrix}-1 & 0\\ -0.8 & -1\end{bmatrix}, \tag{37}$$

and the uncertainties are described by inequality (4) with

$$\|\triangle A(t)\|\le 0.2,\ \ \|\triangle A_d(t)\|\le 0.2,$$
$$L=\begin{bmatrix}\sqrt{0.2} & 0\\ 0 & \sqrt{0.2}\end{bmatrix},\ G_1=G_2=\begin{bmatrix}\sqrt{0.2} & 0\\ 0 & \sqrt{0.2}\end{bmatrix}. \tag{38}$$

When $\underline{\tau}=0$ and $\mu=0$, the upper bounds on time delay obtained from Corollary 1 are shown in Table 2. Note that the results in Ref. [49] are obtained by using Lemma 1 in Ref. [49] with Lemma 4 in this paper. Obviously, it can be seen from Table 2 that even in the special case of Theorem 1, namely, the lower bound is equal to 0, Corollary 1 can still give much better results than those obtained by the methods in Refs. [48]–[50].

Table 2. Comparisons among maximum allowable upper bounds of $\overline{\tau}$ in Example 2.

Methods	Ref. [48]	Ref. [49]	Ref. [50]	Our result
$\overline{\tau}$	0.3570	0.6811	0.6822	0.8545

Example 3 Consider the system (1), with the following parameters:

$$A=\begin{bmatrix}2.5 & 0.1\\ 5 & 3.8\end{bmatrix},\ A_d=\begin{bmatrix}1.1 & 0.2\\ 0.2 & -0.3\end{bmatrix},\ A_f=\begin{bmatrix}0.2 & 0.2\\ -0.2 & 0.9\end{bmatrix},$$
$$B_1=\begin{bmatrix}0.3 & 0.2\\ 0.1 & 0.4\end{bmatrix},\ B_w=\begin{bmatrix}0.5 & 0.01\\ -0.2 & 0.2\end{bmatrix},\ E=\begin{bmatrix}0.4 & 0.1\\ 0.01 & 0.6\end{bmatrix},$$
$$E_d=\begin{bmatrix}0.3 & 0.01\\ -0.3 & -0.1\end{bmatrix},\ E_f=\begin{bmatrix}0.1 & -0.01\\ 0.01 & 0.1\end{bmatrix},$$
$$C=\begin{bmatrix}0.1 & 0.5\\ 0.3 & -0.01\end{bmatrix},\ D=\begin{bmatrix}-0.15 & 0.1\\ 0.5 & -0.5\end{bmatrix},$$
$$L=\begin{bmatrix}0.2 & -0.1\\ 0.15 & 0.2\end{bmatrix},\ E_1=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix},\ E_2=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix},$$
$$E_3=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ E_4=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ E_5=\begin{bmatrix}1 & 0\\ 0 & 0.5\end{bmatrix},$$
$$\text{and}\quad E_6=\begin{bmatrix}1 & 0\\ 0 & 0.5\end{bmatrix}.$$

The prescribed H_∞ performance level is chosen as $\gamma=0.5$. In order to design a robust delay-dependent H_∞ control, taking $u=0.5$, $\underline{\tau}=0.6$, $\overline{\tau}=0.7$, and $h=0.5$, and using the Theorem 3 in the MATLAB LMI Toolbox the feasible solutions

are

$$\overline{X} = \begin{bmatrix} 2.2326 & -0.1517 \\ -0.1517 & 1.5555 \end{bmatrix}, \quad \overline{R} = \begin{bmatrix} 136.4141 & 4.3252 \\ 4.3252 & 56.4246 \end{bmatrix},$$

$$\overline{V} = \begin{bmatrix} 191.2014 & -2.9087 \\ -2.9087 & 98.2527 \end{bmatrix}, \quad \overline{Z}_1 = \begin{bmatrix} 24.6193 & 0.4660 \\ 0.4660 & 3.2148 \end{bmatrix},$$

$$\overline{Z}_2 = \begin{bmatrix} 94.0162 & 0.1255 \\ 0.1255 & 20.3758 \end{bmatrix}, \quad \overline{Z}_3 = \begin{bmatrix} 113.7728 & 8.1198 \\ 8.1198 & 35.9445 \end{bmatrix},$$

$$\overline{Z}_4 = \begin{bmatrix} 508.5523 & 76.5671 \\ 76.5671 & 490.2001 \end{bmatrix}, \quad \text{and} \quad \epsilon_1 = \epsilon_2 = 101.2060.$$

By Theorem 3, we can obtain the desired state-feedback controller as follows:

$$K = \begin{bmatrix} -91.2179 & -129.5677 \\ -93.1386 & -133.3792 \end{bmatrix}.$$

Therefore, the system of interest is robustly asymptotically stable.

6. Concluding remarks

In this work, we study the H_∞ control for uncertain stochastic systems with interval time-varying and distributed delays. Based on Lyapunov–Krasoviskii functional method, a new scheme of delay-dependent robust H_∞ controller design is presented in terms of LMIs. It has been shown that a desired state-feedback controller can be constructed when the given LMI is feasible. Numerical examples are given to demonstrate the effectiveness of the proposed method. From the numerical results, it follows that our results are more significantly improved than those from the existing methods.[48–50]

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