# A new model for film bulk acoustic wave resonators*

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(Received 5 March 2014; revised manuscript received 12 July 2014; published online 24 September 2014)

Based on cavity resonance and sandwich composite plate theory, this paper presents a universal three-dimensional (3D) theoretical model for frequency dispersion characterization and displacement profile shapes of the film bulk acoustic resonator (FBARs). This model provides results of FBAR excited thickness-extensional and flexure modes, and the result of frequency dispersion is proposed in which the thicknesses and impedance of the electrodes and the piezoelectric material are taken into consideration; its further simplification shows good agreement with the modified Butterworth–Van-Dyke (MBVD) model. The displacement profile reflects the vibration stress distribution of electrode shapes and the lateral resonance effect, which depends on the axis ratio of the electrode shapes $a/b$. The results are consistent with the 3D finite element method modeling and laser interferometry measurement in general.

**Keywords:** film bulk acoustic wave resonators, acoustic field vibration, cavity resonance, piezoelectric composite

**PACS:** 46.40.–f, 68.60.Bs, 62.25.Jk **DOI:** 10.1088/1674-1056/23/11/114601

## 1. Introduction

Film bulk acoustic wave resonators (FBARs) are electromechanical resonators based on electroacoustic resonance, with high performance and compatibility in the low and medium GHz frequency range, and the extremely high-$Q$ factor makes them stable frequency-control devices.[1–5] Most piezoelectric FBARs are of the thickness mode type, and a typical FBAR structure consists of a resonant acoustic cavity formed by a piezoelectric film sandwiched between two electrode films, as shown in Fig. 1.

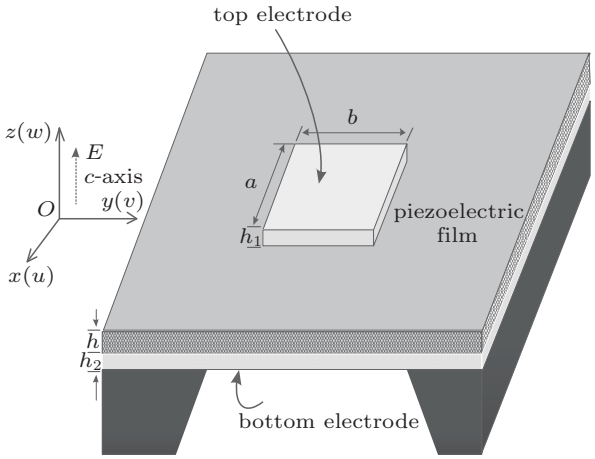



**Fig. 1.** The typical configurations of FBARs.

Characterization and accurate modelling of FBAR comprise, in general, the equivalent circuit approach[6] and the finite element method (FEM). The traditional piezoelectric theory using the equivalent circuit approach is applied to calculate the main resonant frequency of a one-dimensional (1D) resonator. For instance, the modified Butterworth–Van-Dyke (MBVD) model[7] allows one to obtain a more realistic representation of FBARs and acoustic resonators, thus it is the most accepted model for parameter extraction purposes. The transfer matrix method,[8] also based on the equivalent circuit approach, for which the impedance resonant spectrum of the FBAR needs to be obtained, can analyze multilayer composite resonators. However, these equivalent circuit approaches are not adequate to detect the existence of acoustic modes with a lateral vibration and meanwhile are performed through successive measurement. The FEM method can be employed in two-dimensional (2D) and three-dimensional (3D) modeling. The 2D models are adequate for configurations with narrow and long electrodes of infinitesimal thickness and mass, and the effects of the inertia and stiffness of metal electrodes on piezoelectric film need to be considered in high-frequency resonators. To calculate spurious responses observed in FBAR structures, full 3D FEM simulation that can accurately model the coupling and spurious modes is required.[9] The simulated displacement profiles can be validated by measurements of laser interferometry.[10]

## 2. Basic equations of new model and its solution

In this paper, we study the vibrations of the FBAR through a translational equation, and the mechanics analysis was performed using the exact equations of piezoelectricity. Our analysis is valid for 6-mm crystals (for example, ZnO, AlN). The $c$-axis is perpendicular to the plate surface, the piezoelectric film $c$-axis and the direction of the electric field is along the $z$ axis. Based on the resonant cavity theory, we

*Project supported by the National Natural Science Foundation of China (Grant No. 61275081).

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obtained the exact longitudinal frequency formula taking account of the electrode effect, analyzed the lateral vibration using the Prysakov sandwich theory[11], which supposes that the face sheet is a thin plate, taking into account the transverse deformation of the core. In this paper, the core was replaced by the piezoelectric material, and the force from the piezoelectric effect was introduced into the sandwich theory.

The FBAR typical structure is shown in Fig. 1. The piezoelectric film is of thickness $h$ and mass density $\rho_0$ with the $c$-axis normal to its surface. The electrodes are with thicknesses $h_1$ and $h_2$, and mass densities $\rho_1$ and $\rho_2$ respectively. In rectangular Cartesian coordinates, considered a FBAR vibrating material particle of elementary cubes, inertial and elastic restoring forces (or stresses) in the FBAR vibrating system are thus related through the translational equation of motion

$$\boldsymbol{\nabla}\cdot\boldsymbol{T}=T_x\cdot\hat{x}+T_y\cdot\hat{y}+T_z\cdot\hat{z}=\rho_m\frac{\partial^2 U}{\partial t^2}, \tag{1}$$

where

$$\begin{aligned}T_x&=\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{xy}}{\partial y}+\frac{\partial T_{xz}}{\partial z},\\ T_y&=\frac{\partial T_{yx}}{\partial x}+\frac{\partial T_{yy}}{\partial y}+\frac{\partial T_{yz}}{\partial z},\\ T_z&=\frac{\partial T_{zx}}{\partial x}+\frac{\partial T_{zy}}{\partial y}+\frac{\partial T_{zz}}{\partial z},\end{aligned}$$

$\rho_m$ $(m=0,1,2)$ are the densities of piezoelectric material, top and bottom electrodes, respectively, and the body-force has been ignored. The piezoelectric crystal film $c$-axis and the electric field direction is along the $z$ axis, and $T_{xx}=T_{yy}=0$. Thickness extensional (TE) mode is essentially employed in FBAR, while the thickness-shear mode is left out of the account, and therefore, $T_{xy}=T_{yx}=0$. According to Kirchhoff's assumptions, the sandwich core is relatively soft, so the sandwich stress parallel to the $xy$ plane can be ignored, thus $\partial T_{xz}/\partial z=0$, $\partial T_{yz}/\partial z=0$, and therefore, $T_{xz}$ and $T_{yz}$ are just functions of $x$ and $y$. Approximating the displacement $\boldsymbol{U}$ as $\boldsymbol{U}(x,y,z,t)=u(z,t)+w(x,y,t)$, where $u(z,\ t)$ is the longitudinal displacement along the $z$ axis and $w(x,y,t)$ is transverse deflection, respectively arising from $\partial T_{zz}/\partial z$ and $\partial T_{zx}/\partial x+\partial T_{zy}/\partial y$, equation (1) can be expressed as

$$\frac{\partial T_{zz}}{\partial z}=\rho_m\frac{\partial^2 u(z,t)}{\partial t^2}, \tag{2a}$$

$$\frac{\partial T_{zx}}{\partial x}+\frac{\partial T_{zy}}{\partial y}=\rho_m\frac{\partial^2 w(x,y,t)}{\partial t^2}, \tag{2b}$$

which represents a longitudinal wave propagating in the $z$ direction with displacement normal to the $xy$ plane. Consider a coordinate system with the $z$ axis in the direction of longitudinal wave propagation, then $T_{zz}$ in Fig. 1 of the top and bottom electrodes and the piezoelectric film region can be expressed as

$$c_1\frac{\partial u(z,t)}{\partial z},\quad c_2\frac{\partial u(z,t)}{\partial z},\quad c_{33}^{\mathrm{E}}\frac{\partial u(z,t)}{\partial z}-eE_z,$$

respectively, where $c_1$, $c_2$, and $c_{33}^{\mathrm{E}}$ denote the elastic constants. In addition, let $u(z,t)=u(z)\exp(\mathrm{j}\omega t)$, we obtain

$$\frac{\partial^2 u(z)}{\partial z^2}+k_i^2u(z)=0, \tag{3}$$

where $k_i=\omega\sqrt{\rho_i/c_i}$ $(i=1,2)$ and $k_0=\omega\sqrt{\rho_0/c^{\mathrm{D}}}$ are the propagation constants in the top and bottom electrodes and the piezoelectric film region, $\omega$ is the angular frequency, and $c^{\mathrm{D}}=c_{33}^{\mathrm{E}}\left(1+e/c_{33}^{\mathrm{E}}\varepsilon^{\mathrm{s}}\right)$, termed the piezoelectric stiffened elastic constant, where $\varepsilon^{\mathrm{s}}$ is the dielectric constant at constant stain, and $e$ is the piezoelectric stress constant.

To obtain the $\omega$ or frequency dispersion equation that reflects the properties of FBAR through the six boundary conditions is difficult, so we use the resonant cavity theory to achieve it. The absorption, diffraction and scattering will not be discussed, and the electrodes only change the phase, leading to a phase delay. We assumed that the round trip phase delay caused by the top and bottom electrodes are $\delta_1=2k_1h_1$ and $\delta_2=2k_2h_2$. Total refection happens on the top and bottom electrode surfaces. Reflection and transmission coefficients are $r_1$, $r_2$, and $t_1$, $t_2$ at the boundaries between the piezoelectric film and the electrodes. Taking the bottom electrode as an example, we sum up the first several passes and use the formula for an infinite geometrical series to obtain the displacement reflectivity, using $r_2$=$-r_2'$[6] and $t_2t_2'-r_2r_2'=1$, after which we obtain

$$\begin{aligned}R_2&=r_2+t_2t_2'\mathrm{e}^{\mathrm{i}\delta_2}+t_2t_2'r_2'\mathrm{e}^{2\mathrm{i}\delta_2}+t_2t_2'r_2'^2\mathrm{e}^{3\mathrm{i}\delta_2}+\cdots\\ &\quad+t_2t_2'r_2'^{(n-1)}\mathrm{e}^{\mathrm{i}n\delta_2}+\cdots\\ &=\frac{r_2+\mathrm{e}^{\mathrm{i}\delta_2}}{1+r_2\mathrm{e}^{\mathrm{i}\delta_2}},\end{aligned}$$

let $r_2=(Z-Z_2)/(Z+Z_2)$, where $Z$ and $Z_2$ are the piezoelectric film acoustic impedance and the bottom electrode impedance, receptively, then the formula becomes

$$\begin{aligned}R_2&=\frac{Z^2\cos^2\delta_2+\mathrm{i}Z_2^2\sin^2\delta_2}{Z^2\cos^2\delta_2-\mathrm{i}Z_2^2\sin^2\delta_2}=\mathrm{e}^{\mathrm{i}\varphi_2};\\ \varphi_2&=2\arctan\left(\frac{Z_2}{Z}\tan\frac{\delta_2}{2}\right).\end{aligned} \tag{4}$$

Similarly, the longitudinal wave reflectivity of the top electrode can be written as

$$\begin{aligned}R_1&=\frac{r_1+\mathrm{e}^{\mathrm{i}\delta_1}}{1+r_1\mathrm{e}^{\mathrm{i}\delta_1}}=\mathrm{e}^{\mathrm{i}\varphi_1};\\ \varphi_1&=2\arctan\left(\frac{Z_1}{Z}\tan\frac{\delta_1}{2}\right).\end{aligned} \tag{5}$$

By using the same method, we can obtain the transmittances of the top and the bottom electrode surfaces

$$R_1 = \frac{t_1}{1 + r_1 \mathrm{e}^{\mathrm{i}\delta_1}}, \quad R_2 = \frac{r_2}{1 + r_2 \mathrm{e}^{\mathrm{i}\delta_2}}.$$

A stable standing wave pattern will be established in the piezoelectric film, and the resonance frequencies of a given spatial mode are determined with the requirement that the complete round trip phase shift through the cavity should be some multiples of $2\pi$

$$2k_0 h + \varphi_1 + \varphi_2 = 2q\pi, \quad q = 1,3,5,\ldots \tag{6}$$

Using Eqs. (4) and (5) and substituting $k_1 h_1$ and $k_2 h_2$, for $\delta_1$ and $\delta_2$ into Eq. (6), it follows that

$$k_0 h + \arctan\left(\frac{ZZ_1 \tan(k_1 h_1) + ZZ_2 \tan(k_2 h_2)}{Z^2 + Z_1 Z_2 \tan(k_2 h_2)\tan(k_1 h_1)}\right) = q\pi,$$
$$q = 1,3,5,\ldots, \tag{7}$$

which is a transcendental equation with the angular eigenfrequency. Solving Eq. (7) is difficult, yet it could be dealt with by using numerical simulation or the graphical method.

Then, with the displacement and resultant force boundary conditions, the harmonic displacement solution is given by

$$\begin{aligned} u_q(z) &= C_q \frac{t_1 \exp(\mathrm{i}k_1 h_1/2)}{1 + r_1 \exp(2\mathrm{i}k_1 h_1)} \cos\left(k_1\left(z + \frac{h_1}{2}\right)\right), \\ &\quad h/2 < z < h/2 + h_1, \\ &= C_q \left(\exp(-\mathrm{i}(k_0 z - \varphi_1)) + \exp(\mathrm{i}(k_0 z + \varphi_2))\right), \\ &\quad z < |h/2|, \\ &= C_q \frac{t_2 \exp(\mathrm{i}k_2 h_2/2)}{1 + r_2 \exp(2\mathrm{i}k_2 h_2)} \cos\left(k_2\left(z + \frac{h_2}{2}\right)\right), \\ &\quad -h/2 - h_2 < z < -h/2, \end{aligned} \tag{8}$$

where the constant $C_q$ is the maximum displacement. Equation (9) represents the vertical displacement caused by $T_{zz}$, and if the thicknesses and materials of the top and bottom electrodes are the same, then $u_q(z) = -u_q(-z)$, symmetric relative to the mean plane. This solution is known as the symmetric wave, with the maximum of displacement appears at the electrode surface.

Equation (2b) represents the transverse displacement, which is also called transverse deflection and can be expressed as $w(x,y,t)$ in Cartesian coordinates. The ratio of total thickness to width of FBAR is usually less than 1/6, so the transverse deflection of FBAR can be solved by the thin plane sandwich bending theory. We follow the Prysakov analysis model, and take the impact of piezoelectric film into consideration, then the shearing forces $T_{zx}$ and $T_{zy}$ are given by $T_{zx} = Q_x/(h_1 + h_2 + h)$ and $T_{zy} = Q_y/(h_1 + h_2 + h)$. The total shearing forces $Q_x$, $Q_y$, the bending moments $M_x$, $M_y$, and the twisting moments $M_{xy}$ are related by the following equations:

$$\begin{cases} \dfrac{\partial M_x}{\partial x} + \dfrac{\partial M_{xy}}{\partial y} + Q_x = 0, \\ \dfrac{\partial M_{xy}}{\partial x} + \dfrac{\partial M_y}{\partial y} + Q_y = 0. \end{cases} \tag{9}$$

Substituting Eq. (9) into Eq. (2b) yields

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \bar{\rho}\frac{\partial^2 w(x,y,t)}{\partial t^2} = 0, \tag{10}$$

where $\bar{\rho} = \rho_1 h_1 + \rho_2 h_1 + \rho_0 h$. When considering the piezoelectric effect, the electromechanical constitutive equations for the piezoelectric are

$$\begin{aligned} S_1 &= s_{11}^D T_1 + s_{12}^D T_2 + g_{31} D_3, \\ S_2 &= s_{12}^D T_1 + s_{22}^D T_2 + g_{31} D_3, \\ E &= -g_{31}(T_1 + T_2) + \beta_{33}^T D_3, \end{aligned} \tag{11}$$

where $T_i$ $(i = 1,2)$ is the stress, $S_i$ $(i = 1,2)$ is the strain, $E$ is the electric field, and $D$ is the electric displacement. The parameters $s$, $g$, $\beta$, etc. represent the elastic, piezoelectric, and dielectric impermeability constants for the piezoelectric material. From Eq. (11), we obtain

$$\begin{aligned} T_1 &= \frac{1}{s_{11}^{\mathrm{D}}(1-\sigma^2)}(S_1 + \sigma S_2) - \frac{g_{31}}{s_{11}^{\mathrm{D}}(1-\sigma)} D_3, \\ T_2 &= \frac{1}{s_{11}^{\mathrm{D}}(1-\sigma^2)}(\sigma S_1 + S_2) - \frac{g_{31}}{s_{11}^{\mathrm{D}}(1-\sigma)} D_3, \\ E &= -\frac{g_{31}}{s_{11}^{\mathrm{D}}(1-\sigma)}(S_1 + S_2) + \bar{\beta}_{33} D_3, \end{aligned} \tag{12}$$

where $\sigma$ is the piezoelectric Poisson's ratio, $\bar{\beta}_{33} = \beta_{33}^{\mathrm{T}}/(1 - k_p^2)$, and $k_{\mathrm{p}}$ is the electro–mechanical coupling coefficient. According to Kirchhoff deformation theory, the nonzero linear strains associated with the displacement fields in Eq. (12) are

$$S_1 = -\frac{\partial^2 w}{\partial x^2} z, \quad S_2 = -\frac{\partial^2 w}{\partial y^2} z, \quad S_6 = -\frac{\partial^2 w}{\partial x \partial y} z,$$

where $S_1$ and $S_2$ are the normal strains, and $S_6$ is the shear strain. Substituting them into Eq. (12), we can obtain the piezoelectric moments

$$\begin{aligned} M_x^c &= \int_{-\frac{h}{2}}^{\frac{h}{2}} T_1 z \mathrm{d}z = \frac{-h^3}{12 s_{11}^{\mathrm{D}}(1-\sigma^2)}\left(\frac{\partial^2 w}{\partial x^2} + \sigma\frac{\partial^2 w}{\partial y^2}\right) \\ &\quad - \frac{g_{31} h^2}{4 s_{11}^{\mathrm{D}}(1-\sigma)} D_3, \\ M_y^c &= \int_{-\frac{h}{2}}^{\frac{h}{2}} T_2 z \mathrm{d}z = \frac{-h^3}{12 s_{11}^{\mathrm{D}}(1-\sigma^2)}\left(\sigma\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\ &\quad - \frac{g_{31} h^2}{4 s_{11}^{\mathrm{D}}(1-\sigma)} D_3, \\ M_{xy}^c &= \int_{-\frac{h}{2}}^{\frac{h}{2}} T_6 z \mathrm{d}z = \frac{-h^3(1-\sigma)}{12 s_{11}^{\mathrm{D}}(1-\sigma^2)}\frac{\partial^2 w}{\partial x \partial y}. \end{aligned}$$

According to the Prysakov analysis model theory, the top and bottom electrode moments are

$$
\begin{aligned}
M_x^{f_1} &= -D_{f1}\left(\frac{\partial^2 w}{\partial x^2}+\sigma_{f1}\frac{\partial^2 w}{\partial y^2}\right),\\
M_y^{f_1} &= -D_{f1}\left(\sigma_{f1}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right),\\
M_{xy}^{f_1} &= D_{f1}(1-\sigma_{f1})\frac{\partial^2 w}{\partial x\partial y},\\
M_x^{f_2} &= -D_{f2}\left(\frac{\partial^2 w}{\partial x^2}+\sigma_{f2}\frac{\partial^2 w}{\partial y^2}\right),\\
M_y^{f_2} &= -D_{f2}\left(\sigma_{f2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right),\\
M_{xy}^{f_2} &= D_{f2}(1-\sigma_{f2})\frac{\partial^2 w}{\partial x\partial y},
\end{aligned}
$$

where $D_{f1}$ and $D_{f2}$ are the bending rigidity of the top and bottom electrodes, while $\sigma_{f1}$ and $\sigma_{f2}$ are the Poisson's ratios of the top and bottom electrodes. Then the total moments can be expressed as

$$
\begin{aligned}
M_x &= M_x^c + M_x^{f_1} + M_x^{f_2},\\
M_y &= M_y^c + M_y^{f_1} + M_x^{f_2},\\
M_z &= M_{xy}^c + M_{xy}^{f_1} + M_x^{f_2}.
\end{aligned}
\tag{13}
$$

Substituting Eq. (13) into Eq. (10), we obtain the equation of the deflection $w(x,y,t)$ as

$$
\begin{aligned}
&\left(D_c + D_{f1} + D_{f2}\right)\nabla^2\nabla^2 w\\
&-\frac{g_{31}h^2}{4s_{11}^{\mathrm{D}}(1-\sigma)}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)D_3+\bar{\rho}\frac{\partial^2 w}{\partial t^2}=0,
\end{aligned}
\tag{14}
$$

where $D_c$ is the piezoelectric bending rigidity. Carrying out the indicated integration in the third formula of Eq. (11) over the core region, we obtain

$$
D_3 = \frac{2V}{h\bar{\beta}_{33}} - \frac{g_{31}h}{4s_{11}^{\mathrm{D}}(1-\sigma)\bar{\beta}_{33}}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right). \tag{15}
$$

Substituting Eq. (15) into Eq. (14), and letting $\partial V/\partial x = \partial V/\partial y = 0$, we obtain

$$
\begin{aligned}
&\left(D_c + D_{f1} + D_{f2} - \frac{g_{31}^2 h^3}{16 s_{11}^{\mathrm{D}2}(1-\sigma)^2\bar{\beta}_{33}}\right)\\
&\times\nabla^2\nabla^2 w + \bar{\rho}\frac{\partial^2 w}{\partial t^2} = 0.
\end{aligned}
\tag{16}
$$

For calculating normal modes on the membranes, it is well-known that the normal vibrations of an elastic linear system are harmonic. Therefore, the deflection in normal vibrations of FBAR can be assumed to be $w(x,y,t) = w(x,y)\exp(\mathrm{j}\omega t)$, and substitution of it into Eq. (16) yields a four-order partial differential equation involving the natural mode $w(x,y)$

$$
\bar{D}\nabla^2\nabla^2 w(x,y) - \bar{\rho}\omega^2 w(x,y) = 0, \tag{17}
$$

where

$$
\bar{D} = D_c + D_{f1} + D_{f2} - \frac{g_{31}^2 h^3}{16 s_{11}^{\mathrm{D}2}(1-\sigma)^2\bar{\beta}_{33}},
$$

and $\nabla^2$ is the Laplacian. Equation (17) is the governing equation of the transverse vibration eigenvalue problem of FBAR. Since the electrodes are known to be responsible for strong energy trapping of vibration modes in FBAR, and the stable vibration does not exist outside the top electrode, it can be assumed that the deflection $w(x,y)$ along the top electrode edge is zero, so the boundaries of FBAR are simply supported edges. The stretched rectangular top electrode is at $0 < x < a$, $0 < y < b$, as shown in Fig. 1, then the solution to Eq. (17) takes the Navier double series form

$$
w_{mn}(x,y) = C_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \tag{18}
$$

where the constant $C_{mn}$ ($m, n = 1,2,3,\ldots$) represents the maximum displacement of FBAR in the transverse direction, which is inversely proportional to $\bar{D}$. Substituting Eq. (18) into Eq. (17), the characteristic lateral frequencies for FBAR are

$$
f_{mn} = \frac{1}{2\pi}\sqrt{\frac{\bar{D}}{\bar{\rho}}\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]}. \tag{19}
$$

## 3. Results analysis and discussion

The solution to Eq. (7) is a transcendental equation with angular eigenfrequency. We have dealt with a special case: $\delta_1 = \delta_2 = p\pi$ ($p = 1,2,3,\ldots$), and the displacement reflectivity $R_1 = R_2 = 1$. The electrodes do not affect longitudinal wave propagation in piezoelectric materials, so we can obtain $\omega_q$ or frequency $f_q$ from Eq. (7)

$$
f_q = \frac{\omega_q}{2\pi} = \frac{qc_0}{2h}, \quad q = 1,3,5,\ldots, \tag{20}
$$

where $c_0 = \sqrt{c^{\mathrm{D}}/\rho_0}$. Based on ANSI/IEEE Std. 176-1987, equation (20) is the antiresonance frequency. Another special case dealt is $Z^2 = Z_1 Z_2$, for which equation (7) becomes

$$
k_0 h + k_1 h_1 + k_2 h_2 = q\pi, \quad q = 1,3,5,\ldots
$$

The frequency $f_q$ of FBAR is

$$
f_q = \frac{q}{2}\left(h_1\sqrt{\frac{\rho_1}{c_1}}+h_2\sqrt{\frac{\rho_2}{c_2}}+h\sqrt{\frac{\rho_0}{c^{\mathrm{D}}}}\right)^{-1}.
$$

In this case, the acoustic impedance of the electrode and the piezoelectric film do not affect $f_q$. Complete transmission takes place between the piezoelectric film and the electrodes, thus the reflection at the boundaries of the two electrodes can be completely eliminated by choosing a quarter-wave layer between them with impedance equal to the geometric mean

of the impedances of two electrode layers. A similar phenomenon will be observed at $k_0h = 3\pi/2, 5\pi/2$, etc.

As typical values for the thickness of the electrode are on the micron or nanometer level, under which circumstances $\delta_1$ and $\delta_2$ are by $10^{-3}$ order of magnitude, so the tangent function and cotangent function can be replaced by Taylor series expansion. In this case, equation (7) reduces to

$$k_0h + \frac{Z_1}{Z}k_1h_1 + \frac{Z_2}{Z}k_2h_2 = q\pi, \quad q = 1,3,5,\ldots \tag{21}$$

It follows that the frequency $f_q$ of FBAR is

$$f_q = \frac{qc_0}{2h}\left(1 + \frac{\rho_1h_1 + \rho_2h_2}{\rho h}\right)^{-1/2} = \frac{qc_0}{2h\sqrt{1+R}}, \tag{22}$$

where the mass ratio $R = (\rho_1h_1 + \rho_2h_2)/\rho_0h$.

Based on ANSI/IEEE Std. 176-1987, equation (22) is the series resonance frequency, and this result is in good agreement with the literature.[8] From Eqs. (20) and (22), the effective coupling factor $k^2_{\text{eff}}$ and the resonance quality factor $Q_s$ can be obtained. Equation (22) shows that as the thicknesses of the electrodes $h_1$ and $h_2$ increase, while the working frequency of FBAR decreases. Therefore, the smaller the thicknesses of the electrodes are, the better the performance is. However, when the value of the thickness is small enough, the values of contact resistance and capacitive become larger.

The frequency dispersion formula of FBAR, equation (7), has some advantages in reflecting the physics nature of an FBAR. The factors considered include: thickness, impedance of the electrodes, and piezoelectric materials. These factors are not contained in MBVD and the Mason model that are derived from the equivalent circuit approach. Moreover, the characteristic lateral frequencies for FBAR can be obtained from Eq. (19), while the other two models are not capable.

According to the principle of rectangular resonant cavity, disregard the piezoelectric crystal anisotropy, and the frequency of the FBAR is given by

$$f_{qmn} = \sqrt{f_q^2 + f_{mn}^2} \approx f_q + \frac{f_{mn}^2}{2f_q}, \tag{23}$$

which shows that the spurious mode always exists. For $f_p \gg f_{mn}$, the higher the FBAR frequency is, the nearer the spurious appears,[12] which must be avoided. One method is reducing lateral resonance $f_{mn}$ numbers by an irregular top electrode.[13] Another method is to increase $f_q$ using materials with stronger bending rigidity to enlarge the lateral mode spacing.

Equation (18) represents transverse displacement caused by $T_{zx}$ and $T_{zy}$. The lateral resonance and transverse displacement depend on the axis ratio $a/b$ of the top electrode shapes. The modes displacement and lateral resonance are labeled by the ordered pair $(m, n)$. If the ratio $a/b$ is a rational number, degenerate vibrations occurs, for instance, (1, 2) and (2, 1).

Substituting $m = n = 1$ into Eq. (18), the fundamental mode is obtained. The electromechanical fields concentrate at the middle of an electrode where there is a stress concentration, not at the edge of the electrode.[14] Laser interferometer measurements can probe vibration displacement fields on the top electrode surface, which now can be theoretically explained by Eq. (18). This agrees well with the results in many other literatures.[10,14,15]

Considering the damage of the film residual stress in FBAR, the maximum of the displacement field should be avoided. Irregular top electrode shapes with no parallel sides can decrease the strength of the spurious modes. That is to say, if the solution of Eq. (18) is not analytical, the maximum can be avoided by focusing on a certain region. In the solution of Eq. (18), we assumed that the deflection region is the overlapping section of the top electrode and the bottom electrode. According to the energy trapping analysis, displacement decays exponentially, thus the deflection region is wider than the electrode overlapping section, therefore, the deflection region can only be obtained by experiment.

## 4. Conclusion

In summary, using the resonant cavity theory and the Prysakov sandwich model considering the piezoelectric effect, a direct method to characterize the piezoelectric film with two electrodes was presented. We obtained the frequency dispersion formula of FBARs for parallel and series resonance frequencies, where the effects of the material parameters, thickness and impedance on the resonance frequencies were explicitly expressed. The results of this simplification are in good agreement with those of the MBVD model, which means that the features of the resonance frequency of FBAR can be analyzed directly with our model.

The results for frequency dispersion and displacement profiles are useful for optimizing FBAR designs and performances, providing a theoretical foundation for FBARs. We admit that there are still some limits to this method: the conclusions above are obtained by theoretical analysis, and the thickness of the film and region of energy trapping need to be quantified accurately. Besides, it is difficult to precisely control the thickness of the piezoelectric films and the electrodes. However, results of the universal model can determine resonator layers thickness that correspond to the objective resonant frequency, and investigate the effect of the electrode shape on the lateral spurious modes.

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