Enhancement of electromagnetically induced transparency cooling by an optical cavity*

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We theoretically investigate an enhanced electromagnetically induced transparency (EIT) cooling method by introducing a high finesse cavity. We find that the quantum destructive interference that is induced by the EIT effect and the cavity coupling can eliminate all of the heating effects in the cooling process by choosing appropriate parameters. Compared with the EIT cooling scheme, a lower final temperature can be obtained under the same conditions in our scheme.

Keywords: electromagnetically induced transparency cooling, quantum interference, high finesse cavity

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1. Introduction

Motional ground-state atoms are of great value[1] in many fields, such as, high-precision spectroscopy measurement,[2] high-accuracy atomic clocks,[3,4] quantum computation based on trapped cold atoms.[5] Therefore, many kinds of cooling methods have been proposed to cool the trapped systems to their ground states, such as feedback cooling,[6] standing wave cooling,[7] Raman sideband cooling,[8,9] dark-state cooling,[10–12] resolved-sideband cooling,[13] and so on.[14]

One of the most popular methods for cooling trapped ions to the ground state is sideband cooling. When an ion is cooled to the Lamb–Dicke regime (LDR), which indicates that the possible motional range of the ion is smaller than the optical wavelength of the cooling lasers, three kinds of possible processes may be excited by the laser fields. Carrier transitions excite the internal degrees of freedom (DOF) of the ion without changing the motional quantum number. Blue (red) sideband transitions excite the internal DOF with increasing (decreasing) motional quantum number. In the strong confinement condition that trap frequency ν is much greater than the effective linewidth[15] of the optical transition $\Gamma_{\rm eff}$, that is, $\nu \gg \Gamma_{\rm eff}$, the red sideband can annihilate a phonon, when the red sideband transition rate is higher than the heating rates (carrier and blue sideband transitions), the ion can be cooled to its ground state. However, carrier and blue sideband transitions still exist, which are the obstacles for reaching lower phonon numbers.

For the sake of suppressing the carrier and blue sideband transitions, destructive interference is utilized. As a typical cooling method that employs the quantum interference, electromagnetically induced transparency (EIT)[16–18] has become a predominant cooling scheme for trapped ions because no resolved sideband is needed. The EIT cooling employs a pair of Raman lasers to couple two ground states to an excited dissipative state of a Λ-configuration ion. In the dressed state picture, one dark state is found, which can cancel the carrier transition. The theoretical final phonon number of the EIT cooling can be given by[17]

$$\langle n\rangle_{\rm st} = (\gamma/2\Delta)^2, \tag{1}$$

where the spontaneous emission rate is 2γ, and Δ is the detuning of the lasers. Although the EIT cooling has canceled the carrier transition and suppressed blue sideband transition to some extent, the blue sideband transition still exists.

Several methods have been proposed to further suppress the blue sideband transition and improve the EIT cooling.[19–21] Cavity cooling schemes have been proven to enhance the EIT cooling in an atom system.[21] Compared with the EIT cooling in free space, introducing the cavity makes the cooling more robust and more efficient. Cavity cooling schemes also have been proposed in ion systems. These schemes employ quantum correlation[22–29] to overcome the heating effects caused by the quantum interference between the laser fields and the cavity. With the help of a cavity, a motional ground state could be reached.[30–38] These cavity cooling schemes rely on the large cooperativity of the optical cavity; that is, cooperativity parameter $\mathcal{C}$ should meet the condition

$$\mathcal{C} = \frac{\tilde{g}^2}{\kappa\gamma} \gg 1, \tag{2}$$

where the parameter $\tilde{g}$ is the effective atom–cavity Rabi frequency, 2γ is the rate of spontaneous emitting process, and 2κ is the cavity leak rate. When lasers are resonant with cavity and the leak rate of the cavity is very small, the population of the excited state is nearly zero and the diffusion almost vanishes.[24] On the other hand, by tuning the frequency of the laser and cavity fields to a proper value, heating absorption

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can be suppressed by destructive quantum interference.[25,26] Therefore, introducing an optical cavity provides us with a promising way to enhance the EIT cooling and reach a lower phonon number for a trapped ion.

In this article, we provide a scheme to enhance the EIT cooling of a trapped three-level Λ-configuration ion by introducing a high-finesse optical cavity. We find that the diffusion can be canceled by the EIT effect, and the blue sideband transition can be suppressed by the destructive quantum interference between the cavity and ion. Analytical calculation shows that the cooling limit of this new scheme is

$$\langle n\rangle_{\rm st} = \frac{1}{\mathcal{C}}, \tag{3}$$

which is lower than that of the EIT cooling under same parameters.

The structure of this paper is arranged as follow. In Section 2, the model and basic equations of the enhanced EIT cooling are given. In Section 3, the analytical results of the ground state of the new scheme are derived and the cooling processes are discussed. Finally, numerical simulation is performed to show that the numerical results are in good agreement with the analytical results.

2. Model and basic equation

A Λ-configuration ion with mass M is trapped in a Paul trap. We consider a situation where the radial confinement is much stronger than that of the axial direction, so only one-dimensional movement along the axis should be concerned. The focused energy levels of the ion are an excited state $|\mathrm{e}\rangle$ and two ground states $|\mathrm{g}\rangle$ and $|\mathrm{r}\rangle$, and the corresponding frequencies are $\omega_{\rm e}$, $\omega_{\rm g}$, and $\omega_{\rm r}$. Excited state $|\mathrm{e}\rangle$ couples with both of the ground states by two laser beams with detunings. One laser beam that drives the transition $|\mathrm{e}\rangle \leftrightarrow |\mathrm{g}\rangle$ with a Rabi frequency $\Omega_{\rm g}$ and detuning $\Delta_{\rm g}$ is called the "coupling" laser. Another laser beam that drives the transition $|\mathrm{e}\rangle \leftrightarrow |\mathrm{r}\rangle$ with a Rabi frequency $\Omega_{\rm r}$ and detuning $\Delta_{\rm r}$ is called the "driven" laser. Here we make the assumption that the parameter setup can lead to the EIT effect; that is, $\Delta_{\rm g} = \Delta_{\rm r} = \Delta$. For an enhanced EIT cooling scheme, the ion is also coupled to a high-finesse cavity with a frequency $\omega_{\rm c}$, as shown in Fig. 1(a), another transition is induced between the ion and cavity mode with an effective cavity Rabi frequency $\tilde{g}$, as shown in Fig. 1(b).

In the following, we consider that the ion is already in the LDR, and the Hamiltonian of the trapped system can be written as follows:

$$\mathcal{H} = \mathcal{H}_{\rm free} + \mathcal{H}_{\rm EIT} + \mathcal{H}_{\rm cavity}, \tag{4}$$

where $\mathcal{H}_{\rm free}$ denotes the free Hamiltonian of the system, including ion, phonon, and cavity mode ($\hbar = 1$)

$$\mathcal{H}_{\rm free} = \nu b^{\dagger} b - \delta_{\rm c} a^{\dagger} a - \Delta_{\rm g} |\mathrm{e}\rangle\langle \mathrm{e}|, \tag{5}$$

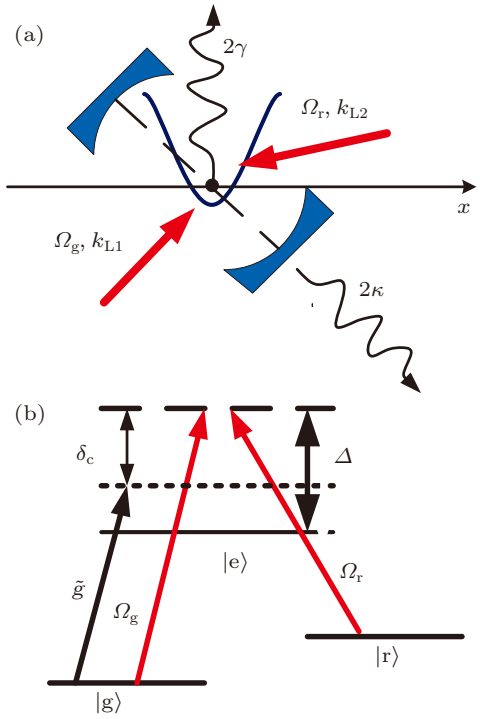


Fig. 1. (color online) (a) Two ground states of a Λ-configuration ion are coupled to the excited state by two laser beams with the same detuning, which can lead to the EIT effect. Cavity decay and spontaneous emission are the two possible ways to dissipate the system. (b) The three focused energy levels of the ion are the excited state $|\mathrm{e}\rangle$, the ground states $|\mathrm{g}\rangle$, and the ground state $|\mathrm{r}\rangle$. Cavity field and "cooling" laser drive the transition $|\mathrm{e}\rangle \leftrightarrow |\mathrm{g}\rangle$, and "driven" laser drives the transition $|\mathrm{e}\rangle \leftrightarrow |\mathrm{r}\rangle$. The other parameters are defined in Section 2.

where the Hamiltonian is described in the reference frame rotating at the laser frequency ω_{L1}; a and $a^{\dagger}$ stand for the annihilation and creation operators for the cavity photon states; and, b and $b^{\dagger}$ are the annihilation and creation operators of the phonon states. The detunings are defined by

$$\begin{aligned} \delta_{\rm c} &= \omega_{\rm L1} - \omega_{\rm c}, \\ \Delta_{\rm g} &= \omega_{\rm L1} - (\omega_{\rm e} - \omega_{\rm g}). \end{aligned} \tag{6}$$

$\mathcal{H}_{\rm EIT}$ is the interaction induced by the lasers, which is given by

$$\begin{aligned} \mathcal{H}_{\rm EIT} &= \Omega_{\rm g} |\mathrm{e}\rangle\langle g| [1 + \mathrm{i}\eta_{\rm g}(a + a^{\dagger})] \\ &\quad + \Omega_{\rm r} |\mathrm{e}\rangle\langle r| [1 + \mathrm{i}\eta_{\rm r}(a + a^{\dagger})] + \mathcal{H}.c., \end{aligned} \tag{7}$$

$\eta_{\rm g}$ and $\eta_{\rm r}$ are the Lamb–Dicke parameters,

$$\begin{aligned} \eta_{\rm g} &= \boldsymbol{k}_{\rm L1} \cdot \boldsymbol{e}_x \sqrt{\frac{1}{2M\nu}}, \\ \eta_{\rm r} &= \boldsymbol{k}_{\rm L2} \cdot \boldsymbol{e}_x \sqrt{\frac{1}{2M\nu}}, \end{aligned} \tag{8}$$

where $\boldsymbol{k}_{\rm L1}$ and $\boldsymbol{k}_{\rm L2}$ are the wave vectors of the two laser beams, and $\boldsymbol{e}_x$ is the unit vector of the motional axis. $\mathcal{H}_{\rm cavity}$ denotes the interaction between the ion and the cavity mode

$$\mathcal{H}_{\rm cavity} = g(a|\mathrm{e}\rangle\langle \mathrm{g}| + a^{\dagger}|\mathrm{g}\rangle\langle \mathrm{e}|)[\sin\phi + \eta_{\rm c}\cos\phi(b + b^{\dagger})]. \tag{9}$$

Here, ϕ is a phase that is related to the position of the trap

center. η_c is the Lamb–Dicke parameter in the form of

$$\eta_c = \boldsymbol{k}_c \cdot \boldsymbol{e}_x \sqrt{\frac{1}{2M\nu}} \cot\phi, \tag{10}$$

where $\boldsymbol{k}_c$ is the wave vector of the cavity.

Now we can write the master equation of the system as

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[\mathcal{H},\rho] + \mathcal{L}\rho, \tag{11}$$

where $\mathcal{L}$ denotes the Liouvillian operator of the system

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}^{\gamma} + \mathcal{L}^{\kappa},\\
\mathcal{L}^{\gamma}\rho &= \gamma_g(|g\rangle\langle e|\tilde{\rho}|e\rangle\langle g| - |e\rangle\langle e|\rho - \rho|e\rangle\langle e|)\\
&\quad + \gamma_r(|r\rangle\langle e|\tilde{\rho}|e\rangle\langle r| - |e\rangle\langle e|\rho - \rho|e\rangle\langle e|),\\
\mathcal{L}^{\kappa}\rho &= 2\kappa(a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a),
\end{aligned} \tag{12}$$

where $\mathcal{L}^{\gamma}$ is the dissipation by spontaneous emission; γ_g and γ_r denote the decay rate from state $|e\rangle$ to states $|g\rangle$ and $|r\rangle$, respectively; and, $\mathcal{L}^{\kappa}$ denotes the dissipation by cavity decay.

$$\tilde{\rho} = \int_{-1}^{1} \mathrm{d}\cos\theta_0 W(\theta_0)\, \mathrm{e}^{\mathrm{i}k_j x\cos\theta_0}\rho\, \mathrm{e}^{-\mathrm{i}k_j x\cos\theta_0} \tag{13}$$

describes the motional recoil at the angle θ_0 by emitting a photon, with a probability $W(\theta_0)$, which is related to the geometry of the setup.

Given that we have assumed that the ion is in the LDR, that is, the Lamb–Dicke parameter is very small, the internal and external DOF are weakly coupled, adiabatically eliminating the internal DOF as the steady state is acceptable. We further assume that the states $|g\rangle$ and $|e\rangle$ are weakly coupled by the "cooling" laser, namely, $\Omega_g \ll \max\{|\Delta|,\gamma\}$. Meanwhile, the coupling of the cavity field and the "driven" laser are much stronger; that is,

$$\Omega_g \ll \Omega_r, \tilde{g}. \tag{14}$$

In this case, the initial internal state, the starting point of the heating/cooling processes, is $|g,0_c\rangle$ and the Fock state of the photon in the cavity is represented by m_c. Because of small Lamb–Dicke parameter and weak coupling assumption, second- and higher-order transitions are ignored, we only need to consider the lowest cavity excitation $|g,1_c\rangle$ and the internal DOF; namely, $|g,0_c\rangle, |r,0_c\rangle$ and $|e,0_c\rangle$. We can then regard the concerned states as two groups: one is the ground-state group $|g,0_c\rangle$, and the other is excited-state group, which consists of the following states: $|r,0_c\rangle$, $|e,0_c\rangle$, and $|g,1_c\rangle$. Now we can rewrite Eq. (4) as

$$\mathcal{H} = \mathcal{H}_c + \mathcal{V}. \tag{15}$$

Then, $\mathcal{H}_c$ is

$$\begin{aligned}
\mathcal{H}_c &= \nu b^{\dagger}b - \delta_c a^{\dagger}a - \Delta_g|e\rangle\langle e| + g(a\sigma^{\dagger} + a^{\dagger}\sigma)\\
&\quad + \Omega_r(|e\rangle\langle r| + |r\rangle\langle e|),
\end{aligned} \tag{16}$$

where $\sigma = |g\rangle\langle e|$ and $\sigma^{\dagger}$ denotes the adjoint. The $\mathcal{V}$ is the optical pump transition, which includes the carrier transitions $\mathcal{V}_{0I}^{0}$, sideband transitions $\mathcal{V}_{0I}^{1}$ of the ground-state group to excited-state group, and the excited-state group sideband transitions $\mathcal{V}_{1I}$,

$$\begin{aligned}
\mathcal{V} &= \mathcal{V}_{0I}^{0} + \mathcal{V}_{0I}^{1} + \mathcal{V}_{1I},\\
\mathcal{V}_{0I}^{0} &= \Omega_g(\sigma + \sigma^{\dagger}),\\
\mathcal{V}_{0I}^{1} &= \eta_g\Omega_g(b^{\dagger} + b)\sigma_x^{g},\\
\mathcal{V}_{1I} &= \eta_c\tilde{g}(a\sigma^{\dagger} + a^{\dagger}\sigma)(b + b^{\dagger}) + \eta_r\Omega_r(b + b^{\dagger})\sigma_x^{r},
\end{aligned} \tag{17}$$

where $\tilde{g} = g\sin\phi$ $(\phi \neq 0)$ is the effective atom–cavity Rabi frequency, and $\sigma_x^{j} = \mathrm{i}(|j\rangle\langle e| - |e\rangle\langle j|)$ $(j = g, r)$.

3. Analytical results and cooling processes

3.1. Explicit form of the scattering rate

In the LDR, because of the small Lamb–Dicke parameters, the multi-phonon processes can be ignored. The rate equation[7] of the cooling process is

$$\frac{\mathrm{d}}{\mathrm{d}t}p_n = p_{n+1}\Gamma_{n+1\to n} + p_{n-1}\Gamma_{n-1\to n} - p_n(\Gamma_{n\to n-1} + \Gamma_{n\to n+1}), \tag{18}$$

where $p_n = \langle n|\rho|n\rangle$ is the possibility of the n-th phonon state and the notation $\Gamma_{n\to n'}$ is the transition rate from the n-th phonon state to the n'-th phonon state.

By calculating all of the scattering processes from the n-th phonon state to the $(n\pm1)$-th phonon state in the first order of Lamb–Dicke parameters,[24,25,27] we obtain the transition rate $\Gamma_{n\to n\pm1}$ in the form of

$$\Gamma_{n\to n\pm1} = 2\alpha\gamma|\mathcal{T}_{\mathcal{L}}^{\mathrm{D}}|^2 + 2\gamma|\mathcal{T}_{\mathcal{L}}^{\gamma,\pm}|^2 + 2\kappa|\mathcal{T}_{\mathcal{K}}^{\kappa,\pm}|^2, \tag{19}$$

where α is the geometry factor, which denotes the momentum dispersion of the ion due to the spontaneous emission process. $\mathcal{T}_{\mathcal{L}}^{\mathcal{D}}$ is the recoil of a photon emitting from the excited state with n phonons. Here, $\mathcal{T}_{\mathcal{L}}^{\mathcal{D}}$ vanishes due to the EIT effect, and the diffusion (which will not be calculated below) can be suppressed. $|\mathcal{T}_{\mathcal{L}}^{\gamma,\pm}|^2$ and $|\mathcal{T}_{\mathcal{K}}^{\kappa,\pm}|^2$ describe scattering by spontaneous emitting a photon and cavity leak with increasing/decreasing one phonon, respectively,

$$\begin{aligned}
\mathcal{T}_{\mathcal{L}}^{\gamma,\pm} &= \langle\psi_{n\pm1}|W_{\gamma}^{0}G(E_n)\mathcal{V}_{0I}^{1}|\psi_n\rangle\\
&\quad + \langle\psi_{n\pm1}|W_{\gamma}^{0}G(E_n)\mathcal{V}_{1I}G(E_n)\mathcal{V}_{0I}^{0}|\psi_n\rangle,\\
\mathcal{T}_{\mathcal{K}}^{\kappa,\pm} &= \langle\psi_{n\pm1}|W_{\kappa}G(E_n)\mathcal{V}_{0I}^{1}|\psi_n\rangle\\
&\quad + \langle\psi_{n\pm1}|W_{\kappa}G(E_n)\mathcal{V}_{1I}G(E_n)\mathcal{V}_{0I}^{0}|\psi_n\rangle.
\end{aligned} \tag{20}$$

Here, the Green function $G(z)$ is defined by[38]

$$G(z) = \frac{1}{z - \mathcal{H}_{\mathrm{eff}}}, \tag{21}$$

where the effective Hamiltonian is

$$\mathcal{H}_{\mathrm{eff}} = \mathcal{H}_c - \mathrm{i}\kappa a^{\dagger}a - \mathrm{i}\gamma|e\rangle\langle e|, \tag{22}$$

and the energy of the initial state $|\psi_n\rangle$ is E_n. W_{γ}^{0}, W_{γ}^{1}, and W_{κ} are dissipative quantum jump operators of the system,

$$\begin{aligned}
W_{\gamma}^{0} &= |g\rangle\langle e|,\\
W_{\gamma}^{1} &= \mathrm{i}\eta_g(b^{\dagger} + b)|g\rangle\langle e|,\\
W_{\kappa} &= a.
\end{aligned} \tag{23}$$

Now we can obtain the transition rates by calculating Eq. (20),

$$\Gamma_{n\to n+1} = (n+1)A_+, \quad \Gamma_{n\to n-1} = nA_-. \tag{24}$$

Here, $A_\pm$ are the heating and cooling parameters,

$$A_\pm = 2\gamma|\mathcal{T}^{\gamma,\pm}|^2 + 2\kappa|\mathcal{T}^{\kappa,\pm}|^2, \tag{25}$$

with

$$\mathcal{T}^{\gamma,\pm} = \pm \mathrm{i}\Omega_\mathrm{g}\eta \frac{\nu(\mathrm{i}\kappa \mp \nu + \delta_\mathrm{c})}{f(\mp\nu)}, \tag{26}$$

$$\mathcal{T}^{\kappa,\pm} = \pm \mathrm{i}\Omega_\mathrm{g}\eta \frac{\nu\tilde{g}}{f(\mp\nu)}, \tag{27}$$

where

$$f(x) = (\mathrm{i}\kappa + x + \delta_\mathrm{c})[x(x+\mathrm{i}\gamma+\Delta) - \Omega_\mathrm{r}^2] - \tilde{g}^2 x, \tag{28}$$

$$\eta = \eta_\mathrm{r} - \eta_\mathrm{g}. \tag{29}$$

The corresponding $A_\pm$ are

$$A_\pm = 2\eta^2\Omega_\mathrm{g}^2\nu^2 \frac{\gamma[\kappa^2 + (\nu \mp \delta_\mathrm{c})^2]}{|f(\mp\nu)|^2}, \tag{30}$$

with

$$\begin{aligned} |f(\pm\nu)|^2 = {} & [(\delta_\mathrm{c} \pm \nu)(\nu^2 \pm \nu\Delta - \Omega_\mathrm{r}^2) \mp \nu(\kappa\gamma + \tilde{g}^2)]^2 \\ & + [\kappa(\nu^2 \pm \nu\Delta - \Omega_\mathrm{r}^2) \pm \nu\gamma(\delta_\mathrm{c} \pm \nu)]^2. \end{aligned} \tag{31}$$

By substituting Eq. (24) into the rate equation, we obtain another form of the rate equation

$$\frac{\mathrm{d}}{\mathrm{d}t}p_n = (n+1)A_- p_{n+1} - [(n+1)A_+ + nA_-]p_n + nA_+ p_{n-1}, \tag{32}$$

and the average phonon number takes the form of

$$\langle\dot{n}\rangle = -(A_- - A_+)\langle n\rangle + A_+. \tag{33}$$

The cooling rate is defined as

$$W = A_- - A_+. \tag{34}$$

If $W > 0$, the phonon number of the ion can reach

$$\langle n\rangle_\mathrm{st} = \frac{A_+}{A_- - A_+}. \tag{35}$$

3.2. Ground state cooling limit

In this subsection we will study the enhanced EIT cooling dynamics of the ion in a very good cavity. First let us assume that the resonator is lossless, namely, $\kappa = 0$. We obtain the cooling/heating parameters

$$A'_\pm = \frac{2\gamma\eta^2\nu^2\Omega_\mathrm{g}^2(\delta_\mathrm{c} \mp \nu)^2}{\gamma^2\nu^2(\delta_\mathrm{c} \mp \nu) + [(\delta_\mathrm{c} \mp \nu)(\nu^2 \mp \Delta\nu - \Omega_\mathrm{r}^2)^2 \pm \tilde{g}^2\nu]^2}. \tag{36}$$

It can be found that under the conditions

$$\delta_\mathrm{c} = \nu, \tag{37}$$

and

$$\Delta_\mathrm{g} = \Delta_\mathrm{r} = \Delta, \tag{38}$$

all of the heating transitions can be suppressed. Since the diffusion is also eliminated by the EIT effect, then

$$\langle n\rangle_\mathrm{st}|_{\kappa=0} = o(\eta^2). \tag{39}$$

This is the exact result for $\kappa = 0$.

For our experimental requirements, we should take care of the decay rate κ of the cavity, which is related to the finesse of the cavity,[39]

$$\kappa = \frac{c\pi}{2LF}, \tag{40}$$

where c is the speed of light, L is the length of cavity, and F is the finesse of the cavity. Therefore, we consider a high-finesse cavity and introduce corrections to the heating parameter

$$A'_+ \approx \frac{2\kappa\eta^2\Omega_\mathrm{g}^2}{\tilde{g}^2}. \tag{41}$$

Correspondingly, the final average phonon number of the ion is

$$\langle n\rangle_\mathrm{st} = \frac{4\gamma^2\nu^2 + [2(\nu^2 + \Delta\nu - \Omega_\mathrm{r}^2) - \tilde{g}^2]^2}{4\gamma^2\nu^2}\frac{1}{C}. \tag{42}$$

It can be seen that the minimum of the phonon number can be reached, when we choose the value of Δ as follows:

$$\Delta = \frac{\tilde{g}^2 + 2(\Omega_\mathrm{r}^2 - \nu^2)}{2\nu}. \tag{43}$$

In this case, the final mean phonon number is

$$\langle n\rangle_\mathrm{st} = \frac{1}{C}. \tag{44}$$

The corresponding cooling rate is

$$W = 2\eta^2\Omega_\mathrm{g}^2(\tilde{g}^2 - \kappa\gamma)/\tilde{g}^2\gamma. \tag{45}$$

3.3. Cooling and heating scattering processes

In the subsection, we will give a physical explanation for the elimination of the carrier and blue sideband heating.

A schematic illustration of suppressing heating transition is shown in Fig. 2. Initially, the system is in $|\mathrm{g},0_\mathrm{c},n\rangle$, and the three possible transition processes are: the carrier transition $|\mathrm{g},0_\mathrm{c},n\rangle \to |\mathrm{e},0_\mathrm{c},n\rangle$, the red sideband transition $|\mathrm{g},0_\mathrm{c},n\rangle \to |\mathrm{e},0_\mathrm{c},n-1\rangle$, and the blue sideband transition $|\mathrm{g},0_\mathrm{c},n\rangle \to |\mathrm{e},0_\mathrm{c},n+1\rangle$. If we set the dutuning as Eq. (38), then the EIT effect, which occurs between the transition $|\mathrm{g},0_\mathrm{c},n\rangle \to |\mathrm{e},0_\mathrm{c},n\rangle$ and $|\mathrm{r},0_\mathrm{c},n\rangle \to |\mathrm{e},0_\mathrm{c},n\rangle$, leads to the vanishment of the carrier transition. For the case of no cavity loss, there is no heating scattering because A_+ is zero, and the final phonon number is given by Eq. (39). However, for the finite cavity loss rate κ, the transition $|\mathrm{g},1_\mathrm{c},n+1\rangle \to |\mathrm{e},0_\mathrm{c},n+1\rangle$ and $|\mathrm{g},0_\mathrm{c},n\rangle \to |\mathrm{e},0_\mathrm{c},n+1\rangle$ still exists. By setting Eq. (37), the heating effect from blue sideband transition can be suppressed by quantum interference between the laser and cavity, which exists between $|\mathrm{g},1_\mathrm{c},n+1\rangle \to |\mathrm{e},0_\mathrm{c},n+1\rangle$ and $|\mathrm{g},0_\mathrm{c},n\rangle \to |\mathrm{e},0_\mathrm{c},n+1\rangle$, as shown by the blue solid lines in Fig. 2. Compared with the EIT cooling method, the enhanced EIT cooling scheme can suppress both the blue sideband transition and carrier transition.

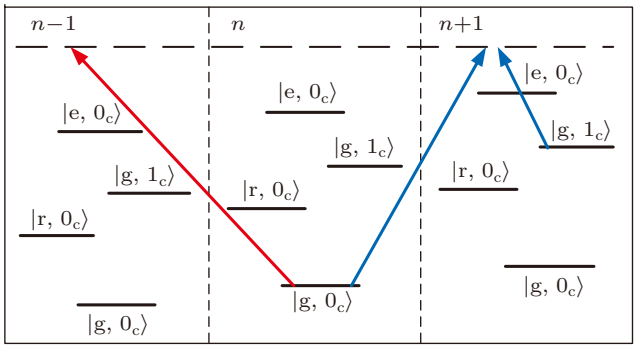


Fig. 2. (color online) Illustration of the cooling and heating processes. The carrier transition $|g,0_c,n\rangle \longrightarrow |e,0_c,n\rangle$ is canceled by the EIT effect. The blue sideband transition $|g,0_c,n\rangle \longrightarrow |e,0_c,n+1\rangle$ can destructively interfere with the transition $|e,0_c,n+1\rangle \longrightarrow |g,1_c,n+1\rangle$, leading to the elimination of the transition, which is shown by the blue solid lines. The red solid line shows the red sideband transition.

4. Numerical simulation and discussion

In order to verify the cooling effect of the enhanced EIT cooling scheme, we compare it with the EIT cooling scheme, in which the ion only couples to two laser beams (standard EIT cooing). We numerically simulate the cooling dynamics of the new cooling scheme by solving the master equation (11). For dipole transition, we have

$$W(\theta_0) = \frac{3}{4}(1+\cos^2\theta_0), \tag{46}$$

in an ordinary geometry.[40] For experimental consideration, we choose a high-finesse cavity with small decay rate parameter $\kappa = 0.02\nu$.

The result is shown in Fig. 3(a). Here we choose the same parameters for both of the two cooling schemes, and the final phonon of the enhanced EIT cooling method can reach the order of $O(10^{-3})$. Figure 3(b) shows the average phonon number of the standard EIT cooling scheme at a steady state as a function of cooling time, whose analytical cooling limit is given by Eq. (1). It can be seen that the average phonon number for this method can only reach an order of $O(10^{-2})$. As shown by Fig. 3, the new cooling scheme can reach a lower phonon number than the standard EIT scheme, which means that the standard EIT cooling is enhanced by introducing an optical cavity.

We then plot the final number of the enhanced EIT cooling as a function of δ_c, as shown in Fig. 4. As expected, the minimum phonon number of $\langle n\rangle_{st}$ is at $\delta_c = \nu$, which coincides with the quantum interference between the ion and cavity.

We should also notice that the enhancement of the EIT cooling comes from the coupling with high finesse cavity. If we set the atom–cavity Rabi frequency to be zero, that is, $\tilde{g} = 0$ and $\kappa = 0$, the heating/cooling parameters given by Eq. (30) are reduced to be

$$A''_{\pm} = \frac{2\gamma\eta^2\nu^2\Omega_g^2}{\gamma^2\nu^2 + (\nu^2 \mp \Delta\nu - \Omega_r^2)^2}. \tag{47}$$

From the cooling limit of Eq. (35), we obtain the final phonon number as

$$\langle n\rangle_{st} = (\gamma/2\Delta)^2, \tag{48}$$

which is identical with the cooling limit of the standard EIT cooling. This clearly shows that the EIT cooling is enhanced by introducing a high-finesse optical cavity.

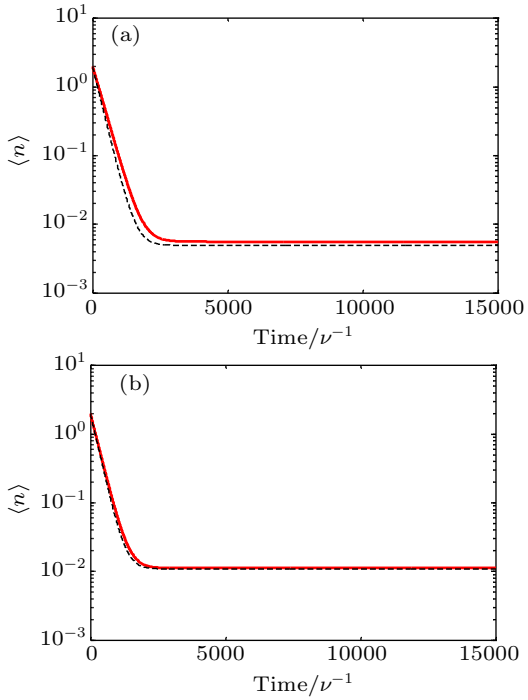


Fig. 3. (color online) Numerical simulation of cooling a trapped ion with two methods. (a) Enhanced EIT cooling scheme in the cooling conditions of (38) and (37). The parameters are $\eta_g = -0.05$, $\eta_r = \eta_c = 0.05$, $\phi = \pi/6$, $\gamma = 5\nu$, $\kappa = 0.02\nu$, $\Omega_g = \nu$, and $\Omega_r = 5\nu$. (b) Standard EIT scheme with $\tilde{g} = 5\nu$. The other parameters are the same as the enhanced EIT cooling scheme. The dashed line corresponds to the analytical result.

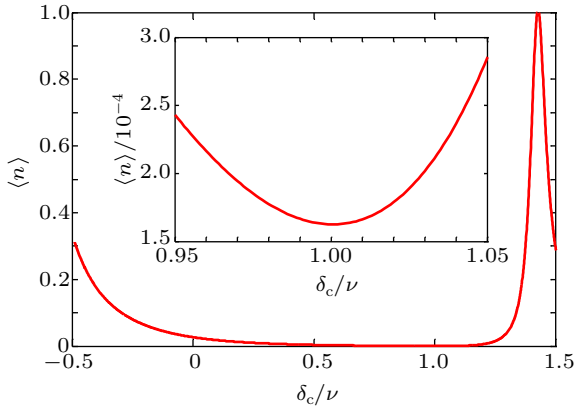


Fig. 4. (color online) Numeric simulation of phonon number $\langle n\rangle$ of enhanced EIT cooling as a function of δ_c under the condition of Eq. (43). The other parameters are $\eta_g = -0.05$, $\eta_r = \eta_c = 0.05$, $\phi = \pi/6$, $\gamma = 5\nu$, $\kappa = 0.02\nu$, $\Omega_g = \nu$, $\Omega_r = 5\nu$, and $g = 10\nu$. The inset shows the trend of the phonon number near ν.

5. Conclusions

Above all, we have presented an enhanced EIT cooling scheme for a three-level Λ-configuration trapped ion by in-

troducing another optical cavity, which is based on the cooling scheme of an atom system.[21] The heating effects induced by carrier transition and blue sideband heating transition can be suppressed by the EIT effect and quantum interference, respectively. Analytical results and numerical simulation show that the final phonon number of the enhanced EIT cooling scheme can reach the order of $O(1/\mathcal{C})$, which enhances the standard EIT cooling with the help of a high-finesse cavity.

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