Equivalent comparison and analysis between different nominal frequencies*

Li Zhi-Qi(李智奇)†, Wei Zhong(韦 中), Zhou Wei(周 渭), Song Hui-Min(宋慧敏), and Lu Wei-Hao(鲁伟昊)

Department of Measurement and Instrumentation, Xidian University, Xi'an 710071, *China*

(Received 6 April 2014; revised manuscript received 8 May 2014; published online 24 September 2014)

We analyze the phenomena of phase group synchronization between the different nominal frequency signals and propose a new theory of the equivalent comparison between them. The exact expression of the equivalent comparison is deduced. High resolution frequency measurement and phase comparison can be realized using this theory with the divider. For avoiding the frequency mixing, multiplication and synthesis, the system phase noise is improved and the higher resolution comparison and measurement are achieved between the different nominal frequencies by theory.

Keywords: phase difference, group synchronization, equivalent comparison, frequency measurement

PACS: 06.30.Ft **DOI:** 10.1088/1674-1056/23/11/110602

1. Introduction

The frequency source measurement and frequency phase comparison are key technologies to guarantee precision satellite navigation and positioning, and time synchronization.[1,2] The time comparison and measurement techniques that are usually used are the beat method and the double mixing time difference method. Because of the dead zone and nonlinearity, the development of the beat method is limited. Based on the double mixing time difference method, Mochizuki *et al.* used the developing digital processing technology to improve the measurement resolution, which is widely used in frequency stability measurement worldwide.[3] However, the double mixing time difference method needs the same nominal frequency signals, so the frequency synthesis is necessary in the method to make the two frequency signals have the same frequency. Due to the use of frequency synthesis technology, the noise is introduced inevitably.

Phase difference period shift plays a significant role in the time and frequency area, and it shows a kind of quantization relation.[4] The comparison between the same nominal frequency is ordered and simple as shown in Fig. 1.[5,6]

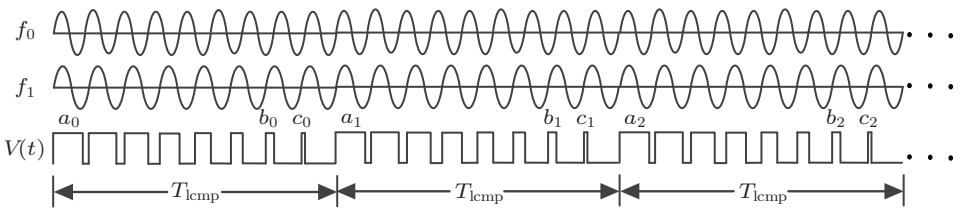


Fig. 1. The phase comparison between signals with the same nominal frequency.

In Fig. 1, f_0 and f_1 are the same nominal frequency, and $V(t)$ is the phase difference between them. We can see that the values of phase differences in the lowest common multiple period $T_{\rm lcmp}$ change from large to small, and the phase differences between different $T_{\rm lcmp}$ are corresponding. What is more, the phase differences in a $T_{\rm lcmp}$ form an arithmetic progression, and the common difference is the quantization phase shift resolution $T_{\rm qpsr}$. By the way, when the nominal frequencies of the comparison signals have a multiple number relationship, the situation is similar.

The variables used are as follows: $f_{\rm gcff}$ is the greatest common factor frequency of f_0 and f_1; $T_{\rm lcmp} = 1/f_{\rm gcff}$; $f_0 = Af_{\rm gcff}$; $f_1 = Bf_{\rm gcff}$; $T_{\rm qpsr} = 1/ABf_{\rm gcff}$.[5]

The phase difference φ and its corresponding interval T between two signals with the same nominal frequency f_0 have a linear relation as

$$\phi(t) = 2\pi f_0 T, \tag{1}$$

where the unit of ϕ is radian. The interval will be used as the phase difference in the following.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10978017 and 61201288) and the Fundamental Research Funds for the Central Universities, China (Grant No. JB140413).

†Corresponding author. E-mail: zhiqili_9@126.com

© 2014 Chinese Physical Society and IOP Publishing Ltd

The frequency deviation Δf also has a linear relation with the accumulative phase difference ΔT in the sample interval τ[7,8]

$$\frac{\Delta f}{f_0} = \frac{\Delta T}{\tau}. \tag{2}$$

The frequency measurement can be realized using the following equation:

$$f = f_0 - \Delta f = f_0 - \frac{f_0}{\tau}\Delta T. \tag{3}$$

According to the analysis of the phase difference group between arbitrary periodic signals, we proposed the theory of an equivalent comparison between them. Using this theory, high resolution comparison and measurement between different nominal frequency signals can be achieved through simple frequency division.

2. Comparison between two different nominal frequencies

However, for the comparison between two different nominal frequencies, the phase differences in a $T_{\rm lcmp}$ is unordered and complex, as shown in Fig. 2.[6,9]

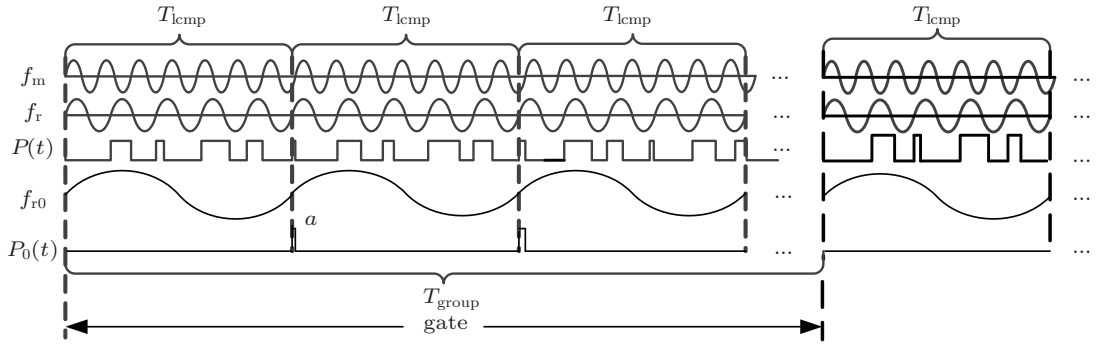


Fig. 2. The phase comparison between two different nominal frequencies.

In Fig. 2, $f_{\rm m}$ and $f_{\rm r}$ are two different nominal frequencies, and $P(t)$ is the phase difference between them. We can see that the phase differences in $T_{\rm lcmp}$ change irregularly, but the phase differences between different $T_{\rm lcmp}$ are corresponding. What is more, the phase differences in a $T_{\rm lcmp}$ form an arithmetic progression after ranking, and the common difference is the quantization phase shift resolution $T_{\rm qpsr}$. The nominal frequency of $f_{\rm m}$ is $f_{\rm m0}$, and $f_{\rm r0}$ is the greatest common factor frequency $f_{\rm gcff}$ of $f_{\rm m0}$ and $f_{\rm r}$. We call $f_{\rm r0}$ the equivalent comparison signal, $P_0(t)$ is the phase difference between $f_{\rm m}$ and $f_{\rm r0}$, which is regular in a group period $T_{\rm group}$. In addition, the phase differences between different $T_{\rm group}$ are corresponding.

We discover that the phase comparison can be directly done between the signals with different nominal frequencies: Δf is the frequency difference between $f_{\rm m0}$ and $f_{\rm m}$, and it is equal to $f_{\rm m0} - f_{\rm m}$, while Δf is usually very small. The phase differences between $f_{\rm m}$ and $f_{\rm r}$ in a $T_{\rm lcmp}$ can be taken as a unit. Corresponding phase differences can be extracted from every unit as $P_0(t)$ in Fig. 2. These phase differences are also ranked from small to large in a $T_{\rm group}$ because of the existance of Δf. Finally, the phase differences reach to synchronization.[6] The two compared signals are also synchronous.

The comparison between signals with different nominal frequencies is expanded to $T_{\rm lcmp}$, which means that the comparison is between $f_{\rm m}$ and $f_{\rm r0}$. Let $\Delta t_{\rm q}$ be the first phase difference between them after phase coincidence as a shown in Fig. 2. In fact, $\Delta t_{\rm q}$ is the accumulative phase difference between $f_{\rm m0}$ and $f_{\rm m}$ in a $T_{\rm lcmp}$, and its equation is

$$\Delta t_{\rm q} = \frac{f_{\rm m0}}{f_{\rm r0}}(T_{\rm m} - T_{\rm m0}). \tag{4}$$

According to Eqs. (2) and (3), we can obtain the following equation:

$$\begin{aligned} &\frac{\Delta f}{f_{\rm m0}} = \frac{\Delta t_{\rm q}}{\tau} = f_{\rm r0}\Delta t_{\rm q} \Longrightarrow \Delta f = f_{\rm m0} f_{\rm r0}\Delta t_{\rm q} \\ \Longrightarrow\ & f_{\rm m} = f_{\rm m0} - \Delta f = f_{\rm m0} - f_{\rm m0} f_{\rm r0}\Delta t_{\rm q} = f_{\rm m0}(1 - f_{\rm r0}\Delta t_{\rm q}). \end{aligned} \tag{5}$$

According to Eq. (5), we can obtain the frequency measuring $\Delta t_{\rm q}$.

The phase differences of $P_0(t)$ in a $T_{\rm group}$ form an arithmetic progression group, and the common difference is $\Delta t_{\rm q}$, $P_0(t)$ is made up of such arithmetic progressions. We can obtain the following equations:

$$\begin{aligned} &P_0(t) = \{\text{group}|\text{group}_0,\ \text{group}_1, \ldots, \text{group}_m\}, \\ &\text{group}_0 = \{\Delta T|\Delta T_{11},\ \Delta T_{12}, \ldots,\ \Delta T_{1n}\}, \\ &\text{group}_1 = \{\Delta T|\Delta T_{21},\ \Delta T_{22}, \ldots,\ \Delta T_{2n}\}, \\ &\ldots \\ &\text{group}_m = \{\Delta x|\Delta T_{m1},\ \Delta T_{m2}, \ldots,\ \Delta T_{mn}\}, \end{aligned} \tag{6}$$

where ΔT is the phase difference of $P_0(t)$, $\Delta T_{11} = 0$, $\Delta T_{a,b+1} - \Delta T_{a,b} = \Delta t_{\rm q}$, $\Delta T_{m+1,n} - \Delta T_{m,n} = n\Delta t_{\rm q}$, $n = T_{\rm m}/\Delta t_{\rm q}$ (n is taken as

an integer), $nT_{\rm lcmp} \leq 1/(f_{\rm m}, f_{\rm r}) < (n+1)T_{\rm lcmp}$, $(f_{\rm m}, f_{\rm r})$ is the $f_{\rm gcff}$ of $f_{\rm m}$ and $f_{\rm r}$. Although $\Delta T_{m+1,n} - \Delta T_{m,n}$ is equal to $n\Delta t_{\rm q}$, $\Delta T_{m+1,n}$ and $\Delta T_{m,n}$ are almost the same in fact.

In this case, the comparison between $f_{\rm m}$ and $f_{\rm r}$ can be taken as the comparison between $f_{\rm m}$ and $f_{\rm r0}$, so we achieve the equivalent comparison between signals with different nominal frequencies. In addition, the comparison between signals with the same nominal frequency is the special case, and every $T_{\rm lcmp}$ is a $T_{\rm group}$.

3. Test

Analyzing the above theory, there are two means to achieve frequency measurement by the equivalent comparison between the different nominal frequencies. One is based on Eq. (5), and the theory can be acquired by the phase difference measurement. The other is according to counting in the gate.[9–11] In the paper, the experiment of frequency measurement by the coincidence detection connects with the counting is adopted to prove the theory, and the functional block diagram is shown in Fig. 3.

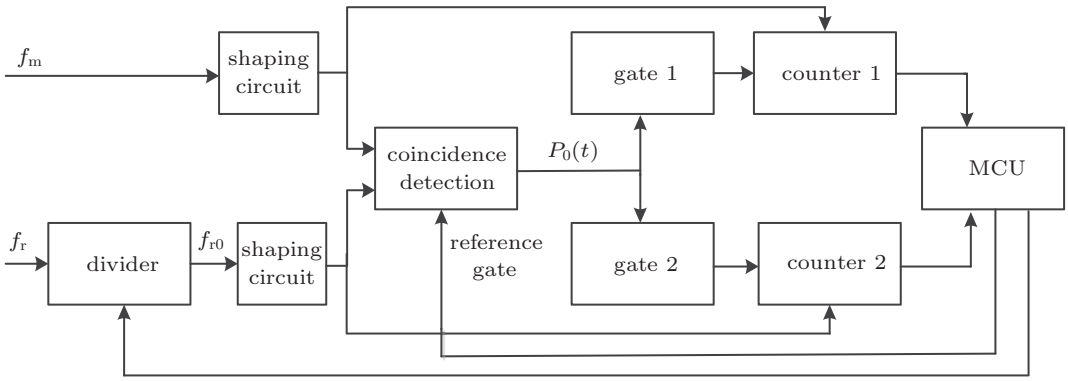


Fig. 3. The functional block diagram of frequency measurement based on the comparison between the different nominal frequencies.

In Fig. 3, $f_{\rm m}$ and $f_{\rm r}$ are two different nominal frequencies. The equivalent signal $f_{\rm r0}$ is acquired by setting the frequency division in the MCU. $P_0(t)$ is achieved by sending $f_{\rm m}$ and $f_{\rm r0}$ to the shaping and phase coincidence detection circuits. According to the reference gate produced by MCU, the phase coincidence detection circuit generates two real gates as the gate shown in Fig. 2, and the two counters count the cycle numbers of $f_{\rm m}$ and $f_{\rm r}$ in the gates. The frequency values can be calculated by the numbers. In addition, the exact phase coincidence position is not certain as the frequency is variable. Therefore, the rough frequency measurement is done first to estimate the $T_{\rm group}$. Then, a reference gate, which is a little smaller than $T_{\rm group}$, is produced by MCU. When a phase coincidence pulse is detected in the reference gate, the measurement gates are open. When the reference gate is closed and a phase coincidence pulse is detected, the measurement gates are closed. The measurement gates are the integer multiple of $T_{\rm group}$.

In this experiment, a frequency synthesizer HP8662 is used to output the measured signal, and its inner frequency standard is used to output the reference signal, which is 10 MHz. Because the gates open and close when the phase differences between $f_{\rm m}$ and $f_{\rm r0}$ reach synchronization, the counting error can be eliminated, and high precision can be obtained. Here the signals of 20.5 MHz, 18.6 MHz, 10.1 MHz, 17.05 MHz, and 18.22 MHz are chosen as the measured signals. At this time, the reference signal needs to be divided with different values. In the same condition, we use a DDS to take the place of the divider to measure the frequency. The DDS is used to do frequency consistency processing and makes the frequency of the reference signal equal to the nominal frequency value of the measured signal, and a frequency meter FCA 3000 produced by Tektronix is used to measure the frequencies, whose frequency resolution is 12 bit/s.

Table 1 shows the frequency stabilities of 20.5 MHz, 18.6 MHz, 10.1 MHz, 17.05 MHz, 10.22 MHz, and 18.22 MHz measured by equivalent comparison (with the division processing), frequency synthesis (with the DDS) and Tektronix FCA 3000. In addition, the frequency stabilities are calculated by the Allan variance formula.[12,13] Several measurements have been done for every frequency chosen using every mean, and the measurement data in Table 1 are the average values.

According to Table 1, it is obvious that the frequency stability obtained by the method of equivalent comparison is better than that obtained by the method of frequency synthesis or Tektronix FCA 3000. In the same condition, frequency measurement using the equivalent comparison shown in Fig. 3 can obtain a better resolution than using frequency synthesis or Tektronix FCA 3000, and the frequency divider value does not affect the measurement precision by the equivalent com-

parison. It also shows that the phase noise which is brought in by the divider is smaller than that brought in by the DDS. In addition, using the equivalent comparison to measure frequency is better than some existing technologies.

Table 1. The frequency stabilities (in units of s^{-1}).

Measured frequency	Equivalent comparison	Frequency divider value	Frequency synthesis	Tektronix FCA 3000
20.5 MHz	5.8×10^{-12}	20	1.2×10^{-11}	1.4×10^{-10}
18.6 MHz	1.0×10^{-11}	50	1.2×10^{-11}	1.4×10^{-10}
10.1 MHz	1.2×10^{-11}	100	1.4×10^{-11}	1.6×10^{-10}
17.05 MHz	1.0×10^{-11}	200	1.4×10^{-11}	1.8×10^{-10}
18.22 MHz	9×10^{-12}	500	1.1×10^{-11}	1.6×10^{-10}

4. Conclusion

The comparison and measurement between arbitrary periodic signals are significant in the time and frequency area, such as atomic clock frequency signal processing chain, frequency source measurement and calibration, time scale, time and frequency transfer, etc.[14] We have proved that our theory of the comparison between signals with different nominal frequencies is very useful in improving the frequency and phase comparison and measurement precision, so it is very important to do further thorough researches and many new experiments based on our theory.

Acknowledgment

We acknowledge the valuable suggestions and comments from Senior Engineer Xuan Zong-Qiang and Professor Yu Jian-Guo.

References

[1] Getting I A 1993 *IEEE Spectrum* **30** 36
[2] Weiss M A, Yao J and Li J 2012 *Proc. 44th PTTI Meeting*, November 26–29, 2012 Reston, Virginia, pp. 179–186
[3] Mochizuki K, Uchino M and Morikawa T 2007 *IEEE Trans. Instrum. Meas.* **56** 1887
[4] Bosco G C, Garcocz M, Lind K, Pogliano U, Rietveld G, Tarasso V, Voljc B and Zachovalová V N 2011 *IEEE Trans. Instrum. Meas.* **60** 2359
[5] Yu J G, Zhou W, Du B J, Dong S F and Fan Q Y 2012 *Chin. Phys. Lett.* **29** 070601
[6] Li Z Q, Zhou W, Zhou H, Zhang X P and Zhao J 2013 *Rev. Sci. Instrum.* **84** 025106
[7] Zhou W, Ou X, Zhou H, Wang H and Qu B Y 2006 *The Measurement and Control Technology of Time and Frequency* (Vol. 1) (Shaanxi: Xidian University Press) pp. 118–120 (in Chinese)
[8] Zhou W and Zhu G F 1986 *Frequency and Time Measurement* (Vol. 1) (Shaanxi: Science and Technology Press) pp. 63–65 (in Chinese)
[9] Miao M, Zhou W and Wang B 2012 *Rev. Sci. Instrum.* **83** 024706
[10] Li Z Q, Zhou W, Chen F X and Liu C G 2010 *Chin. Phys. B* **19** 090601
[11] Atsushi Y, Nobuyasu S, Shigeo N, Ying L, Hiroshi I, Hidekazu H, Motohiro K and Tetsuya I 2012 *Appl. Phys. Express* **5** 022701
[12] Sesia I, Galleani L and Tavella P 2011 *IEEE Trans. Aerosp. Electron. Syst.* **47** 884
[13] Jain S, Nandy S, Chakraborty G, Kumar C S, Ray R and Shome S N 2011 *IEEE Int. Conf. on Signal Processing, Communications and Computing*, September 14–16, 2011 New York, USA, pp. 1–4
[14] Anne L C, Jürgen A, Jelmer J R, Daniel O, Niels K and Eugene S P 2010 *New J. Phys.* **12** 065032