State-dependent event-triggered control of multi-agent systems*

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Event-triggered control has been recently proposed as an effective strategy for the consensus of multi-agent systems. We present an improved distributed event-triggered control scheme that remedies a shortcoming of some previous event-triggered control schemes in the literature. This improved distributed event-triggered method has no need for continuously monitoring each agent' neighbors. Moreover, each agent in the multi-agent systems will not exhibit the Zeno behavior. Numerical simulation results show the effectiveness of the proposed consensus control.

Keywords: distributed event-triggered control, consensus, rendezvous, multi-agent systems

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1. Introduction

The distributed cooperative control of multi-agent systems has attracted much attention in various distributed cooperative control scenarios, such as consensus,[1–4] formation control,[5,6] flocking,[7,8] and rendezvous.[9,10] The consensus problems are the most fundamental ones in the distributed cooperative control of multi-agent systems and have been actively studied in recent years.[1–4,11–13] To deal with the limited resource problem in communication networks, sampled-data consensus strategies have been used in Refs. [14] and [15]. However, since all agents in multi-agent systems need to obtain information actively and periodically from their neighbors and to update their controllers synchronously, a higher instantaneous communication processing capacity is required in the sampled-data consensus algorithms.

Event-triggered (event-based) sampling is an alternative to the traditional periodic sampling.[16,17] Event-triggered sampling means that it is the occurrence of an event rather than the passing of time determining the time of sampling. The occurrence of an event is determined by the output measurement exceeding a specified value. Åström and Bernhardsson[16] pointed out that event-based sampling gives better performance than periodic sampling. Tabuada[18] considered the event-triggered feedback sampled-data systems where the control task is triggered to execute whenever a certain error becomes large compared with the state norm, and excluded the Zeno behavior by ensuring the existence of a lower bound for the inter-execution time. Note that the implementation of event-triggered sampling in Ref. [18] is highly centralized and such a centralization is impractical in large-scaled systems. Wang and Lemmon[19] proposed a distributed approach for the implementation of event-triggered sampling in networked control systems where a subsystem broadcasts its state information only when the local state error exceeds a given threshold. More literature on the distributed event-triggered feedback scheme can be found in Refs. [20] and [21].

The distributed event-triggered control scheme has also been shown to be suitable for the cooperative control of multi-agent systems over the networks with limited resources.[22–24] Dimarogonas *et al.*[22] explored a centralized event-triggered formulation for the average consensus problems of multi-agent systems and extended the results to the distributed counterpart. Seyboth *et al.*[23] analyzed the average consensus of multi-agent systems with a time-dependent trigger function. Fan *et al.*[24] discussed the rendezvous problem of multi-agent systems by using distributed event-triggered controllers. A shortcoming among the above literatures is that the event-triggered condition (see for instance, Eq. (11) in Ref. [22] and Eq. (4) in Ref. [24]) for any agent requires the real-time information of its neighbors rather than the last event time information of its neighbors. This condition disobeys the original purpose of introducing the event-triggered control as a means for reducing the communication requirements. Though all the above works have realized that the extensive application of the embedded micro-processor with limited resources motivates researchers to develop distributed event-triggered control schemes, they may neglect that the state-dependent event-triggered conditions should also be subject to the limited resources of the embedded micro-processor. Moreover, recently, more and more researchers paid attention to the cooperative control of multi-agent systems with event-triggered control strategies.[25–27] Weng and Yue[25] investigated the distributed event-triggered cooperative attitude control of multiple rigid bodies with a leader–follower architecture. Guo *et al.*[26] studied the dis-

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tributed event-triggered transmission strategy for the sampled-data consensus of multi-agent systems. Zhou *et al.*[27] investigated the formation control of multi-agent systems with an event-triggered strategy.

The purpose of this paper is to remedy the drawback of the proposed distributed event-triggered control scheme and present an improved distributed state-dependent event-triggered approach with no need to continuously monitor the states of each agent's neighbors. The functions describing the state-dependent triggering conditions are developed in such a way that the triggering events supervising the broadcast by an agent only depend on the local information at the triggering time.

The rest of the paper is organized as follows. Section 2 presents the problem treated in this paper along with a number of preliminary notions on the graph theory. The improved state-dependent distributed event-triggered scheme is proposed in Section 3. Simulation results are presented in Section 4. The last section contains the conclusion of this paper.

2. Problem formulation and preliminaries

2.1. Algebraic graph theory

Some concepts and facts in the algebraic graph theory are given as follows. An undirected graph G is a pair (V,E), where $V=\{v_1,v_2,\ldots,v_n\}$ is the set of nodes, and $E\subseteq V\times V$ is the set of edges of the graph. Two nodes v_i and v_j $(i,j=1,2,\ldots,n)$ are adjacent or neighboring, if (v_i,v_j) is an edge of graph G. The set of all neighbor nodes of v_i is denoted as $N_i=\{v_j \mid (v_i,v_j)\in E, i,j=1,2,\ldots,n\}$. The number of the neighbor nodes of v_i is denoted as $|N_i|$, $i=1,2,\ldots,n$. The adjacency matrix $\boldsymbol{A}=[a_{ij}]\in\mathbf{R}^{n\times n}$ of an undirected graph with node set $V=\{v_1,v_2,\ldots,v_n\}$ is defined such that $a_{ij}=a_{ji}$ is a positive weight if $(v_i,v_j)\in E$, and $a_{ij}=a_{ji}=0$ if $(v_i,v_j)\notin E$. Moreover, we assume $a_{ii}=0$ for all $i=1,2,\ldots,n$. For convenience, we always let the positive connected weight be $a_{ij}=a_{ji}=1$. Define the in-degree of node v_i as $d_i=\sum_{j=1}^{n}a_{ij}$ and the in-degree matrix as $\boldsymbol{D}=\mathrm{diag}\{d_i\}\in\mathbf{R}^{n\times n}$. Then the Laplacian matrix corresponding to the undirected graph is defined as $\boldsymbol{L}=\boldsymbol{D}-\boldsymbol{A}$. For an undirected graph, the adjacency matrix $\boldsymbol{A}$ and the Laplacian matrix $\boldsymbol{L}$ are symmetric. A path in the undirected graph is a sequence of ordered edges of the form (v_{i1},v_{i2}), (v_{i2},v_{i3}), (v_{i3},v_{i4}), $\ldots$, where $v_{ij}\in V$. If there is a path between any distinct pair of nodes, then the undirected graph is said to be connected. For a connected undirected graph, the Laplacian matrix $\boldsymbol{L}$ is positive semi-definite, $\boldsymbol{L}\boldsymbol{x}=0$ if and only if $\boldsymbol{x}\in\mathrm{span}\{1\}$, where $\boldsymbol{L}\in\mathbf{R}^{n\times n}$ and $\boldsymbol{x}\in\mathbf{R}^n$.

2.2. Problem formulation

Consider a group of n identical agents moving with a continuous-time dynamics described by

$$\dot{x}_i(t)=u_i(t),\quad i=1,2,\ldots,n, \tag{1}$$

where $x_i(t)\in\mathbf{R}$ is the state of the i-th agent, and $u_i(t)\in\mathbf{R}$ is the control input. When suitable control input $u_i(t)$, $i=1,2,\ldots,n$, is designed such that $\lim_{t\to+\infty}|x_i(t)-x_j(t)|=0$ for all $i,j\in\{1,2,\ldots,n\}$, then the multi-agent system is said to achieve consensus asymptotically. Meanwhile, if $\lim_{t\to+\infty}u_i(t)=0$ for all $i=1,2,\ldots,n$, then the multi-agent system is also said to achieve rendezvous asymptotically. Note that the framework proposed in this paper can be extended to multi-agent systems with higher-dimensional agents.

In this paper, we consider a distributed event-triggered control scheme for the multi-agent systems where each agent broadcasts its state to its neighbors only when its local state error exceeds a threshold, i.e., the triggering event for the agent to broadcast happens. The time at which an event happens is called the event time. The event time sequence for the i-th agent is denoted as $t_0^i,t_1^i,\ldots,t_k^i,\ldots$, and the latest broadcast state of the i-th agent is denoted as $\hat{x}_i(t)=x_i(t_k^i)$, $t\in[t_k^i,t_{k+1}^i)$, $k=0,1,2,\ldots$. Note that each agent can monitor its own state continuously, however, it cannot monitor the states of its neighbors continuously. Each agent can obtain the states of its neighbors (the latest broadcast states) only when the events of its neighbors happen.

The distributed control protocol for the i-th agent is given by

$$u_i(t)=-\sum_{j\in N_i}(x_i(t)-\hat{x}_j(t)),\quad i=1,2,\ldots,n, \tag{2}$$

where $\hat{x}_j(t)=x_j(t_{k'(t)}^j)$, and $k'(t)\triangleq\arg\ \min_{l\in N,t\geq t_l^j}\{t-t_l^j\}$, the $\arg\ \min$ over $l\in N,t\geq t_l^j$ denotes the value of l that minimizes $(t-t_l^j)$. From Eq. (2), we can see that the control protocol updating of the i-th agent relies on its own real-time state and its neighbors' latest broadcast states. The measurement error for the i-th agent is defined as

$$e_i(t)=\hat{x}_i(t)-x_i(t),\quad t\in[t_k^i,t_{k+1}^i). \tag{3}$$

If the norm of $e_i(t)$ exceeds the threshold, i.e., the event-triggered condition of the i-th agent is fulfilled, then the i-th agent will update (broadcast its state) and reset $e_i(t)$ to 0.

It is worth pointing out that a connected communication topology here does not mean that any two connected agents communicate continuously, it actually means that a path exists to be used for broadcasting between the two connected agents. Based on the idea in Ref. [10], the whole controller of the i-th agent in this paper can be partitioned into two parts with both time-triggered and event-triggered samplings. One

part is the controller decided by Eq. (2), and the other is the event detector decided by the following Eqs. (4a) and (4b). The event-triggered control structure of the i-th agent is shown in Fig. 1. In Fig. 1, the dashed lines inside the box indicate the event-triggered broadcasting, the solid lines indicate the time-triggered broadcasting, and the dashed lines outside the box indicate the broadcasting between the connected agents.

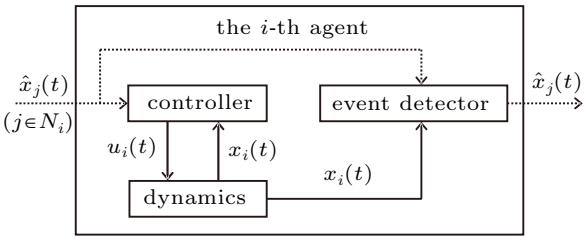


Fig. 1. Event-triggered control structure of the i-th agent.

The following lemma will be used in the next section.

Lemma 1[28] Let $\varepsilon > 0$ be a given scalar and $\boldsymbol{\Xi} \in \mathbf{R}^{p\times q}$ be a matrix such that $\boldsymbol{\Xi}^{\mathrm{T}}\boldsymbol{\Xi} \leq \boldsymbol{I}$, where $\boldsymbol{I}$ is an identity matrix with appropriate dimensions, then

$$2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{y} \leq \varepsilon\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x} + \varepsilon^{-1}\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y}$$

for all $\boldsymbol{x} \in \mathbf{R}^p$ and $\boldsymbol{y} \in \mathbf{R}^q$.

3. Distributed event-triggered control

The main goal is to propose an improved distributed event-triggered control mechanism to reduce the communication among the neighboring agents while preserving the asymptotic property of consensus. The event detector of each agent in Fig. 1 is configured to determine the time of updating the control action of itself. The event-triggered condition for the i-th agent is given by

$$f_i(t) = e_i^2 - \frac{\sigma_i}{a}y_i^2 = 0, \quad y_i(t) \neq 0, \tag{4a}$$

$$\text{or} \quad f_i(t) = e_i^2 - \frac{\sigma_i}{a}y_i^2 > 0, \tag{4b}$$

where $y_i(t) = \sum_{j\in N_i}(x_i - \hat{x}_j)$, $0 < \sigma_i < 1$, and $a > 0$ such that $\boldsymbol{A}^2 - a\boldsymbol{I} \leq 0$.

Remark 1 For the triggered condition (4a) and (4b), it is worth pointing out the reason to restrict $y_i(t) \neq 0$ in Eq. (4a) and the time when the triggered condition (4a) or (4b) holds. If triggered condition (4a) holds and $y_i(t) = 0$, then at this moment, the i-th agent updates and broadcasts. However, the broadcast information is the same as its last broadcast information, therefore, this event triggering is redundant. Moreover, after the achievement of the consensus, we have to end the event-driven behavior by manual operations if not to restrict $y_i(t) \neq 0$ in Eq. (4a). Now let us discuss the other restriction. Without loss of generality, we assume that the i-th agent can obtain the update information from its neighbors between two consecutive event times. Note that between two consecutive event times of the i-th agent, $e_i(t)$ is continuous, while $y_i(t)$ maybe discontinuous at the time when the i-th agent obtains information from its neighbors, $y_i(t)$ is right-continuous then. Therefore, without considering the time delay, if the i-th agent's next event time differs from its neighbors' event time, then the i-th agent's next event time must satisfy the triggered condition (4a). In other words, the triggered condition (4b) means that the i-th agent's next event time must equal to the latest one of its neighbors' event times. Of course, if the i-th agent's next event time equals to the latest one of its neighbors' event times, the triggered condition (4a) also maybe holds.

Remark 2 In protocol (2), for the i-th agent, its state $x_i(t)$ determined by its own dynamics can be assumed to be known by itself in real-time. Therefore, in this paper, the sampling of the i-th agent just means the i-th agent is to obtain information from its neighbors. Moreover, for the i-th agent, the cost of resource for its necessary running (decided by its own dynamics) is much less than that for broadcasting its state to its neighbors (or sampling its neighbors' states actively). Monitoring the event-triggering conditions continuously is necessary for each agent in the event-triggered control scenarios. It also can be regarded as one part of the dynamics of the agents, as the above statement where the cost of resource for its necessary running is small. However, for each agent, it may cost a considerable amount of resource to obtain its neighbors' information which is used in the event-triggering conditions.

In this paper, according to Eqs. (4a) and (4b), the event detector needs only the instantaneous broadcast information from the neighbors. However, the event-triggered condition of the decentralized event-triggered scheme of Dimarogonas *et al.*[22] and that of Fun *et al.*[24] need continuous local information from the neighbors, which is contradictory to the original purpose of using an event-triggered control to reduce communication. The difference facilitates our implementation of event detectors in a real event-triggered fashion.

Theorem 1 Consider multi-agent system (1) over a connected communication graph with protocol (2) driven by event-triggered condition (4a) and (4b). If each agent broadcasts its update information to its neighbors, then all agents achieve rendezvous asymptotically.

Proof Consider a candidate Lyapunov function

$$V(t) = \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{x},$$

where $\boldsymbol{x} = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$. Taking the time derivative of $V(t)$ yields

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n}\sum_{j\in N_i}(x_i - x_j)\Big(-\sum_{j\in N_i}(x_i - \hat{x}_j)\Big)\\ &= \sum_{i=1}^{n}\sum_{j\in N_i}(x_i - \hat{x}_j + e_j)\Big(-\sum_{j\in N_i}(x_i - \hat{x}_j)\Big)\end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{n}\Big(-\sum_{j\in N_i}(x_i-\hat{x}_j)\Big(\sum_{j\in N_i}(x_i-\hat{x}_j)\Big)-\sum_{j\in N_i}e_j\Big(\sum_{j\in N_i}(x_i-\hat{x}_j)\Big)\Big)\\
&= \sum_{i=1}^{n}\Big(-\Big(\sum_{j\in N_i}(x_i-\hat{x}_j)\Big)^2-\Big(\sum_{j\in N_i}(x_i-\hat{x}_j)\Big)\Big(\sum_{j\in N_i}e_j\Big)\Big)\\
&= -\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}-\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{E}, \qquad (5)
\end{aligned}
$$

where $\boldsymbol{Y}(t) = [y_1(t), y_2(t), ..., y_n(t)]^{\mathrm{T}}$ and $\boldsymbol{E}(t) = [e_1(t), e_2(t), ..., e_n(t)]^{\mathrm{T}}$.

According to Lemma 1 and $\boldsymbol{A}^2 - a\boldsymbol{I}_n \le 0$, let $\varepsilon = 1/\sqrt{a}$, we have

$$
\begin{aligned}
-\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{E} &= \sqrt{a}\boldsymbol{Y}^{\mathrm{T}}\Big(-\frac{1}{\sqrt{a}}\boldsymbol{A}\Big)\boldsymbol{E} \le \frac{\sqrt{a}}{2}(\varepsilon\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}+\varepsilon^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E})\\
&= \frac{1}{2}(\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}+a\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}).
\end{aligned}
$$

Then we can obtain from Eq. (5) that

$$
\dot{V} \le -\frac{1}{2}(\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}-a\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}) = -\frac{1}{2}\sum_{i=1}^{n}(y_i^2-ae_i^2).
$$

By enforcing the condition

$$
e_i^2 \le \frac{\sigma_i}{a}y_i^2, \qquad (6)
$$

for all $i = 1, 2, \ldots, n$, $0 < \sigma_i < 1$, we have

$$
\dot{V} \le -\frac{1}{2}\sum_{i=1}^{n}(1-\sigma_i)y_i^2,
$$

which is negative for $y_i \neq 0$, $i = 1, 2, \ldots, n$. Hence, for the i-th agent, if the event-triggered condition (4a) or (4b) is fulfilled, then an event happens and triggers the i-th agent to update and reset $e_i(t)$ to 0. Therefore, in the event-triggered control scheme, we have

$$
\dot{V} \le -\sum_{i=1}^{n}(1-\sigma_i)y_i^2 \le 0,
$$

for all $t > 0$. Together with the fact $V(t) \ge 0$ for all $t > 0$, we obtain $\lim_{t\to\infty}\dot{V} = 0$. Thus we have $\lim_{t\to\infty} y_i = 0$, i.e.,

$$
\lim_{t\to\infty}\sum_{j\in N_i}(x_i-\hat{x}_j) = \lim_{t\to\infty}\sum_{j\in N_i}(x_i-x_j-e_j) = 0, \qquad (7)
$$

for all $i = 1, 2, \ldots, n$.

Using inequality (6) and $\lim_{t\to\infty} y_i(t) = 0$, we have $\lim_{t\to\infty} e_i(t) = 0$ for all $i = 1, 2, \ldots, n$. As a result, equation (7) can be rewritten as $\lim_{t\to\infty}\sum_{j\in N_i}(x_i - x_j) = 0$ for all $i = 1, 2, \ldots, n$, which is equivalent to $\lim_{t\to\infty}\boldsymbol{L}\boldsymbol{x}(t) = 0$. Since the communication topology is connected, $\lim_{t\to\infty}\boldsymbol{x}(t) \in \mathrm{span}\{1\}$, i.e., $\lim_{t\to\infty}|x_i - x_j| = 0$ for all $i, j = 1, 2, \ldots, n$, which means that all agents achieve the consensus asymptotically. On the other hand, since $u_i(t) = -y_i(t)$, we have $\lim_{t\to\infty} u_i(t) = -\lim_{t\to\infty} y_i(t) = 0$ for all $i = 1, 2, \ldots, n$, which means that all agents tend to be static asymptotically. Therefore, all agents asymptotically achieve rendezvous. The proof is completed.

The following theorem is proposed to exclude the Zeno behavior and the method to prove the theorem is inspired by Wang and Lemmon.[19]

Theorem 2 Consider multi-agent system (1) over a connected communication graph with protocol (2) driven by event-triggered condition (4a) and (4b). If each agent broadcasts its update information to its neighbors, then each agent will not exhibit Zeno triggering.

Proof Assume that the i-th agent updates at time t_k^i and the next update time t_{k+1}^i. Without loss of generality, we can assume that the i-th agent receives broadcasts from its neighbors during the time interval $[t_k^i, t_{k+1}^i]$. Let m be the number of broadcasts. We can order the time at which the i-th agent receives the broadcasts as $\zeta_1 < \zeta_2 < \cdots < \zeta_m$, where ζ_s $(s = 1, 2, \ldots, m)$ denotes the time at which the i-th agent receives the s-th broadcast from one of its neighbors during the time interval $[t_k^i, t_{k+1}^i]$. For convenience, we denote $t_k^i = \zeta_0 < \zeta_1 < \zeta_2 < \cdots < \zeta_m \le t_{k+1}^i$. Then there exist two cases for time t_{k+1}^i, one case is $\zeta_m < t_{k+1}^i$ and the other is $\zeta_m = t_{k+1}^i$.

We first consider the case $\zeta_m < t_{k+1}^i$ and further denote $t_k^i = \zeta_0 < \zeta_1 < \zeta_2 < \cdots < \zeta_m < \zeta_{m+1} = t_{k+1}^i$. From Eq. (3), we have

$$
\dot{e}_i(t) = -\dot{x}_i(t) = \sum_{j\in N_i}(x_i(t)-\hat{x}_j(t)) = \sum_{j\in N_i}(\hat{x}_i(t)-\hat{x}_j(t)) - |N_i|e_i.
$$

Denote $z_i(t) = |e_i(t)|$, we then have

$$
\dot{z}_i(t) \le |\dot{e}_i(t)| \le |\sum_{j\in N_i}(\hat{x}_i(t)-\hat{x}_j(t))| + |N_i|z_i(t). \qquad (8)
$$

Since the measured states $\hat{x}_i(t)$ and $\hat{x}_j(t)$ are constants for any $j \in N_i$ during the time interval $[\zeta_s, \zeta_{s+1})$, $s = 0, 1, 2, \ldots, m$, from inequality (8), we have

$$
\mathrm{e}^{-\int_{\zeta_s}^{t}|N_i|\,\mathrm{d}\tau}(\dot{z}_i(t) - |N_i|z_i(t)) \le \omega_s^i\,\mathrm{e}^{-\int_{\zeta_s}^{t}|N_i|\,\mathrm{d}\tau} \le \omega_s^i, \qquad (9)
$$

where $t \in [\zeta_s, \zeta_{s+1})$, and $\omega_s^i = \omega_s^i(t) = |\sum_{j\in N_i}(\hat{x}_i(t) - \hat{x}_j(t))|$ for $t \in [\zeta_s, \zeta_{s+1})$, $s = 0, 1, \ldots, m$. Then from inequality (9), we have

$$
\begin{cases}
z_i(\zeta_1)\,\mathrm{e}^{-\int_{t_k^i}^{\zeta_1}|N_i|\,\mathrm{d}\tau} - z_i(t_k^i) \le \omega_0^i(\zeta_1 - t_k^i),\\
z_i(\zeta_2)\,\mathrm{e}^{-\int_{\zeta_1}^{\zeta_2}|N_i|\,\mathrm{d}\tau} - z_i(\zeta_1) \le \omega_1^i(\zeta_2 - \zeta_1),\\
\qquad\qquad\qquad\qquad\vdots\\
z_i(t)\,\mathrm{e}^{-\int_{\zeta_m}^{t}|N_i|\,\mathrm{d}\tau} - z_i(\zeta_m) \le \omega_m^i(t - \zeta_m),
\end{cases} \qquad (10)
$$

where $t \in [\zeta_m, \zeta_{m+1})$. After a simple calculation, inequalities (10) can be rewritten as

$$
\begin{cases}
z_i(\zeta_1)\,\mathrm{e}^{-\int_{t_k^i}^{\zeta_1}|N_i|\,\mathrm{d}\tau} - z_i(t_k^i) \le \omega_0^i(\zeta_1 - t_k^i),\\
z_i(\zeta_2)\,\mathrm{e}^{-\int_{t_k^i}^{\zeta_2}|N_i|\,\mathrm{d}\tau} - z_i(\zeta_1)\,\mathrm{e}^{-\int_{t_k^i}^{\zeta_1}|N_i|\,\mathrm{d}\tau} \le \omega_1^i(\zeta_2 - \zeta_1),\\
\qquad\qquad\qquad\qquad\vdots\\
z_i(t)\,\mathrm{e}^{-\int_{t_k^i}^{t}|N_i|\,\mathrm{d}\tau} - z_i(\zeta_m)\,\mathrm{e}^{-\int_{t_k^i}^{\zeta_m}|N_i|\,\mathrm{d}\tau} \le \omega_m^i(t - \zeta_m),
\end{cases} \qquad (11)
$$

where $t \in [\zeta_m, \zeta_{m+1})$.

Summing all inequalities in inequality (11) and after some calculations, we obtain

$$z_i(t) = |e_i(t)| \leq \omega^i_{\max}(t - t^i_k)\, \mathrm{e}^{|N_i|(t-t^i_k)}, \tag{12}$$

where $\omega^i_{\max} = \max\{\omega^i_s, s = 0, 1, \ldots, m\}$, $t \in [\zeta_m, \zeta_{m+1})$. We can see that $\omega^i_{\max} \neq 0$. This is because if $\omega^i_{\max} = 0$, from inequality (12) we have $z_i(t) = |e_i(t)| = 0$ for $t \in [t^i_k, t^i_{k+1}]$, according to the triggered condition (4a), it will not trigger the i-th agent to update, which contradicts the assumption that the i-th agent updates at time t^i_{k+1}.

According to inequality (12), if $|N_i|(t - t^i_k) \geq 1$, i.e., $t - t^i_k \geq 1/|N_i|$, then Theorem 2 is proved. In the sequel, we only need to consider $0 \leq |N_i|(t - t^i_k) < 1$. Then, from inequality (12), we have

$$z_i(t) = |e_i(t)| \leq e\omega^i_{\max}(t - t^i_k), \quad t \in [\zeta_m, t^i_{k+1}). \tag{13}$$

From Eq. (4a), the i-th agent updates at time t^i_{k+1} if

$$|e_i(t^i_{k+1})| = \gamma_i \left| \sum_{j \in N_i} (x_i(t^i_{k+1}) - \hat{x}_j(t^i_{k+1})) \right| > 0, \tag{14}$$

where $\gamma_i = \sqrt{\sigma_i/a} > 0$, and $e_i(t^i_{k+1})$ is the error at time t^i_{k+1} before the i-th agent updating and resetting $e_i(t^i_{k+1})$ to zero.

Before the i-th agent updating at time t^i_{k+1}, according to Eq. (3), we have $x_i(t^i_{k+1}) = x_i(t^i_k) - e_i(t^i_{k+1})$. Then equation (14) can be rewritten as

$$|e_i(t^i_{k+1})| = \gamma_i \left| \sum_{j \in N_i} (x_i(t^i_k) - \hat{x}_j(t^i_{k+1})) - |N_i| e_i(t^i_{k+1}) \right| > 0. \tag{15}$$

Since the i-th agent does not receive any broadcasts from its neighbors at time t^i_{k+1}, we have $\sum_{j \in N_i} \hat{x}_j(t^i_{k+1}) = \sum_{j \in N_i} \hat{x}_j(t)$ for any $t \in [\zeta_m, t^i_{k+1})$. Therefore, for any $t \in [\zeta_m, t^i_{k+1})$, we have

$$\begin{aligned} \left| \sum_{j \in N_i} (x_i(t^i_k) - \hat{x}_j(t^i_{k+1})) \right| &= \left| \sum_{j \in N_i} (x_i(t^i_k) - \hat{x}_j(t)) \right| \\ &= \left| \sum_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t)) \right| = \omega^i_m. \end{aligned}$$

Then from Eq. (15), we have

$$|e_i(t^i_{k+1})| \geq \gamma_i \omega^i_m - \gamma_i |N_i| \times |e_i(t^i_{k+1})|. \tag{16}$$

Thus when $\omega^i_m \neq 0$ (without loss of generality, we only need to guarantee that $\gamma_i |N_i| \neq 1$, then from Eq. (15), we always have $\omega^i_m \neq 0$), combining inequalities (13) and (16) yields

$$t^i_{k+1} - t^i_k \geq \frac{\gamma_i \omega^i_m}{e(1 + \gamma_i |N_i|)\omega^i_{\max}} > 0.$$

Specially, if $m = 0$, we have

$$t^i_{k+1} - t^i_k \geq \frac{\gamma_i}{e(1 + \gamma_i |N_i|)} > 0,$$

which means that the i-th agent does not exhibit the Zeno behavior for $\zeta_m < t^i_{k+1}$.

Now consider the second case $\zeta_m = t^i_{k+1}$. Similar to inequality (13), we can obtain

$$z_i(t) = |e_i(t)| \leq e\omega^i_{\max'}(t - t^i_k), \tag{17}$$

where $\omega^i_{\max'} = \max\{\omega^i_s, s = 0, 1, \ldots, m-1\} \neq 0$, $t \in [\zeta_{m-1}, t^i_{k+1})$.

According to Remark 1, if $t^i_{k+1} = \zeta_m$, then at this time either the event-triggered condition (4a) or (4b) will be satisfied. If the event-triggered condition (4a) is satisfied for $t^i_{k+1} = \zeta_m$, then the following analysis will be similar to that in the case of $\zeta_m < t^i_{k+1}$. Therefore, in the following, we only consider that the event-triggered condition (4b) is satisfied for $t^i_{k+1} = \zeta_m$. Then from Eq. (4b), we have

$$\begin{aligned} |e_i(t^i_{k+1})| &> \gamma_i \left| \sum_{j \in N_i} (x_i(t^i_{k+1}) - \hat{x}_j(\zeta_m)) \right| \\ &= \gamma_i \left| \sum_{j \in N_i} (x_i(t^i_k) - \hat{x}_j(\zeta_m)) - |N_i| e_i(t^i_{k+1}) \right|, \end{aligned} \tag{18}$$

where $e_i(t^i_{k+1})$ is the error at time t^i_{k+1} before the i-th agent updates and resets $e_i(t^i_{k+1})$ to zero.

Denote $\omega^i_{m'} = |\sum_{j \in N_i} (x_i(t^i_k) - \hat{x}_j(\zeta_m))|$. Thus if $\omega^i_{m'} \neq 0$, combining inequalities (17) and (18) yields

$$t^i_{k+1} - t^i_k > \frac{\gamma_i \omega^i_{m'}}{e(1 + \gamma_i |N_i|)\omega^i_{\max'}} > 0.$$

When $\omega^i_{m'} = 0$, inequality (18) can be rewritten as

$$|e_i(t^i_{k+1})| > \gamma_i |N_i| \times |e_i(t^i_{k+1})|. \tag{19}$$

On the other hand, according to the event-triggered scheme, we have

$$\begin{aligned} |e_i(t)| &< \gamma_i \left| \sum_{j \in N_i} (x_i(t) - \hat{x}_j(\zeta_{m-1})) \right| \\ &= \gamma_i \left| \sum_{j \in N_i} (x_i(t^i_k) - \hat{x}_j(\zeta_{m-1})) - |N_i| e_i(t) \right|, \end{aligned} \tag{20}$$

for $t \in [\zeta_{m-1}, \zeta_m)$. Since $e_i(t)$ is continuous in the time interval $[\zeta_{m-1}, \zeta_m]$ before the i-th agent updating at time $t^i_{k+1} = \zeta_m$, from inequality (20), we have

$$|e_i(t^i_{k+1})| \leq \gamma_i \left| \sum_{j \in N_i} (x_i(t^i_k) - \hat{x}_j(\zeta_{m-1})) - |N_i| e_i(t^i_{k+1}) \right|. \tag{21}$$

According to inequalities (18) and (21), there exists a number μ, which can be decided by $\sum_{j \in N_i} \hat{x}_j(\zeta_{m-1})$ and $\sum_{j \in N_i} \hat{x}_j(\zeta_m)$ and then denoted as $\mu = \mu(\sum_{j \in N_i} \hat{x}_j(\zeta_{m-1}), \sum_{j \in N_i} \hat{x}_j(\zeta_m))$, such that

$$\begin{aligned} |e_i(t^i_{k+1})| = \gamma_i \Bigg| & \sum_{j \in N_i} x_i(t^i_k) - \mu\Big(\sum_{j \in N_i} \hat{x}_j(\zeta_{m-1}), \sum_{j \in N_i} \hat{x}_j(\zeta_m) \Big) \\ & - |N_i| e_i(t^i_{k+1}) \Bigg|. \end{aligned} \tag{22}$$

Denote

$$\omega^i_{m''} = \left| \sum_{j\in N_i} x_i(t^i_k) - \mu\Big(\sum_{j\in N_i} \hat{x}_j(\zeta_{m-1}), \sum_{j\in N_i} \hat{x}_j(\zeta_m) \Big) \right|,$$

from Eqs. (19) and (22), we can see that $\omega^i_{m''} \neq 0$. Thus when $\omega^i_{m'} = 0$, combining Eqs. (17) and (22) yields

$$t^i_{k+1} - t^i_k \geq \frac{\gamma_i \omega^i_{m''}}{e(1+\gamma_i|N_i|)\omega^i_{\max'}} > 0.$$

Therefore, the i-th agent does not exhibit the Zeno behavior for $\zeta_m = t^i_{k+1}$. The proof is completed.

Remark 3 In the proof of Theorem 2, we assume that the i-th agent can receive the broadcasts from its neighbor agents during the time interval $[t^i_k, t^i_{k+1}]$ and no Zeno behavior implicitly happens in the neighbor agents of the i-th agent during the same time interval $[t^i_k, t^i_{k+1}]$. In the following proof, we show that if no broadcasts are received from its neighbor agents, the time between consecutive updatings of the i-th agent must be greater than zero, i.e., $t^i_{k+1} - t^i_k \geq \frac{\gamma_i}{e(1+\gamma_i|N_i|)} > 0$, $i = 1, 2, \ldots, n$, which means that no Zeno behavior happens in any agent when it receives no broadcasts from its neighbor agents. Furthermore, according to the proof of Theorem 2, for any agent receiving broadcasts from its neighbors, the Zeno behavior can also be excluded. Therefore, for the discussion of excluding the Zeno behavior of the i-th agent, it is sufficient to assume implicitly that no Zeno behavior happens in the neighbor agents of the i-th agent during the time interval $[t^i_k, t^i_{k+1}]$.

4. Numerical simulation

The proposed distributed event-triggered control scheme in Section 3 is illustrated by the following numerical simulations.

Consider a scenario where five agents will move to a single position in a two-dimensional (2D) space. Figure 2 shows the corresponding communication topology among these agents. The adjacency matrix is given by

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Randomly choose $\boldsymbol{x}_1(0) = [-2, 8]^{\mathrm{T}}$, $\boldsymbol{x}_2(0) = [4, 6]^{\mathrm{T}}$, $\boldsymbol{x}_3(0) = [8, 4]^{\mathrm{T}}$, $\boldsymbol{x}_4(0) = [3, 2]^{\mathrm{T}}$, and $\boldsymbol{x}_5(0) = [0, 5]^{\mathrm{T}}$ as the initial states of the agents and make all the agents update firstly at time $t = 0$. The parameters in the event-triggered condition (4a) and (4b) are given by $a = 6$ and $\sigma_i = 0.99$, $i = 1, 2, \ldots, 5$. Hence, the five agents under protocol (2) will achieve rendezvous according to Theorem 1. Figure 3 shows the state trajectories of all the agents. Let us take agent 5 as an example to show the evolution of the error norm of an agent. Figure 4 shows the evolution of the error norm of agent 5 which embodies the distributed event-triggered scheme, i.e., once the error norm of agent 5 reaches or exceeds the threshold $\gamma_5|y_5(t)|$, the error norm of agent 5 is reset to 0. It is seen from Fig. 4 that at time $t = T$, agent 5 receives the broadcasting of its neighbor agent 1 and at the same time, the error norm of agent 5 exceeds the threshold $\gamma_5|y_5(T)|$, which causes agent 5 to update at the same time $t = T$ as its neighbor agent 1. The time instants when the events occur for each agent are shown in Fig. 5.

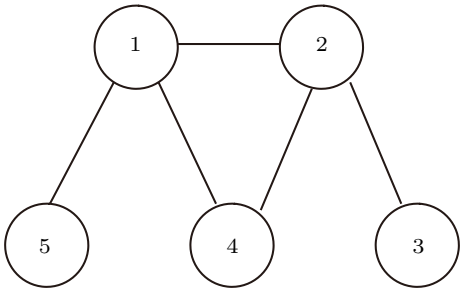


Fig. 2. Communication graph of five agents.

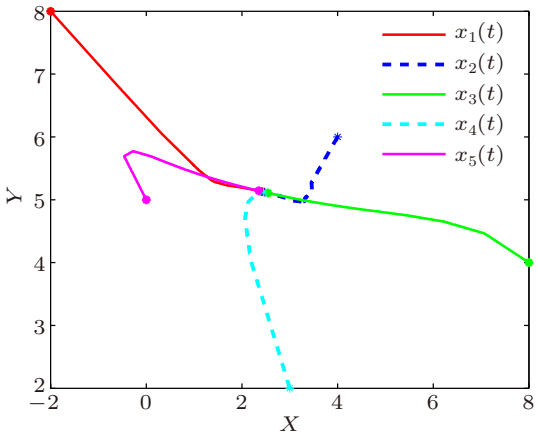


Fig. 3. (color online) State trajectories of all agents with event-triggered control.

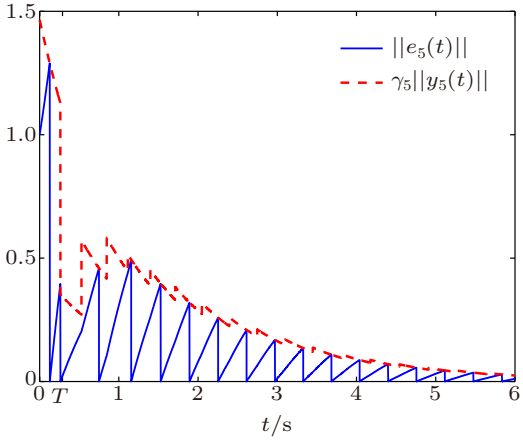


Fig. 4. (color online) Evolution of the error norm of agent 5.

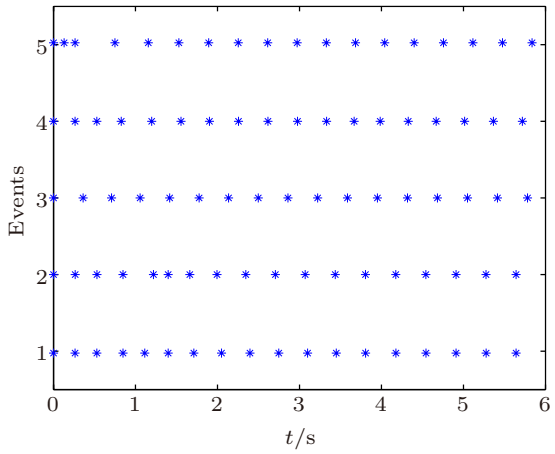


Fig. 5. (color online) Event time of the five agents.

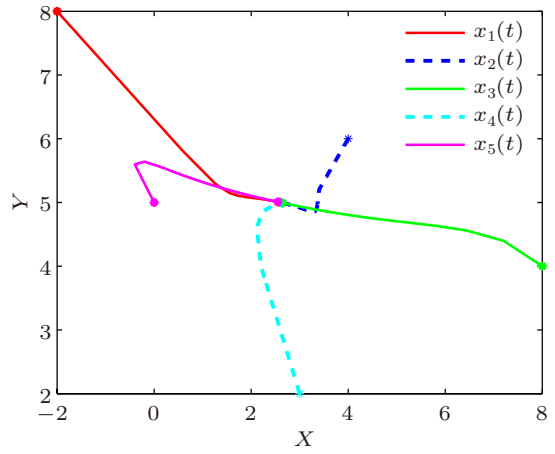


Fig. 6. (color online) State trajectories of all agents with period sampling control, where the sampling period is $h = 0.2$ s.

Compared with the consensus problem via the period sampling control, our distributed event-triggered method exhibits better performance. For the same system in the above example, we use the period sampling protocol proposed in Ref. [15], i.e., $u_i(t) = \sum_{j\in N_i}(x_j(kh) - x_i(kh))$, where $h > 0$ is the sampling period, and $t \in [kh, kh+h)$, $k = 0,1,2,\ldots$, $i = 1,2,\ldots,5$. Then, according to Proposition 1 in Ref. [15], all agents with the same initial conditions will asymptotically achieve rendezvous if and only if $0 < h < 0.47$. We can see that when $h = 0.2$, the state trajectories and the rendezvou time of the period sampling are similar to those of the event-triggered control, i.e., both period sampling control and event-triggered control have the similar performances. Figure 6 shows the state trajectories of the period sampling of all agents with the sampling period $h = 0.2$. Figure 7 shows the sampling instants every 0.2 s. While the average inter-event time of the event-triggered control in the above example is $t_{\rm av} = 0.33$, which is larger than $h = 0.2$. Moreover, the samplings of all agents in the period sampling control are centralized, while the information broadcasts of all agents in the event-triggered control are distributed. Therefore, the multi-agent systems with the period sampling control should be equipped with a higher instantaneous communication processing capacity (more communication resources) than the same multi-agent systems with the event-triggered control. These illustrate the advantages of the event-triggered control on saving the communication resources compared with the period sampling control.

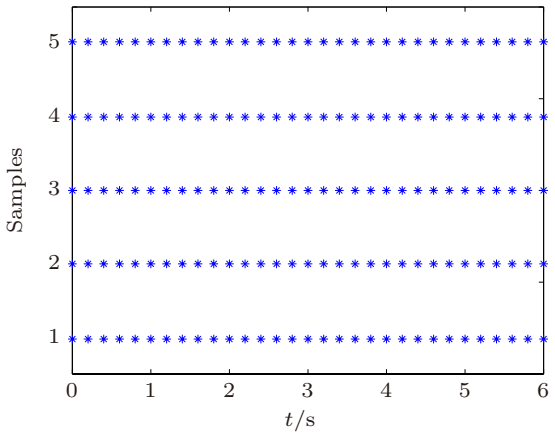


Fig. 7. (color online) Period sampling time of the five agents.

5. Conclusion

In this paper, an improved state-dependent distributed event-triggered scheme was proposed to make multi-agent systems contractive in the sense of consensus. A Lyapunov function was introduced and the time derivative of the Lyapunov function was made negative semi-definite through a proper choice of the event-triggered condition. Without continuously monitoring the states of each agent's neighbors, the proposed event-triggered control mechanism overcomes the drawback of requiring continuous local information in Refs. [22] and [24]. Finally, the simulation results were presented to support the effectiveness of the theoretical results.

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