Elementary analysis of interferometers for wave–particle duality test and the prospect of going beyond the complementarity principle*

Li Zhi-Yuan(李志远)†

Laboratory of Optical Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, *China*

(Received 2 September 2014; revised manuscript received 9 September 2014; published online 17 October 2014)

A distinct method to show a quantum object behaving both as wave and as particle is proposed and described in some detail. We make a systematic analysis using the elementary methodology of quantum mechanics upon Young's two-slit interferometer and the Mach–Zehnder two-arm interferometer with the focus placed on how to measure the interference pattern (wave nature) and the which-way information (particle nature) of quantum objects. We design several schemes to simultaneously acquire the which-way information for an individual quantum object and the high-contrast interference pattern for an ensemble of these quantum objects by placing two sets of measurement instruments that are well separated in space and whose perturbation of each other is negligibly small within the interferometer at the same time. Yet, improper arrangement and cooperation of these two sets of measurement instruments in the interferometer would lead to failure of simultaneous observation of wave and particle behaviors. The internal freedoms of quantum objects could be harnessed to probe both the which-way information and the interference pattern for the center-of-mass motion. That quantum objects can behave beyond the wave–particle duality and the complementarity principle would stimulate new conceptual examination and exploration of quantum theory at a deeper level.

Keywords: wave–particle duality, complementarity principle, atom interferometer

PACS: 03.65.Ta, 03.75.Dg, 42.50.Xa **DOI:** 10.1088/1674-1056/23/11/110309

1. Introduction

Quantum theory has enabled brilliant successes in predicting the physical and chemical properties of microscopic objects such as electrons, atoms, and molecules, as well as macroscopic objects such as crystals, insulators, semiconductors, and superconductors. There is no doubt that quantum theory is one of the most deliberate and successful physical theories in history. However, the interpretation of quantum theory has been an issue ever since its founding more than 100 years ago[1–18] and has raised extensive hot controversies, e.g., between Einstein and Bohr.[1–3,5] The wave–particle duality of quantum objects, and more generally, the complementarity principle, stands at the conceptual core of quantum theory. According to the orthodox Copenhagen interpretation, all quantum objects, including massless and massive particles, exhibit mutually exclusive behaviors of two intrinsic attributes — they behave either as waves or as particles, depending on how they are observed and measured, but never as both.[1–3,9,10,12–18]

In recent years, there are still many theoretical and experimental discussions and investigations of the conceptual implications of the wave–particle duality of various quantum objects, such as photons, electrons, atoms, molecules, and even macromolecules and heavy particles.[19–27] Besides, a wide variety of experimental setups, both gerdanken (thought) and practical, have been adopted.[24–42] All results, without any exception, just confirm again and again the old wisdom that the wave–particle duality stands solidly and firmly at the conceptual center of quantum theory. In this paper, we will show a different perspective, by making an elementary quantum mechanical analysis of the previous gedanken or realistic wave–particle duality experiments, that it is possible to design experiments to demonstrate that a quantum object can behave both as wave and as particle. We focus our analysis on the two most popular experimental schemes in history to test the wave–particle duality: Young's two-slit interferometer and the Mach–Zehnder two-arm interferometer. The perspective relies on the design of two simultaneous sets of good measurements well separated in space, where one works to acquire the which-way information (particle behavior) and the other works separately to acquire the high-contrast interference pattern (wave behavior), and they cause only negligible perturbation to each other. We discuss in detail how to make good measurements of the which-way information and the interference pattern of quantum objects and analyze the effect of proper or improper arrangement and cooperation of these two sets of measurements on the outcome of a wave–particle duality test.

2. Analysis of classical interferometers for wave–particle duality test

A standard Young's two-slit interferometer experiment is illustrated schematically in Fig. 1. This scheme is widely adopted to illustrate the wave nature of classical waves, such

*Project supported by the National Natural Science Foundation of China, the Ministry of Science and Technology of China, and Chinese Academy of Sciences.

†Corresponding author. E-mail: lizy@aphy.iphy.ac.cn

© 2014 Chinese Physical Society and IOP Publishing Ltd

as water waves, sonic waves, and light waves, and it is used equally a lot to test the wave nature of quantum objects. In both the classical and the quantum views, the interference of waves relies closely on the degree of coherence, which describes mathematically the mutual correlation status of the waves in two space–time points. Physically, the interference pattern is directly related with the superposition of two secondary waves exiting the two slits of an interferometer and transporting to the observation screen. The state of motion or transport of these waves can be uniformly described by the spatial function $\psi(\boldsymbol{r},z)$. For optical waves, $\psi(\boldsymbol{r},z)$ refers to the electric field (more precisely, one of its three vector components), which satisfies Maxwell's equations, while for massive particles such as electrons and atoms, $\psi(\boldsymbol{r},z)$ refers to the probability wave function, which satisfies Schrödinger's equation. Yet, one important common thing is that in free space $\psi(\boldsymbol{r},z)$ has a simple wave motion (e.g., plane wave) form. Thus, in the following and all throughout this paper we use the term quantum object to represent all massless and massive particles, such as photons, electrons, and atoms.

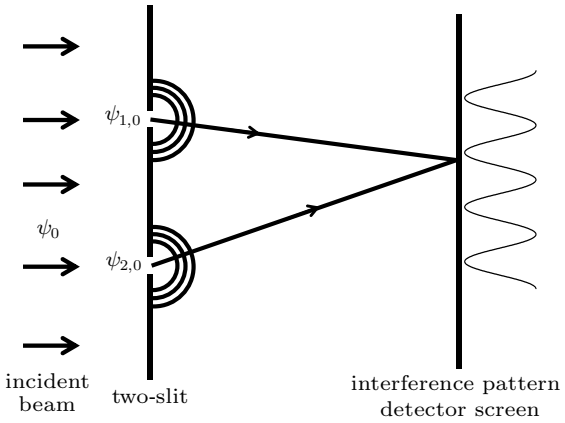


Fig. 1. Principle of Young's two-slit interferometer for wave–particle duality test of quantum objects. The interferometer consists of three components: the incident beam of quantum object, assumed to have high coherence and described by wave function ψ_0; an opaque screen encoded with two parallel slits separated by some distance; a screen directly decorated with a detector or through other indirect mechanisms to observe and measure the interference pattern formed by the secondary waves ($\psi_{1,0}$ and $\psi_{2,0}$) diffracting from the two slits. When the slits are fixed, a high-contrast interference pattern can be observed while the which-way information, i.e., which slit the quantum objects pass through, is unknown completely. When the slits can recoil freely to allow for recording the which-way information, the secondary waves are strongly disturbed so that the high-contrast interference pattern disappears.

When a wave with spatial distribution $\psi(\boldsymbol{r},z)$ impinges upon the two slits of a Young's interferometer, diffraction and interference take place, and the pattern depends on the correlation between the wave states at the two slits ($\psi_{1,0}$ and $\psi_{2,0}$), which is just the way the spatial coherence status of $\psi(\boldsymbol{r},z)$ is defined. Mathematically, in the paraxial approximation, the overall wave function $\psi_{\rm t}(\boldsymbol{r},z)$ at the observation screen after the two-slit diffraction can be written as

$$\begin{aligned}\psi_{\rm t}(\boldsymbol{r},z) &= \psi_1(\boldsymbol{r},z)+\psi_2(\boldsymbol{r},z)\\ &= A(z)\left[\psi_{1,0}\,{\rm e}^{{\rm i}\varphi_1(\boldsymbol{r},z)}+\psi_{2,0}\,{\rm e}^{{\rm i}\varphi_2(\boldsymbol{r},z)}\right].\end{aligned} \tag{1}$$

Here $\psi_1(\boldsymbol{r},z)$ and $\psi_2(\boldsymbol{r},z)$ are the wave functions evolved from the secondary sources ($\psi_{1,0}$ and $\psi_{2,0}$) at the exits of the two slits, $\varphi_1(\boldsymbol{r},z)$ and $\varphi_2(\boldsymbol{r},z)$ are the phases accumulated during the wave transport process, and $A(z)$ is the single-slit diffraction factor. The signal intensity at the observation screen is then calculated by

$$\begin{aligned}I_{\rm t}(\boldsymbol{r},z) &= |\psi_{\rm t}(\boldsymbol{r},z)|^2\\ &= |A(z)|^2\left[|\psi_{1,0}|^2+|\psi_{2,0}|^2+2|\psi_{1,0}||\psi_{2,0}|\cos(\Delta\varphi)\right].\end{aligned} \tag{2}$$

Here the phase difference $\Delta\varphi(\boldsymbol{r},z)=\varphi_1(\boldsymbol{r},z)-\varphi_2(\boldsymbol{r},z)$ is the key factor for any interference experiment, which determines the peak and valley positions of the interference fringe. However, whether stable (temporal) and clear (spatial) interference patterns can be observed strongly depends on the temporal and spatial coherence nature of the wave function $\psi(\boldsymbol{r},z)$ and its values $\psi_{1,0}$ and $\psi_{2,0}$ at the two slits. This point has been well established in classical optics, where to manifest a good two-slit interference experiment, a laser source is usually used to guarantee both high temporal and spatial coherence. When the coherence condition is well satisfied, a perfect interference fringe pattern is observed at the observation screen, following the formalism of Eq. (2).

Another experimental scheme popularly used for illustrating the wave nature of classical and quantum objects is the Mach–Zehnder two-arm interferometer, as depicted in Fig. 2. A coherent optical beam or particle beam described by the wave function $\psi(\boldsymbol{r},z)=\psi_0(\boldsymbol{r})\,{\rm e}^{{\rm i}\varphi(z)}$ passes through the first beam splitter (BS 1) and is separated into two beams, which then pass along two paths (path x and path y). Here $\boldsymbol{r}$ is the transverse position and z is the transport position of the beam. In practice, the diffraction of the beam is neglected so that $\psi_0(\boldsymbol{r})$ is a constant over $\boldsymbol{r}$ and the transport phase $\varphi(z)=kz$ (with k being the wave number and z the transport path) is the only relevant quantity. When the two beams pass through the second beam splitter (BS 2), interference occurs in both paths, and both detector x and detector y can record the periodic variation of the signal intensity when the path length difference of the two arms is changed continuously, measuring the wave nature of the object. Mathematically, the signal intensity in both arms is given by

$$I_{\rm t}(\Delta\varphi)=|\psi_0|^2[1+\cos(\Delta\varphi)]/2, \tag{3}$$

where $\Delta\varphi$ is the phase difference of the two beams passing across the two arms before finally reaching detector x or y, and perfect 50:50 beam splitting has been assumed in both BS

1 and BS 2. Similar to the situation of Young's two-slit interferometer experiment, good temporal coherence of the beam $\psi(\boldsymbol{r},z)$ is the condition for this type of interference experiment.

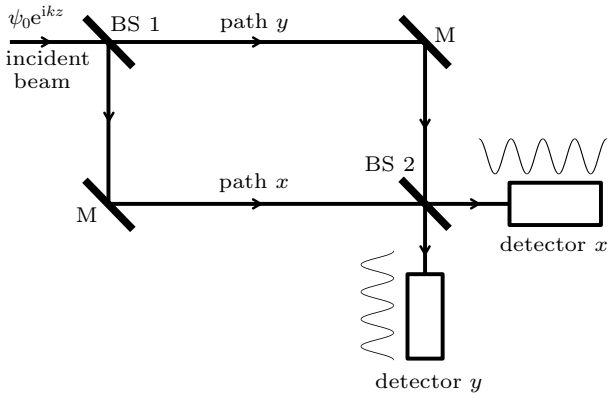


Fig. 2. Principle of Mach–Zehnder two-arm interferometer for wave–particle duality test of quantum objects. The incident wave beam of quantum object described passes through the first beam splitter (BS 1) and is separated into two beams, which transport along two paths (path x and path y). When they pass through the second beam splitter (BS 2), both detector x and detector y can record the periodic variation of the signal intensity when the path length difference of the two arms is changed continuously by moving the two reflection mirrors (M), detecting the wave nature of the quantum object. When BS 2 is absent, neither detector observes a periodic variation of signal, but rather a constant value of signal, detecting the particle nature of the quantum object.

The above two experimental setups are popularly used for exclusively illustrating the wave nature of photons and massive quantum objects. Simple modification can be made to both setups for exclusively revealing the particle nature of photons and massive quantum objects. In the Young's two-slit interferometer setup, one slit is closed to allow the beam to pass only through the other slit. The transport path of a particle is then determined while letting the interference fringe at the observation disappear, because now the signal intensity is $I_{\rm t}(\boldsymbol{r},z) = |A(z)|^2[|\psi_{1,0}|^2 + |\psi_{2,0}|^2]/2$. In the Mach–Zehnder interferometer, the BS 2 is removed, so that the path of the particle can be unambiguously determined by looking at the signal recorded by detectors x and y. However, the signal observed by both detectors is now $I_{\rm t}(\Delta\varphi) = |\psi_0|^2/2$, losing the periodic variation of signal and thus unable to tell the wave nature of the particle. Obviously, neither the old nor the new version of Young's or the Mach–Zehnder interferometer can simultaneously tell the wave and particle nature of the quantum object; instead they tell only one or the other nature (i.e., wave or particle).

The above description is more or less based on conceptual argument. Strictly speaking, the behavior of optical and quantum particle beams in all these setups can be perfectly predicted by quantum theory itself. Some very simple elementary analysis and calculation alone can already do the job well. The results in the old and the new setups of interferometer are very different just because the environments within which the quantum object transports are very different. In some sense, quantum theory operates very well in these problems in the framework of its own methodology, but this methodology in itself just is not able to get people closer to the conceptual insight of the nature of quantum objects. However, just based on the above analysis (simple but classical, adopted by all people now and throughout modern history) for two classical interferometer setups, it is unfair to jump too hurriedly to the conclusion that quantum theory completely excludes the possibility that the wave and particle nature of quantum object can be simultaneously observed by some very different, while brand new, experimental setups.

Note that equation (2) is the basic formalism for describing the interference pattern in the observation screen that has been used almost exclusively in all Young's interferometers. Yet, in the context of quantum mechanics, there should exist a certain operation of measurement in reality (whether perfect or imperfect, at least in principle) that can well correspond to this formalism and whose outcome can be described by this formalism. Otherwise the measurement is not working to allow for unambiguous acquisition of the interference pattern. In classical optics, this formalism has been proven to be true based on a very solid physical mechanism of how the optical interference pattern is observed: $I_{\rm t}(\boldsymbol{r},z)$ in Eq. (2) is just the optical intensity distribution pattern. Yet, great caution and care should be made for more general quantum particles. It might not be a safe assumption for arbitrary operation of measurement without thinking deeply of the corresponding physics underlying such a measurement. This point will become clear in later discussion.

It is worthwhile to examine and address various interferometer experiments for the wave–particle duality and complementarity principle tests through mathematical solutions based on the orthodox operation methodology of quantum theory, instead of purely conceptual argument and justification. Solutions of quantum mechanical problems for quantum objects passing through these interferometers based on the Schrödinger equation (or Maxwell's equations) should rely on both the environment, which determines the Hamiltonian, and the initial condition of the wave function (the coherence status). After the final wave function of the quantum objects is solved, the outcome of the quantum system under various operations of observation and measurement can also be calculated and determined via standard formulations of quantum mechanics, such as perturbation theory and scattering theory, used to handle the transition between the initial and the final quantum states. In this regard, the experimental setups for acquiring the wave and the particle behaviors in the above standard Young's and Mach–Zehnder interferometers offer completely different environments for quantum objects to transport and evolve. The totally different and mutually exclusive

outcomes, i.e., either wave or particle, but never both, in these experiments are thus a natural result of quantum mechanics. If more complicated situations such as various "delayed-choice" experiments, as first conceptually proposed by Wheeler, are concerned,[36–46] then the great details of how the environment (thus the Hamiltonian) changes in time and space must be first clarified and determined to allow for the precise prediction of the outcome of these experiments. Usually a good physical model faithfully representing the experimental operation details needs to be placed into the Schrödinger equation in order to obtain good theory–experiment agreement. Unfortunately, this reasonable roadmap of problem solution (other than purely conceptual argument and deduction) has rarely been adopted to reveal the puzzling wave–particle duality in these delayed-choice experiments.

3. New design of Mach–Zehnder interferometer

In the modified Mach–Zehnder two-arm interferometer, BS 1 remains in the path while BS 2 has been removed. It is well-known that this setup can unmistakably reveal the particle nature of a quantum object but not the wave nature. Let us temporarily follow the transport path of quantum objects within the new Mach–Zehnder interferometer rather than hurrying at once to the final destinations of the path, detector x and detector y. Notice that the initial coherent beam of quantum objects is described by the wave function $\psi(\boldsymbol{r},z)=\psi_0(\boldsymbol{r})\,\mathrm{e}^{\mathrm{i}\varphi(z)}$. However, in the above analysis made by ourselves (to be fair, also by all others now and in the past), the transverse function $\psi_0(\boldsymbol{r})$ is completely omitted, assuming its irrelevance. Yes, it is true that $\psi_0(\boldsymbol{r})$ is irrelevant to the final outcomes of detector x and detector y, either in the original or in the modified setup of Mach–Zehnder interferometer. Yet, do not forget that $\psi_0(\boldsymbol{r})$ is an indispensible part of the wave function $\psi(\boldsymbol{r},z)$ and it also carries information of the coherence of the beam. This information would manifest itself in the cross region of the two beams in space. It is worth making a more mathematical analysis on the behavior of the wave transport in this region.

For simplicity, both beams are approximately modeled as a plane wave with wave vector $\boldsymbol{k}_x=(k_0,0,0)$ for the horizontal beam (path x) and $\boldsymbol{k}_y=(0,-k_0,0)$ for the vertical beam (path y). Then the wave function in the cross region is given by

$$\psi(x,y)=\psi_0(\,\mathrm{e}^{\mathrm{i}k_0x}+\mathrm{e}^{-\mathrm{i}k_0y+\mathrm{i}\Delta\varphi})/\sqrt{2}. \tag{4}$$

The corresponding wave intensity is simply

$$I(x,y)=|\psi(x,y)|^2=|\psi_0|^2[1+\cos(k_0x-k_0y+\Delta\varphi)]. \tag{5}$$

Obviously, a volume interference pattern forms in the crossing region of the two coherent beams. Therefore, the general picture of this new Mach–Zehnder interferometer is that the two quantum-object beams pass along their own paths, cross and interfere, leave each other without changing their original paths, and then finally reach their own destinations, detector x and detector y, respectively.

When summing up all the above information and thinking over from a new conceptual view rather than the standard one, it seems natural for us to immediately point out a very strong conclusion that the modified setup of Mach–Zehnder interferometer, as illustrated in Fig. 3, can simultaneously record the wave and particle nature of quantum objects, although in different temporal and spatial domains, by using two different sets of measurement instruments at the same time, one in the two-beam cross region for acquiring the wave nature, and the other at the destinations for acquiring the particle nature. Having said this, we still need to go one step further to make the principle idea of the above gedanken experiment operationally practical and leave no loopholes for doubt. The key then is to design a smarter instrument to observe the interference pattern in the two-beam cross region while not significantly modifying the environment of the quantum object transport paths. In optics, this is not difficult to do. One can place a glass plate coated with a very thin (say, a few nanometers thick) scattering or absorption material with its surface inclined in some angle (say 30°) with the x axis. In the scheme of weak scattering, the scattering intensity by the thin film would be proportional to the intensity profile of light at the surface, which can then be collected by an optical telescope outside the path of the Mach–Zehnder interferometer. In the absorption scheme, the interference pattern is directly recorded by the thin film. In both schemes, the scattering or absorption must be sufficiently strong to allow the interference pattern to be recorded by state-of-the-art sensing materials or devices with very high peak–valley contrast, and at the same time must be sufficiently weak so that most photons (say $>99\%$) can pass through the thin film and follow their original path to reach the detector so that its particle nature is faithfully recorded. In both weak scattering and absorption schemes, the measured optical pattern is proportional to the spatial interference distribution and thus has a perfect peak-to-valley contrast under ideal conditions.

Notice that the above description of the operational mechanism of the weak detection scheme for the interference pattern works well not only in the framework of classical optics for light waves, but also works equally well in the framework of quantum mechanics for photons. The reason is the same as for the beam splitter used in this interferometer and commonly used in many other quantum systems. It is well known that the functionality of a beam splitter, such as the amplitude and phase of the two split beams, can be designed and described exactly based on the knowledge of classical physical optics. Yet, it equally works for a single photon, where the probability amplitude and phase of the wave function for the two split

beams of photon are the same as those designed or predicted by classical optics. In fact, many other devices involved in quantum systems also work exactly in the way described by classical physics, because they are essentially classical. This means that the above weak measurement scheme for interference pattern observation, although designed and discussed basically in the context of classical physics, can be described equally well in terms of quantum mechanics and should yield equal experimental results followed not only by classical light waves, but also by an ensemble of photons, or equivalently by a single photon in the context of probability.

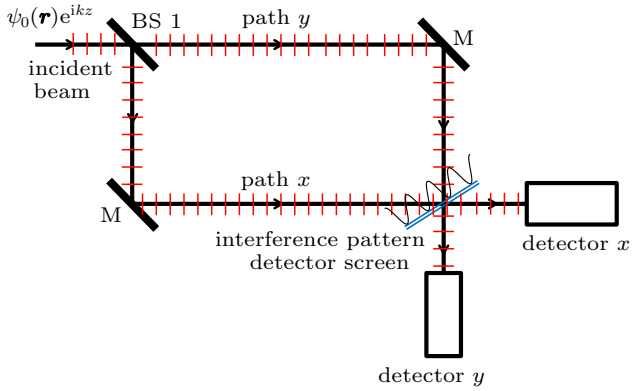


Fig. 3. New design of Mach–Zehnder interferometer allowing for simultaneous acquisition of which-way information and high-contrast interference fringe of quantum objects. The incident quantum object wave beam described by $\psi_0(\boldsymbol{r})e^{i\varphi(z)}$ with longitudinal phase $\varphi(z)$ and transverse wave front $\psi_0(\boldsymbol{r}) = \psi_0(x,y)$ (denoted by the red short line segments) will pass through the first beam splitter (BS 1) and is separated into two beams, which transport along two paths (path x and path y). When they intersect in space, the interference pattern of waves will appear in the intersection region due to the overlapping of their transverse wave fronts. A highly transparent (99% transmittance) and weakly scattered or absorbed (1% scattered or absorbed) screen (denoted by the thick blue lines, unlike BS 2 in Fig. 2) inserted into this region can be used to record the interference pattern of the two wave beams, while imposing only a little influence on the propagation of the quantum object. The two wave beams of the quantum object will transmit through the screen along their original path and be detected by particle detectors x and y, yielding the which-path information of the quantum object. The simultaneous observation of the wave and particle nature of quantum objects is made in different space–time domains without the troublesome mutual repulsion effect seen in the conventional Mach–Zehnder interferometers, as illustrated in Fig. 2.

The modified Mach–Zehnder interferometer designed to manifest both the wave and particle nature simultaneously for quantum objects can work, not only in principle (in gedanken experiment) but also in practice (using modern technology), and it can be immediately brought to experimental test for photons. As to other massive particles such as electrons and atoms, the key to successful experimental demonstration of their new wave–particle duality also relies on design and fabrication of instruments or materials like the above thin film for photons that allow for weak measurement of the interference pattern in the two-beam cross region. This proposed scheme is drastically different from many previous schemes based on the Mach–Zehnder interferometer for wave–particle duality test. The previous schemes, like Wheeler's delayed-choice gedanken experiment scheme, try to operate on each single quantum object to reveal both the wave and particle nature; however, two sets of instruments are proposed or actually used (namely, inserting BS 2 or removing BS 2) but not simultaneously, which obviously are incompatible with each other. In the methodology of quantum theory, such an arrangement of two sets of instruments corresponds to two mutually repulsive quantum mechanical problems (corresponding to two different setups of Mach–Zehnder interferometer), whose solution can never lead to simultaneous wave and particle observation no matter how clever the two setups for measurement are arranged in space–time. In contrast, our proposed scheme uses only a single set of measurements involving two instruments simultaneously placed in different spatial and temporal domains, avoiding strong mutual influence; thus it allows simultaneously observing the wave and particle nature of quantum objects without contravening any standard methodology of quantum theory.

4. Young's interferometer for quantum objects with internal freedom

In the history of quantum theory, Young's two-slit interferometer has become a popular gedanken experiment used to debate and clarify the concepts and philosophies of quantum mechanics, among which Einstein's recoiling slit,[3] Heisenberg's γ-ray microscope,[4] and Feynman's light–electron scattering scheme[12] are several prominent examples. The failure to obtain simultaneous definite information of wave and particle by using these old experimental setups is closely related with Heisenberg's uncertainty principle. The act of measuring the which-way information inevitably disturbs the particle in such a manner that coherence is lost and the interference pattern disappears. Mathematically, the wave function diffracted by the two slits at the presence of strong perturbation can be written as

$$\begin{aligned}\psi_{\rm t}(\boldsymbol{r},z) &= \psi_1(\boldsymbol{r},z)e^{i\delta\varphi_1} + \psi_2(\boldsymbol{r},z)e^{i\delta\varphi_2} \\ &= A(z)[e^{i\delta\varphi_1}\psi_{1,0}e^{i\varphi_1(\boldsymbol{r},z)} + e^{i\delta\varphi_2}\psi_{2,0}e^{i\varphi_2(\boldsymbol{r},z)}], \quad (6)\end{aligned}$$

under the simplified model where the perturbation induces large random phases $\delta\varphi_1$ and $\delta\varphi_2$ to each secondary wave function from the two slits. Then the signal recorded at the observation screen is given by

$$\begin{aligned}I_{\rm t}(\boldsymbol{r},z) &= |A(z)|^2\big[|\psi_{1,0}|^2 + |\psi_{2,0}|^2 \\ &\quad +2|\psi_{1,0}||\psi_{2,0}|\langle e^{-i\delta\varphi_1}\rangle\langle e^{i\delta\varphi_2}\rangle\cos(\Delta\varphi)\big] \\ &= |A(z)|^2(|\psi_{1,0}|^2 + |\psi_{2,0}|^2). \quad (7)\end{aligned}$$

Here bracket $\langle\ldots\rangle$ means the ensemble average. Obviously, the methodology of quantum theory once again successfully predicts the outcomes of the strongly perturbed Young's two-slit interferometer gedanken experiments. On the other hand,

if the which-way measurement does not induce a large random phase shift, but instead induces a constant phase shift (or a random but negligibly small phase shift) for every particle, then equation (2) still holds and the which-way information acquisition operation does not destroy the interference pattern. Of course, to achieve this goal in practice rather than just in a thought experiment, the observation-induced perturbation must be sufficiently weak so as to maintain the spatial coherence of the two-slit secondary waves, as understood in the language of wave optics or wave mechanics.

To this end, Scully and coworkers have proposed a modified scheme of Young's two-slit interferometer involving a micromaser-cavity which-way detector to interact with the internal state of long-lived Rydberg atoms.[28,29] Although the which-way detector induces only a negligibly small but random phase shift to the center-of-mass wave function of atoms,[30–33] Scully's team concluded that their scheme still cannot acquire both the wave and particle information simultaneously due to the entanglement between the which-way detector and the atomic internal states.[28,29] Later experiments seem to justify this scheme of entanglement, upholding the complementarity principle.[30,31] In the following, we will revisit and generalize this type of gedanken experiment and make an elementary quantum mechanical analysis on the outcome under various situations from a new conceptual angle, which eventually leads to a conclusion that differs totally from the historically popular one.

The first thing that we need to do before going anew into the details of discussing the wave–particle duality in an atomic system is to clarify the object of examination. This could become a problem because, as we learned so long ago, an atom itself is not an elementary particle but rather a very complicated composite system. So it is the motion of the atom as a whole, more precisely, the atom's center-of-mass, that should be considered as the most appropriate representation of the atom as a quantum object. As is increasingly well known, each atom possesses some internal structures and freedoms. For example, it consists of a nucleus at its center and some electrons around the nucleus. The nucleon consists of many protons and neutrons, which in turn are made up of quarks. When observed, the atom might interact with the environment (e.g., which-way detectors) through some of its internal freedoms during the process of its center-of-mass motion. It is therefore obvious that when we talk about the interference of atoms, we should refer only to the atom's center-of-mass, rather than the interference of any of these internal components. Otherwise, ambiguity inevitably occurs, and more seriously, incoherence in any internal freedom (such as electrons and quarks) will force one to conclude that the atomic wave cannot be coherent, which is of course not the case. Essentially, the effects of the internal freedoms in an atom interferometer should be reflected only through their influence on the atomic center-of-mass motion; otherwise, they are just unobserved by the operation of an atom interferometer.

The motion of atomic center-of-mass can also be described by quantum state $\psi(\boldsymbol{r},z)$. In free space, $\psi(\boldsymbol{r},z)$ can be approximately described by plane wave $\psi(\boldsymbol{r},z) = \psi_0 \mathrm{e}^{\mathrm{i}kx}$ when a high-coherence atomic beam is concerned, and the atomic center-of-mass momentum is given by $p = \hbar k$. When only considering Young's two-slit interferometer for the atomic center-of-mass, the formalism (Eqs. (1)–(3)) applies equally well, and the coherence of the atomic beam is the key to observing a high-quality interference pattern. However, it is also impossible to acquire the wave and particle information simultaneously using the conventional scheme where the which-way detector works directly on the atomic center-of-mass. Things might appear very different when the internal freedom of an atom is adopted to interact with the which-way detector and thus serves as a means of probing the which-way information.

In the new Young's two-slit interferometer setup for quantum objects with internal freedoms described by quantum state φ, the total wave function of the atomic system after passing through the two slits and being subject to diffraction and interference effects is given by

$$\Psi(\boldsymbol{R}) = \psi_1(\boldsymbol{R})\varphi_1 + \psi_2(\boldsymbol{R})\varphi_2. \tag{8}$$

If one directly uses the methodology of quantum theory literally without going into details of its physical meaning, the interference pattern is given by

$$\begin{aligned} I(\boldsymbol{R}) &= |\Psi(\boldsymbol{R})|^2 \\ &= |\psi_1(\boldsymbol{R})|^2 + |\psi_2(\boldsymbol{R})|^2 + 2\mathrm{Re}[(\varphi_1^*\varphi_2)\psi_1^*(\boldsymbol{R})\psi_2(\boldsymbol{R})]. \end{aligned} \tag{9}$$

This formulation has been overwhelmingly and popularly adopted in the analysis of atomic interferometers, and it implies that the interference fringe pattern is modulated by both the center-of-mass wave motion (denoted by coordinate $\boldsymbol{R}$) and the internal quantum states. Here we have assumed that φ_1 and φ_2 are normalized wave functions defined in a certain Hilbert sub-space so that $\langle\varphi_1|\varphi_1\rangle = 1$ and $\langle\varphi_2|\varphi_2\rangle = 1$. Yet, equation (9) needs more careful examination. Usually in practice, φ_1 and φ_2 are quantum states in their specific Hilbert subspaces, rather than in the usual real space, unlike the center-of-mass motion states $\psi_1(\boldsymbol{R})$ and $\psi_2(\boldsymbol{R})$. Therefore, it seems hard to interpret the total state in Eq. (8) according to the probability definition of orthodox quantum mechanics in terms of Born's interpretation of the wave function for a quantum state, namely, the probability of finding a particle at position $\boldsymbol{R}$. Similarly, the definition of the interference pattern of the atomic center-of-mass according to Eq. (9) is also problematic. This issue will make the wave–particle duality more complicated to describe and harder to imagine and understand. That this issue can have a very significant consequence will become clear from the following examples.

In general experiments about wave diffraction and interference of quantum objects, the internal quantum state does not play an active role; in other words, the internal quantum state is not intentionally prepared in some specific states. This means that the quantum states in the two slits are generally not coherent, so $\langle\varphi_1^*\varphi_2\rangle = \langle\varphi_1^*\rangle \times \langle\varphi_2\rangle = 0$. Then according to Eq. (9), $I(\boldsymbol{R}) = |\psi_1(\boldsymbol{R})|^2 + |\psi_2(\boldsymbol{R})|^2$ and the interference fringe completely disappears, as the internal state modulation amplitude is zero. If special care is taken to prepare the internal quantum state to be highly coherent, then equation (9) holds true, but whether the interference pattern appears or not sensitively depends on the value of $\langle\varphi_1|\varphi_2\rangle$. If the two internal states are orthogonal, then the interference pattern disappears but leaving the possibility to acquire the which-way information by looking at the internal state. Otherwise, if $\varphi_1 = \varphi_2$, then the interference pattern appears but making it impossible to acquire the which-way information.

Yet, when the above conceptual argument and methodology operation extend without any constraint to all the internal states of electrons and the nucleus comprising the atom, then all these internal states cannot be coherent simultaneously. If we apply Eq. (9) literally, then no interference pattern can be observed in any Young's two-slit interferometer for composite quantum objects with internal freedoms, even if one does not trouble to observe simultaneously the which-way information for the quantum object transporting in the interferometer. This of course is not the case, as numerous experiments have demonstrated the wave nature of quantum objects through diffraction and interference. So something must be wrong physically in directly and literally applying the orthodox definition that the interference pattern is given simply by $I(\boldsymbol{R}) = |\Psi(\boldsymbol{R})|^2$, even if $\Psi(\boldsymbol{R})$ is the total wave function for the composite quantum system involving multiple quantum states defined in various Hilbert sub-spaces.

5. The role of which-way detectors interacting with quantum objects

Things will become even more complicated when the which-way detectors are further introduced to form an even larger coupled quantum system together with the atomic system. Through direct and literal application of $I(\boldsymbol{R}) = |\Psi(\boldsymbol{R})|^2$ to define the interference pattern, many conceptual conclusions can be, and indeed have been, deduced. Although they may be obviously counter-intuitive and seemingly unphysical in many situations, more and more new and fashionable concepts and methodologies such as entanglement, delayed-choice, quantum eraser, just to name a few important ones, are introduced by defenders and proponents of the orthodox Copenhagen interpretation of quantum theory to argue that these conceptual conclusions reflect the truth in the quantum worldview. A prominent example is the micromaser which-way detector gedanken experiment proposed and analyzed by Scully.[28,29] A simplified version of the gedanken experiment is illustrated in Fig. 4, where a coherent beam of Rydberg atoms with excited internal state passes through two micromaser cavities each placed just ahead of one of Young's two slits. The micromaser cavity serves as the which-way detector by interacting with the internal quantum states of the atoms.

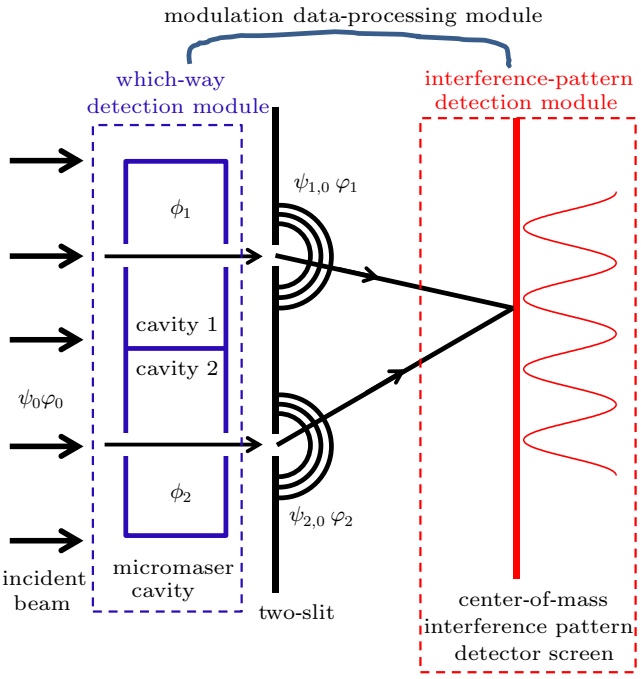


Fig. 4. Analysis of Young's two-slit interferometer setup for atoms with internal freedom coupled with the micromaser which-way detectors. The interferometer consists of four components. The first part is the incident Rydberg atomic beam described by composite wave function $\psi_0\varphi_0$, where ψ_0 is the atomic center-of-mass wave function, assumed to have high coherence, and $\varphi_0 = \varphi_e$ is the initial excited quantum state of atomic internal freedom, which is assumed to be the same for all atoms. The second part is the micromaser cavity placed before and collimated with the two slits. They are coupled with the excited quantum state of atom and serve as the which-way detector from their photon quantum state described by ϕ. By looking at the emitted microwave photon number in each cavity and the corresponding quantum state of field (ϕ_1 and ϕ_2), one can tell which cavity the atom passes through. The third part is an opaque screen encoded with two parallel slits separated by some distance, and the secondary atom waves at the exits of the two slits are described by $\psi_{1,0}\varphi_1$ and $\psi_{2,0}\varphi_2$, where $\varphi_1 = \varphi_2 = \varphi_g$ is the final state of the atom internal freedom (ground state), while $\psi_{1,0}$ and $\psi_{2,0}$ are the atomic center-of-mass wave functions. The fourth part is a screen decorated with detectors that allow only for observing and measuring the interference pattern reflecting the atomic center-of-mass positioning probability without the complexity induced by incorporation and interaction with the internal freedom of atom. In a practical experiment, the second and the fourth parts can be categorized as the which-way detector module (blue box) and the interference-pattern detection module (red box), respectively. These two modules act together in a well-separated space–time domain without mutual influence and allow one to simultaneously acquire the wave and the particle behaviors of the atoms. However, if another module, called the modulation data-processing module, is used to artificially modulate the atomic center-of-mass interference pattern with the coincidence-counting outcome of cavity photon numbers in the which-way detection module, the outcome of this joint data-convolution action will lead to a seeming disappearance of the interference pattern because it just corresponds to the mathematical interference term of $\mathrm{Re}[(\varphi_1^*\varphi_2)(\phi_1^*\phi_2)\psi_1^*(\boldsymbol{R})\psi_2(\boldsymbol{R})] = 0$. This result has been popularly described as the entanglement enforced complementarity principle by Scully and many others. Besides, such an artificial connection of the which-way detection module, where many changes and modifications can be made, with the interference-pattern detection module is also the origin of so-called delayed-choice, quantum eraser, and many other counter-intuition conceptual arguments and deductions.

Let us tentatively follow Scully by going beyond the atom itself and considering simultaneously the detector. In other words, we now need to consider the atom–detector as a whole system. Suppose the quantum states of the two which-way detectors are described by ϕ_1 and ϕ_2 in another Hilbert subspace, respectively, corresponding to atoms passing through either slit. In addition, assume that the detector interacts only with the internal quantum state of the particle and neglect the direct interaction of the detector with the center-of-mass motion of the atom. Then the total wave function of the coupled atom–detector quantum system is given by

$$\Psi(\boldsymbol{R}) = \psi_1(\boldsymbol{R})\varphi_1\phi_1 + \psi_2(\boldsymbol{R})\varphi_2\phi_2. \tag{10}$$

Again one uses the operation methodology of orthodox quantum theory and finds that the interference pattern is given by

$$\begin{aligned} I(\boldsymbol{R}) &= |\Psi(\boldsymbol{R})|^2 \\ &= |\psi_1(\boldsymbol{R})|^2 + |\psi_2(\boldsymbol{R})|^2 \\ &\quad + 2\mathrm{Re}\big[(\varphi_1^*\varphi_2)(\phi_1^*\phi_2)\psi_1^*(\boldsymbol{R})\psi_2(\boldsymbol{R})\big]. \end{aligned} \tag{11}$$

When the internal quantum state of the atom, φ, interacts with the micromaser cavity quantum state ϕ so that the which-way information of the atom is unambiguously transferred to and recorded by the which-way detector, one has $\varphi_1 = \varphi_2 = \varphi$ and $\langle\phi_1|\phi_2\rangle = 0$, thus the interference term in Eq. (10) $\mathrm{Re}[(\varphi_1^*\varphi_2)(\phi_1^*\phi_2)\psi_1^*(\boldsymbol{R})\psi_2(\boldsymbol{R})]$ is equal to zero. This means that the unambiguous recording of the which-way information in the Young's interferometer will definitely sweep away the interference pattern for the atomic center-of-mass motion. This thought outcome would thus seemingly uphold the principle of complementarity.

In the modified two-slit interferometer proposed here, the acquisition of the which-way information does not suffer from Heisenberg's uncertainty principle induced by the uncontrollable interaction of the which-way detector with the atomic internal freedoms, because the interaction energy is much smaller than the center-of-mass momentum energy.[28–33] This which-way detector seems to be smarter than the old ones such as Einstein's recoiling slit.[3] In the scheme of Einstein's recoiling slit, one needs to directly operate on the particle center-of-mass motion in order to acquire the which-way information, and the interaction length scale must be comparable with the slit width, leading to an interaction sufficiently strong to induce uncontrollable perturbation upon the center-of-mass motion and destroy the interference pattern. Obviously, it is Heisenberg's uncertainty principle that works to greatly deteriorate the coherence of the atomic beam. In contrast, in the scheme of Scully's micromaser interferometer, the interaction length scale of the micromaser with the atomic internal freedoms is on the scale of the wavelength of microwave photon and comparable with the cavity size, which could be much larger than the slit width. So the emission of microwave photons and the subsequent recoil of the subject atom induce only negligible influence on the coherence of the atomic center-of-mass. However, it was argued and generally agreed that the entanglement between the which-way information and the micromaser quantum state would enforce a destructive factor in the atom–detector quantum state, as described in Eq. (11), to sweep away the interference pattern.[28,29] Notice that these arguments about the entanglement-enforced complementarity principle are more or less based on the confidence and belief that the conceptual basis for the orthodox Copenhagen interpretation of quantum theory, the principle of complementarity, is always correct, no matter how much time passes and how much scientific technologies develop.

In some sense, the analysis is like looking for supporting materials based on an *a priori* conceived conclusion, i.e., the complementarity principle is always true, rather than deducing conclusions from new data in normal logic. The logical way of argument about the role of entanglement in the wave–particle duality is counter-intuitive for many people. Yet, many further arguments and outcomes as a consequence of this analysis, including the concept of delayed-choice experiment and quantum erasers, look even more counter-intuitive and harder to swallow for people who are not hot-faith proponents of orthodox conceptual quantum theory.[5–8,11–18,47] Being a realist rather than a doctrinaire of quantum theory, I try to make a more comprehensive analysis of such a type of gedanken experiment, following the well-established regular methodology of quantum theory and using regular logic, in a hope to offer an alternative and common-sense solution to this old, classical, and puzzling problem of wave–particle duality. I find that the key lies in how to measure the interference pattern.

6. Analysis of interference pattern measurement

The above discussions largely focus on how to acquire the which-way information of an atomic system. The Scully scheme has offered, at least in principle, a smarter way to achieve this goal without suffering from the constraint of Heisenberg's uncertainty principle. Yet, there is another issue that is equally important in the wave–particle duality test by using an atomic interferometer, namely, the acquisition of the interference pattern. Obviously, much less attention has been paid to handling this issue, not only in the Scully work, but also in many other gedanken and practical experiments. In some sense, the situation of much less intensive and extensive analysis, or even negligence and ignorance, attending to the vital details of measuring the interference pattern formed by quantum objects in a wave–particle duality test might be the reason why so many concepts and interpretations that are apparently counter-intuitive, seemingly unphysical or metaphysical, but are still overwhelmingly thought to be correct

quantum mechanically, are raised and more or less cause great confusion in conceptual understanding of these gedanken or practical wave–particle duality test experiments. Thus, it is invaluable to make a detailed analysis on how to measure the interference pattern in various interferometers.

First, let us go to the simplest situation — a quantum object without internal freedom. The virtual interference pattern of the two-slit interferometer is given by Eqs. (1) and (2). I use the term "virtual" here because the interference pattern at this step is largely only of mathematical or conceptual meaning. To bring it into reality, some certain experimental technique or measurement (a term popularly used by the proponents of orthodox quantum theory) must be performed to transfer the "virtual" probability distribution profile of the quantum object into a form that can be sensed by human beings or recorded through some practical instruments. However, the paramount principle is that the measurement should faithfully reflect the "virtual" profile. In optics, a simple flat rough-cast screen or glass can be used to record this interference pattern as the scattering intensity of light off the screen as visualized by human eyes is just proportional to the intensity profile at the screen, thus the "virtual" interference pattern is recorded by the screen. In atomic optics, a position sensitive detector can be used to record the distribution of atoms. On the other hand, the measurement device should not deviate much from this linear operational regime, otherwise, the "virtual" interference pattern would not be faithfully recorded. In optics, a black surface obviously cannot implement such a recording functionality as it smears off all patterns irrespective of their profiles.

The importance of talking about the measurement process for the interference pattern will become clear when it comes to the Young's interferometer for composite quantum objects with internal freedoms. A good example is the Rydberg atom beam with its internal quantum states interacting with the micromaser cavity which-way detector, as discussed in Fig. 4. The total wave function for the composite quantum object when reaching the observation screen is given by Eq. (8) as $\Psi(\boldsymbol{R}) = \psi_1(\boldsymbol{R})\varphi_1 + \psi_2(\boldsymbol{R})\varphi_2$. Only $\psi_1(\boldsymbol{R})$ and $\psi_2(\boldsymbol{R})$ are related with the center-of-mass motion of the quantum object in real space. They are subject to phase shifts depending on their transport paths and are responsible for the formation of the interference pattern. Of course, this is just the "virtual" interference pattern. On the other hand, the quantum states for internal freedoms φ_1 and φ_2 are defined in another Hilbert subspace. They will evolve by themselves, say, having a phase shift or other changes, during their transport accompanying their master quantum object. Usually the center-of-mass motion and the internal freedoms are independent of each other during the transport of a quantum object in free space, unless an indirect interaction happens when one freedom or all of them are subject to some external perturbation.

The paramount principle of faithful measurement should also be followed to record the interference pattern — in our proposed method, for detecting the center-of-mass of the quantum object. Frankly speaking, this is not an easy task that any measurement instrument would conveniently work to achieve. This task requires great caution and care so as not to induce ambiguity, confusion, or error without being aware of them. I would not attempt to reexamine how measurement of the interference pattern was made in all the wave–particle duality gedanken and practical experiments performed in history, as this seems to be impossible. Instead, I will here argue and discuss the principles of how to make a good measurement, considering various situations.

Generally, the measurement can be modeled as an operation M working on the wave function of the composite system $\Psi(\boldsymbol{R}) = \psi_1(\boldsymbol{R})\varphi_1 + \psi_2(\boldsymbol{R})\varphi_2$. The new wave function after the measurement is then

$$\Psi_M(\boldsymbol{R}) = M\{\Psi(\boldsymbol{R})\} = M\{\psi_1(\boldsymbol{R})\varphi_1\} + M\{\psi_2(\boldsymbol{R})\varphi_2\}. \quad (12)$$

In general situations, the operator M could work on the center-of-mass motion or on the internal freedom or on both. In the first case, M only works on $\psi_1(\boldsymbol{R})$ and $\psi_2(\boldsymbol{R})$; then the internal quantum state does not participate in the measurement process, or in other words, the measurement is insensitive to them. In this case, the composite quantum object behaves the same as a pure quantum object. Then equation (12) transforms to

$$\Psi_M(\boldsymbol{R}) = M\{\Psi(\boldsymbol{R})\} = A[\psi_1(\boldsymbol{R}) + \psi_2(\boldsymbol{R})]. \quad (13)$$

Here a good measurement has been assumed, where the measurement introduces only a linear amplitude factor. Besides, the measurement should also yield a signal intensity given by

$$\begin{aligned} I_M(\boldsymbol{R}) &= |\Psi_M(\boldsymbol{R})|^2 \\ &= |A|^2\{|\psi_1(\boldsymbol{R})|^2 + |\psi_2(\boldsymbol{R})|^2 + 2\mathrm{Re}[\psi_1^*(\boldsymbol{R})\psi_2(\boldsymbol{R})]\}. \end{aligned} \quad (14)$$

Obviously, a perfect interference pattern representing the positioning probability of the quantum object center-of-mass has been recorded by this good measurement, which is the term $\mathrm{Re}[\psi_1^*(\boldsymbol{R})\psi_2(\boldsymbol{R})]$. Of course, in practice there are many other possibilities for bad measurements that deteriorate the interference pattern by either introducing a random phase shift, or imposing nonlinear or more complicated modulation to $\psi_1(\boldsymbol{R})$ and $\psi_2(\boldsymbol{R})$, although these measurements also completely neglect the internal freedoms. These bad measurements should be avoided in practical experiments.

In the second case, the measurement operator M works only on the internal freedom and induces a transition of φ_1 and φ_2 to new states $\varphi_{1,M}$ and $\varphi_{2,M}$. This scheme has been adopted in most atom interferometers for wave–particle duality tests, such as those reported in Refs. [34] and [35]. For instance, light emission when atom jumps from a higher electronic state to a lower electronic state under the dipole transition can be

used to probe the position of an atom by recording the emission site optically. In this situation, equation (12) transforms to

$$\Psi_M(\boldsymbol{R}) = M\{\Psi(\boldsymbol{R})\} = \psi_1(\boldsymbol{R})\varphi_{1,M} + \psi_2(\boldsymbol{R})\varphi_{2,M}. \quad (15)$$

It is seen that the center-of-mass wave function just imposes a position dependent probability modulation to the internal state transition. Suppose that the recorded signal intensity is proportional to the transition probability of the quantum state, as is the case in light emission experiments. The signal is given mathematically by

$$\begin{aligned} & I_M(\boldsymbol{R}) \propto \langle \Psi(\boldsymbol{R})|M|\Psi_M(\boldsymbol{R})\rangle \\ = & |\psi_1(\boldsymbol{R})|^2\langle\varphi_1|M|\varphi_{1,M}\rangle + |\psi_2(\boldsymbol{R})|^2\langle\varphi_2|M|\varphi_{2,M}\rangle \\ & + \psi_1^*(\boldsymbol{R})\psi_2(\boldsymbol{R})\langle\varphi_1|M|\varphi_{2,M}\rangle \\ & + \psi_1(\boldsymbol{R})\psi_2^*(\boldsymbol{R})\langle\varphi_2|M|\varphi_{1,M}\rangle. \end{aligned} \quad (16)$$

Obviously, the best way to record a perfect interference pattern with $I_M(\boldsymbol{R}) \propto |A|^2\{|\psi_1(\boldsymbol{R})|^2 + |\psi_2(\boldsymbol{R})|^2 + 2\mathrm{Re}[\psi_1^*(\boldsymbol{R})\psi_2(\boldsymbol{R})]\}$ is to make $\langle\varphi_1|M|\varphi_{2,M}\rangle = \langle\varphi_2|M|\varphi_{1,M}\rangle$, and the simplest way to satisfy this is to have $\varphi_1 = \varphi_2$ and $\varphi_{1,M} = \varphi_{2,M}$. On the other hand, if $\langle\varphi_1|M|\varphi_{2,M}\rangle = \langle\varphi_2|M|\varphi_{1,M}\rangle = 0$, then the interference pattern disappears even if the measurement imposes no uncontrollable perturbation to the quantum object's center-of-mass. The analysis clearly shows that the interference pattern obtained by this type of measurement is a convolution of the internal state transition intensity and the quantum object center-of-mass positioning probability. Both contribute equally to the apparent interference pattern. Strictly speaking, this composite-measurement scheme is subject to serious ambiguity, as it strongly depends on the internal state evolution of the atom under probe, and essentially it is also the origin of many delayed-choice outcomes of wave–particle duality test, like those presented in Refs. [34] and [35]. This point should be kept in mind when revisiting the wave–particle duality test experiments (either gedanken or practical) of history.

In the above, we discussed how to make a good measurement of the interference pattern formed by quantum objects, either those without internal freedoms or those with internal freedoms. As the two quantities exactly overlap in space, a probe over them can reveal the interference feature of the quantum object, albeit caution and care should be taken. On the other hand, in our proposed method, the probe over the which-way information is made by another set of measurement instruments that is separated in space from the interference measurement instrument. These two sets of measurements could arguably be independent with proper care, as a result of which it is then possible to acquire both the which-way information and the interference pattern simultaneously. However, in many examples of wave–particle duality test experiments, these two sets of measurements are assumed to be closely correlated or entangled, even if they are separated in space, rendering it impossible to acquire both the wave and the particle behaviors simultaneously, and enabling one to uphold the consistency of the complementarity principle.[28,29]

Let us take a closer look at the Scully gedanken experiment[28,29] and focus on the measurement process. The total wave function of the coupled atom–detector quantum system is given by $\Psi(\boldsymbol{R}) = \psi_1(\boldsymbol{R})\varphi_1\phi_1 + \psi_2(\boldsymbol{R})\varphi_2\phi_2$ when the atom has just passed through the two slits. This type of wave function represents a typical entanglement state and it is the atom–detector interaction that creates this entanglement. After the interaction ends, the two sub-systems evolve by themselves without disturbing each other and each is subject to measurement. The atom, with its center-of-mass carrying an internal quantum state, transports in free space and reaches the observation screen where the measurement over the interference pattern is implemented. By its wall, the micromaser cavity will eventually absorb the microwave photon emitted by the transmitted atom and fulfill the measurement of the which-way information by enabling the detection of the photon number (0 or 1) every time in the two cavities. If we generalize the atom–detector system as a quantum object (atom) with internal freedom (the state of the which-way detector), then according to the above discussion on the measurement process, one can simply adopt a measurement that either operates directly on the atomic center-of-mass motion or indirectly operates on the atom's internal state (which imposes no trouble as $\varphi_1 = \varphi_2 = \varphi$), from which it is not difficult to acquire the interference pattern, either appearing or disappearing. In either way, the signal intensity at the screen is given by Eq. (14), where a nearly perfect interference pattern should be observed as $\psi_1(\boldsymbol{R})$ and $\psi_2(\boldsymbol{R})$ are nearly identical (more precisely, with the addition of only a negligibly small random phase factor) to the perfect function without interaction between the atom and the maicromaser cavity. On the other hand, because of the excellent separation of the screen from the micromaser cavity, the measurement of the interference pattern will have no influence upon the which-way information measurement, i.e., the cavity photon number counting. In this way, the wave and the particle behaviors of the atom could be simultaneously illustrated by using this experimental scheme.

If, on the other hand, a very extraordinary, and strange in some sense, measurement operation is made to examine the wave–particle behavior in the coupled atom system, things can be very different and favor the upholding of the complementarity principle. As depicted in Fig. 4, if one modulates the atomic center-of-mass wave function (exhibiting interference pattern) with the outcome of a special measurement made upon the internal freedom of the atom (the micromaser cavity state), which is just a simple coincidence counting of the photon numbers in the two cavities, then this modulation factor is exactly zero, destroying the interference pattern, as im-

plied by Eq. (11). Mathematically, this coincidence counting measurement is equal to determine the term $\langle\phi_1|\phi_2\rangle$, which should be exactly zero as expected. In some sense, this coincidence counting is a good measurement for which-way information acquisition, but it is a bad measurement for interference pattern observation when it is inappropriately used to modulate the interference pattern. The interaction of the good and the bad measurements leads to the failure of the simultaneous illustration of wave and particle natures in this special Young's two-slit interferometer and inadvertent upholding of the complementarity principle of the orthodox quantum theory. Although the two sets of measurements are mutually exclusive (either wave or particle, but not both), they are just two very special interrelated measurements among so many possible operations of measurement that can work on our proposed two-slit interferometer. One can easily go around them and take other good measurements for illustrating both natures simultaneously, like the ones mentioned earlier. Yet, if one wishes to uphold the complementarity principle and the conceptual orthodox Copenhagen interpretation of quantum theory, then the two special sets of measurements in cooperation via artificial data processing are the only way to achieve this goal, no matter how counter-intuitive, extraordinary, and even absurd such an artificial action of measurement might look. However, such an action necessarily leaves the obvious loophole of conceptual justification.

The inappropriate and artificial connection of the which-way detector measurement and the interference pattern measurement as illustrated in Fig. 4 can be be used to explain the outcome of so many delayed-choice experiments and quantum eraser experiments that have been further deduced to be intrinsically involved and manifested in this type of interferometer.[36–46] The reason is that one can now connect whatever operation on the which-way detector (instant or delayed-choice) with the distant interference pattern measurement operation and generate any outcome of experiment, removing or restoring the interference pattern. In some sense, this connection is not a physical one that instantly occurs in the interferometer but just a mathematical operation that can be arbitrarily manipulated in a way according to one's subjective desire via post management and processing (e.g., convolution and modulation) of two sets of measurement data and can be implemented at an arbitrary time, long after the experiment.

7. Further discussion

The above discussions show that the key to acquire both the wave and particle information for quantum objects in a good interferometer lies in the simultaneous use of two sets of good measurement devices that are well separated in space and do not perturb each other. In addition, all the analyses are made based on elementary methodology of quantum mechanics rather than abstract and counter-intuitive conceptual arguments and justifications. These gedanken experimental schemes can be readily tested by using modern quantum optics or atomic physics technologies.[27,41–46,48–51]

The wave nature of a quantum object manifests in the appearance or disappearance of a distinct peak–valley contrast interference pattern, and the formation of this pattern requires an ergodic ensemble of quantum objects to participate either via time-accumulation or via sample-accumulation. The key in experiments is the maintenance of the coherence status of the ensemble. A single quantum object passing through an interferometer cannot illustrate a sufficiently fine interference pattern in experiments. This point should hold true in both the Mach–Zehnder interferometer as discussed in Section 3 and Young's interferometer as discussed in Sections 4–6. Actually, such assumption and concept have been justified in numerous experiments that were performed to test the wave nature of massive particles such as electrons and massless particles such as photons. For instance, in experiments of particle diffraction and interference, the observed pattern always gradually evolves from some randomly kicking points in the observation screen for the initial one, two, three, and small numbers of particles and events, to a final stable and well-defined diffraction or interference pattern for a sufficiently large number of particles and events. This feature can be well understood by the probability nature of the wave function used to describe the motion of these particles in orthodox quantum mechanics. Therefore, any experimental setup, including the interference pattern detector screen in the Mach–Zehnder interferometer as illustrated in Fig. 3, that is used to observe the interference pattern of quantum objects, must operate and should only operate in such a probabilistic way for a sizable ensemble of particles in the practical operation of measurement. Nobody has ever designed or even imagined a technological means to demonstrate and observe the interference pattern for only one single quantum object.

On the other hand, the particle nature of quantum objects manifests in the which-way information detection of each individual particle. This can be easily done by a wide variety of modern technologies. The difficulty here is to introduce as little as possible perturbation to the particle passing through the two slits and maintain the coherence of the ensemble of particles as a whole. In history, many schemes for interferometers failed to pass such a two-fold criterion. But now the internal freedom of quantum objects, which is intrinsic to almost all quantum particles, can be exploited to probe the which-way information of every particle while maintaining the high coherence status of the whole ensemble of particles' center-of-mass in various interferometers. In other words, it is now possible to design good schemes of measurement to acquire both the particle and wave nature of a quantum object. Although this would seem to violate the popular wave–particle duality and complementarity principle that stand at the center

conceptual stage of orthodox quantum theory, it strictly follows the orthodox methodology of quantum theory about the probability nature of the wave function and its evolution following Schrödinger's equation, either under a fixed environment or under interaction with detectors.

Another interesting issue is whether it is possible experimentally to test the wave–particle duality for an atomic beam with different internal freedoms transporting within the Young's two-slit interferometer. In the Scully scheme, both the incident atomic beam going into the micromaser cavity (excited state) and the transmission beam passing through the two slits (lower state) have the same quantum states of internal freedom in the two paths. When the atomic beam reaches the screen, the internal freedom is allocated at the same quantum state of $\varphi_1 = \varphi_2 = \varphi$, therefore, one can directly operate on the atomic center-of-mass motion or the internal quantum state to measure the interference pattern. The which-way information is acquired via another measurement in the micromaser cavity. Yet, if the atomic beams passing through the two slits are in different internal quantum states, say $\varphi_1 \neq \varphi_2$, which can be done by placing a micromaser cavity only before the upper slit while leaving the lower slit open, then according to Eq. (12), one can adopt a measurement scheme that operates only on the atomic center-of-mass motion and misses the internal freedom to acquire the interference pattern of the ensemble of quantum object.

The schematic setup of this new Young's two-slit interferometer can be envisioned by making only a small change to the setup illustrated in Fig. 4. The which-way information can still be acquired by the single micromaser cavity by looking at whether there is emission of photon in the upper cavity. Whether the atom passes through the lower slit cannot be known directly, but this does not impose any trouble that changes the outcome, because logically, if one knows that the atom passes (or does not pass) the upper slit, then it must not pass (or must pass) the lower slit. There is something that one would know without making a direct measurement, but just through many physical laws, such as energy conservation, momentum conservation, or particle number conservation, which suffice as an indirect partial measurement. On the other hand, if one can find an even smarter scheme of measurement that allows for simultaneous determination of not only the atomic center-of-mass position, but also its internal state (either φ_1 or φ_2), then it is capable of acquiring the which-way information of every individual atom by such an operation working purely upon the atom itself, without the trouble to count the photon number (0 or 1) in the micromaser cavities. This, of course, brings more flexibility in designing good measurement schemes to test the wave–particle duality with high fidelity and precision.

In principle, almost all particles, including massless particles such as photons and massive particles such as electrons, protons, neutrons, atoms, and molecules have intrinsic internal freedoms. For instance, photons have polarizations, electrons have spins, protons and neutrons have nucleus spins, while atoms and molecules have many subtle electronic states. Various interferometers constructed and operated based on the above-mentioned schemes can be employed to make a wave–particle duality test experimentally rather than just gedankenly by using state-of-the-art technologies. Subtle differences between massless particles (known as the media for interaction force) and massive particles (known as the origin of interaction force) might be revealed by such a high fidelity and precise wave–particle duality experimental test. They would undoubtedly open a new door to explore the nature of various elementary particles and their composite objects in the quantum world, more than 100 years after the first door of the quantum world was uncovered by the pioneers of quantum theory, including Einstein, Bohr, Heisenberg, Schrödinger, Dirac, and many others.

8. Conclusions and perspectives

We have made a systematical analysis of various Young's two-slit interferometers and Mach–Zehnder two-arm interferometers for testing the wave–particle duality and the principle of complementarity by adopting only an elementary methodology of quantum mechanics rather than purely conceptual arguments and justifications. In particular, we focused on the key issues of how to make good measurements of the which-way information (particle nature) and the interference pattern (wave nature) of quantum objects in these interferometers. We have realized that the internal freedoms of quantum objects could be used to construct both which-way detectors and interference-pattern detectors for the center-of-mass motion of atoms. However, the way of arrangement and cooperation of these two sets of measurement instruments in the interferometer would lead to completely different results. We have found that arrangement of good and bad measurements on either aspect (wave or particle), or inappropriate arrangement of good measurements on both aspects (wave and particle) would lead to failure of simultaneous observation of wave and particle behaviors in numerous past gedanken and realistic experiments and seem to uphold the orthodox conceptual knowledge about the wave–particle duality and the principle of complementarity. Based on such a new view, we are able to design several schemes that allow for simultaneously acquiring the which-way information for an individual quantum object and the high-contrast interference pattern for an ensemble of these quantum objects by placing two sets of measurement instruments that are well separated in space and whose perturbation of each other is negligibly small within a single Young's interferometer or Mach–Zehnder interferometer.

Our new designs of interferometer are drastically different from many interferometers in history and current time in

their concept and motivation. As the focus is placed on how to handle the issue of making simultaneous good measurement of the wave (via interference pattern) and the particle (via which-way information) nature of quantum objects, rather than on the philosophical and conceptual argument of how to observe them, our schemes offer a more practical means to go beyond the complementarity principle, namely, a way to simultaneously observe the wave and the particle behaviors of quantum objects. In addition, our schemes allow for a continuous approach to the ideal realm of perfect wave and particle observation by adopting state-of-the-art weak measurement technologies in quantum optics and atomic physics. In this regard, the key issue is no longer the technology for making things finer, but rather the courage for bringing change and revolution of concept.

That quantum objects can behave beyond the wave–particle duality and the complementarity principle would stimulate new conceptual examination and exploration of quantum theory at a deeper level, more than 100 years after its foundation. Of course, such an important consequence in fundamental physics involved in our theoretical analysis should be subject to the highest level strict experimental tests, based on advanced quantum optics and atomic physics technologies. Nonetheless, even if these experimental tests would bring new understanding on the wave–particle duality and the complementarity principle in the quantum world, other more subtle issues such as whether Heisenberg's uncertainty principle can be violated in an Einstein–Podolsky–Rosen type experiment,[5] whether localism and realism strictly hold in the quantum world, whether one can observe simultaneously the wave and particle nature of a single quantum object, and many others, still largely remain mysterious and will continue to puzzle human beings. Yet, the step of human being to explore unknown frontiers and landscapes in science and nature will never come to a stop, and the new understanding of wave–particle duality and complementarity principle surely will shed new light and bring new insights for answering these fundamental questions in the quantum world.

Acknowledgement

The author acknowledges the helpful discussions with many colleagues ever since the first idea of the present work was raised in 1999.

References

[1] Bohr N 1928 *Naturwissenschaften* **16** 245
[2] Bohr N 1928 *Nature* **121** 580
[3] Bohr N 1983 *Quantum Theory and Measurement* (Wheeler J A and Zurek W H, ed.) (Princeton: Princeton Univ. Press) pp. 9–49
[4] Heisenberg W 1930 *The Physical Principles of the Quantum Theory* (Chicago: University of Chicago Press)
[5] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[6] Schrödinger E 1935 *Proc. Camb. Philos. Soc.* **31** 555
[7] Bohm D A 1952 *Phys. Rev.* **85** 166; **85** 180
[8] de Brogliem L 1960 *Non-linear Wave Mechanics, a Causal Interpretation* (Amsterdam: Elsevier)
[9] Von Neumann J 1955 *Mathematical Foundations of Quantum Mechanics* (Princeton: Princeton University Press)
[10] Stapp H P 1972 *Am. J. Phys.* **40** 1098
[11] Everett III H 1957 *Rev. Mod. Phys.* **29** 454
[12] Feynman R, Leighton R and Sands M 1965 *The Feynman Lectures on Physics* Vol. III, Chapter 1 (Reading: Addison Wesley)
[13] Jammer M 1974 *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective* (New York: Wiley)
[14] Bell J S 1988 *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press)
[15] Wootters W K and Zurek W H 1979 *Phys. Rev. D* **19** 473
[16] Wheeler J A and Zurek W H 1983 *Quantum Theory and Measurement* (Princeton: Princeton University)
[17] Zurek W H 2003 *Rev. Mod. Phys.* **75** 715
[18] Schlosshauer M 2004 *Rev. Mod. Phys.* **76** 1267
[19] Jönsson C 1961 *Z. Phys.* **161** 454
[20] Summhammer J, Badurek G, Rauch H, Kischko U and Zeilinger A 1983 *Phys. Rev. A* **27** 2523
[21] Zeilinger A, Gahler R, Shull C G, Theimer W and Mampe W 1988 *Rev. Mod. Phys.* **60** 1067
[22] Carnal O and Mlynek J 1991 *Phys. Rev. Lett.* **66** 2689
[23] Eichman U, Bergquist J C, Bollinger J J, Gilligan J M, Itano W M and Wineland D J 1993 *Phys. Rev. Lett.* **70** 2359
[24] Arndt M, Nairz O, Vos-Andreae J, Keller C, van der Zouw G and Zeilinger A 1999 *Nature* **401** 680
[25] Andrews M R et al 1997 *Science* **275** 637
[26] Hackermuller L, Uttenthaler S, Hornberger K, Reiger E, Brezger B, Zeilinger A and Arndt M 2003 *Phys. Rev. Lett.* **91** 090408
[27] Cronin A D, Schmiedmayer J and Pritchard D E 2009 *Rev. Mod. Phys.* **81** 1051
[28] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge Univ. Press)
[29] Scully M O, Englert B G and Walther H 1991 *Nature* **351** 111
[30] Storey P, Tan S, Collett M and Walls D 1994 *Nature* **367** 626
[31] Englert B G, Scully M O and Walther H 1995 *Nature* **375** 367
[32] Storey E P, Tan S M, Collett M J and Walls D F 1995 *Nature* **375** 368
[33] Wiseman H and Harrison F 1995 *Nature* **377** 584
[34] Durr S, Nonn T and Rempe G 1998 *Nature* **395** 33
[35] Bertet P, Osnaghi S, Rauschenbeutel A, Nogues G, Auffeves A, Brune M, Raimond J M and Haroche S 2001 *Nature* **411** 166
[36] Wheeler J A 1978 *Mathematical Foundations of Quantum Theory* (Marlow E R, ed.) (New York: Academic Press) pp. 9–48; 1979 *Problems in the Formulations of Physics* (di Francia G T, ed.) (Amsterdam: North-Holland)
[37] Scully M O and Druhl K 1982 *Phys. Rev. A* **25** 2208
[38] Hellmuth T, Walther H, Zajonc A and Schleich W 1987 *Phys. Rev. A* **35** 2532
[39] Lawson-Daku B J, Asimov R, Gorceix O, Miniatura Ch, Robert J and Baudon J 1996 *Phys. Rev. A* **54** 5042
[40] Herzog T J, Kwiat P G, Weinfurter H and Zeilinger A 1995 *Phys. Rev. Lett.* **75** 3034
[41] Jacques V, Wu E, Grosshans F, Treussart F, Grangier P, Aspect A and Roch J F 2007 *Science* **315** 966
[42] Jacques V, Wu E, Grosshans F, Treussart F, Grangier P, Aspect A and Roch J F 2008 *Phys. Rev. Lett.* **100** 220402
[43] Ionicioiu R and Terno D R 2011 *Phys. Rev. Lett.* **107** 230406
[44] Tang J S, Li Y L, Xu X Y, Xiang G Y, Li C F and Guo G C 2012 *Nat. Photon.* **6** 600
[45] Kaiser F, Coudreau T, Milman P, Ostrowsky D B and Tanzilli S 2012 *Science* **338** 637
[46] Peruzzo A, Shadbolt P, Brunner N, Popescu S and O'Brien J L 2012 *Science* **338** 634
[47] Jaynes E 1980 *Foundations of Radiation Theory and Quantum Electrodynamics* (Barut A O, ed.) (New York: Plenum) pp. 37–43
[48] Leggett A J 2008 *Reports on Progress in Physics* **71** 022001
[49] Haroche S 2013 *Rev. Mod. Phys.* **85** 1083
[50] Wineland D J 2013 *Rev. Mod. Phys.* **85** 1103
[51] Shadbolt P, Mathews J C F, Laing A and O'Brien J L 2014 *Nat. Phys.* **10** 278