Quantum correlation switches for dipole arrays*

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We investigate the characteristics of three kinds of quantum correlations, measured by pairwise quantum discord (QD), geometric measure of quantum discord (GMQD), and measurement-induced disturbance (MID), in the systems of three- and four-dipole arrays. The influence of the temperature on the three quantum correlations and entanglement of the systems is also analyzed numerically. It is found that novel quantum correlation switches called QD, GMQD, and MID respectively can be constructed with the qubits consisting of electric dipoles coupled by the dipole–dipole interaction and oriented along or against the external electric field. Moreover, with the increase of temperature, QD, GMQD, and MID are more robust than entanglement against the thermal environment. It is also found that for each dipole pair of the three- and four-dipole arrangements, the MID is always the largest and the GMQD the smallest.

Keywords: dipole arrays, quantum discord, geometric measure of quantum discord, measurement-induced disturbance

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1. Introduction

As a prominent feature of quantum mechanics, quantum entanglement plays an essential role not only in testing quantum nonlocality, but also in performing a variety of quantum information tasks.[1,2] However, entanglement is not the only type of quantum correlation. There are some quantum correlations that are different from entanglement[3–6], which also offer some specific advantages as a resource for quantum information processing. Therefore, it is a crucial problem to explore efficient methods to characterize and quantify the nonclassicality or quantumness of correlations in quantum systems. In 2001, Ollivier and Zurek[7] introduced the concept of the so called quantum discord (QD) to describe the difference between the total and classical correlations for a given state, as measured by von Neumann entropy. It has been proven that QD still exists in some mixed separable states even if they are not entangled. At present, it is commonly accepted that QD is the best potential candidate to measure quantum correlations. Some algorithms to analytically evaluate the QD were derived only for a limited set of two-qubit quantum states[8–11] and explicit expressions for more general quantum states are still unknown. The reason lies in the calculation of QD involving a complicated maximization procedure, which is not easy to perform. To overcome the maximization problem, a geometric measure of quantum discord (GMQD) was proposed based on the Hilbert–Schmidt norm.[12] Further, Luo and Fu[13] presented an equivalent but simplified version of the geometric quantum discord. In addition to QD and GMQD, measurement-induced disturbance (MID) was used to characterize quantum correlation by means of the quantum mutual information difference between a given state and its closest classical state from a different angle.[14] In a word, QD, GMQD, and MID are three different measurement tools of quantum correlation for a quantum system.

On the other hand, recent rapid progresses have been made in methods and technologies for producing, trapping, and controlling cold polar molecules.[15–19] Since polar molecules integrate the advantages of both neutral atoms and trapped ions, i.e., a long coherence time and strong dipole–dipole interactions, respectively, they were chosen as a promising new platform for hybrid quantum computing and simulation protocols concerning solid-state device, microwave field, and molecular ensembles simultaneously.[20–23] Furthermore, some quantum computing schemes with polar molecules serving as the physical carrier of quantum information were suggested in Refs. [24]–[29]. Especially, DeMille[24] put forward the seminal proposal for the physical realization of a quantum computer by using the qubits composed of electric dipole moments of ultracold polar molecules, trapped in a one-dimensional optical lattice and coupled by dipole–dipole interactions. In DeMille's proposal, the molecular electric dipole moments can only orient along ($|0\rangle$) or against ($|1\rangle$) the external electric fields, and each qubit is one dipole molecule with equal spacing along the axis.

Recently, Wei *et al.*[30] examined the pairwise concurrence for different arrangements of the dipoles. Motivated by the work,[30] in this paper we study the properties of the three

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physical quantities (pairwise QD, GMQD, and MID) in the systems of three- and four-dipole arrays. Here, we present a new method to construct three kinds of quantum correlation switches, namely, the QD switch, GMQD switch, and MID switch, by varying the ratio of the Rabi frequency to the dipole–dipole coupling strength. Moreover, the influence of the temperature variance on the three physical quantities and the entanglement of the systems are also numerically discussed. It is found that as the temperature increases, QD, GMQD, and MID are more robust than entanglement against a thermal environment.

This paper is organized as follows. We introduce our model and provide a brief review of the concept of QD, GMQD, MID, and concurrence in Section 2. In Section 3, we study the properties of the three physical quantities (pairwise QD, GMQD, and MID) in the arrangements of three dipoles. In Section 4, we describe the characteristics of the three physical quantities in the four dipole arrays. The temperature effects on pairwise QD, GMQD, MID, and concurrence for a one-dimensional dipole chain are examined in Section 5. Finally, we summarize our conclusions in Section 6.

2. Theory

2.1. Model

To study the quantum correlations of dipole arrays, we consider the cases that the qubits are characterized with the molecular electric dipole moments only orienting along ($|0\rangle$) or against ($|1\rangle$) the external electric field. The Hamiltonian of N-trapped dipoles in an external electric field[30,31] is described by

$$H=\hbar\sum_{i=1}^{N}\omega_i S_i^z+\hbar\sum_{i\neq j}^{N}g_{ij}S_i^+S_j^-, \tag{1}$$

where $S_i^z=\begin{pmatrix}1&0\\0&0\end{pmatrix}$ is the dipole operator, $S_i^+=\begin{pmatrix}0&1\\0&0\end{pmatrix}$ and $S_j^-=\begin{pmatrix}0&0\\1&0\end{pmatrix}$ represent dipole excitation and de-excitation, respectively, and ω_i is the transition frequency of the dipole on site i with $\hbar\omega_i=|\boldsymbol{d}\cdot\boldsymbol{E}|$. Here, $\boldsymbol{d}$ is the electric dipole moment which is assumed to be the same for each site and $\boldsymbol{E}$ is the external electric field. In Eq. (1), g_{ij} is the dipole–dipole coupling constant between sites i and j given by

$$g_{ij}=\frac{|\boldsymbol{d}|^2\left(1-3\cos^2\theta\right)}{\left|\boldsymbol{r}_{ij}\right|^3}, \tag{2}$$

where θ is the angle between the direction of the external electric field and the distance vector $\boldsymbol{r}_{ij}$ orienting from dipole i to dipole j. For the sake of simplicity, we set the angle $\theta\in[0,\pi/2]$ in the rest of this paper. Clearly, g_{ij} is determined by the strength of electric dipole moment, the external electric field orientation and the lattice constant of the dipole array.

In general, the temperature-dependent density matrix for a system in thermal equilibrium at temperature T takes the form $\rho=\mathrm{e}^{-\beta H}/Z$ with $\beta=1/kT$, in which k is the Boltzmann constant. The partition function of the system is given by

$$Z(T)=\sum_i\delta_i\,\mathrm{e}^{-\beta\chi_i}, \tag{3}$$

where χ_i is the i-th eigenvalue and δ_i is the degeneracy, and the corresponding density matrix can be written as

$$\rho(T)=\frac{1}{Z}\sum_i^N\mathrm{e}^{-\beta\chi_i}\left|\phi_i\right\rangle\left\langle\phi_i\right|, \tag{4}$$

where $|\phi_i\rangle$ is the i-th eigenfunction. After deriving the bipartite reduced density matrix, we can discuss the properties of quantum correlations and entanglement for the dipole arrangements with respect to the temperature and the ratio of the dipole transition frequency to the dipole–dipole coupling strength.

2.2. Three measurement tools for quantum correlation

For a bipartite quantum system, its total correlation can be quantified by the following quantum mutual information:[32–34]

$$\Gamma(\rho)=S(\rho_A)+S(\rho_B)+S(\rho), \tag{5}$$

where ρ denotes the density operator of the composite bipartite systems A and B, ρ_A and ρ_B denote the reduced density operator of subsystems A and B, respectively, and $S(\rho)=-\mathrm{tr}(\rho\log_2\rho)$ is the von Neumann entropy. Moreover, quantum mutual information can also be written as a sum of classical correlation $C(\rho)$ and quantum correlation $Q(\rho)$,[7,35] namely,

$$\Gamma(\rho)=C(\rho)+Q(\rho). \tag{6}$$

The quantum part $Q(\rho)$ is called QD, which is in general a difficult task to perform. Depending on the maximal information of ρ by measuring one of the subsystems, classical correlation (CC) is defined as $C(\rho)=S(\rho_A)-\min_{B_i^+B_i}\sum_i q_i S\left(\rho_A^i\right)$, in which $B_i^+B_i$ is a positive-operator-valued measure performed on the subsystem B, $q_i=\mathrm{tr}_{AB}\left(B_i\rho B_i^+\right)$ is the probability of obtaining the outcome i, and $\rho_A^i=\mathrm{tr}_B\left(B_i\rho B_i^+\right)/q_i$ is the postmeasurement state of subsystem A by tracing over particle B. Once $C(\rho)$ is in hand, QD is simply gained by subtracting it from the quantum mutual information.

Geometric measure of quantum discord (GMQD)[12,13] is defined as the squared Hilbert–Schmidt closest distance between the quantum state ρ and the classical correlated state χ_0

$$\mathrm{GMQD}(\rho)=\min_{\chi\in\chi_0}\|\rho-\chi\|^2, \tag{7}$$

where $\|A\|^2 = \mathrm{tr}(A^+A)$ is the Hilbert–Schmidt norm. Any bipartite states ρ can be represented in the Bloch basis as

$$\rho = \frac{1}{4}\left[\boldsymbol{I}\otimes\boldsymbol{I} + \sum_{i=1}^{3}(x_i\sigma_i\otimes\boldsymbol{I} + y_i\boldsymbol{I}\otimes\sigma_i) + \sum_{i,j=1}^{3}(R_{ij}\sigma_i\otimes\sigma_j)\right]. \quad (8)$$

Here, σ_i and σ_j $(i,j=1,2,3)$ are three Pauli matrices, $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{x} = (x_1,x_2,x_3)^{\mathrm{T}}$ and $\boldsymbol{y} = (y_1,y_2,y_3)^{\mathrm{T}}$ are the bloch vectors, and R_{ij} denote the elements in the correlation matrix $\boldsymbol{R}$. Then GMQD can be re-expressed as[36–38]

$$\mathrm{GMQD}(\rho) = \frac{1}{4}\left(\left\|\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}\right\|^2 + \|\boldsymbol{R}\|^2 - k_{\max}\right), \quad (9)$$

where $k_{\max}$ is the largest eigenvalue of the matrix $\boldsymbol{K} = \boldsymbol{x}\boldsymbol{x}^{\mathrm{T}} + \boldsymbol{R}\boldsymbol{R}^{\mathrm{T}}$.

The measurement-induced disturbance (MID)[6,14] can reveal the properties of a classical system without modifying the system. It is defined as the quantum mutual information difference between the given state ρ and its closest classical state $\Pi\rho$

$$\mathrm{MID}(\rho) = I(\rho) - I(\Pi\rho). \quad (10)$$

Here, $\Pi\rho = \sum_{a,b}(\Pi_a\otimes\Pi_b)\rho(\Pi_a\otimes\Pi_b)$ denotes the local measurements induced by the spectral decomposition of the reduced states $\rho_A = \sum_a p_a\Pi_a$ and $\rho_B = \sum_b p_b\Pi_b$. Due to the fact that the reduced states are not affected by this measurement, MID is just the entropic cost on this measurement basis

$$\mathrm{MID}(\rho) = S(\Pi\rho) - S(\rho). \quad (11)$$

2.3. Concurrence

It is well established that the entanglement of a two-qubit system can be analytically quantified with concurrence[39,40] defined by

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (12)$$

where ρ is the density matrix of the general two-qubit state, and λ_i are the eigenvalues in decreasing order of the Hermitian matrix $R = \rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y)$ with ρ^* being the complex conjugate of ρ under the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

3. Three dipole arrays

In what follows, we study the quantum correlations for the arrangements of three dipoles. To be specific, we analyze the behavior of the three physical quantities (pairwise QD, GMQD, and MID) in the system of three dipoles, which are arranged into a one-dimensional linear chain and a triangle. Without loss of generality, we first take the system of three dipoles constituting an equiangular triangle with the external electric field perpendicular to the triangle plane as an example to describe the characteristics of the three quantum correlations. According to Eq. (1), we can obtain the Hamiltonian of the system under consideration, which can be expanded under the basis vector $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}_{123}$ as

$$\begin{aligned} H' = {} & \hbar\omega_1 S_1^z + \hbar\omega_2 S_2^z + \hbar\omega_3 S_3^z + \hbar g_{12} S_1^+\otimes S_2^- \\ & + \hbar g_{13} S_1^+\otimes S_3^- + \hbar g_{23} S_2^+\otimes S_3^- + \hbar g_{21} S_1^-\otimes S_2^+ \\ & + \hbar g_{31} S_1^-\otimes S_3^+ + \hbar g_{32} S_2^-\otimes S_3^+. \end{aligned} \quad (13)$$

For simplicity, we assume that all the dipole transition frequencies are identical, i.e., $\omega_1 = \omega_2 = \omega_3 = \omega$. Since the external electric field is perpendicular to the two-dimensional triangle array, we have $\cos\theta_1 = \cos\theta_2 = \cos\theta_3 = 0$. Due to $r_{12} = r_{23} = r_{13}$ for the equiangular triangle, we can learn from Eq. (2) that $g_{12} = g_{13} = g_{23} = g_{21} = g_{31} = g_{32} = |\boldsymbol{d}|^2/|\boldsymbol{r}_{12}|^3$. In the subsequent discussions, we use g to replace $|\boldsymbol{d}|^2/|\boldsymbol{r}_{12}|^3$. Therefore, the Hamiltonian H' can be rewritten in the following matrix form:

$$H' = \hbar\begin{pmatrix} 3\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\omega & g & 0 & g & 0 & 0 & 0 \\ 0 & g & 2\omega & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega & 0 & g & g & 0 \\ 0 & g & g & 0 & 2\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & g & 0 & \omega & g & 0 \\ 0 & 0 & 0 & g & 0 & g & \omega & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (14)$$

By diagonalizing the above Hamiltonian matrix, we can obtain all the eigenvalues λ_i and their corresponding eigenstates $|\phi_i\rangle$. After substituting λ_i and $|\phi_i\rangle$ into Eq. (4), the density matrix $\rho'(T)$ of the system can be given by

$$\rho'(T) = \frac{1}{Z}\begin{pmatrix} \rho_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 & \rho_{23} & 0 & 0 & 0 \\ 0 & \rho_{23} & \rho_{33} & 0 & \rho_{35} & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{44} & 0 & \rho_{46} & \rho_{46} & 0 \\ 0 & \rho_{23} & \rho_{35} & 0 & \rho_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{46} & 0 & \rho_{66} & \rho_{67} & 0 \\ 0 & 0 & 0 & \rho_{46} & 0 & \rho_{67} & \rho_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{88} \end{pmatrix}, \quad (15)$$

with

$$\begin{aligned} \rho_{11} &= \exp(-3\omega/kT), \\ \rho_{22} &= \frac{1}{3}\exp((-2g-2\omega)/kT) + \exp((g-2\omega)/kT), \\ \rho_{33} &= \frac{1}{3}\exp((-2g-2\omega)/kT) + \frac{1}{2}\exp((g-2\omega)/kT), \\ \rho_{44} &= \frac{1}{3}\exp((-2g-\omega)/kT) + \exp((g-\omega)/kT), \\ \rho_{66} &= \frac{1}{3}\exp((-2g-\omega)/kT) + \frac{1}{2}\exp((g-\omega)/kT), \\ \rho_{88} &= 1, \\ \rho_{23} &= \frac{1}{3}\exp((-2g-2\omega)/kT) - \frac{1}{2}\exp((g-2\omega)/kT), \end{aligned}$$

$$
\begin{aligned}
\rho_{35} &= \frac{1}{3}\exp((-2g-2\omega)/kT),\\
\rho_{46} &= \frac{1}{3}\exp((-2g-\omega)/kT)-\frac{1}{2}\exp((g-\omega)/kT),\\
\rho_{67} &= \frac{1}{3}\exp((-2g-\omega)/kT),\\
Z &= 2\exp((g-\omega)/kT)+\exp((-g-2\omega)/kT)\\
&\quad+\exp((-2g-2\omega)/kT)+\exp((-3\omega)/kT)\\
&\quad+2\exp((g-2\omega)/kT)+1.
\end{aligned}
$$

It is easy to get the following reduced density matrix for the two particles 1 and 2 by tracing over particle 3 from the state $\rho'(T)$ shown in Eq. (15)

$$
\rho_{12}=\mathrm{tr}_3(\rho'(T))=\frac{1}{Z}\begin{pmatrix}\chi_{11}&0&0&0\\0&\chi_{22}&\chi_{23}&0\\0&\chi_{23}&\chi_{33}&0\\0&0&0&\chi_{44}\end{pmatrix}, \tag{16}
$$

with

$$
\begin{aligned}
\chi_{11} &= \exp(-3\omega/kT)+\frac{1}{3}\exp((-2g-2\omega)/kT)\\
&\quad+\exp((g-2\omega)/kT),\\
\chi_{22} &= \frac{1}{3}\exp((-2g-2\omega)/kT)+\frac{1}{2}\exp((g-2\omega)/kT)\\
&\quad+\frac{1}{3}\exp((-2g-\omega)/kT)+\exp((g-\omega)/kT),\\
\chi_{33} &= \frac{1}{3}\exp((-2g-2\omega)/kT)+\frac{1}{2}\exp((g-2\omega)/kT)\\
&\quad+\frac{1}{3}\exp((-2g-\omega)/kT)+\frac{1}{2}\exp((g-\omega)/kT),\\
\chi_{44} &= \frac{1}{3}\exp((-2g-\omega)/kT)+\frac{1}{2}\exp((g-\omega)/kT)+1,\\
\chi_{23} &= \frac{1}{3}\exp((-2g-2\omega)/kT)+\frac{1}{3}\exp((-2g-\omega)/kT)\\
&\quad-\frac{1}{2}\exp((g-\omega)/kT).
\end{aligned}
$$

Here, tr_3 is the partial trace over particle 3. Due to the structure symmetry of the equiangular triangle and the exchange invariability of the three dipoles 1, 2, and 3, it is not hard to learn that the three reduced density matrices ρ_{12}, ρ_{13}, and ρ_{23} have the identical form, namely, $\rho_{12}=\rho_{13}=\rho_{23}$. According to Eq. (16), it is clear to see that the two-qubit state ρ_{12} takes the form of the so-called X state, i.e., the density matrix ρ_{12} only contains non-zero elements along the main diagonal and anti-diagonal.

Through numerical calculations, in Fig. 1, we plot the three physical quantities (pairwise QD, GMQD, and MID) versus ω/g for ρ_{12} in the array of three dipoles constituting an equiangular triangle at $kT=0.004\hbar g$. From the figure, we can see that with the increase of ω/g, the three physical quantities keep their nonzero constant till ω/g approaches 1, above this value they almost disappear rapidly. Moreover, QD and MID have the same nonzero value remaining invariable.

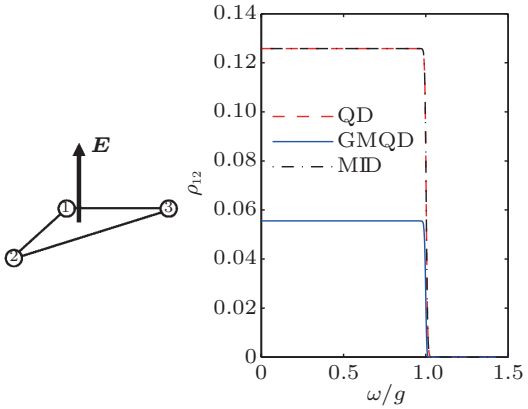


Fig. 1. (color online) For neighboring dipole pairs in the system of three dipoles constituting an equiangular triangle, pairwise QD, GMQD, and MID are plotted as a function of ω/g at $kT=0.004\hbar g$ with $\rho_{12}=\rho_{13}=\rho_{23}$ and $g_{12}=g_{13}=g_{23}=g$. Here, the external electric field is perpendicular to the triangle plane.

Figure 2 depicts the properties of the three physical quantities (pairwise QD, GMQD, and MID) for each dipole pair in the arrangement of a one-dimensional linear chain consisting of three dipoles. It can be observed from the figure that with the increase of ω/g, the MID of ρ_{13} in Fig. 2(b) displays the variation behavior similar to those in Fig. 1, whereas the other physical quantities in Fig. 2 first increase to high values sharply, then preserve the nonzero values over a long ratio ω/g, and finally decay to zero asymptotically. As a result, we can learn from Figs. 1 and 2 that the quantum correlation switches including the QD switch, GMQD switch, and MID switch can be constructed by appropriately tuning the value of the ratio ω/g.

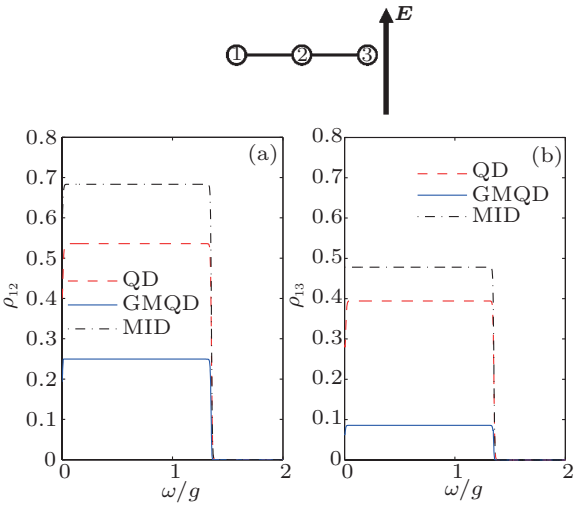


Fig. 2. (color online) For ρ_{12} (a) and ρ_{13} (b) in the system of a one-dimensional three dipole linear chain, pairwise QD, GMQD, and MID are plotted versus ω/g at $kT=0.004\hbar g$ with $g_{12}=g_{23}=8g_{13}=g$. Here, the external electric field is perpendicular to the linear chain.

4. Four dipole arrays

In this section, we turn to investigate the behavior of the three quantum correlations in the arrays of four dipoles using similar methods to Section 3. The properties of QD, GMQD,

and MID versus ω/g for each pair of the four dipoles constituting a one-dimensional linear chain and a two-dimensional square array are plotted in Figs. 3 and 4, respectively. It can be seen from the two figures that the three quantum correlations exhibit the similar variance trend, i.e., with ω/g increasing, the three physical quantities first keep constant, then jump to another constant promptly, and decay rapidly to almost zero in the end. It is worth mentioning that the pairwise QD, GMQD, and MID of ρ_{14} have a tiny extreme point at $\omega/g = 0.64$, see Fig. 3(c). Moreover, the three quantum correlations of ρ_{12} in the square array shown in Fig. 4(a) also exist at a minimal value of about $\omega/g = 0.85$.

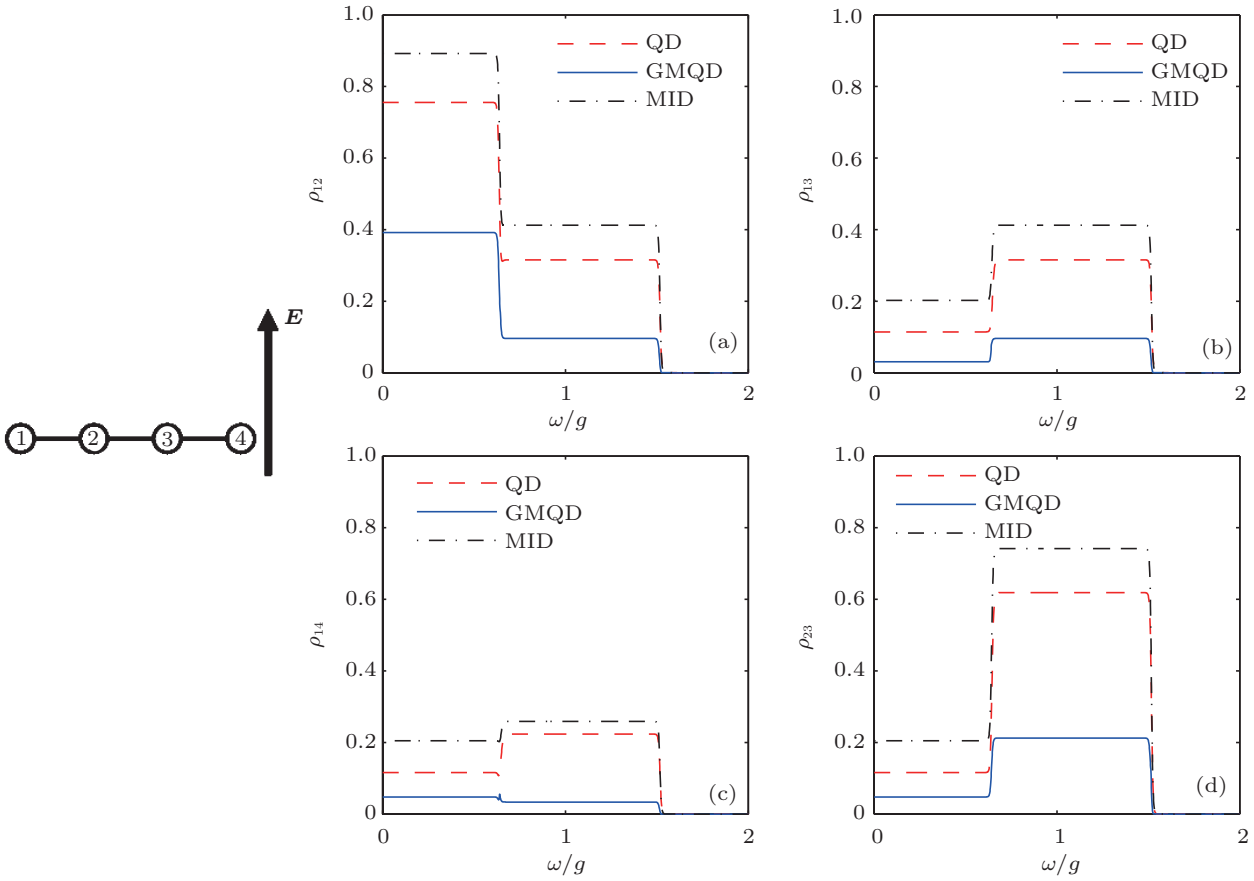


Fig. 3. (color online) For each dipole pair ρ_{12} (a), ρ_{13} (b), ρ_{14} (c), and ρ_{23} (d) in the system of a one-dimensional four dipole linear chain, pairwise QD, GMQD, and MID are plotted as a function of ω/g at $kT = 0.004\hbar g$ with $g_{12} = g_{23} = g_{34} = 8g_{13} = 8g_{24} = 27g_{14} = g$. Here, the external electric field is perpendicular to the linear chain.

Figure 5 depicts the characteristics of QD, GMQD and MID in the system of a rectangular tetrahedron consisting of four dipoles. From the figure, we can see that when $\omega/g \sim 0$, the three quantum correlations of ρ_{12} and ρ_{13} are close to zero while QD and MID of both ρ_{14} and ρ_{23} are approximate to 1 (the GMQD of ρ_{14} and that of ρ_{23} are almost equal to 0.5). Furthermore, with ω/g increasing from the value of 0.11, the three quantum correlations corresponding to ρ_{12}, ρ_{13}, ρ_{14}, and ρ_{23} keep their respective different nonzero constant over a long ratio ω/g, and diminish rapidly to approximately, but never reaching, zero. From Figs. 3–5, we find that under certain conditions, the three quantum correlations (pairwise QD, GMQD, and MID) can keep their stable nonzero values for each dipole pair when ω/g is in moderate ranges. In addition, for all the dipole pairs in the arrangements of four dipoles, the value of MID is the largest while the value of GMQD is the smallest.

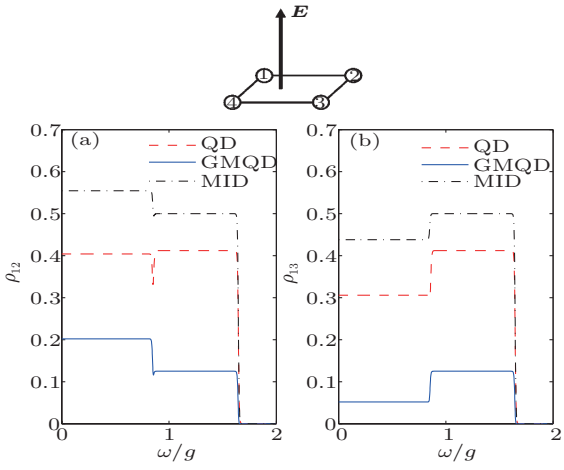


Fig. 4. (color online) For each dipole pair ρ_{12} (a) and ρ_{13} (b) in the system of four dipoles constituting a square arrangement, pairwise QD, GMQD, and MID are plotted as a function of ω/g at $kT = 0.004\hbar g$ with $g_{12} = g_{23} = g_{34} = g_{14} = 2\sqrt{2}g_{13} = 2\sqrt{2}g_{24} = g$. Here, the external electric field is perpendicular to the square plane.

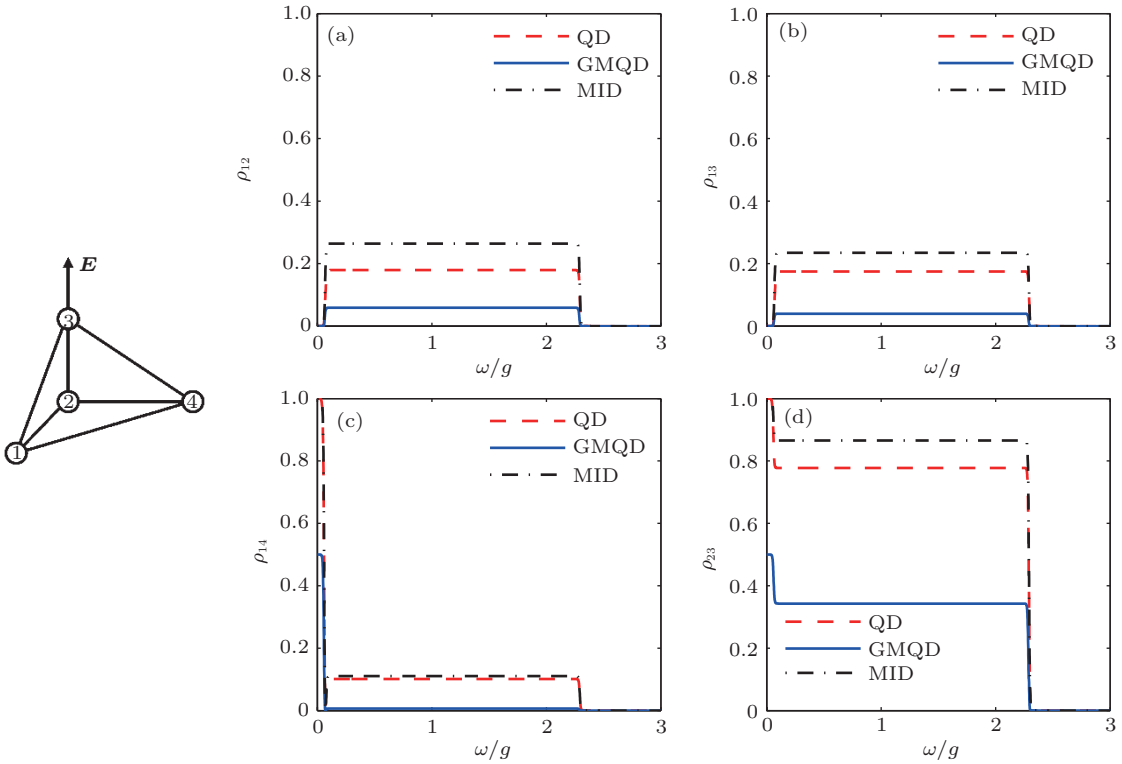


Fig. 5. (color online) For each dipole pair ρ_{12} (a), ρ_{13} (b), ρ_{14} (c), and ρ_{23} (d) in the system of four dipoles constituting a rectangular tetrahedron, pairwise QD, GMQD, and MID are plotted as a function of ω/g for $kT = 0.004\hbar g$ and $g_{12} = -4\sqrt{2}g_{13} = 2\sqrt{2}g_{14} = -g_{23}/2 = g$ with the external electric field along dipoles 2 to 3.

5. Temperature-dependent quantum correlation and concurrence

In this section, we discuss the properties of three quantum correlations and concurrence in the linear chain arrangement of four dipoles varying with the temperature, which are illustrated in Fig. 6. Here, we take the nearest neighboring dipole pair (say, dipoles 1 and 2) in the chain array as an example. As displayed in Fig. 6(a), when the temperature is very low, the pairwise QD, GMQD, MID, and concurrence first keep unchanged versus ω/g, then jump to their respective small stable values after a critical point of $\omega/g \approx 0.64$, finally decaying to zero asymptotically. Numerical analysis shows that when $\omega/g < 0.64$, the ground state at $kT = 0.004\hbar g$ is $|\phi\rangle = 0.193|1100\rangle - 0.510|1010\rangle + 0.450|1001\rangle + 0.450|0110\rangle - 0.510|0101\rangle + 0.193|0011\rangle$, nevertheless, when $0.64 < \omega/g < 1.51$, the corresponding ground state becomes $|\phi\rangle = 0.360|0001\rangle - 0.609|0010\rangle + 0.609|0100\rangle - 0.360|1000\rangle$. Due to the existence of different ground states before and after the point $\omega/g \approx 0.64$, QD, GMQD, MID and concurrence for the four-dipole linear chain exhibit the behavior of sudden transition. We notice that our result regarding the entanglement of ρ_{12} in Fig. 6(a) is in good agreement with that of Ref. [30]. From Figs. 6(b)–6(d), it can be seen that as the temperature increases, the curves of three quantum correlations and concurrence become smoother, and concurrence relatively decreases the most rapidly. For the temperature $kT = 0.4\hbar g$ (see Fig. 6(c)), concurrence becomes small and the three quantum correlations gradually decay to zero with ω/g increasing. Moreover, for the temperature $kT = 2.0\hbar g$ (see Fig. 6(d)), entanglement vanishes while QD, GMQD and MID still survive for small ω/g and then diminish to zero in an asymptotical way for large ω/g. In addition, by numerical calculations, we find that when $kT > 1.035\hbar g$, concurrence disappears no matter what the value of ω/g is, whereas the three quantum correlations with respect to ω/g are always larger than zero. Thus to some extent QD, GMQD and MID quantifying nonclassical correlation is more robust than entanglement against the thermal environment. As we know, the three physical quantities (pairwise QD, GMQD, and MID) and concurrence are independent measures of quantum correlation, generally they do not have definite ordering relations between them.[41,42] It needs emphasizing that the influence of the temperature on the variance trend of the quantum correlations and concurrence versus ω/g for all the other dipole pairs in this four-dipole linear array is similar to that for dipoles 1 and 2 described above, so too in the other arrangements of the dipoles mentioned in this paper. In a word, with the increase of temperature, the curves of QD, GMQD, and MID against the ratio ω/g become smoother, namely, the three quantum correlation switches may be realizable only under the situation that the dipoles are cooled to quite a low temperature and the ratio ω/g is in the appropriate ranges.

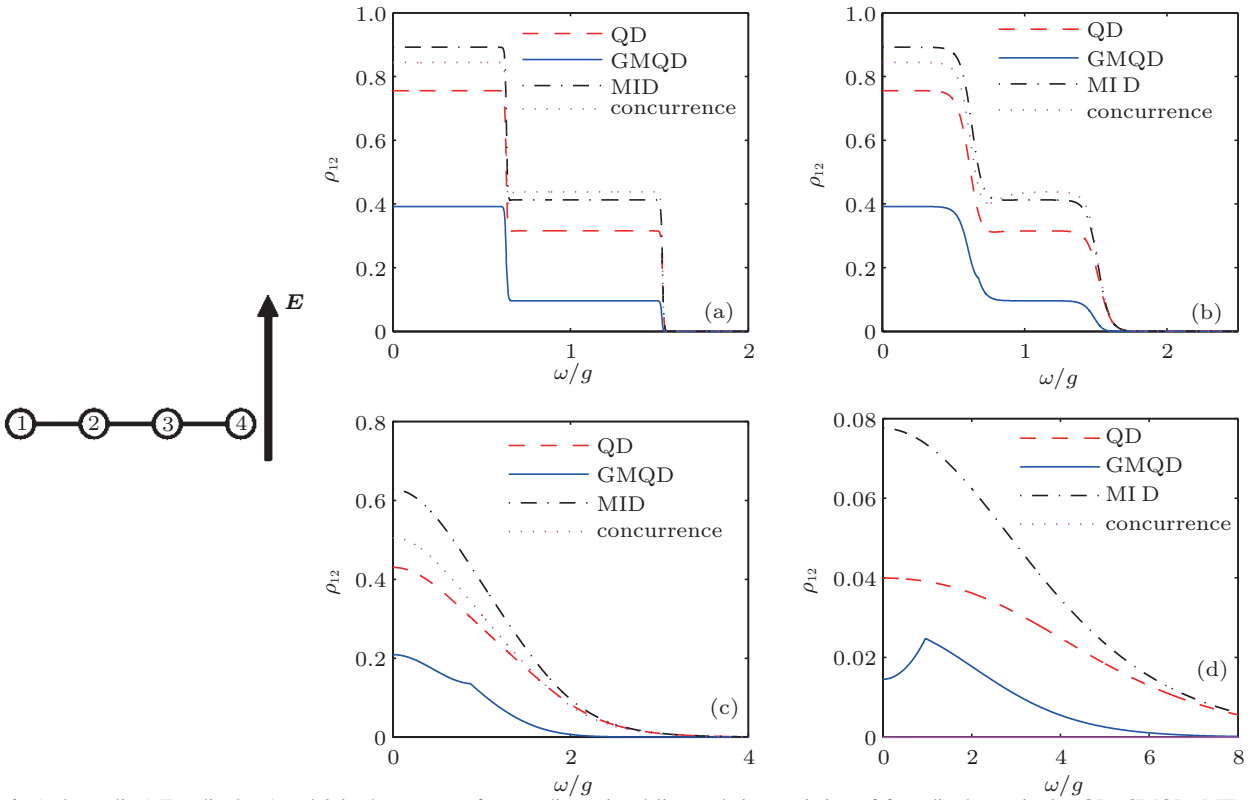


Fig. 6. (color online) For dipoles 1 and 2 in the system of a one-dimensional linear chain consisting of four dipoles, pairwise QD, GMQD, MID, and concurrence are plotted versus ω/g with $kT = 0.004\hbar g$ (a), $kT = 0.04\hbar g$ (b), $kT = 0.4\hbar g$ (c), and $kT = 2\hbar g$ (d). Here, the external electric field is perpendicular to the dipole chain.

6. Conclusions

In summary, we have studied the properties of the three quantum correlations (pairwise QD, GMQD, and MID) in the systems of three- and four-dipole arrays. The influence of the environmental temperature on the three quantum correlations and the entanglement measured by concurrence has also been numerically analyzed. In particular, we have proposed a new way to construct three types of quantum correlation switches called QD, GMQD, and MID, with the qubits consisting of the electric dipoles coupled by the dipole–dipole interaction and oriented along or against the external electric field. It is found that the three quantum correlation switches can be manipulated by tuning the value of the ratio ω/g. It should be pointed out that as for the given dipole arrangements, there exists the identical range of ω/g for the three quantum correlations preserving the nonzero constant even for different dipole pairs. Furthermore, for each dipole pair of the three- and four-dipole arrangements, the MID is always the largest but the GMQD is the smallest. It is also found that with the increase of the temperature, the QD, GMQD, and MID are more robust than entanglement against a thermal environment.

The recent developments in cooling, trapping and manipulating atomic[43,44] and molecular dipoles[16–19,45,46] provide potential opportunities to experimentally realize the quantum correlation switches described above, which requires the dipoles to be cooled to a temperature corresponding to a fraction of the dipole–dipole interaction energy.[30,47] It should be mentioned that the range of ω/g for the three quantum correlations keeping nonzero values is about 0–3 in the present paper. For a dipole moment d of a few debyes and a typical electric field (about 10^5 V/m), the dipole–dipole separation r (about 10 nm) is needed for $\omega/g \sim 1$. Such short r can be achieved with the arrangements of nanoscale plasmon-enhanced electric or electro–optical traps recently reported in Refs. [48] and [49]. With respect to such r, the electric dipole–dipole interaction $g = d^2/4\pi\varepsilon_0 r^3$ is approximately 0.1 K. Thus cooling the dipoles to such a temperature level is available with the current experimental technology. Besides, ω/g can be experimentally adjusted by the external electrical field and/or by r related to the microtrap configuration. Therefore, we expect that our proposed scheme for quantum correlation switches with dipole arrays might be feasible in the near future.

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