Non-Markovianity of the Heisenberg XY spin environment with Dzyaloshinskii–Moriya interaction*

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Using the effective non-Markovian measure proposed by Breuer *et al.* recently, we study the memory effect of a central qubit system coupled to a spin chain environment with Dzyaloshinskii–Moriya interaction in a transverse field. It is discovered that the central qubit system presents different memory effects in different environment phases with the different oscillatory behaviors of the decoherence factor. Moreover, it is revealed that the Dzyaloshinskii–Moriya interaction has a prominent influence on the memory effect of a central qubit system via modifying the amplitude and period of the decoherence factor under certain conditions.

Keywords: non-Markovinity, qubit dynamics, spin chain

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1. Introduction

As is known, a realistic quantum system inevitably interacts with an ambient environment.[1,2] Dynamical evolutions of open quantum systems can usually be classified into Markovian processes and non-Markovian processes. In a Markovian process, the information of the system monotonously flows into its environment, while in a non-Markovian process, the information flowing into the environment will return to the system in the future,[3] i.e., the reflow of information, the so called memory effect, is the characteristic behavior of the non-Markovian process. In general, the Markovian approximation is sufficient to describe the dynamical processes when the open system interacts with a memoryless environment or the interaction between the system and its environment is very weak.[2] However, when taking account of strong system-environment couplings or short time scales, Markovian approximation loses validity and dynamical processes present complex non-Markovian properties.

In the past few years, non-Markovian behaviors have become increasingly important, and have been widely studied in many physical systems such as quantum dots,[4,5] solid state systems,[6] quantum optical systems[7,8] and biological systems.[9] In quantum information processing, non-Markovianity has been nominated as a critical resource to improve quantum protocols such as quantum key distribution and quantum metrology.[10–13] As is known, decoherence of a quantum system is the main obstacle for practicing quantum computation tasks. It is found that non-Markovian environments can significantly refrain from decoherence of qubit systems.[14] Moreover, Bellomo *et al.*[15] found that the phenomenon of "entanglement sudden death" does not only rely on the purity or the degree of entanglement of the initial state, but also it relies on the degree of non-Markovianity of the system.

In order to clarify non-Markovian processes, the primary task is to propose a scheme for defining and measuring non-Markovian processes. Several definitions and measures for the non-Markovianity have been proposed based on different mathematical and physical concepts.[3,16–18] Breuer *et al.*[3] have proposed a method for measuring non-Markovian processes based on the trace distance between quantum states. This method has been applied in experimental investigation of non-Markovian behavior.[19] Lu *et al.*[16] also proposed a measure of non-Markovianity based on quantum Fisher information flow. Besides, Rivas *et al.*[17] proposed a scheme for measuring non-Markovianity based on entanglement between the system and ancilla. The advantage of the scheme proposed by Breuer *et al.* is the fact that exploring experimentally non-Markovianity is allowed even without knowing the properties of the environment or the structure of the system-environment interaction. However, in this method, it was very troublesome to optimize the initial states; fortunately, this shortcoming has been resolved.[20]

The non-Markovianity of open systems is closely related to fundamental aspects of quantum theory and practical aspects of engineering. Hence, more and more efforts are devoted to exploring non-Markovian processes of open systems.[21–25] Interacting many-body spin environments are the representative non-Markovian environments which present many interesting non-Markovian behaviors.[26–28] Haikka *et al.*[28] have studied the dephasing process of a qubit system coupled to an Ising chain environment, and revealed that the

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qubit system is shown to have purely Markovian dynamics only at the critical point. But for some systems, such as quantum-Hall systems[29] and cavity QED systems,[30] the Ising Hamiltonian is insufficient to describe the relation of qubits, instead, the Heisenberg XY Hamiltonian is a more suitable model.[31]

In this paper, by employing the non-Markovian measure in Ref. [3], we will mainly investigate the non-Markovian behavior of a single qubit coupled to the Heisenberg XY spin environment with Dzyaloshinskii–Moriya (DM) interaction. The DM interaction[32] arises from the spin–orbit coupling, which leads to many interesting quantum phenomena.[33,34] We will analyze how environmental parameters influence the non-Markovianity. The close relationship between oscillation behaviors of the decoherence factor and the non-Markovianity will be studied in detail. Contrary to previous studies, our approach does not depend on either the Born or Markov approximation.

This paper is organized as follows. In Section 2, methods are introduced about diagonalizing the total Hamiltonian and calculating the degree of non-Markovianity. In Section 3, by the numerical calculation, we analyze the effects of the transverse field intensity and the Dzyaloshinskii–Moriya interaction intensity on the degree of non-Markovianity. Section 4 gives a summary for our main results.

2. Methods

In this paper, we consider a single qubit coupled to a Heisenberg XY spin chain environment of N interacting spin-1/2 particles with DM interaction. The total Hamiltonian is given by ($\hbar$ is taken to be one)

$$H = H_{\mathrm{E}}^{(\lambda)} + H_{\mathrm{I}}, \tag{1}$$

where $H_{\mathrm{E}}^{(\lambda)}$ denotes the Hamiltonian of the spin chain environment and is given by

$$\begin{aligned} H_{\mathrm{E}}^{(\lambda)} &= -\sum_{l}^{N}\left(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z\right) \\ &\quad + \sum_{l}^{N} D\left(\sigma_l^x\sigma_{l+1}^y - \sigma_l^y\sigma_{l+1}^x\right), \end{aligned} \tag{2}$$

where H_{I} describes the interaction between the qubit spin and the spin chain environment and is given by

$$H_{\mathrm{I}} = -g\sigma^z\sum_{l}^{N}\sigma_l^z. \tag{3}$$

In Eqs. (2) and (3), σ^α and σ_l^α ($\alpha = x,y,z$) are used to describe the central qubit spin and the environmental chain spin at site l, respectively. The γ is the anisotropy parameter which characterizes anisotropy in-plane interaction. Parameter λ measures the intensity of the transverse field along with the Z direction and parameter D measures the intensity of the DM interaction. The periodic boundary condition $\sigma_{N+1}^\alpha = \sigma_1^\alpha$ is satisfied.

By utilizing the Jordan–Wigner transformation,[31] Fourier transforms, and Bogoliubov transformation, $H_{\mathrm{E}}^{(\lambda_i)}$ is finally digonalized as

$$H_{\mathrm{E}}^{(\lambda_i)} = \sum_k \Lambda_k^{(\lambda_i)}\left(b_{k,\lambda_i}^\dagger b_{k,\lambda_i} - \frac{1}{2}\right), \tag{4}$$

where $k = -(N-1)/2,\ldots,(N-1)/2$, $\lambda_{0(1)} = \lambda \pm g$, $\Lambda_k^{(\lambda_i)}$ is the energy spectrum with $\Lambda_k^{(\lambda_i)} = 2\varepsilon_k^{(\lambda_i)} + 4D\sin(2\pi k/N)$, b_{k,λ_i} is the corresponding Bogoliubov transformed fermion operators with

$$b_{k,\lambda_i} = \cos\frac{\theta_{k,\lambda_i}}{2}d_k - \mathrm{i}\sin\frac{\theta_{k,\lambda_i}}{2}d_{-k}^\dagger.$$

Here,

$$\varepsilon_k^{(\lambda_i)} = \sqrt{\gamma^2\sin^2\left(\frac{2\pi k}{N}\right) + \left[\lambda_i - \cos\left(\frac{2\pi k}{N}\right)\right]^2},$$

$$\theta_{k,\lambda_i} = \arctan\left(\frac{\gamma\sin\dfrac{2\pi k}{N}}{\lambda_i - \cos\dfrac{2\pi k}{N}}\right).$$

Next, we presume that the product state is designated as the initial state of the composite system and reads $\rho_{\mathrm{tot}}(0) = \rho_A(0)\otimes\rho_{\mathrm{E}}(0)$, where $\rho_A(0)$ and $\rho_{\mathrm{E}}(0)$ are the initial states of the qubit system and the spin chain environment respectively. Furthermore, the environment initial state is given by $\rho_{\mathrm{E}}(0) = |\psi_{\mathrm{E}}\rangle\langle\psi_{\mathrm{E}}|$, where $|\psi_{\mathrm{E}}\rangle$ is the groundstate of the environment. The A qubit system's initial state is given by $\rho_A(0) = p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1| + q|0\rangle\langle 1| + q^*|1\rangle\langle 0|$ where $p\in\mathscr{R}$, $q\in\mathscr{C}$, $0\le p\le 1$, $|q|^2\le p(1-p)$, in order to make sure that $\rho_A(0)$ is a real quantum state. The evolutional state of the composite system is resolved by $\rho_{\mathrm{tot}}(t) = U(t)\rho_{\mathrm{tot}}(0)U^\dagger(t)$. Then, by tracing out the environment, we can obtain the reduced state of the qubit A

$$\begin{aligned} \rho_A(t) &= \mathrm{tr}_{\mathrm{E}}[\rho_{\mathrm{tot}}(t)] \\ &= \sum_{ij} F_{ij}(t)\langle\varphi_i|\rho_A(0)|\varphi_j\rangle|\varphi_i\rangle\langle\varphi_j| \\ &= \begin{pmatrix} p & qF_{01} \\ q^*F_{01}^* & 1-p \end{pmatrix}, \end{aligned} \tag{5}$$

where $F_{ij}(t)$ is named decoherence factor with $F_{ij}(t) = \langle\psi_{\mathrm{E}}(0)|U_{\mathrm{E}}^{\dagger(\lambda_j)}(t)U_{\mathrm{E}}^{(\lambda_i)}(t)|\psi_{\mathrm{E}}(0)\rangle$ and $U_{\mathrm{E}}^{(\lambda_i)}(t) = \exp[-\mathrm{i}H_{\mathrm{E}}^{(\lambda_i)}t]$ is the projected time evolution operator driven by $H_{\mathrm{E}}^{(\lambda_i)}$. Actually, this dynamical process is called the dephasing process. In order to provide support for the calculation of Section 3, we calculate the exact expression of $|F_{ij}(t)|$. By using the method reported in Ref. [35], we obtain

$$\begin{aligned} &|F_{ij}(t)| \\ &= \prod_{k>0}\left[1 - \sin^2\left(2\omega_k^{(\lambda_i)}\right)\sin^2\left(\Lambda_k^{(\lambda_i)}t\right) - \sin^2\left(2\omega_k^{(\lambda_j)}\right)\right. \end{aligned}$$

$$\times \sin^2\left(\Lambda_k^{(\lambda_j)}t\right)+2\sin\left(2\omega_k^{(\lambda_i)}\right)\sin\left(2\omega_k^{(\lambda_j)}\right)$$
$$\times \sin\left(\Lambda_k^{(\lambda_i)}t\right)\sin\left(\Lambda_k^{(\lambda_j)}t\right)\cos\left(\Lambda_k^{(\lambda_i)}t-\Lambda_k^{(\lambda_j)}t\right)$$
$$-4\sin\left(2\omega_k^{(\lambda_i)}\right)\sin\left(2\omega_k^{(\lambda_j)}\right)\sin^2\left(\omega_k^{(\lambda_i)}-\omega_k^{(\lambda_j)}\right)$$
$$\times \sin^2\left(\Lambda_k^{(\lambda_i)}t\right)\sin^2\left(\Lambda_k^{(\lambda_j)}t\right)\Big]^{1/2}, \quad (6)$$

where $\omega_k^{(\lambda_i)}=(\theta_{k,\lambda_i}-\theta_{k,\lambda})/2$. According to Eq. (5), we can find that the value of F_{01} close to 1 indicates an immune qubit state and the value of F_{01} close to 0 indicates a strong suppression of the system qubit coherence and entanglement.

Recently, Breuer *et al.*[3] put forward an effective method to measure the non-Markovianity of the quantum dynamical process in an open system based on the trace distance[36] between the two states. The trace distance of the two states ρ_1 and ρ_2 can be defined as

$$D(\rho_1,\rho_2)=\frac{1}{2}\mathrm{tr}|\rho_1-\rho_2|, \quad (7)$$

where $|A|=\sqrt{A^\dagger A}$ and $0\le D\le 1$. The trace distance indicates the distinguishability between the two states. If $D=1$, the two states are totally distinguishable, whereas if $D=0$, the two states are the same. In a quantum process $\rho(0)$ is the initial state, and $\rho(t)$ is the evolutional state. One defines the rate of change of the trace distance of a couple of states by means of

$$\sigma=\frac{\mathrm{d}}{\mathrm{d}t}D[\rho_1(t),\rho_2(t)]. \quad (8)$$

On the one hand $\sigma\le 0$ indicates that the increasing distinguishability of the couple of states corresponds to all dynamical semigroups and all Markovian processes. On the other hand, $\sigma>0$ means that the decreasing distinguishability of the pair of states corresponds to non-Markovian dynamics processes. According to Eq. (8), one can define the degree of non-Markvianity of a quantum dynamics process

$$R=\max_{\rho_{1,2(0)}}\int_{\sigma>0}\sigma\,\mathrm{d}t, \quad (9)$$

where the time integration is extended over all time intervals in which $\sigma>0$ and the maximum is taken over all couples of initial states. In fact, the non-Markovian measure based on fidelity applies to continuous-variable systems.[37] The spin system researched by us is a discrete-variable system and the following calculation can be simplified by a non-Markovian measure based on the trace distance. According to Eq. (5), in some interval $[a_n,b_n]$, R can be rewritten as

$$R=\max_{\rho_{1,2(0)}}\sum_n\{D[\rho_1(b_n),\rho_2(b_n)]-D[\rho_1(a_n),\rho_2(a_n)]\}, \quad (10)$$

noteworthily, σ is positive in the interval $[a_n,b_n]$. We set R_n as the increase of the trace distance in each time interval with $t>0$. R_n reads

$$R_n=\max_{\rho_{1,2(0)}}\{D[\rho_1(b_n),\rho_2(b_n)]-D[\rho_1(a_n),\rho_2(a_n)]\}. \quad (11)$$

In a general way, it is complex to make the maximization running all over the initial states. In order to calculate R_n, we need the two specific initial states which make the maximization of R_n in every time interval. We set the two initial qubit states as $\rho_1(0)=\alpha|0\rangle\langle 0|+(1-\alpha)|1\rangle\langle 1|+\beta|0\rangle\langle 1|+\beta^*|1\rangle\langle 0|$ and $\rho_2(0)=\mu|0\rangle\langle 0|+(1-\mu)|1\rangle\langle 1|+\nu|0\rangle\langle 1|+\nu^*|1\rangle\langle 0|$. According to Eq. (5), the evolutional states are given by $\rho_1(t)=\alpha|0\rangle\langle 0|+(1-\alpha)|1\rangle\langle 1|+\beta F_{01}|0\rangle\langle 1|+\beta^* F_{01}^*|1\rangle\langle 0|$ and $\rho_2(t)=\mu|0\rangle\langle 0|+(1-\mu)|1\rangle\langle 1|+\nu F_{01}|0\rangle\langle 1|+\nu^* F_{01}^*|1\rangle\langle 0|$. According to Eqs. (5) and (7), we give the expression of the trace distance at any time $t>0$

$$D[\rho_1(t),\rho_2(t)]=\sqrt{(\alpha-\mu)^2+F|\beta-\nu|^2}, \quad (12)$$

where $F=|F_{01}|^2$. In addition, the parameterized condition in $\rho_1(0)$ and $\rho_2(0)$ is equivalent to $\alpha-\mu=r\cos\theta$, $\beta-\nu=re^{i\varphi}\sin\theta$, ($r\le 1$, $\theta\in[0,2\pi]$, and $\varphi\in[0,\pi]$). Applying these parameterized conditions to Eq. (11), equation (11) is written in the following form:

$$R_n=\max_{r,\theta}\left[r\left(\sqrt{\cos^2\theta+B\sin^2\theta}-\sqrt{\cos^2\theta+A\sin^2\theta}\right)\right], \quad (13)$$

where $B=F(b_n)$, $A=F(a_n)$ and $1>B>A>0$. Via analysis, when $\theta=\pi/2$ or $3\pi/2$ and $r=1$, the maximum of R_n is given by

$$R_{n,\max}=\sqrt{B}-\sqrt{A}. \quad (14)$$

Actually, the optimal initial states are the so called equatorial antipodal states, which have some special geometrical characteristics. Finally combining Eq. (10) with Eq. (14), the degree of non-Markovianity can be given by the following form

$$R=\sum_n\sqrt{F(b_n)}-\sqrt{F(a_n)}. \quad (15)$$

3. The effect of environmental parameters on the non-Markovianity

In this section, we will discuss the effects of the various environmental parameters on the degree of non-Markovianity.

From above discussion, we can see that the non-monotonous F_{01} corresponds to a non-Markovian dynamics process, while the monotonous damping corresponds to a Markovian dynamics process. In light of Eq. (15), the degree of non-Markovianity R depends on the oscillatory behaviors of decoherence factor F_{01}. On account of the complex formation of decoherence factor F_{01}, we will calculate the degree of the non-Markovianity resorting to the numerical method based on Eq. (15). In fact, in some fixed time interval high oscillatory frequency and amplitude of decoherence, factor F_{01} signifies

a quick and vast information exchange between a qubit system and the spin chain environment, and it leads to a strong non-Markovianity.

According to Ref. [38], in the case of $\gamma = 0$, the spin chain environment goes into the XX style and decoherence factor F_{01} maintains unity during the dynamical process. In general, the information flowing out from the qubit system precedes the information back to the qubit system in the qubit system dynamics process. Therefore, F_{01} maintaining unity indicates that the qubit system does not begin with the information flowing out and does not have information back later. In this case, the measure of non-Markovianity is zero regardless of the variation of other parameters. It is not difficult to understand that the spin chain environment loses the ability to obtain information from the central qubit, needless to say backing information to it.

In Fig. 1, we provide a result of the effects of λ and γ on the qubit system's memorability without DM interaction. Apparently, $\lambda = 1$ is the critical point of the spin chain environment in which the decaying behavior of the central qubit is enhanced strongly[39] and reveals a complete memoryless dynamical process, where there is a concurrence between this result and Ref. [27]. Furthermore, we have a luminous discovery that there is a distinction about the central qubit system's dynamical process at two flanks of the critical point. Obviously, the zone of $0 < \lambda < 1$ (called zone I in the remainder of this paper) exhibits a powerful non-Markovianty while the zone of $\lambda > 1$ (called zone II in the remainder of this paper) verges to an extremely weak non-Markovianty. In light of decoherence factor $|F_{01}|$'s oscillatory behavior, we explain this phenomenon in Fig. 2. In our fixed parameter space, decoherence factor $|F_{01}|$'s oscillatory behavior of zone I is different from zone II. In Fig. 2, decoherence factor $|F_{01}|$ of $\lambda = 0.5$ (solid line) and $\lambda = 0.6$ (dotted line) have a distinct periodic oscillatory behavior with a degenerative amplitude and an increscent period, which indicates frequent information exchange between the central qubit system and the spin chain environment, while decoherence factor $|F_{01}|$ of $\lambda = 1.1$ (dashed line) and $\lambda = 1.2$ (dot-dashed line) have reached an approximate stabilization after undergoing a rapid decay and a following faint increase. Obviously, in our fixed parameter space, the point like $\lambda = 0.5$ and $\lambda = 0.6$ bearing distinct periodic oscillatory behaviors leads to a powerful non-Markovianty because this point's decoherence factor has much more increasing intervals than others in a fixed time interval.

In this section, we reveal how the DM interaction influences the central qubit system's memory effect. We show our results in Figs. 3–5. In Fig. 3, zone I still presents a strong non-Markovian behavior, nevertheless, zone II approaches a Markovianity process. Zone I is more sensitive than zone II for the DM interaction. Further, in zone I, one can see that, as D increases, firstly the non-Markovianity measure R proximately linearly increases to the maximum value, then after a slight decrease, reaches a stable value. We explain this behavior in Figs. 4 and 5. In Fig. 4, the decoherence factor $|F_{01}|$ changes with time t at the different D cases, and we give the reason that the non-Markovianity measure R increases before the maximum value. Observing Fig. 4, the oscillatory period of $|F_{01}|$ decreases as D increases, however, the decay rate of $|F_{01}|$'s amplitude is nearly immune to a mutative D, in which the shortened oscillatory period is dominant to increase the non-Markovianity measure R before the maximum value. But the case is different from the above after the maximum value. Observing Fig. 5, the oscillatory period of $|F_{01}|$ decreases as D increases as before, however, the decay of $|F_{01}|$'s amplitude is speeded by the enhanced DM interaction and the accelerated damped amplitude is dominant to decrease the non-Markovianity measure R after the maximum value.

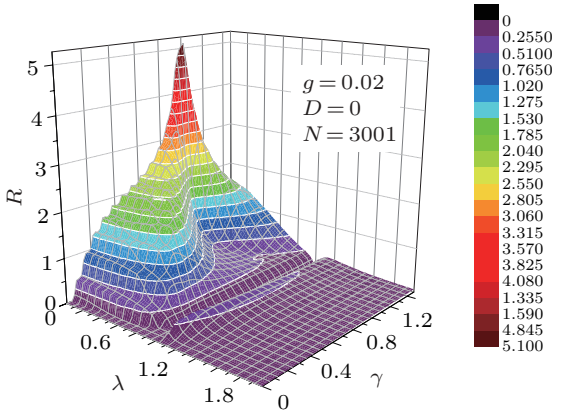


Fig. 1. (color online) We set $N = 3001$ and $g = 0.02$, non-Markovianity measure R varies with external magnetic field λ and anisotropy parameter γ without DM interaction. The critical point $\lambda = 1$ approaches the zero measure of non-Markovianity and presents a Markovianity process. Zone I exhibits a powerful non-Markovianty while zone II verges to an extremely weak non-Markovianty.

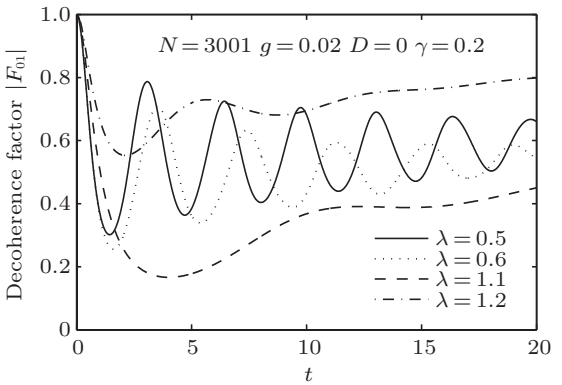


Fig. 2. We set $N = 3001$ and $g = 0.02$, decoherence factor F_{01} varies with time without DM interaction. Obviously, the two points of $\lambda = 0.5$ (solid line) and $\lambda = 0.6$ (dotted line) have a distinct periodic oscillatory behavior with degenerative amplitude and increscent period while decoherence factor $|F_{01}|$ of $\lambda = 1.1$ (dashed line) and $\lambda = 1.2$ (dotted-dashed line) have no clear oscillatory behavior.

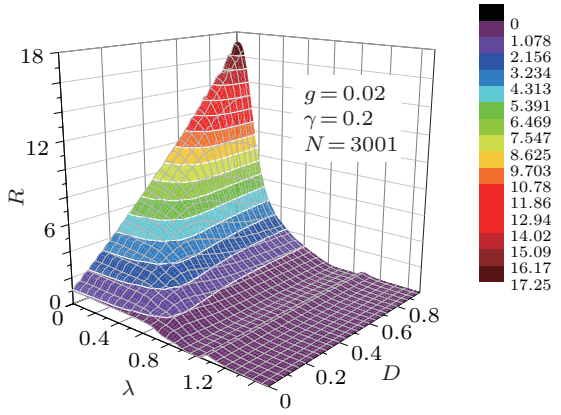


Fig. 3. (color online) We set $N = 3001$ and $g = 0.02$, decoherence factor F_{01} varies with time without DM interaction. Obviously, the two points of $\lambda = 0.5$ (solid line) and $\lambda = 0.6$ (dotted line) have a distinct periodic oscillatory behavior with degenerative amplitude and increscent period while decoherence factor $|F_{01}|$ of $\lambda = 1.1$ (dashed line) and $\lambda = 1.2$ (dotted–dashed line) have no clear oscillatory behavior.

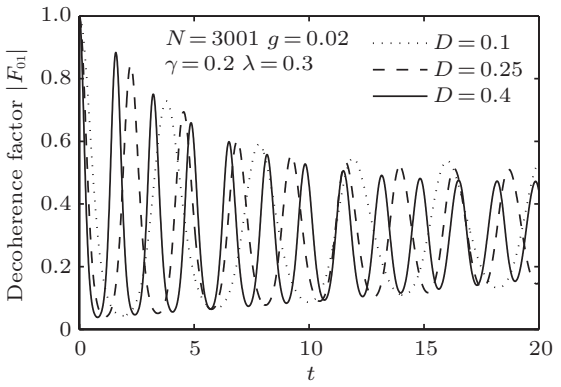


Fig. 4. In the fixed parameter space with $N = 3001$, $g = 0.02$, $\gamma = 0.2$, and $\lambda = 0.3$, we plot the decoherence factor $|F_{01}|$ of $D = 0.1$ (dotted line), $D = 0.25$ (dashed line), and $D = 0.4$ (solid line). The oscillatory period of $|F_{01}|$ decreases as D increases, however, the amplitude of $|F_{01}|$ is nearly immune to a mutative DM interaction D.

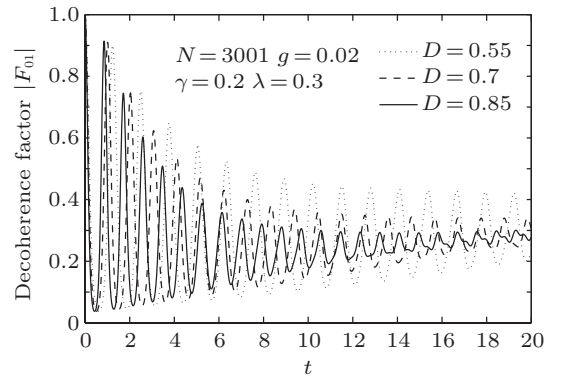


Fig. 5. In the fixed parameter space with $N = 3001$, $g = 0.02$, $\gamma = 0.2$, and $\lambda = 0.3$, we plot the decoherence factor $|F_{01}|$ of $D = 0.85$ (solid line), $D = 0.7$ (dashed line), and $D = 0.55$ (dotted line). The oscillatory period of $|F_{01}|$ decreases as D increases as before, however, the decay of $|F_{01}|$'s amplitude is speeded by the enhanced DM interaction.

4. Conclusion

In summary, we have studied the non-Markovianity of the single central spin qubit coupled to a Heisenberg XY spin chain environment with DM interaction. We have analyzed the effect of the environment parameters on the non-Markovianity of a single central qubit system. In the case of $\gamma = 0$, the information exchange between the qubit system and the spin chain environment does not exist, and the degree of the non-Markovianity is zero. On both sides of the critical point of the spin chain environment, the central qubit system demonstrates a greatly different memory effect on account of the different oscillatory behaviors of the decoherence factor. In the spin chain phase I ($\lambda < \lambda_C$), the decoherence factor has a prominent oscillatory behavior, which leads to the strong non-Markovianity, while in the phase II ($\lambda > \lambda_C$) the oscillatory behavior is not significant. Then, we have analyzed the effect of the DM interaction on the non-Markovianity. It is found that the spin chain phase I is more sensitive than the spin chain phase II for the DM interaction. In the spin chain phase I, the DM interaction changes the memory effect of the qubit system via modifying the amplitude and period of the decoherence factor in some specific parameter space. In the increasing zone of the non-Markovian degree, the shortened oscillatory period plays a dominant role, while in the decreasing zone of the non-Markovian degree, the accelerated damped amplitude is the dominant reason. Our results may serve as a theoretical reference for tuning the non-Markovianity in the spin environment experimentally, and may have potential applications in the quantum information field.

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