Promote entanglement trapping in photonic band gaps*

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We study the entanglement trapping of two entangled qubits, each of which is in its own photonic band gap, based on the weak measurement and quantum measurement reversal. An almost maximal entanglement of the two-qubit system can be trapped by using a certain weak measurement strength. Furthermore, we find that the optimal entanglement enhancing is not only dependent on the weak measurement strength but also on the different initial states. The outcomes in our scheme are completely different from that without any measurement on the studied system.

Keywords: quantum information, entanglement trapping, weak measurement, quantum measurement reversal

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1. Introduction

Entanglement is a fundamental resource of quantum information, especially the bipartite entanglement between remote parties, which is useful to realize quantum teleportation,[1] quantum key distribution,[2,3] quantum computing,[4,5] and so on. In reality, entanglement is quite fragile in the presence of decoherence[6–8] due to the fact that no system can be completely isolated. Therefore, the control and the protection of entanglement from decoherence become one of the crucial issues in quantum optics and quantum information science. In the past few decades, many schemes have been proposed to protect entanglement. For example, the microcavity[9,10] as a structured environment storing the entangled qubits can suppress the spontaneous emission. Another strategy is to use dynamic manipulations such as the mode modulation[11] and the quantum Zeno effect.[12] The decoherence-free subspace[13,14] and the entanglement distillation[15] can also be used to suppress decoherence and generate entanglement under some specific situations. In particular, it is pointed out that the entangled qubits can be placed in a photonic band gap to protect their entanglement.[16,17] In reality, entanglement trapping would appear in the band gap of a photonic crystal, where the density of states presents a dip. The dip can inhibit the spontaneous emission of the qubit.[18–20]

Recently, it has been shown that the weak measurement and quantum measurement reversal can effectively suppress the decoherence of a qubit and the entanglement decay of qubits.[21–24] For the case of two qubits, the weak measurement and quantum measurement reversal can remarkably increase the entanglement, and can even avoid the entanglement sudden death.[24,25] Consequently, the weak measurement and quantum measurement reversal locally on the individual qubits in the photonic band gaps may induce a larger entanglement trapping. In this paper, we consider two initially entangled qubits, each of which is in its own photonic band gap. We carry out the weak measurement as a pre-measurement on each qubit before it undergoes decoherence. When the qubits interact with their own band gaps, the proper post-measurements are performed on both qubits. In our scheme, we can obtain a better promotion of entanglement trapping of the two qubits. Our proposed approach works well in promoting entanglement trapping compared with the trapping obtained without performing any measurement. Besides, we study the influences of different initial states and the pre-measurement strength on the optimal entanglement enhancing. We note that our scheme does not need frequent measurements compared with the quantum Zeno effect in suppressing decoherence.

The paper is organized as follows. Firstly, we describe our physical model and the two-qubit dynamics in the independent photonic band gaps (Section 2). In Section 3, we introduce the weak measurement and quantum measurement reversal to obtain an optimal concurrence[26] of entanglement trapping. In Section 4, we give our calculation results and discussion. Finally, we conclude this paper in Section 5.

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2. Physical model and dynamics

Consider an initially entangled system consisting of two qubits A and B, where each interacts with its own zero-temperature band gap reservoir, denoted as a and b, respectively. That is, each qubit–reservoir subsystem is isolated. The interaction between a qubit and a multi-mode reservoir can be described through the Hamiltonian (under the rotating wave approximation, $\hbar = 1$)

$$H_j = \omega_j \hat{\sigma}_+^j \hat{\sigma}_-^j + \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k + \sum_k g_k (\hat{\sigma}_+^j \hat{b}_k + \text{h.c.}), \quad (1)$$

where $\hat{b}_k^\dagger$ and $\hat{b}_k$ are the creation and the annihilation operators of the k-th reservoir mode with frequency ω_k, g_k stands for the coupling strength between the j-th qubit ($j = \text{A}, \text{B}$) and the k-th reservoir mode, and $\hat{\sigma}_+^j = |1_j\rangle\langle 0_j|$ and $\hat{\sigma}_-^j = |0_j\rangle\langle 1_j|$ are the inversion operators of the j-th qubit with transition frequency ω_j, for simplicity, here we choose $\omega_\text{A} = \omega_\text{B} = \omega_0$.

Initially, we assume the j-th qubit in the excited state $|1_j\rangle$ and its corresponding reservoir in the vacuum state $|\bar{0}\rangle$ ($|\bar{0}\rangle = \prod_{k=1} |0_k\rangle$). For an ideal photonic band gap as the reservoir, the spectral density is

$$D(\omega) = \frac{W_1 \Gamma_1}{(\omega - \omega_\text{c})^2 + (\Gamma_1/2)^2} - \frac{W_2 \Gamma_2}{(\omega - \omega_\text{c})^2 + (\Gamma_2/2)^2}, \quad (2)$$

where the weights of the two Lorentzians satisfy $W_1 - W_2 = 1$, Γ_1 and Γ_2 are the full widths at half maximum of the two Lorentzians, $\Gamma_2 < \Gamma_1$ ensures the positivity of $D(\omega)$, and ω_c is the center frequency of the spectrum. There are two poles located at $\omega_\text{c} - \text{i}\Gamma_1/2$ and $\omega_\text{c} - \text{i}\Gamma_2/2$ in $D(\omega)$, and the residues of the two poles have different signs.

Based on the above analysis and the pseudomode method,[27] the photonic band gap has two pseudomodes a_1 and a_2 decaying with decay rates $\Gamma_1' = W_1\Gamma_2 - W_2\Gamma_1$ and $\Gamma_2' = W_1\Gamma_1 - W_2\Gamma_2$, respectively. The effective Hamiltonian of the qubit–reservoir subsystem can be rewritten as

$$\begin{aligned} H_j^\text{eff} = {} & \omega_0 \hat{\sigma}_+^j \hat{\sigma}_-^j + \omega_\text{c} \hat{a}_1^\dagger \hat{a}_1 + \omega_\text{c} \hat{a}_2^\dagger \hat{a}_2 \\ & + \Omega(\hat{a}_2^\dagger \sigma_-^j + \hat{a}_2 \sigma_+^j) + V(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger), \end{aligned} \quad (3)$$

where $\hat{a}_1^\dagger$ ($\hat{a}_1$), $\hat{a}_2^\dagger$ ($\hat{a}_2$) are the creation (annihilation) operators of the two pseudomodes, and $V = (\Gamma_1 - \Gamma_2)\sqrt{W_1 W_2}/2$ is the coupling strength of the two pseudomodes. The qubit interacts coherently with the second pseudomode (with coupling strength Ω), which is in turn coupled to the first one. By using the pseudomode approach, the time evolution of the state of the j-th qubit and its corresponding reservoir, which initially is $|1_j\rangle \otimes |\bar{0}\rangle$, can be represented as

$$\begin{aligned} |1_j\rangle \otimes |\bar{0}\rangle = {} & |1_j 0_1 0_2\rangle \\ \Longrightarrow {} & c_1(t)|1_j 0_1 0_2\rangle + b_1(t)|0_j 1_1 0_2\rangle \\ & + b_2(t)|0_j 0_1 1_2\rangle, \end{aligned} \quad (4)$$

where $c_1(t)$, $b_1(t)$, and $b_2(t)$ are the evolution coefficients for the state with one excited qubit and one excited mode which is the first pseudomode or the second pseudomode. Due to the initial conditions $c_1(0) = 1$ and $b_1(0) = b_2(0) = 0$, we can acquire the exact evolution coefficients by solving the differential equations.

According to the above discussion, for an initial state of the form

$$\begin{aligned} |\Psi(0)\rangle_\text{ABab} &= |\Psi(0)\rangle_\text{AB} \otimes |\bar{0}\rangle_\text{a} |\bar{0}\rangle_\text{b} \\ &= (\cos\theta|11\rangle + \sin\theta|00\rangle)_\text{AB} \otimes |\bar{0}\rangle_\text{a} |\bar{0}\rangle_\text{b}, \end{aligned} \quad (5)$$

the time evolution of the total system is

$$\begin{aligned} |\Psi(t)\rangle_\text{ABab} = {} & \cos\theta(c_1(t)|10_1 0_2\rangle + b_1(t)|01_1 0_2\rangle \\ & + b_2(t)|00_1 1_2\rangle)_\text{Aa} (c_1(t)|10_1 0_2\rangle \\ & + b_1(t)|01_1 0_2\rangle + b_2(t)|00_1 1_2\rangle)_\text{Bb} \\ & + \sin\theta|00\rangle_\text{AB} |0_1 0_2\rangle_\text{a} |0_1 0_2\rangle_\text{b}. \end{aligned} \quad (6)$$

By tracing over the freedom degree of the two pseudomodes, we obtain the reduced density matrix of the two qubits

$$\begin{aligned} \rho_\text{AB}(t) = {} & [\sin^2\theta + \cos^2\theta(1 - |c_1(t)|^2)^2]|00\rangle_\text{AB}\langle 00| \\ & + \cos^2\theta|c_1(t)|^4|11\rangle_\text{AB}\langle 11| \\ & + \cos^2\theta|c_1(t)|^2(1 - |c_1(t)|^2)|01\rangle_\text{AB}\langle 01| \\ & + \cos^2\theta|c_1(t)|^2(1 - |c_1(t)|^2)|10\rangle_\text{AB}\langle 10| \\ & + \cos\theta\sin\theta c_1^2(t)|11\rangle_\text{AB}\langle 00| \\ & + \cos\theta\sin\theta(c_1^2(t))^*|00\rangle_\text{AB}\langle 11|, \end{aligned} \quad (7)$$

which determines the dynamics of the two-qubit system. It is well known that an initially entangled two-qubit system will have entanglement trapping in the weak-coupling regime (as shown in Fig. 4). In this paper, we set the parameters to be $W_1 = 1.1$, $W_2 = 0.1$, $\Gamma_1 = 11\Omega$, and $\Gamma_2 = \Omega$, which lie in the weak-coupling regime.

3. Scheme for promoting qubits' entanglement trapping

Now, let us describe the scheme to promote entanglement trapping of the above two-qubit system. In our scheme, two effective operations on the two qubits are weak measurement and quantum measurement reversal. For simplicity, here we discuss the state maps in the single-qubit case. We assume that the j-th qubit's computational bases are $|0_j\rangle$ and $|1_j\rangle$, and the environment is initially in $|\bar{0}\rangle$. Theoretically, a weak measurement on a qubit can be done by monitoring its environment via an ideal detector. Whenever there is an excitation in the environment, the result is discarded. In the case of no excitation, the qubit state is just renormalized and allowed to evolve. A null-result weak measurement on a qubit maps as follows:

$$\begin{aligned} |0_j\rangle|\bar{0}\rangle &\to |0_j\rangle|\bar{0}\rangle, \\ |1_j\rangle|\bar{0}\rangle &\to \sqrt{1-p}|1_j\rangle|\bar{0}\rangle. \end{aligned} \quad (8)$$

The corresponding non-unitary quantum operation can be written as

$$M_{\rm wk}(p) = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \quad (9)$$

where p ($0 \le p \le 1$) is the weak measurement strength. Because the weight of the component $|1_j\rangle$ is reduced and the computational basis state $|0_j\rangle$ does not couple to the environment, the qubit after the weak measurement is less vulnerable to decoherence.

Contrarily, the quantum measurement reversal on a qubit maps as

$$\begin{aligned} |0_j\rangle|\bar{0}\rangle &\to \sqrt{1-p_{\rm r}}|0_j\rangle|\bar{0}\rangle, \\ |1_j\rangle|\bar{0}\rangle &\to |1_j\rangle|\bar{0}\rangle, \end{aligned} \quad (10)$$

and the reversing measurement is also a non-unitary operation which can be written as

$$M_{\rm rev}(p_{\rm r}) = \begin{pmatrix} \sqrt{1-p_{\rm r}} & 0 \\ 0 & 1 \end{pmatrix}, \quad (11)$$

where $p_{\rm r}$ ($0 \le p_{\rm r} \le 1$) is the quantum measurement strength.

At $t = 0$ (before the qubits undergo decoherence), we perform weak measurements as pre-measurements on both qubits A and B with the same measurement strength $p_{\rm A} = p_{\rm B} = p$, the initial state of the two qubits in Eq. (5) becomes

$$|\Psi(p,0)\rangle_{\rm AB} = \frac{\cos\theta(1-p)|11\rangle_{\rm AB} + \sin\theta|00\rangle_{\rm AB}}{\sqrt{\cos^2\theta(1-p)^2 + \sin^2\theta}}. \quad (12)$$

Then we let the two qubits enter their independent photonic band gaps (decoherence quantum channels). The total state evolves to

$$\begin{aligned} \rho_{\rm AB}^{\rm wk}(p,t) = \frac{1}{P(p,t)}&[a(p,t)|00\rangle_{\rm AB}\langle 00| + b(p,t)|11\rangle_{\rm AB}\langle 11| \\ &+ c(p,t)(|01\rangle_{\rm AB}\langle 01| + |10\rangle_{\rm AB}\langle 10|) \\ &+ d(p,t)|11\rangle_{\rm AB}\langle 00| + d^*(p,t)|00\rangle_{\rm AB}\langle 11|], \end{aligned} \quad (13)$$

where

$$\begin{aligned} a(p,t) &= \sin^2\theta + \cos^2\theta(1-p)^2(1-|c_1(t)|^2)^2, \\ b(p,t) &= \cos^2\theta(1-p)^2|c_1(t)|^4, \\ c(p,t) &= \cos^2\theta(1-p)^2|c_1(t)|^2(1-|c_1(t)|^2), \\ d(p,t) &= \cos\theta\sin\theta(1-p)(c_1(t))^2, \\ P(p,t) &= a(p,t) + b(p,t) + 2c(p,t). \end{aligned} \quad (14)$$

As discussed in Ref. [25], if one wants to gain as much entanglement as possible, the first thing that should be done is to determine which post-measurement should be taken by comparing the values of $a(p,t)$ and $b(p,t)$.

In the case of $a(p,t) < b(p,t)$, which means the component of $|11\rangle$ is larger than that of $|00\rangle$, we should reduce the weight of $|11\rangle$ to balance the two components $|11\rangle$ and $|00\rangle$ for the purpose of increasing the entanglement. So a post weak measurement operator $M_{\rm wk}(p_{\rm r})$ should be performed on individual qubits A and B respectively when they undergo the decoherence process. In order to distinguish the prior and the post weak measurements, the post-measurement strength is donated as $p_{\rm r}$, i.e., $p_{\rm A} = p_{\rm B} = p_{\rm r} \in [0,1]$. So the total state becomes

$$\begin{aligned} \rho_{\rm AB}^{(1)}(p,p_{\rm r},t) = \frac{1}{P^{(1)}(p,p_{\rm r},t)}&[a(p,t)|00\rangle_{\rm AB}\langle 00| \\ &+ b(p,t)(1-p_{\rm r})^2|11\rangle_{\rm AB}\langle 11| \\ &+ c(p,t)(1-p_{\rm r})(|01\rangle_{\rm AB}\langle 01| + |10\rangle_{\rm AB}\langle 10|) \\ &+ d(p,t)(1-p_{\rm r})|11\rangle_{\rm AB}\langle 00| \\ &+ d^*(p,t)(1-p_{\rm r})|00\rangle_{\rm AB}\langle 11|], \end{aligned} \quad (15)$$

where $P^{(1)}(p,p_{\rm r},t) = a(p,t) + b(p,t)(1-p_{\rm r})^2 + 2c(p,t)(1-p_{\rm r})$ is the overall success probability of the combined former and latter weak measurements.

On the other hand, when $a(p,t) > b(p,t)$, i.e., the weight of component $|00\rangle$ is larger than that of $|11\rangle$, we need to apply the quantum measurement reversal operator in Eq. (11) to qubits A and B respectively when the entangled qubits undergo the decoherence process. Here, we also assume $p_{\rm r_A} = p_{\rm r_B} = p_{\rm r}$ for simplicity. Under such conditions, we obain the following evolution state:

$$\begin{aligned} \rho_{\rm AB}^{(2)}(p,p_{\rm r},t) = \frac{1}{P^{(2)}(p,p_{\rm r},t)}&[a(p,t)(1-p_{\rm r})^2|00\rangle_{\rm AB}\langle 00| \\ &+ b(p,t)|11\rangle_{\rm AB}\langle 11| \\ &+ c(p,t)(1-p_{\rm r})(|01\rangle_{\rm AB}\langle 01| + |10\rangle_{\rm AB}\langle 10|) \\ &+ d(p,t)(1-p_{\rm r})|11\rangle_{\rm AB}\langle 00| \\ &+ d^*(p,t)(1-p_{\rm r})|00\rangle_{\rm AB}\langle 11|], \end{aligned} \quad (16)$$

where $P^{(2)}(p,p_{\rm r},t) = a(p,t)(1-p_{\rm r})^2 + b(p,t) + 2c(p,t)(1-p_{\rm r})$ is the overall success probability of the combined prior weak measurement and post quantum measurement reversal.

Using the Wootters concurrence[26] to quantify the entanglement, one can exactly derive the form of concurrence of qubits' entanglement from Eqs. (15) and (16)

$$\begin{aligned} &C_{\rm AB}^{(k)}(p,p_{\rm r},t) \\ &= \frac{2}{P^{(k)}(p,p_{\rm r},t)}\max\{0,(1-p_{\rm r})(|d(p,t)| - |c(p,t)|)\}, \end{aligned} \quad (17)$$

with $k = 1,2$ corresponding to the above two situations. In order to obtain the optimal concurrence of entanglement trapping, we can choose a special post-measurement strength $p_{\rm r}$ determined by the conditions $\partial C_{\rm AB}(p,p_{\rm r},t)/\partial p_{\rm r} = 0$ and $\partial^2 C_{\rm AB}(p,p_{\rm r},t)/\partial p_{\rm r}^2 < 0$. In the case of $a(p,t) < b(p,t)$, the optimal result is $p_{\rm r}^{(1)} = 1 - \sqrt{a(p,t)/b(p,t)}$; while for the other case of $a(p,t) > b(p,t)$, it yields $p_{\rm r}^{(2)} = 1 -$

$\sqrt{b(p,t)/a(p,t)}$. The optimal concurrences corresponding to the above two optimal conditions have the same form

$$C_{\text{optimal}}(p,t) = \max\left\{0, \frac{|d(p,t)| - |c(p,t)|}{\sqrt{a(p,t)b(p,t)} + |c(p,t)|}\right\}. \quad (18)$$

The different corresponding success probabilities can be obtained as

$$P^{(1)}_{\text{optimal}}(p,t) = 2a(p,t) + 2c(p,t)\sqrt{\frac{a(p,t)}{b(p,t)}}, \quad (19)$$

$$P^{(2)}_{\text{optimal}}(p,t) = 2b(p,t) + 2c(p,t)\sqrt{\frac{b(p,t)}{a(p,t)}}. \quad (20)$$

Utilizing the optimal concurrence of Eq. (18) and its corresponding success probabilities of Eqs. (19) and (20), we can control the time evolution of the two qubits' entanglement in the independent photonic band gaps by changing the prior weak measurement parameter p. In order to distinguish the above two different equations of success probability under the conditions of $a(p,t) > b(p,t)$ and $a(p,t) < b(p,t)$, we give the following explanation. When $tg^2\theta > 1$, $a(p,t)$ is always greater than $b(p,t)$, we use Eq. (20) as the corresponding success probability in our calculation. When $tg^2\theta < 1$, $a(p,t) - b(p,t)$ may be negative on the condition of $p < 1 - |tg\theta|/\sqrt{2|c_1(t)|^2 - 1}$, then equation (19) must be used in the calculation.

4. Numerical results and discussion

In this section, we analyze the role of the combined pre-measurement and optimal post-measurement on the two-qubit entanglement trapping. First of all, we need to check whether our scheme can promote the concurrence of entanglement trapping, and if it does, then we need to work out how it enhances the concurrence. For this purpose, we illustrate the optimal concurrence and the corresponding success probability as functions of the dimensionless quantity Ωt and the pre-measurement strength p, as shown in Fig. 1. For an initial maximally entangled two-qubit state $\theta = \pi/4$ (Fig. 1(a)), the pre-measurement and the optimal post-measurement indeed work well in taming the two-qubit decoherence and promoting the entanglement trapping of the two qubits. The optimal concurrence C_{optimal} will be arbitrarily close to the initial value of entanglement ($C_{\text{initial}} = 1$) under the asymptotic limit of $p \to 1$. Furthermore, as demonstrated for a non-maximally entangled two-qubit state $\theta = \pi/6$ in Fig. 1(b), we can see that the optimal concurrence increases with increasing p at any time, and $\lim_{p\to 1} C_{\text{optimal}}(p,t) = 1$, which is much larger than the initial concurrence $C_{\text{initial}} = \sqrt{3}/2$. While the corresponding success probability decreases with increasing p, and $\lim_{p\to 1} P_{\text{optimal}}(p,t) = 0$, as shown in Figs. 1(c) and 1(d).

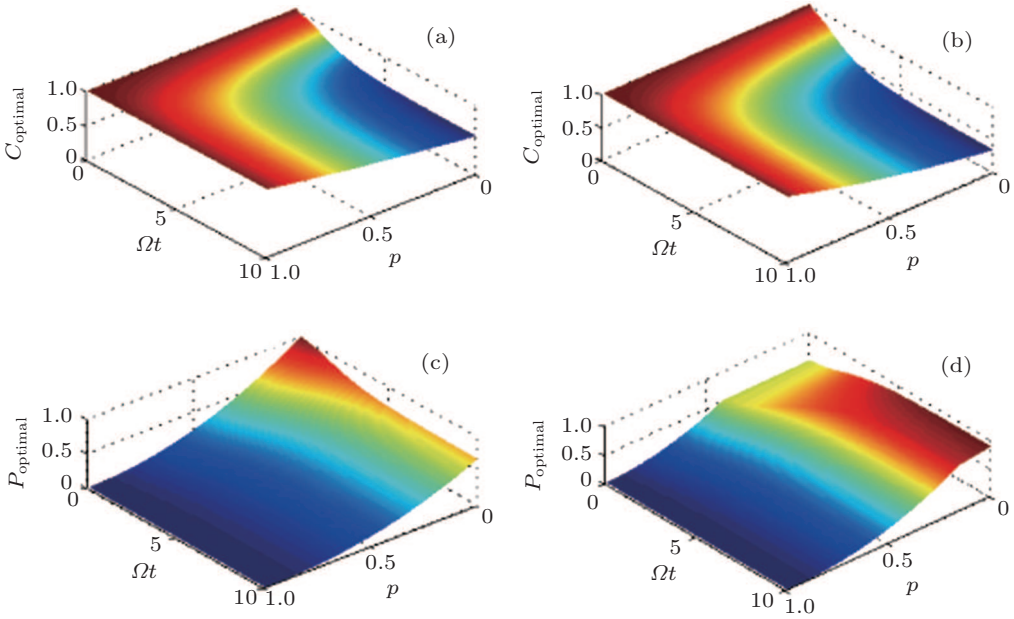


Fig. 1. (color online) (a), (b) The optimal concurrence C_{optimal} and (c), (d) the corresponding success probability P_{optimal} as functions of the dimensionless quantity Ωt and the pre-measurement strength p for two different Bell-like initial states. In panels (a) and (c), $\theta = \pi/4$; in panels (b) and (d), $\theta = \pi/6$. The parameters used are $W_1 = 1.1$, $W_2 = 0.1$, $\Gamma_1 = 11\Omega$, and $\Gamma_2 = \Omega$ lying in the weak-coupling regime.

For clarity, we use two-dimensional graphs to compare the entanglement of trapping between the optimal concurrence and the concurrence without the action of any measurement. Figure 2(a) shows the promotion of entanglement trapping for different initial entangled states ($\theta = \pi/3$, $\theta = \pi/4$, $\theta = \pi/6$) by considering a fixed pre-measurement strength $p = 0.1$. Obviously, the initially maximum entangled qubits state would lead to the most effective entanglement trapping without the action of any measurement during the decoherence process, as shown by the thin lines in Fig. 2(a). However, comparing

with the optimal concurrence represented by the thick lines in Fig. 2(a), we find that the biggest promotion of concurrence cannot occur in the case of $\theta = \pi/4$. Interestingly, the corresponding success probability in Fig. 2(b) seems to be trapping as well as entanglement trapping. In the evolution process, the entanglement trapping means that the weights of components $|00\rangle$ ($a(p,t)$) and $|11\rangle$ ($b(p,t)$) will not change. So the coefficient $c_1(t)$ is independent of time, and the success probabilities can also be independent of time for a fixed pre-measurement by analyzing Eqs. (19) and (20).

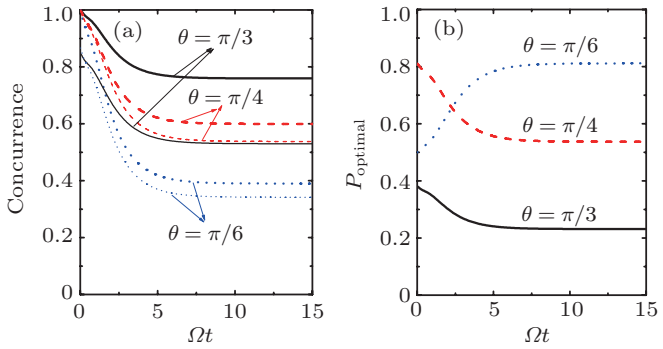


Fig. 2. (color online) Time evolutions of (a) the concurrence and (b) the corresponding success probability of different Bell-like states. The thin lines in panel (a) represent the concurrences without the action of any measurement, while the thick lines in panels (a) and (b) are the optimal values for the concurrence and the corresponding success probability under the optimal conditions, which are given by Eqs. (18)–(20) at a fixed pre-measurement strength $p = 0.1$.

Our scheme elaborates that the initial maximum entangled qubits cannot bring out the most effective entanglement trapping during the decoherence process by the weak measurements and quantum measurement reversals. To find the influences of different initial states on the entanglement trapping, we study the optimal concurrence and its corresponding success probability as functions of the dimensionless quantities Ωt and θ at a fixed pre-measurement strength $p = 0.1$, as shown in Fig. 3. We also illustrate the concurrence of the two-qubit system without performing any measurement derived from Eq. (7), as shown in Fig. 4. A highlight in Fig. 3(a) is that the optimal concurrence increases monotonically with increasing θ in the interval of $\theta \in (0, \pi/2)$, and decreases monotonically as θ increases in $(\pi/2,\pi)$, which is completely different from that without the action of any measurement, as shown in Fig. 4. When $\theta \to \pi/2$, the optimal concurrence can be close to 1, while the corresponding success probability will be close to zero. This is because the initial two-qubit state is changed to $|00\rangle_{\rm AB}$ in the case of $\theta = \pi/2$. So the concurrence of the qubits is always zero and independent of the weak measurement and the evolution process. In our scheme, for the initially entangled state $\cos\theta|11\rangle_{\rm AB} + \sin\theta|00\rangle_{\rm AB}$, the lower component of $|11\rangle$ can result in the higher concurrence of entanglement trapping. In fact, the small weight of excited component $|11\rangle$ makes the qubit more robust against the decoherence. The concurrence of entanglement trapping is proportional to the initial entanglement of the two qubits in the case without performing any measurement as shown in Fig. 4, i.e., the larger initial entanglement can lead to much bigger entanglement trapping. This is very different from the results obtained in our scheme. The corresponding success probability, as shown in Fig. 3(b), first increases and then decreases in the intervals of $\theta \in (0, \pi/2)$ and $(\pi/2, \pi)$. There is a spike in each interval. That is, we have a good choice of θ (for instance, the largest success probability occurs at $\theta = 0.55$) to manipulate the entanglement of the two-qubit system. Moreover, with this scheme, we find a choice compromise of good initial qubits state, with which the obtained concurrence of entanglement trapping is much bigger and the corresponding success probability is not small.

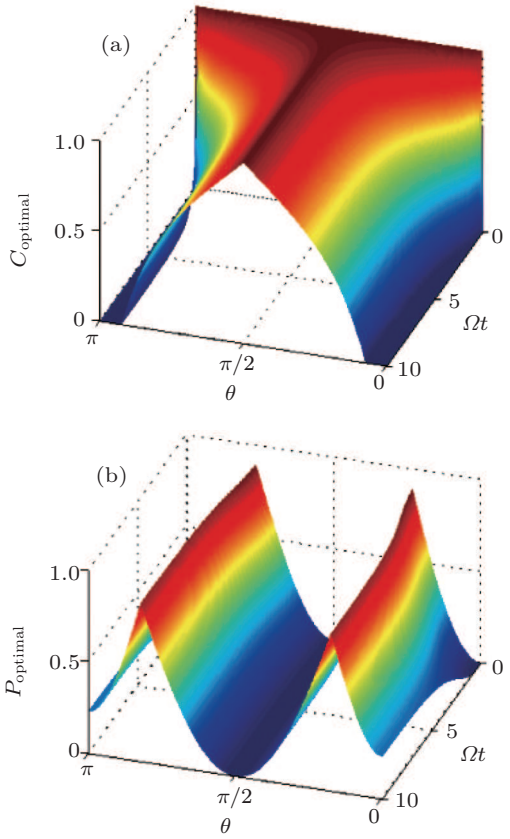


Fig. 3. (color online) (a) The optimal concurrence $C_{\rm optimal}$ and (b) the corresponding success probability $P_{\rm optimal}$ as functions of the dimensionless quantity Ωt and θ at a fixed pre-measurement strength $p = 0.1$.

As we discussed above, when the entanglement trapping occurs, the corresponding success probability will be trapped too, and the following evolution is irrespective of time. We plot the optimal concurrence and the corresponding success probability at the moment that the entanglement trapping has already occured ($\Omega t = 15$) as functions of θ at fixed pre-measurement strengths, as shown in Fig. 5. For a specific θ, the optimal concurrence increases with increasing p, which

is consistent with the results obtained in Fig. 1. The promotions of entanglement trapping are different for different initial states. Furthermore, we find that the corresponding success probability decreases with the increase of the pre-measurement strength p, and the two spikes would separately move with increasing p.

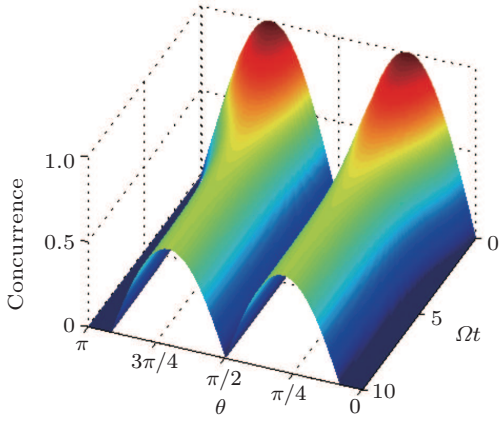


Fig. 4. (color online) The concurrence (without the action of any measurement) of the two-qubit system, derived from Eq. (7), as a function of the dimensionless quantity Ωt and θ.

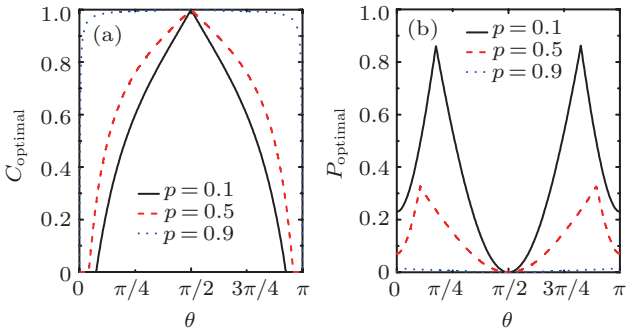


Fig. 5. (color online) (a) The optimal concurrence C_{optimal} and (b) the corresponding success probability P_{optimal}, depending on the different pre-measurement strengths p, as functions of θ at specific time $\Omega t = 15$, at which the entanglement trapping occurs.

5. Conclusion

We have studied entanglement trapping of two entangled qubits, each of which is in its own photonic band gap. Using the pre-measurements (weak measurements) on the qubits before they interact with their own reservoirs and the optimal post-measurements (weak measurements or quantum measurement reversals) when they undergo decoherence, we obtain promoted entanglement trapping. We find that the enhanced degree of entanglement trapping mainly depends on the different initial states and the pre-measurement strength. We highlight that the two-qubit system can be trapped in an almost maximal entangled state by using a certain weak measurement strength. These results are helpful to manipulate the entanglement of a two-qubit system, for example, the atoms embedded in photonic band gap materials.[20,28]

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