Controllable preparation of two-mode entangled coherent states in circuit QED*

Ji Ying-Hua(嵇英华)[a)b)†] and Liu Yong-Mei(刘咏梅)[c)]

[a)] *Department of Physics, Jiangxi Normal University, Nanchang 330022, China*
[b)] *Key Laboratory of Photoelectronics and Telecommunication of Jiangxi Province, Nanchang 330022, China*
[c)] *College of Mathematics and Information Science, Jiangxi Normal University, Nanchang 330022, China*

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Although the multi-level structure of superconducting qubits may result in calculation errors, it can be rationally used to effectively improve the speed of gate operations. Utilizing a current-biased Josephson junction (Λ-type rf-SQUID) as a tunable coupler for superconducting transmission line resonators (TLRs), under the large detuning condition, we demonstrate the controllable generation of entangled coherent states in circuit quantum electrodynamics (circuit QED). The coupling between the TLRs and the qubit can be effectively regulated by an external bias current or coupling capacitor. Further investigations indicate that the maximum entangled state can be obtained through measuring the excited state of the superconducting qubits. Then, the influence of the TLR decay on the prepared entangled states is analyzed.

Keywords: circuit QED, preparation of quantum state, entangled coherent state, concurrence

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1. Introduction

In the past few decades, one of the most important developments in quantum information is the construction of many non-classical states in different quantum systems,[1,2] including entangled coherent states.[3] The entangled coherent states, which are continuous variable entangled states, play an important role in quantum information.[4] They are not only helpful for quantum mechanics to prevail over the local hidden theory,[5] but also valuable in quantum information processes (quantum teleportation,[6] quantum dense coding,[7] and quantum cryptography).[8] By using the standard atom-optical cavity interactions, the entangled coherent states were experimentally produced and probed. References [9] and [10] showed that the entangled coherent states can be generated by the state-selective measurement on a two-level or three-level atom interacting with a two-mode field. Recently, Solano *et al*.[11] proposed a method for generating the entangled coherent states by using a two-level atom cavity QED driven by a strong classic field. However, the optical cavity QED system has no advantage in integration despite the satisfactory experimental platform it provides for quantum information processing.[12]

On the other hand, a variety of qubits have been theoretically proposed and experimentally realized based on superconducting Josephson junctions[13] due to their advantages in design flexibility, large-scale integration, and compatibility with conventional electronics. Therefore, it is a fascinating field in the quantum information world. In the circuit quantum electrodynamics (circuit QED), superconducting qubits can act as artificial atoms, and a one-dimensional superconducting transmission line resonator (TLR) can be used as the microwave cavity field. Most recently, the method of atom-cavity interactions was applied in solid-state circuit QEDs to prepare multi-photon Fock states and generalized binomial states.[14,15] In Ref. [16], a practically feasible scheme to generate the entangled coherent states by coupling two superconducting LC modes with a flux qubit was proposed. However, the scheme has the following disadvantages. (i) In all the superconducting qubits, the superconducting phase differences across the Josephson junctions in the circuit contain the quantum information, i.e., the state of the qubit. Since the superconducting phase is continuous, the superconducting qubits (two-level systems) have to be obtained by truncation of an infinite-dimensional Hilbert space, which indicates that the superconducting qubit is actually an approximate two-level system. For technical reasons, it may be impossible to prepare the initial state with perfect fidelity in the lowest two states,[17] because the erroneous transitions to higher levels (leakage effects) resemble the spontaneous emission of the real atoms. (ii) It is necessary for the two-level superconducting qubit to simultaneously interact with two duplicate TLRs with the same atomic transitions in the scheme, thus it cannot be easily manipulated and recognized. (iii) One circuit cavity should be initially prepared in the single-mode even-odd coherent states, which is difficult to realize.

However, we know that the multi-level structure of the superconducting quantum interference device (SQUID) qubits is advantageous to the exaltation of the calculation speed of

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†Corresponding author. E-mail: ahmxhxtt@aliyun.com
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the qubits.[18] Based on the conventional configuration of the SQUID qubits, a scheme of three-level (Λ-type) rf-SQUID qubit was proposed Ref. [19], where the states of the qubit were two lower states $|0\rangle$ and $|1\rangle$ of the Λ-type system. The manipulation of the qubit was done with two microwave impulses which respectively coupled the two lower states $|0\rangle$ and $|1\rangle$ to the upper state $|2\rangle$. When the transition matrix elements, corresponding to the transitions $|0\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |2\rangle$, were larger than that of the transition $|0\rangle \rightarrow |1\rangle$, the Λ-type superconducting qubit was shown to be significantly faster with a much lower error. The investigation results indicated that the auxiliary level $|2\rangle$ played an important role in improving the speed of the gate operations, prolonging the decoherence time, and reducing the calculation error.

Enlightened by the previous research on the preparation of the entangled coherent states, we propose in this paper an alternative scheme to generate the entangled coherent states using a Λ-type rf-SQUID qubit to induce the interaction between two TLRs. The proposed scheme efficiently avoids the difficulty in integration which exists in the optical cavity scheme. Making use of the multi-level structure of the superconducting qubit not only availably reduces the errors and improves the speed of gate operations, but also makes the manipulation and modulation very convenient, so that an experimental breakthrough is easier to achieve.

This paper is organized as follows. In Section 2, we show the model and present the scheme for preparing the entangled coherent states in circuit QED. The dynamical behaviors of quantum entanglement for the coupled superconducting qubits in circuit QED are investigated in Section 3. In Section 4, the obtained results are summarized and discussed.

2. Model and preparation of entangled coherent state

The coupled circuit QED system is shown in Fig. 1. We choose the superconducting flux qubit because it possesses longer decoherence time. The circle represents a Λ-type rf-SQUID qubit. In Fig. 1(a), a current-biased Λ-type rf-SQUID qubit as a tunable coupler couples two superconducting transmission line oscillators. Through adjusting the physical parameters of the current-biased Josephson junction, the coupling strength between the two oscillators can be efficiently manipulated; the detuning state can be efficiently adjusted as well.

We assume here that the left cavity is connected with a Λ-type rf-SQUID via capacitor $C_{\rm l}$, the coupling is between levels $|0\rangle \leftrightarrow |2\rangle$ with detuning $\delta_{\rm l}$, the coupling constant is $g_{\rm l}$; similarly, the right cavity is connected with a Λ-type rf-SQUID via capacitor $C_{\rm r}$, the coupling is between levels $|1\rangle \leftrightarrow |2\rangle$ with detuning $\delta_{\rm r}$, and the coupling constant is $g_{\rm r}$. The two artificial atomic transitions $|0\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ are also driven by two classical fields with couplings $\Delta_{\rm l}$ and $\Delta_{\rm r}$, and $\Omega_{\rm l}$ and $\Omega_{\rm r}$ are the Rabi frequencies of the two classical fields. For the sake of simplicity, we only consider the case of $\delta_{\rm l} = \delta_{\rm r} = \delta$ and $\Delta_{\rm l} = \Delta_{\rm r} = \Delta$ in this paper.

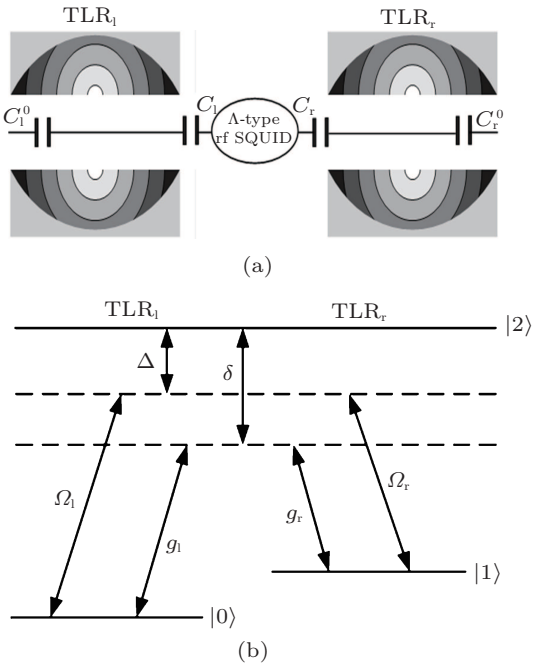


Fig. 1. (a) Setup of two TLRs 1 and 2 coupling with an rf-SQUID qubit. Each cavity here is a one-dimensional superconducting transmission line resonator. The circle represents a Λ-type rf-SQUID qubit, which is coupled to TLR j via a capacitance C_j (j = l, r). (b) Energy level configuration of the Λ-type rf-SQUID qubit.

The Hamiltonian of an rf SQUID (with junction capacitance C and loop inductance L) is given by[20]

$$H_{\rm q} = \frac{Q^2}{2C} + \frac{(\Phi - \Phi_{\rm x})^2}{2L} - E_{\rm J}\cos\left(2\pi\frac{\Phi}{\Phi_0}\right), \tag{1}$$

where Q is the total charge on the capacitor, Φ is the magnetic flux threading the ring, $\Phi_{\rm x}$ is the external magnetic flux applied to the ring, and $E_{\rm J} = I_{\rm c}\Phi_0/(2\pi)$ is the maximum Josephson junction coupling energy with $I_{\rm c}$ being the critical current of the junction and Φ_0 being the flux quantum. The rf SQUID is biased properly to have a Λ-type configuration formed by three lowest levels, denoted as $|0\rangle$, $|1\rangle$, and $|2\rangle$ in Fig. 1(b). Obviously, we can adjust the eigen-frequency of the qubit via adjusting the external flux or current. The one-dimensional transmission line oscillator is a kind of superconducting planar waveguide with a full-wave section, which is fabricated by the method of traditional optical lithography. After quantization, the superconducting transmission line is equivalent to a microwave quantum field.

In the rotating wave approximation, the Hamiltonian for the total quantum circuit can be written as (in units of $\hbar = 1$)

$$\begin{aligned} H \;=\; & \omega_{01}|1\rangle\langle 1| + \omega_{02}|2\rangle\langle 2| + \omega_{\rm l}a_{\rm l}^{+}a_{\rm l} + \omega_{\rm r}a_{\rm r}^{+}a_{\rm r} \\ & + g_{\rm l}(a_{\rm l}^{+}|0\rangle\langle 2| + a_{\rm l}|2\rangle\langle 0|) + g_{\rm r}\left(a_{\rm r}^{+}|1\rangle\langle 2| + a_{\rm r}|2\rangle\langle 1|\right) \end{aligned}$$

$$\times \Omega_{\rm l}\left(\mathrm{e}^{-\mathrm{i}(\omega_{02}-\Delta)t}\left|2\right\rangle\left\langle 0\right|+\mathrm{h.c.}\right)$$
$$+\Omega_{\rm r}\left(\mathrm{e}^{-\mathrm{i}(\omega_{02}-\omega_{01}-\Delta)t}\left|2\right\rangle\left\langle 1\right|+\mathrm{h.c.}\right), \quad (2)$$

where a_j^+ and a_j ($j=$ l, r) are the creation operators of full wave modes in the left and the right TLRs, while ω_{13} and ω_{12} are the Bohr frequencies associated with the two artificial atomic (qubit) transitions $|0\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |2\rangle$, respectively, and they approximate the plasma oscillation frequency at the bottom of the well $\omega_{\rm p}=\sqrt[4]{(2-2I_{\rm b}/I_{\rm c})(2\pi I_{\rm c}/\Phi_0 C_{\rm J})}$ ($I_{\rm b}$ is the bias current of the junction).[21] The resonant frequencies of the left and the right TLRs are

$$\omega_j \approx \frac{2\pi}{X_j\sqrt{L_jC_j}}\left(1-\frac{C_j^0}{X_{\rm J}C_j}-\frac{C_j}{X_{\rm J}C_j}\right), \quad (j=\mathrm{l},\mathrm{r}), \quad (3)$$

where X_j is the length of the TLRs, and L_j and C_j are the inductance and the capacitance per unit length, respectively. The coupling factor is

$$g_j=\frac{\omega_j C_{\rm e}^j}{4X_j^2C_j^2(C_{\rm J}+2C_{\rm e}^j)^2}, \quad (4)$$

where $C_{\rm J}$ is the junction capacitance. From Eq. (4), we know that g_j can be effectively controlled by adjusting the bias current and the coupling capacitor C_j.

If the detuning is sufficiently large,

$$\left\{\frac{\Omega_j}{\Delta},\frac{g_j}{\delta}\right\}<<1,$$

using the adiabatically eliminated method, we can obtain the following effective Hamiltonian:[22,23]

$$H_{\rm eff}=-g\left(a_{\rm l}^+a_{\rm r}\sigma^+ + a_{\rm r}^+a_{\rm l}\sigma^-\right)-\Omega\left(\sigma^+ + \sigma^-\right), \quad (5)$$

with

$$g=\frac{g_{\rm l}g_{\rm r}}{\delta}, \quad \Omega=\frac{\Omega_{\rm l}\Omega_{\rm r}}{\Delta},$$

where $\sigma^+=|1\rangle\langle 0|$ is the raising operator for a qubit (artificial atom). Hamiltonian (5) can reduce to

$$H_{\rm eff}=H_{\rm eff}^0+H_{\rm eff}^{\rm I}, \quad (6)$$

with

$$H_{\rm eff}^0=-\Omega\left(\sigma^+ + \sigma^-\right), \quad H_{\rm eff}^{\rm I}=-g\left(a_{\rm l}^+a_{\rm r}\sigma^+ + a_{\rm r}^+a_{\rm l}\sigma^-\right).$$

Based on this Hamiltonian by performing a unitary transformation $U=\exp(-\mathrm{i}H_0t)$, we can easily find the following state evolution

$$H_{\rm eff}^{\rm int}(t)=-\frac{1}{2}g\left(a_{\rm l}^+a_{\rm r}+a_{\rm r}^+a_{\rm l}\right)\left(\sigma^+ + \sigma^-\right). \quad (7)$$

In order to generate the entangled coherent states, we assume that the qubit is initially in the ground state $|0\rangle = (|+\rangle + |-\rangle)/\sqrt{2}$, where $|+\rangle$ ($|-\rangle$) is the eigenstate of the Pauli operator σ_x with eigenvalue 1 (-1), and the left (right) TLR is prepared in the coherent state $|\alpha\rangle$ ($|\beta\rangle$). Then, the total system is initially in the quantum state

$$|\psi(0)\rangle=|0\rangle\otimes|\alpha\rangle|\beta\rangle. \quad (8)$$

After direct but complex calculations,[24] we can obtain the entangled coherent states

$$|\psi(t)\rangle = \frac{1}{2}|0\rangle\left(\left|\tilde{\alpha},\tilde{\beta}\right\rangle+\left|\tilde{\alpha}^*,\tilde{\beta}^*\right\rangle\right)$$
$$+\frac{1}{2}|1\rangle\left(\left|\tilde{\alpha},\tilde{\beta}\right\rangle-\left|\tilde{\alpha}^*,\tilde{\beta}^*\right\rangle\right), \quad (9)$$

where

$$\tilde{\alpha}=\alpha\cos\left(\frac{1}{2}gt\right)+\mathrm{i}\beta\sin\left(\frac{1}{2}gt\right), \quad (10)$$

$$\tilde{\beta}=\beta\cos\left(\frac{1}{2}gt\right)+\mathrm{i}\alpha\sin\left(\frac{1}{2}gt\right). \quad (11)$$

Through ground and excited state detectors, we implement the measurement on the ground (or excited) state of the superconducting qubit. If the internal state of the atom is detected to be in the state $|0\rangle$ or $|1\rangle$, we obtain the two-mode entangled coherent states

$$|\psi_\pm(t)\rangle=\frac{1}{\sqrt{M}}\left(\left|\tilde{\alpha},\tilde{\beta}\right\rangle\pm\left|\tilde{\alpha}^*,\tilde{\beta}^*\right\rangle\right), \quad (12)$$

with

$$M = 2\pm\left\{\exp\left[-2\mathrm{i}\left(\tilde{\alpha}^*\mathrm{Im}\tilde{\alpha}+\tilde{\beta}^*\mathrm{Im}\tilde{\beta}\right)\right]\right.$$
$$\left.+\exp\left[2\mathrm{i}\left(\tilde{\alpha}\mathrm{Im}\tilde{\alpha}+\tilde{\beta}\mathrm{Im}\tilde{\beta}\right)\right]\right\}, \quad (13)$$

where $\pm$ respectively correspond to the ground and the excited states of the superconducting qubit.

Now we estimate the entanglement of Eq. (12). By using the concept of concurrence for bipartite entangled nonorthogonal states, the concurrence of Eq. (9) is given by[25]

$$C_\pm(t)=\frac{\sqrt{2\left(1-\mathrm{e}^{-4\beta^2\sin^2(gt/2)}\right)\left(1-\mathrm{e}^{-4\alpha^2\sin^2(gt/2)}\right)}}{1\pm\mathrm{e}^{-\left(\alpha^2+\beta^2\right)\sin^2(gt/2)}\cos\left[2\alpha\beta\sin\left(gt\right)\right]}. \quad (14)$$

Here α and β are real. Whenever $C_\pm>0$, state (12) is an entangled state.

The time evolution of the concurrence as a function of time t is plotted in Figs. 2 and 3. From Fig. 2, if the amplitudes of the initial coherent states of the left and the right TLRs are identical, the entanglement degree of the prepared entangled coherent state is larger. The measurement on the excited states of the superconducting qubits helps to obtain the maximum entangled state. Therefore, entanglement sudden death can be completely avoided in the non-dissipative circuit cavity. From Fig. 3, through the effective adjustment of the bias current and the coupling capacitor C_j, the coupling factor between the TLRs and the qubits takes a smaller value, which is more beneficial for the maintenance of the entanglement state. In a word, we reveal that the two TLRs can be prepared in two-mode entangled coherent states through measuring the superconducting qubits. A further numerical analysis indicates that the entanglement dynamics of the two entangled coherent states obtained by measuring the ground state and the excited state are significantly different.

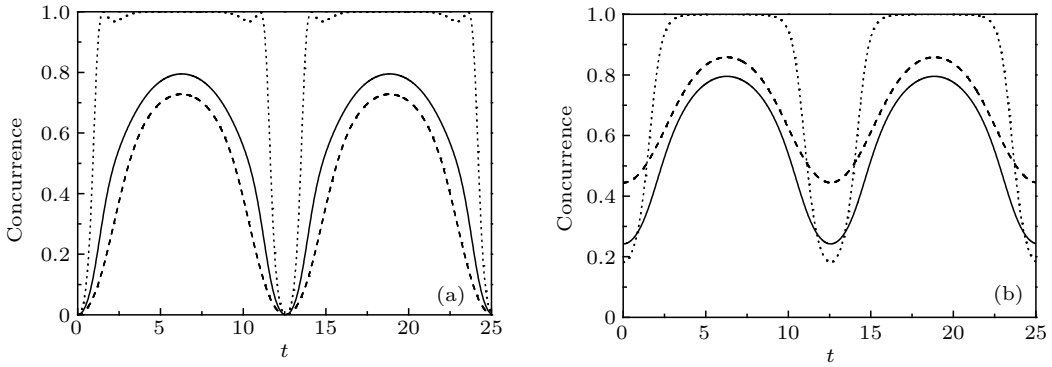


Fig. 2. Time evolution of the degree of entanglement with $g = 0.5$. The other parameters are $\alpha = 0.5$ and $\beta = 2.0$ (solid line), $\alpha = 1.0$ and $\beta = 0.5$ (dashed line), $\alpha = 1.5$ and $\beta = 1.5$ (dotted line). (a) Implement measurement on the ground state of the superconducting qubit. (b) Implement measurement on the excited state of the superconducting qubit.

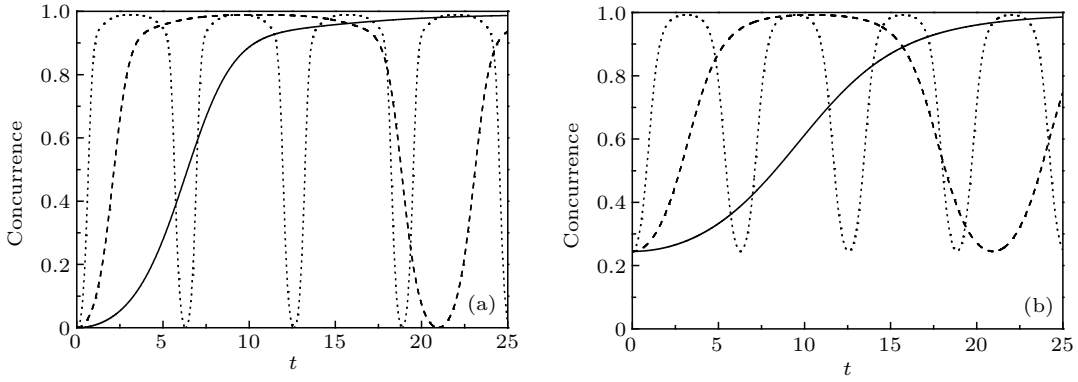


Fig. 3. Time evolution of the degree of entanglement with $\alpha = 1.5$ and $\beta = 1.0$. The other parameters are $g = 0.1$ (solid line), $g = 0.3$ (dashed line), and $g = 1$ (dotted line). (a) Implement measurement on the ground state of the superconducting qubit. (b) Implement measurement on the excited state of the superconducting qubit.

3. Effect of TLR decay on the concurrence of entangled coherent states

In this section, we discuss the effect of the TLR decay on the generation of the entangled coherent states. In order to obtain an analytic solution of the density matrix, here we do not consider the qubit level leakage effects. The evolution of the decay system is described by the master equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\mathrm{i}\left[H_{\mathrm{eff}},\rho(t)\right] + \frac{\gamma_{\mathrm{l}}}{2}\left(2a_{\mathrm{l}}\rho a_{\mathrm{l}}^{+} - a_{\mathrm{l}}^{+}a_{\mathrm{l}}\rho - \rho a_{\mathrm{l}}^{+}a_{\mathrm{l}}\right) + \frac{\gamma_{\mathrm{r}}}{2}\left(2a_{\mathrm{r}}\rho a_{\mathrm{r}}^{+} - a_{\mathrm{r}}^{+}a_{\mathrm{r}}\rho - \rho a_{\mathrm{r}}^{+}a_{\mathrm{r}}\right), \tag{15}$$

where γ_{l} and γ_{r} are the decay rates of the left and the right circuit QEDs, respectively. For simplicity, we assume that the two modes have the same cavity decay rate γ in the following. The concurrence is

$$C_{\pm}^{\gamma}(t) = \frac{2|\eta|}{M_{\gamma}}\sqrt{\left(1-\mathrm{e}^{-4\beta^2\sin^2(gt/2)-2\gamma t}\right)\left(1-\mathrm{e}^{-4\alpha^2\sin^2(gt/2)-2\gamma t}\right)}, \tag{16}$$

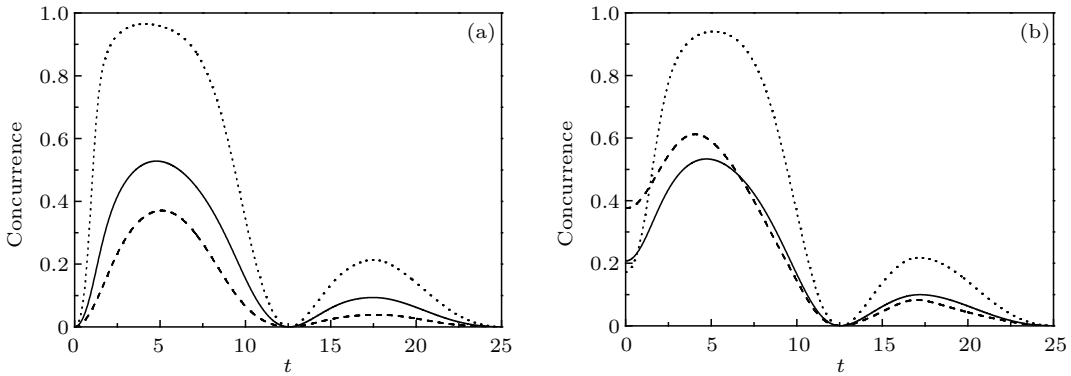


Fig. 4. Time evolution of the degree of entanglement with $g = 0.5$ and $\gamma = 0.2$. The other parameters are $\alpha = 0.5$ and $\beta = 2.0$ (solid line), $\alpha = 1.0$ and $\beta = 0.5$ (dashed line), $\alpha = 1.5$ and $\beta = 1.5$ (dotted line). (a) Implement measurement on the ground state of the superconducting qubit. (b) Implement measurement on the excited state of the superconducting qubit.

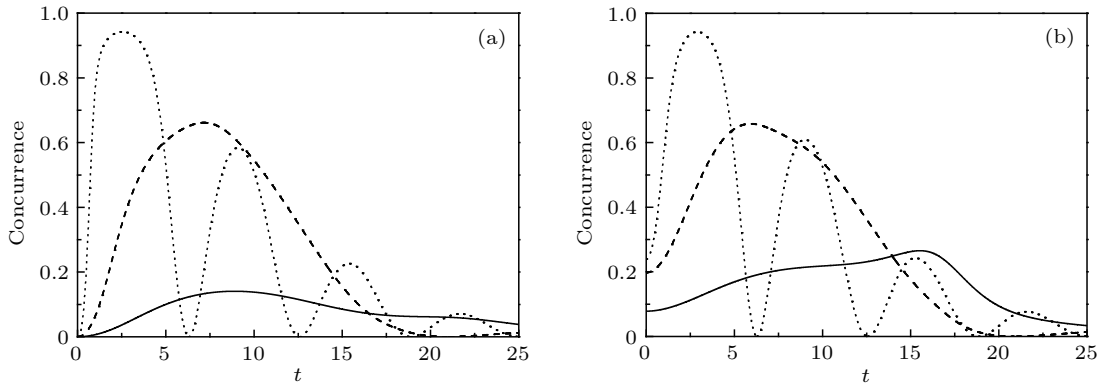


Fig. 5. Time evolution of the degree of entanglement with $\alpha = 1.5$, $\beta = 1.0$, and $\gamma = 0.2$. The other parameters are $g = 0.1$ (solid line), $g = 0.3$ (dashed line), and $g = 1$ (dotted line). (a) Implement measurement on the ground state of the superconducting qubit. (b) Implement measurement on the excited state of the superconducting qubit.

where

$$
\begin{aligned}
M_\gamma &= 2 \pm \mathrm{e}^{-\left(\alpha^2+\beta^2\right)\sin^2(gt/2)-\gamma t} \\
&\quad \times \left[\eta\, \mathrm{e}^{2\mathrm{i}\alpha\beta\sin(gt)} + \eta^*\, \mathrm{e}^{-2\mathrm{i}\alpha\beta\sin(gt)}\right], \\
\eta &= \exp[-4f_1(t)f_2(t) + \left(\alpha^2+\beta^2\right)\left(\mathrm{e}^{-\gamma t}-1\right) \\
&\quad + 2\mathrm{e}^{-\gamma t} f_3(t) f_4(t)], \\
f_1(t) &= -\mathrm{i}\frac{\gamma g\cos(gt) - \gamma^2\sin(gt) - \gamma g\,\mathrm{e}^{-\gamma t}}{2(\gamma^2+g^2)}, \\
f_2(t) &= \alpha\beta\cos(gt) + \mathrm{i}\frac{\alpha^2+\beta^2}{2}\sin(gt), \\
f_3(t) &= \frac{\gamma^2\cos(gt) + \gamma g\sin(gt) - \gamma^2\,\mathrm{e}^{-\gamma t}}{2(\gamma^2+g^2)}.
\end{aligned}
$$

We assume that the two TLRs in the present studied model are independent; figures 4 and 5 display the entanglement of two TLRs measured by the concurrence for $\gamma = 0.2$. One can see that the concurrence of the entangled coherent state $|\psi_+(t)\rangle$, compared with that of the entangled coherent state $|\psi_-(t)\rangle$, is more insensitive to γ. A further numerical analysis indicates that the concurrence of the two TLRs can be maintained for a longer time because of the reduction of the dissipative effects, which is consistent with the expected results.

4. Conclusion

We have proposed a scheme to generate two-mode entangled coherent states by using a current-biased Λ-type rf-SQUID coupling with two TLRs and two classic fields in large detuning. When we perform the measurement on the superconducting qubit state, the two-mode field will collapse into the entangled coherent state if the two TLRs are both initially in the coherent states. From our results, we find that in the Λ-type rf-SQUID qubit with an external circuit, by adopting the three-level structure, one may not only raise the gate speed but also prepare the entangled state of two oscillators through controllably coupling two transmission line oscillators and utilizing the big detuning of the external field. In our scheme, the coupling between the two TLRs and the qubit is realized by externally adjusting the bias current and the coupling capacitor C_j, thus it is easier to implement. In addition, we find that pertinently measuring the excited state of the qubit helps to obtain the maximum entangled state.

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