# Physical analysis on improving the recovery accuracy of the Earth's gravity field by a combination of satellite observations in along-track and cross-track directions*

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The physical investigations on the accuracy improvement to the measurement of the Earth's gravity field recovery are carried out based on the next-generation Pendulum-A/B out-of-plane twin-satellite formation in this paper. Firstly, the Earth's gravity field complete up to degree and order 100 is, respectively, recovered by the collinear and pendulum satellite formations using the orbital parameters of the satellite and the matching accuracies of key payloads from the twin GRACE satellites. The research results show that the accuracy of the Earth's gravity field model from the Pendulum-A/B satellite formation is about two times higher than from the collinear satellite formation, and the further improvement of the determination accuracy of the Earth's gravity field model is feasible by the next-generation Pendulum-A/B out-of-plane twin-satellite formation. Secondly, the Earth's gravity field from Pendulum-A/B complete up to degree and order 100 is accurately recovered based on the orbital parameters of the satellite (e.g., an orbital altitude of 400 km, an intersatellite range of 100 km, an orbital inclination of 89° and an orbital eccentricity of 0.001), the matching accuracies of space-borne instruments (e.g. $10^{-6}$ m in the intersatellite range, $10^{-3}$ m in the orbital position, $10^{-6}$ m/s in orbital velocity, and $10^{-11}$ m/s$^2$ in non-conservative force), an observation time of 30 days and a sampling interval of 10 s. The measurement accuracy of the Earth's gravity field from the next-generation Pendulum-A/B out-of-plane twin-satellite formation is full of promise for being improved by about 10 times compared with that from the current GRACE satellite formation. Finally, the physical requirements for the next-generation Pendulum-A/B out-of-plane twin-satellite formation are analyzed, and it is proposed that the satellite orbital altitude be preferably designed to be close to 400±50 km and the matching precision of key sensors from the Pendulum-A/B mission be about one order of magnitude higher than from the GRACE program.

**Keywords:** twin pendulum-A/B satellites, collinear and pendulum satellite formations, along-cross-track observations, physical analysis, satellite gravity recovery

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## 1. Introduction

Since Galileo, who was a well-known Italian physicist, began to explore gravity at the end of the 16th century, many scholars have carried out extensive and intensive researches on gravity.[1–15] Although the GRACE satellite gravity mission has made numerous important contributions and has provided a better understanding of the global knowledge in the fields of physical geodesy, geophysics, space science, etc. relative to the traditional gravity measurement techniques including car-borne, ship-borne, and air-borne measurements, etc., the inherent limitations of the twin GRACE satellites can no longer meet the urgent requirements of the relevant scientific areas for further improvement in the accuracy of the determination of the Earth's gravity field. For the above-mentioned physical reasons, many international research institutes are now trying to demonstrate the next-generation satellite gravity missions with higher spatial and temporal resolution, such as 1) the collinear satellite formation, involving the American GRACE Follow-On satellite gravity mission,[16–18] the European Next-Generation Gravimetry Mission (NGGM),[19–21] and so on; 2) the cartwheel satellite formation, concerning the American Four-Satellite Cartwheel Formation (FSCF) satellite gravity mission,[22,23] etc.; 3) the multi-satellite constellation, comprising the two pairs of GRACE-like satellites with different orbital inclinations,[24–26] and otherwise; 4) the pendulum satellite formation.[27–29]

As illustrated in Fig. 1, $O_I$–$X_IY_IZ_I$ represents the Earth-centered inertial (ECI) frame, the origin $O_I$ lies in the center of mass of the Earth, the positive direction of $X_I$ axis points to the mean equinox, the $Z_I$ axis points to the direction of the

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Earth North Pole, and the $Y_I$ axis forms a right-handed triad with the $X_I$ axis and the $Z_I$ axis. *o-xyz* denotes the satellite orbit coordinate (SOC) system, the origin *o* is located at the centroid of the satellite, the *x* axis points to the direction of the satellite's orbital motion (along-track), the *y* axis is normal to the orbital plane (cross-track), and the *z* axis is oriented along the radial direction and points away from the geocenter (radial-track). As shown in Fig. 1 and Table 1, the next-generation Pendulum-A/B out-of-plane twin-satellite formation intends to adopt a configuration design almost-circular, near-polar and of low-Earth orbit for a better coverage of the Earth. It aims to continuously run after the low-flying twin Pendulum-A/B satellites with a determination precision of better than $10^{-2}$ m by the high-flying Global Navigation Satellite System (GNSS) constellation (e.g. the American Global Positioning System (GPS), the Russian GLObal NAvigation Satellite System (GLONASS), etc.), high-accurately measure the intersatellite range with a precision of $10^{-6}$ m $\sim 10^{-8}$ m by the Interferometric Laser Ranging (ILR) system, and adequately minimize the physical non-conservative force with a precision of $10^{-11}$ m/s$^2$ $\sim 10^{-13}$ m/s$^2$ by the Drag–Free Control System (DFCS). The next-generation Pendulum-A/B out-of-plane twin-satellite formation is derived from a combination of the collinear formation (along-track component) and the out-of-plane motion (cross-track component). In comparison to the current collinear GRACE formation, the advantages of the next-generation Pendulum-A/B out-of-plane twin-satellite formation are as follows. Firstly, based on the Pendulum-type trajectory design, the maximum intersatellite range can be obtained at the equator and the minimum intersatellite range will be acquired at the poles. Because the along-track and cross-track signals of satellite observations are sufficiently observed, the spatial resolution of mapping the Earth's gravity field is significantly improved. Secondly, the negative influences of satellite observation errors on the accuracy of the Earth's gravity field recovery are highly reduced due to the improvement in the measurement precision of the intersatellite range by the interferometric laser ranging system. The adverse influence of the physical non-conservative force on satellite lifetime is reduced by the drag-free control system, which is propitious to lowering the satellite orbital altitude in a range of 300 km$\sim$400 km and reducing the signal attenuation of the Earth's gravity field. Finally, the longitudinal striping errors can be minimized by a higher resolution of the Earth's time-varying gravity field model in the spatial domain.

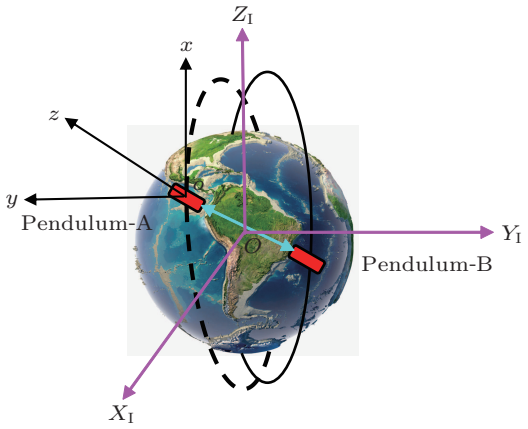



**Fig. 1.** (color online) Measurement principle of the twin Pendulum-A/B satellites.

**Table 1.** A comparison of satellite formation between current GRACE and next-generation Pendulum-A/B.

| Parameters | Satellite formations | |
|---|---|---|
| | GRACE | Pendulum-A/B |
| Orbital altitude | 500 km | 400±50 km |
| Intersatellite range | 220 km | 100±50 km |
| Orbital inclination | 89° | 90±3° |
| Orbital eccentricity | 0.004 | 0.001 |
| Formation flying | Collinear | Pendulum |
| Observation data | Along-track | Along-Cross-track |
| Space-borne instrument | K-band ranging system | Interferometric laser ranging system |
| | accelerometer | drag-free control system |

The benefits of the next-generation Pendulum-A/B out-of-plane twin-satellite formation principally consist in its lower technical demands than the four-satellite cartwheel formation, the less research expenditure compared to the two pairs of GRACE-like satellites with different orbital inclinations, etc. In addition, unlike the collinear twin-satellite formation with the along-track satellite observations, the next-generation Pendulum-A/B out-of-plane twin-satellite formation is propitious to synchronously obtaining gravity information in the along-track and cross-track directions. In Refs. [27]–[29], researches on the pendulum formation flight for future gravity field recovery based on the dynamical method or the short-arc method have been carried out. The model errors of the dynamical method are rapidly cumulated with the increase of the orbital arc length. The short-arc method reduces the enhancement of the accuracy of the Earth's long-wavelength gravity field, owing to adoption of a very short orbital arc length of about 30 minutes. The classical

orbital perturbation method is efficient only near the reference mean values or initial epoch. In the energy conservation method, the influences of satellite observation errors including orbital position and velocity, intersatellite range-rate, and non-conservative forces on the accuracy of the Earth's gravitational field are mixed together. The orbital boundary value method and the semi-analytic method will reduce the recovery accuracy of the Earth's gravitational field, owing to the approximate establishment of satellite observation equation to some extent. The disadvantage of the satellite acceleration method is that the accuracy of the Earth's low-frequency gravity field is reduced, owing to making use of the numerical differential principle. Unlike previous studies, in order to explore the influences of different methods for satellite gravity recovery on the accuracy of the Earth's gravity field, we carry out a series of physical studies to improve the recovery accuracy of the Earth's gravity field around the next-generation Pendulum-A/B out-of-plane twin-satellite formation based on the nine-point intersatellite range interpolation method.

## 2. Method for satellite gravity recovery

The observation equation of the nine-point intersatellite range interpolation is represented as[30]

$$\begin{aligned}
&\boldsymbol{e}_{12}(t_j)\cdot\nabla\boldsymbol{T}_{12}(t_j)\\
&=\boldsymbol{e}_{12}(t_j)\cdot\left\{\frac{1}{(\Delta t)^2}\cdot\left[-\frac{1}{560}\boldsymbol{r}_{\rho 12}(t_{j-4})+\frac{8}{315}\boldsymbol{r}_{\rho 12}(t_{j-3})\right.\right.\\
&\quad-\frac{1}{5}\boldsymbol{r}_{\rho 12}(t_{j-2})+\frac{8}{5}\boldsymbol{r}_{\rho 12}(t_{j-1})\\
&\quad-\frac{205}{72}\boldsymbol{r}_{\rho 12}(t_j)+\frac{8}{5}\boldsymbol{r}_{\rho 12}(t_{j+1})-\frac{1}{5}\boldsymbol{r}_{\rho 12}(t_{j+2})\\
&\quad\left.+\frac{8}{315}\boldsymbol{r}_{\rho 12}(t_{j+3})-\frac{1}{560}\boldsymbol{r}_{\rho 12}(t_{j+4})\right]\\
&\quad+GM\left[\frac{\boldsymbol{r}_2(t_j)}{|\boldsymbol{r}_2(t_j)|^3}-\frac{\boldsymbol{r}_1(t_j)}{|\boldsymbol{r}_1(t_j)|^3}\right]-a_{12}^{\mathrm{C}}(t_j)-f_{12}^{\mathrm{N}}(t_j)\Big\}, \quad (1)
\end{aligned}$$

where $GM$ is the product of the Earth's mass $M$ and gravitational constant $G$;

$$|\boldsymbol{r}_{1(2)}|=\sqrt{x_{1(2)}^2+y_{1(2)}^2+z_{1(2)}^2}$$

are the geocentric radii of the twin satellites, with $x_{1(2)}$, $y_{1(2)}$, and $z_{1(2)}$ denoting three scalar components of orbital position vectors $\boldsymbol{r}_{1(2)}$; $\boldsymbol{e}_{12}=\boldsymbol{r}_{12}/|\boldsymbol{r}_{12}|$ represents the unit vector with the direction from the first to the second satellite, with $r_{12}=r_2-r_1$ being the difference in orbital position vector between the twin satellites; $\boldsymbol{r}_{\rho 12}=\rho_{12}\boldsymbol{e}_{12}+\{\boldsymbol{r}_{12}-[\boldsymbol{r}_{12}\cdot\boldsymbol{e}_{12}]\boldsymbol{e}_{12}\}$, with $\rho_{12}$ being the intersatellite range; $\boldsymbol{a}_{12}^{\mathrm{C}}$ is the physical conservative force difference except for the gravity force of the Earth, involving the solar and lunar gravitation (DE405 model), Earth's solid tide (IERS 2010), oceanic tide (EOT11a model), atmospheric tide (NCEP model) and polar tide (IERS 2010);[31–33] $\boldsymbol{f}_{12}^{\mathrm{N}}$ denotes the difference between physical non-conservative forces acting on the twin Pendulum-A/B satellites, comprising the atmospheric drag, solar radiation pressure, Earth's albedo, and orbital altitude and attitude control forces;[34] $\nabla T_{12}$ expresses the gradient of the Earth's disturbing potential difference acting on the twin satellites

$$\begin{aligned}
T(\boldsymbol{r},\theta,\lambda)=\ &\frac{GM}{R_{\mathrm{e}}}\sum_{l=2}^{L}\left(\frac{R_{\mathrm{e}}}{\boldsymbol{r}}\right)^{l+1}\sum_{m=0}^{l}(\bar{C}_{lm}\cos m\lambda\\
&+\bar{S}_{lm}\sin m\lambda)\bar{\mathrm{P}}_{lm}(\cos\theta), \quad (2)
\end{aligned}$$

where $r$, $\theta$, and $\lambda$ denote geocentric radius, colatitude, and longitude; $R_{\mathrm{e}}$ is the mean radius of the Earth; $\bar{\mathrm{P}}_{lm}(\cos\theta)$ denotes the normalized Legendre polynomials of degree $l$ and order $m$; $L$ is the maximum degree of the spherical harmonic function; $\bar{C}_{lm}$ and $\bar{S}_{lm}$ display the estimated normalized geopotential coefficients.

The Earth's geopotential coefficients degree root-mean-square (RMS) errors are defined as

$$\sigma_l=\sqrt{\frac{\sum_{m=-l}^{l}(\bar{A}_{lm}-\bar{A}_{lm}^{\mathrm{o}})^2}{2l+1}}, \quad (3)$$

where $A_{lm}=(\bar{C}_{lm},\bar{S}_{lm})$ represents the estimated geopotential coefficients of the Earth, and $\bar{A}_{lm}^{\mathrm{o}}$ denotes the reference geopotential coefficients of the Earth's gravity field model EGM2008, truncated at degree and order 100, in this study.

The cumulative geoid height errors are shown as

$$\sigma_N=R_{\mathrm{e}}\sqrt{\sum_{l=2}^{L}\sum_{m=-l}^{l}(\bar{A}_{lm}-\bar{A}_{lm}^{\mathrm{o}})^2}. \quad (4)$$

## 3. Feasibility verification of Pendulum-A/B

As illustrated in Fig. 2, the orbits of the twin Pendulum-A/B satellites are derived from the Runge–Kutta linear single-step in connection with the 12th-order Adams–Cowell linear multi-step integral formulas. The Keplerian orbital parameters applied to numerical simulation are indicated in Table 2. The physical measurement error spectral densities of satellite observations of intersatellite range, orbital position, and non-conservative force are shown in Figs. 3(a)–3(c), respectively.[19]

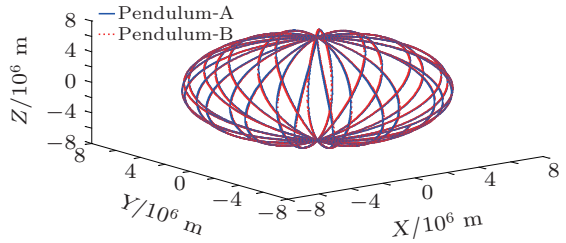



**Fig. 2.** (color online) Three-dimensional orbits of the twin Pendulum-A/B satellites in one day.

**Table 2.** Keplerian orbital elements of the twin Pendulum-A/B satellites.

| Orbital elements | Twin Pendulum-A/B satellites. | | |
|---|---|---|---|
| | Pendulum-A | Pendulum-B | Difference |
| Semimajor axis | 6778 km | 6778 km | $\Delta a = 0$ |
| Orbital inclination | 89° | 89° | $\Delta i = 0$ |
| Orbital eccentricity | 0.001 | 0.001 | $\Delta e = 0$ |
| Right ascension of ascending node | 18.5° | 19.25° | $\Delta\Omega = 0.75°$ |
| Argument of perigee | 90° | 90° | $\Delta w = 0$ |
| Mean anomaly | 0° | 0.4° | $\Delta M = 0.4°$ |

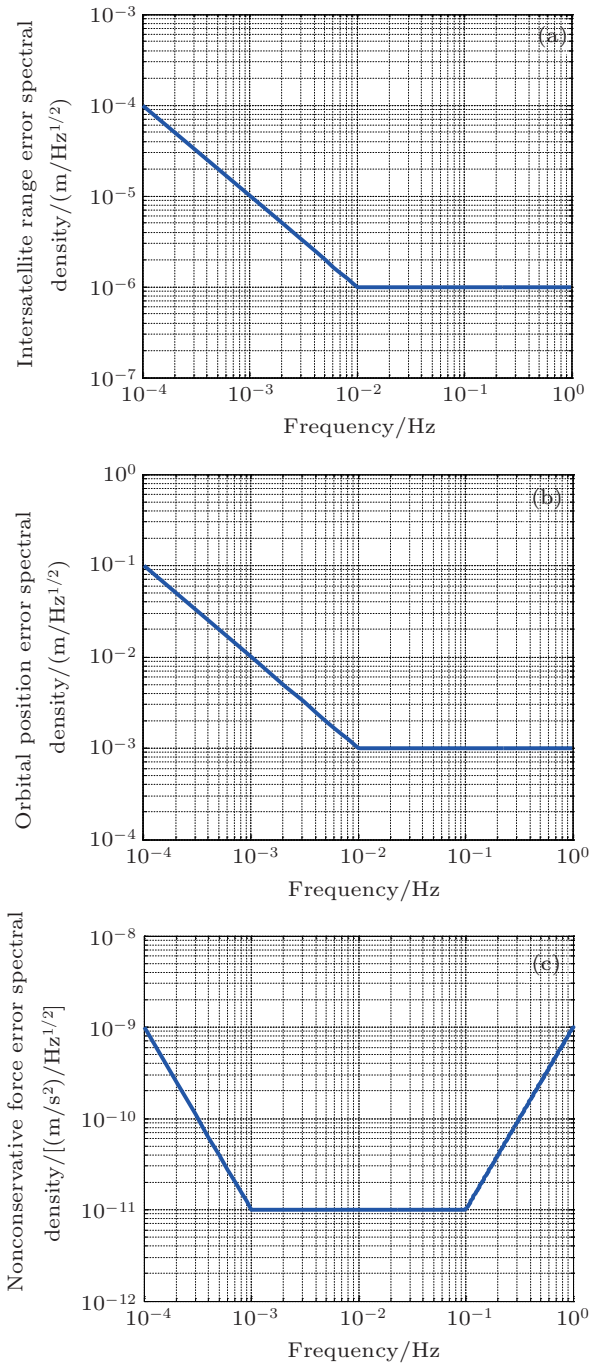



**Fig. 3.** (color online) Measurement error spectral densities from key payloads for three cases: (a) intersatellite range, (b) orbital position, (c) nonconservative force.

Furthermore, we consider the time-variable background force models, consisting of the ocean tide models (FES2004: Finite Element Solution;[35] EOT11a: Empirical Ocean Tide[36]), the non-tidal oceanic models (OMCT: Ocean Model for Circulation and Tides;[37] ECCO: Estimation of the Circulation and Climate of the Ocean[38]), the non-tidal atmospheric models (ECMWF: European Center for Medium-range Weather Forecasts;[39] NCEP: National Centers for Environmental Prediction[40]) and the global continental water storage models (CPC: Climate Prediction Center;[41] GLDAS: Global Land Data Assimilation System[42]), in the orbit integral process and the gravity recovery process, respectively.

As depicted in Fig. 4, the cross line represents the real accuracy of the Earth's gravity field model EIGEN-GRACE02S complete up to degree and order 100 released by the German Research Center for Geosciences (GFZ), and the cumulative geoid height error is $5.756\times10^{-2}$ m at degree 100. The dashed and solid lines denote the simulation accuracies of the Earth's gravity field from the collinear and pendulum formations complete up to degree and order 100 using GRACE orbital parameters (e.g. orbital altitude of 500 km, intersatellite range of 220 km, orbital inclination of 89° and orbital eccentricity of 0.001), and the measurement accuracies of GRACE key payloads (e.g. intersatellite range of $10^{-5}$ m, orbital position of $10^{-2}$ m, orbital velocity of $10^{-5}$ m/s, and non-conservative force of $10^{-10}$ m/s$^2$) based on an observation time of 30 days and a sampling interval of 10 s by the nine-point intersatellite range interpolation method, respectively. The cumulative geoid height errors from the collinear and pendulum formations are $6.205\times10^{-2}$ m and $3.281\times10^{-2}$ m at degree 100, and the statistical results are listed in Table 3. The research results are discussed below.

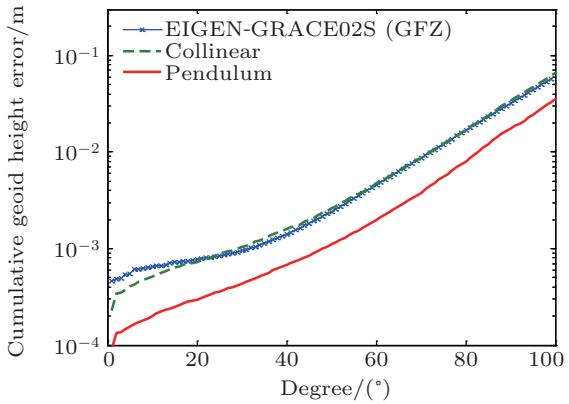



**Fig. 4.** (color online) A comparison of cumulative geoid height error among the Collinear formation, Pendulum formation, and EIGEN-GRACE02S model.

Firstly, in 100 degrees, since the simulated formal errors (dashed line) of the collinear formation derived from this study basically accord with the formal errors (cross line) of the EIGEN-GRACE02S model released by the GFZ, the software reliability of the Earth's gravity field recovery in this study can be verified.

**Table 3.** Statistical results of cumulative geoid height errors from the collinear and pendulum satellite formations.

| Satellite formations | Cumulative geoid height errors/m | | | |
|---|---|---|---|---|
| | Degree 20 | Degree 50 | Degree 80 | Degree 100 |
| EIGEN-GRACE02S | $7.606\times10^{-4}$ | $2.282\times10^{-3}$ | $1.566\times10^{-2}$ | $5.756\times10^{-2}$ |
| Collinear | $7.152\times10^{-4}$ | $2.471\times10^{-3}$ | $1.586\times10^{-2}$ | $6.205\times10^{-2}$ |
| Pendulum | $2.909\times10^{-4}$ | $1.046\times10^{-3}$ | $7.481\times10^{-3}$ | $3.281\times10^{-2}$ |

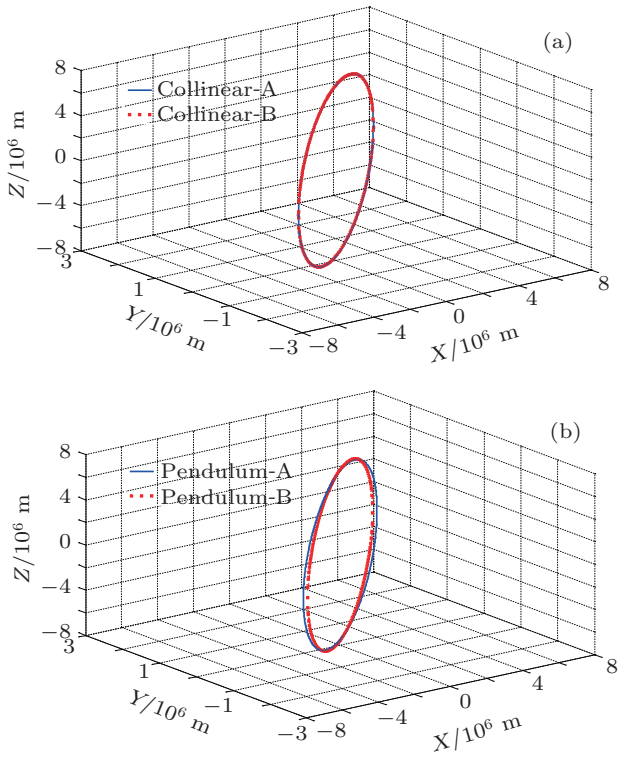



**Fig. 5.** (color online) One-day orbits of the twin Collinear-A/B satellites (a), and the twin Pendulum-A/B satellites (b).

Secondly, based on the same GRACE orbital parameters and the measurement precision of GRACE key payloads, the accuracy of the Earth's gravity field from the pendulum formation is approximately 2 times higher than from the collinear formation. The main physical reasons are analyzed as follows. 1) As shown in Figs. 5(a) and 6(a), the Collinear-A/B twin-satellite formation is not sensitive to the cross-track nor the radial gravity information at the same time, but can observe solely the along-track signals of the Earth's gravity field. Therefore, the longitudinal striping error effect of the Earth's time-dependent gravity field model will arise from the absence of the cross-radial-track gravity signals. 2) As denoted in Figs. 5(b) and 6(b), the signals of the Earth's gravity field in the along-track and cross-track directions can be simultaneously acquired by the next-generation pendulum-A/B out-of-plane twin-satellite formation: (i) the recovery accuracy of the Earth's gravity field from Pendulum is substantially improved because of adding cross-track gravity information; (ii) the longitudinal striping error effect of the Earth's time-variable gravity field will be reduced, owing to the isotropy and uniformity of pendulum observation errors. In a word, the next-generation Pendulum-A/B out-of-plane twin-satellite formation is a better choice for further improving the accuracy and resolution of the Earth's gravity field model.

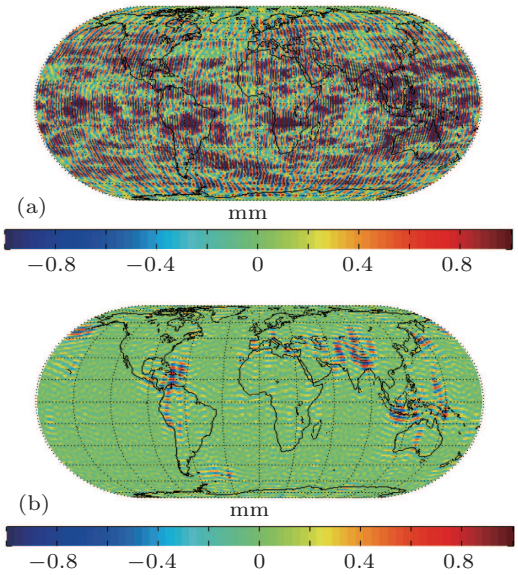



**Fig. 6.** (color online) A comparison of global geoid height errors complete up to degree and order 100 between the Collinear (a) and Pendulum (b) formations.

## 4. Analysis of the physical requirements for the Pendulum-A/B system

As displayed in Fig. 7, the dashed line depicts the simulation accuracies of the Earth's gravity field determination from the current GRACE formation complete up to degree and order 100 and the cumulative geoid height error is $6.205\times10^{-2}$ m at degree 100; the solid line represents the simulated accuracies of the Earth's gravity field recovery from the next-generation Pendulum-A/B out-of-plane formation complete up to degree and order 100 based on the intersatellite range interpolation method by Pendulum-A/B orbital parameters (e.g. orbital altitude of 400 km, intersatellite range of 100 km, orbital inclination of 89°, and orbital eccentricity of 0.001) and the measurement precision of Pendulum-A/B key payloads (e.g. $10^{-6}$ m in intersatellite range, $10^{-3}$ m in orbital position, $10^{-6}$ m/s in orbital velocity, and $10^{-11}$ m/s$^2$ in non-conservative force) using an observation duration of 30 days and a sampling interval of 10 s, and the cumulative geoid height error from

Pendulum-A/B is $6.585\times10^{-4}$ m at degree 100; and the statistical results are listed in Table 4. Figure 8 illustrates the spectral distributions of spherical harmonic coefficient errors complete up to degree and order 100 based on the current GRACE and the next-generation Pendulum twin-satellite formations. The results of the above research suggest that the accuracy of the Earth's gravity field measurement from the next-generation Pendulum-A/B out-of-plane twin-satellite formation is at least one order of magnitude higher than from the present GRACE twin-satellite formation. The analysis of physical requirements for the next-generation Pendulum-A/B out-of-plane twin-satellite formation is performed below.

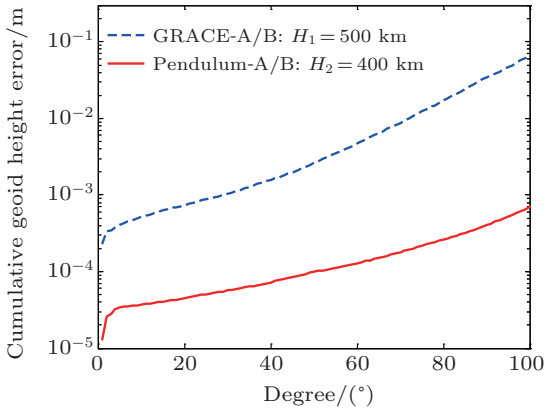



**Fig. 7.** (color online) A comparison of cumulative geoid height error between the current GRACE-A/B and the next-generation Pendulum-A/B missions.

**Table 4.** Statistical results of cumulative geoid height errors from GRACE and Pendulum satellite missions.

| Gravity satellites | Cumulative geoid height errors/m | | | |
|---|---|---|---|---|
| | Degree 20 | Degree 50 | Degree 80 | Degree 100 |
| GRACE-A/B | $7.152\times10^{-4}$ | $2.471\times10^{-3}$ | $1.586\times10^{-2}$ | $6.205\times10^{-2}$ |
| Pendulum-A/B | $4.395\times10^{-5}$ | $9.678\times10^{-5}$ | $2.528\times10^{-4}$ | $6.585\times10^{-4}$ |

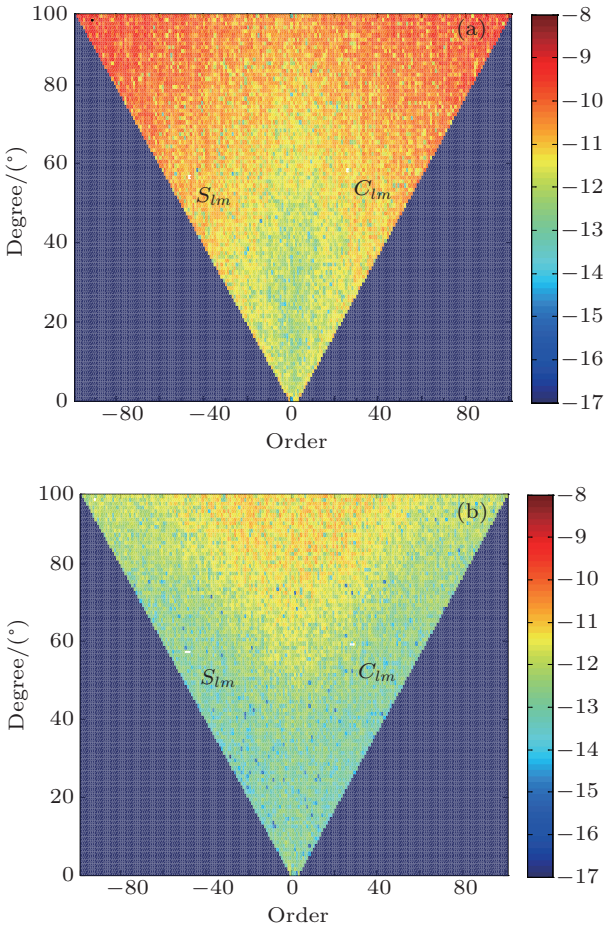



**Fig. 8.** (color online) Error spectra of geopotential coefficients complete up to degree and order 100 from the GRACE (a) and Pendulum (b) satellite formations.

### 4.1. Recommended design for satellite orbital altitude

Due to the exponential attenuation of signal intensity for the Earth's gravity field with the increase of the orbital altitude, properly reducing the satellite orbital altitude is propitious to minimizing the signal loss of the Earth's gravity field determination. However, the physical non-conservative force will be enhanced by about 10 times whenever the orbital altitude of the satellite is reduced by 100 km. Therefore, the instability of the working condition for the satellite platform will extremely degrade the measurement precision of space-borne sensors. In addition, with the decrease of the orbital altitude, due to the increase of the negative influence of the physical non-conservative force and in order to maintain the orbital altitude and three-dimensional attitude of the satellite, the operating frequency of key payloads, including the micro-thruster system, the Attitude and Orbital Control System (AOCS), etc., will be rapidly enhanced and the working lifetime will be shortened as a result of the great consumption of the jet propulsion fuel. Hereby, the tradeoff between the recovery accuracy of the Earth's gravity field and the mission lifespan of the satellite is of importance for preferably selecting the orbital altitude of the future gravity satellites.

A comparison between the dashed and solid lines in Fig. 7 shows that i) if the twin Pendulum-A/B satellites fly at an orbital altitude of 400 km, the accuracy of the Earth's gravity field recovery by the next-generation Pendulum-A/B out-of-plane twin-satellite formation is improved by about 52 times more than by the current GRACE-A/B twin-satellite formation at degree 100, which can meet the scientific objectives that the measurement accuracy of the future Earth's gravity field model is at least one order of magnitude higher than that of the existing gravity models; ii) the orbital altitude of 400 km prolongs the flying lifetime of the satellite, realizing the long-term observation, and improving the accuracy of the Earth's

time-variable gravity field; iii) the appropriate orbital altitude is conducible to minimizing the technical difficulties of making satellite-equipped instrumentations and reducing the control requirements for the in-orbit flight, i.e., orbital control, attitude control, and the like. Consequently, this study proposes to fly at an orbital altitude of 400±50 km for the next-generation Pendulum-A/B out-of-plane twin-satellite formation.

### 4.2. Suggested option for the matching precision of key payloads

For the GRACE satellite formation, the orbital position with a precision of $10^{-2}$ m and orbital velocity with a precision of $10^{-5}$ m/s are tracked based on the GPS satellites, the intersatellite range with a precision of $10^{-5}$ m is measured by the K-Band Ranging (KBR) system, and the non-conservative force with a resolution of $10^{-10}$ m/s$^2$ is determined using the accelerometer. The limitation of the measurement precision from GRACE key payloads not only has a major influence on the longitudinal striping errors which will distort the monthly models of the gravity field, but also results in a poor resolution of the Earth's gravity field model. In order to make up for the deficiencies of the present GRACE satellite formation, many research organizations are actively analyzing the requirements for the next-generation satellite gravity missions from the aspects of improving the measurement precision of key payloads, selecting optimal tracking mode of the satellite, and so on. The next-generation Pendulum-A/B out-of-plane twin-satellite formation plans to measure the intersatellite range with a precision of $10^{-6}$ m $\sim 10^{-8}$ m by the interferometric laser ranging system and compensate for the non-conservative force with a precision of $10^{-11}$ m/s$^2$ $\sim 10^{-13}$ m/s$^2$ by the drag-free control system. However, since it is significantly difficult to reach an orbit determination precision of $10^{-2}$ m $\sim$ $10^{-3}$ m with the GNSS system, this study suggests that the matching accuracies (e.g. intersatellite range of $10^{-6}$ m, orbital position of $10^{-3}$ m, orbital velocity of $10^{-6}$ m/s, and non-conservative force of $10^{-11}$ m/s$^2$) of key payloads from the next-generation Pendulum-A/B out-of-plane twin-satellite formation are about 10 times higher than from the current GRACE satellite formation.

## 5. Conclusions

This study chiefly concentrates on accurately measuring the Earth's gravity field from Pendulum-A/B and comprehensively carrying out the requirement demonstrations of the possible Pendulum-A/B twin-satellite formation for the preferred design of the satellite orbital altitude and the better selection of the matching accuracies of scientific sensors. The specific conclusions are listed below.

Firstly, we verify the possible Pendulum-A/B out-of-plane twin-satellite formation.

(I) The correctness of the applied programming system for gravity recovery is validated by the agreement between the simulation accuracy of the GRACE mission and the real accuracy of the EIGEN-GRACE02S gravity model.

(II) The accuracy improvement of the Earth's gravity field determination based on the possible Pendulum-A/B out-of-plane twin-satellite formation is investigated, showing that by using the pendulum tracking mode, the recovery accuracy of the Earth's gravity field is improved on average by about 2 times compared with using the collinear tracking mode.

Secondly, we demonstrate the physical requirements for the possible pendulum-A/B out-of-plane twin-satellite formation.

a) The Earth's gravity field from Pendulum-A/B complete up to degree and order 100 is precisely recovered by the intersatellite range interpolation method by using orbital parameters (e.g. an orbital altitude of 400 km, an intersatellite range of 100 km, an orbital inclination of 89°, and an orbital eccentricity of 0.001) and the measurement precision of space-borne sensors (e.g. an intersatellite range of $10^{-6}$ m, an orbital position of $10^{-3}$ m, an orbital velocity of $10^{-6}$ m/s, and a non-conservative force of $10^{-11}$ m/s$^2$) based on a 30-day observation time and a 10-second sampling interval. The research results manifest that the determination accuracy of the Earth's gravity field by the possible Pendulum-A/B out-of-plane twin-satellite formation is enhanced at least 10 times more than by the present GRACE twin-satellite formation.

b) It is preferable to design an orbital altitude of 400±50 km and to choose the matching accuracies (e.g., an intersatellite range of $10^{-6}$ m, an orbital position of $10^{-3}$ m, an orbital velocity of $10^{-6}$ m/s, and a non-conservative force of $10^{-11}$ m/s$^2$) of key instrumentations in the possible Pendulum-A/B out-of-plane twin-satellite formation.

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