Description and reconstruction of one-dimensional photonic crystal by digital signal processing theory*

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A method of describing one-dimensional photonic crystals (1DPCs) based on *Z*-domain digital signal processing theory is presented. The analytical expression of the target band gap spectrum in the digital domain is obtained by the autocorrelation of its impulse response. The feasibility of this method is verified by reconstructing two simple 1DPC structures with a target photonic band gap obtained by the traditional transfer matrix method. This method provides an effective approach to function-guided designs of interference-based band gap structures for photonic applications.

Keywords: one-dimensional photonic crystal, digital signal processing, *Z*-transform

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1. Introduction

Photonic crystal (PC)[1] is a periodic nanostructure composed of regularly repeating regions of high and low refractive indices that affect the propagation of light waves in the same way that ionic lattices affect electrons in solids. One-dimensional PC (1DPC), often in the form of periodic multilayer dielectric stacks, is simpler in design and fabrication and has already been extensively used in optics and photonics.[2–7] To design a PC system, the location and width of the photonic band gap is essential to engineer. This engineering feat is often achieved by computational method such as plane wave expansion method, finite difference time domain method, transfer matrix method, etc. The band structure and output spectrum of a PC can be obtained if its structural parameters (such as refractive index and thickness of each layer in a 1DPC) are given. However, in contrast to this forward process, the optimized PC structure is challenging to optimize from a target spectrum or band structure, which is more important for the design of a functional PC system. Hence, effective function-guided PC design methods are urgently needed.

Digital signal processing (DSP) is the mathematical manipulation of a digitized discrete signal to modify or improve it in a specific way (such as filtering). In recent years, the rapid development of computer and microelectronics technology has enabled DSP technology to enter into a new stage and be extensively used in communications, audio/image processing, system control, etc.[8] Based on the similar impulse response nature of optical interference, the DSP algorithm becomes a powerful tool for designing and analyzing optical filters.[9–12] The intrinsic characteristics of this computer-aided method make it more suitable for complex structures and function-guided designs.

In fact, some numerical techniques besides DSP method have been suggested and used to design thick optical thin-film filters,[13] finite length fiber gratings,[14] non-uniform fiber Bragg gratings,[15] and other similar structures[16] previously. These techniques are based on the differential layer-peeling/stripping or inverse scattering algorithm.[17,18] For this design method, although no knowledge of a reasonable starting guess at the final structure is necessary, both the phase and amplitude reflectance data are needed.

In this paper, an effective method of designing a 1DPC filter based on the *Z*-domain DSP technique is presented. The target reflectance/transmittance spectrum with an optical stop-band is described analytically by an accurate rational function and calculated out through its autocorrelation in the digital domain. And then the structural parameters of the 1DPC can be obtained by the polynomial recursion method (a layer-peeling algorithm). It should be pointed out that no phase information of the target spectrum is needed in our calculation, thus it is simpler than the previous methods. The optical parameters of the 1DPC are mathematically described from the DSP point of view and thereby become compatible with common MATLAB programs[19] and digital signal processors.[20]

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2. Theory and numerical methods

The optical multilayer dielectric film can be considered as a special case of 1DPC. The traditional multilayer dielectric mirrors show typical photonic band gap characteristics. An optical multilayer filter can be mathematically synthesized by Fourier transform or Z-transform,[21] which is an important basis of DSP theory. As such, 1DPCs can be described by DSP theory through the multilayer filter. The transmission of light through a certain layer of a multilayer structure and its corresponding Z-domain block diagram are given in Fig. 1;[21] 0, 1, and 2 refer to the incident medium, the layer, and the exit medium respectively. r_{01}, t_{01} and r_{12}, t_{12} are the reflection and transmission coefficients of light on the 0-to-1 and 1-to-2 surfaces respectively. r_{10}, t_{10} are the reflection and transmission coefficients of light on the 1-to-0 surface. The reflection of light incident on the interface from the left and right sides is opposite to each other, i.e., $r_{01} = -r_{10}$. E_i, E_r, and E_t are electric fields of incident, reflected, and transmitted light respectively. z^{-1} represents the time delay or path difference of the light after passing through the layer. $z^{-1} = \mathrm{e}^{-\mathrm{j}2\pi nfd/c} = \mathrm{e}^{-\mathrm{j}\omega nd/c}$, where c is the speed of light in vacuum, n is the refractive index of the layer, f is the frequency of light, d is the thickness of the layer, and + represents the superposition operation.

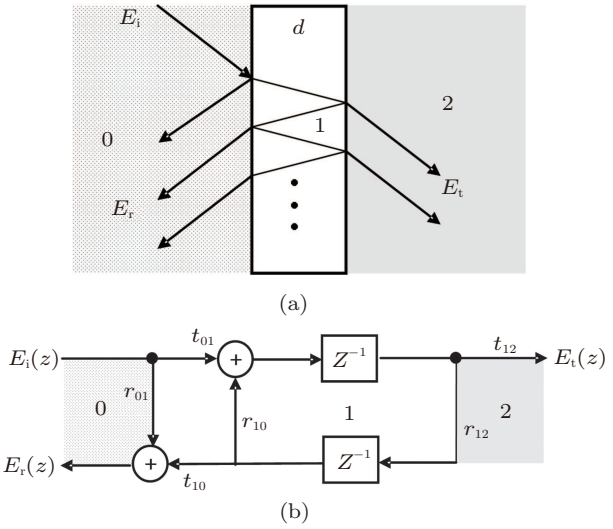


Fig. 1. (a) Transmission of light through a single layer and (b) its Z-domain block diagram.

From the correspondence between the optical and DSP model shown in Fig. 1, the transfer functions of transmitted and reflected light can be obtained as

$$H_\mathrm{t}(z) = \frac{E_\mathrm{t}(z)}{E_\mathrm{i}(z)} = \frac{t_{01}t_{12}z^{-1}}{1 - r_{10}r_{12}z^{-2}}, \tag{1}$$

$$H_\mathrm{r}(z) = \frac{E_\mathrm{r}(z)}{E_\mathrm{i}(z)} = r_{01} + \frac{t_{01}t_{10}r_{12}z^{-2}}{1 - r_{10}r_{12}z^{-2}}. \tag{2}$$

Both of them are in the form of typical rational function, and the numerator and denominator polynomials have the same order of z^{-1} for the reflection transfer function. The transfer functions of the multilayer structure can be calculated by cascading the Z-domain block diagrams and with similar rational function forms.

In the DSP theory, for a rational function in the following form (the numerator and denominator polynomials have the same order of z^{-1}):

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1z^{-1} + b_2z^{-2}\cdots + b_pz^{-p}}{a_0 + a_1z^{-1} + a_2z^{-2}\cdots + a_pz^{-p}}, \ (a_0 = 1) \tag{3}$$

the autocorrelation of its impulse response should satisfy the following relationship:[5]

$$r_{hh}(m) = \begin{cases} -\sum\limits_{k=1}^{p} a_k r_{hh}(m-k), & m > p, \\ -\sum\limits_{k=1}^{p} a_k r_{hh}(m-k) + \sum\limits_{k=0}^{p-m} h(k)b_{k+m}, & 0 \le m \le p, \\ r^*_{hh}(m), & m < 0, \end{cases} \tag{4}$$

where $h(k)$ is the impulse response of $H(z)$.

For the desired filtering spectrum, the autocorrelation of its impulse response can be easily obtained, so the coefficient a_k $(k = 0, 1, \ldots, p)$ can be directly determined from the case of $m > p$ in Eq. (4). Thus, the denominator expression of the desired target reflection transfer function $A(z)$ can be achieved. Then, the frequency response of $B(z)$ can be obtained from the product of the frequency response of the target reflection spectrum and $A(z)$. For the numerator expression of the desired reflection transfer function, its coefficient b_k $(k = 0, 1, \ldots, p)$ cannot be directly obtained from its autocorrelation. The coefficient b_k $(k = 0, 1, \ldots, p)$ can be obtained by constructing a rational function $C(z)$, which has the same frequency response as $B(z)$ and has the following form:

$$C(z) = 1/(c_0 + c_1z^{-1} + c_2z^{-2} + \cdots + c_qz^{-q}), \tag{5}$$

where $q \gg p$. The form of Eq. (5) is the same as that of Eq. (3), in which $b_0 = 1$ and $b_k = 0$, so the coefficient c_k $(k = 0, 1, \ldots, q)$ can be calculated from Eq. (4) in a way similar to that of calculating a_k $(k = 0, 1, \ldots, p)$. Given that $B(z)$ and $C(z)$ have the same frequency response in the Z-domain, then

$$B(z) = 1/\sum_{i=0}^{q} c_\mathrm{i}z^{-i}. \tag{6}$$

Equation (6) shows that the coefficients of $B(z)$ and $C(z)$ are related by a convolution sum. Accordingly, we can construct the square error as

$$\xi = \sum_{k=0}^{p+q}\left[b_0c_k + \sum_{n=1}^{k} b_kc_{k-n}\right]^2 - 1. \tag{7}$$

The coefficients of $B(z)$ can be obtained when ξ has the minimum value. Hence, b_k $(k = 0, 1, \ldots, p)$ can be determined by

the following relationship:

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{pmatrix} = - \begin{pmatrix} r_{cc}(0) & r_{cc}(-1) & \cdots & r_{cc}(-p+1) \\ r_{cc}(1) & r_{cc}(0) & \cdots & r_{cc}(-p+2) \\ \vdots & \vdots & \vdots & \vdots \\ r_{cc}(p-1) & r_{cc}(p-2) & \cdots & r_{cc}(0) \end{pmatrix}^{-1} \times \begin{pmatrix} r_{cc}(1) \\ r_{cc}(2) \\ \vdots \\ r_{cc}(p) \end{pmatrix}, \tag{8}$$

where $r_{cc}(\tau)$ ($\tau = -p+1, -p+2, \ldots, p$) is the autocorrelation of coefficients c_k ($k = 0, 1, \ldots, q$), i.e.,

$$r_{cc}(\tau) = \sum_{k=0}^{q} c_k c_{k+\tau}^{*}, \tag{9}$$

where the superscript $*$ indicates conjugation. Notably, all these coefficients should be normalized.

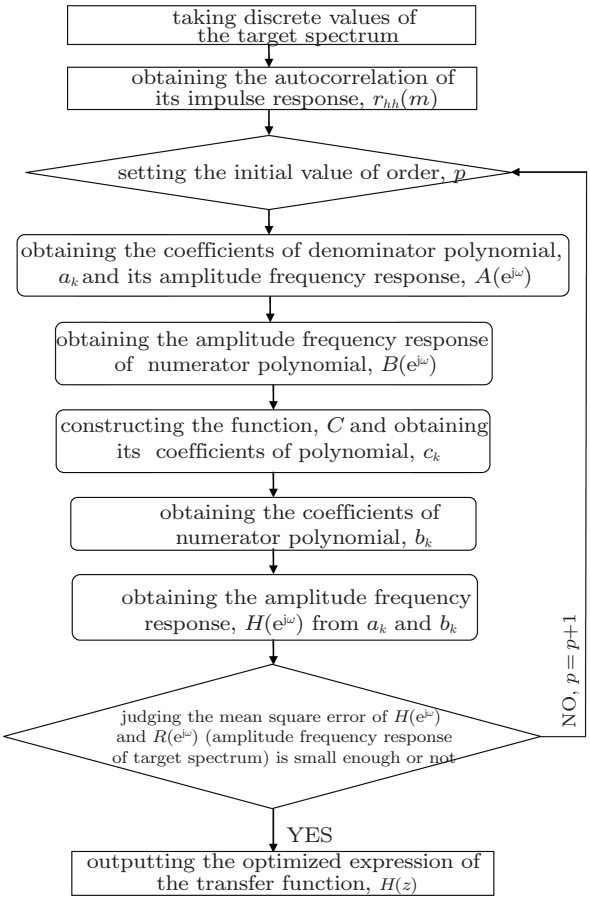


Fig. 2. Flow chart for obtaining the optimized expression of transfer functions.

To date, all coefficients of the denominator and numerator of the target reflection transfer function have been obtained, and the expression of the target reflection spectrum in the digital domain with the order of p is known. We need to optimize the expression to obtain the optimum design result. The optimized expression can be obtained by increasing the order and constantly comparing it with the target reflection transfer function. Once the optimized expression of the target reflection spectrum in the digital domain is achieved, the reflection coefficients of each interface can be obtained by the recursion algorithm.[21] The reflection coefficient of an interface has a certain relationship with the refractive indexes of the two adjacent media.[11] Thus, the refractive index of each layer in the structure can be easily obtained from the reflection coefficients of each interface. The flow diagram for obtaining the optimized expression of the transfer function is shown in Fig. 2. The optical thickness of the medium can be determined by the period of the target spectrum. The period of the target spectrum is related to the parameter z in the digital domain.

3. Design examples

The feasibility of the proposed design method is confirmed by using two simple examples. An 11-layer 1DPC with a band gap near 1.55 μm (fiber communication wavelength) is set as the design target. The solid curve in Fig. 3 shows the transmittance spectrum of the $(HL)^5H$ structure obtained by the traditional transfer matrix method, where the low refractive index material (L) is magnesium fluoride with a refractive index of 1.38, and the high refractive index material (H) is zinc sulfide with a refractive index of 2.35 at 1.55 μm (λ). The optical thickness of each layer is equal to $1/4\lambda$.

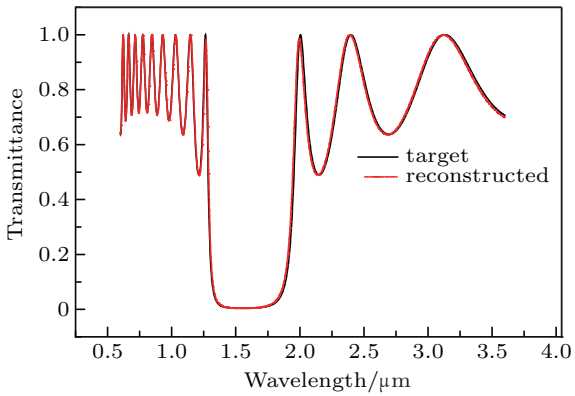


Fig. 3. (color online) Transmittance spectrum of the $(HL)^5H$ 1DPC structure. The inset shows the enlarged view of the bottom part of the band gap.

The reflection transfer function in the digital signal domain of the target spectrum shown in Fig. 3 can be calculated as

$$H(z) = \frac{0.4132 - 0.5570z^{-1} + 0.7711z^{-2} - 0.9643z^{-3} + 1.1064z^{-4} - 1.1828z^{-5}}{1 - 0.8125z^{-1} + 1.0008z^{-2} - 1.1262z^{-3} + 1.1826z^{-4} - 1.1563z^{-5}}$$

$$\frac{+1.1758z^{-6}-1.0944z^{-7}+0.9424z^{-8}-0.7499z^{-9}+0.5341z^{-10}-0.3946z^{-11}}{+1.0574z^{-6}-0.8975z^{-7}+0.7029z^{-8}-0.4973z^{-9}+0.3076z^{-10}-0.1623z^{-11}}. \quad (10)$$

As shown in Eq. (10) and according to the recursion algorithm, the reflection coefficients of each interface can be obtained, which are shown in Table 1. Then, the refractive indexes of each layer can be calculated as shown in Table 2.

Table 1. Reflection coefficients of each interface.

Reflection coefficients	Value	Reflection coefficients	Value
r_{01}	–0.4086	r_{67}	–0.2570
r_{12}	0.2624	r_{78}	0.2592
r_{23}	–0.2614	r_{89}	–0.2556
r_{34}	0.2620	r_{910}	0.2576
r_{45}	–0.2591	r_{1011}	–0.2542
r_{56}	0.2608	r_{1112}	0.3928

Compared with the values of the refractive index of each layer, the structural parameters obtained by the DSP method accord well with the original optical ones. The slight deviation originates from the numerical calculation and almost has no effect on the output stopband spectrum (the transmittance spectrum of the synthesized stopband structure is shown as the dash–dotted curve in Fig. 3). The deviation can be further reduced by sampling the target spectrum with more discrete values, which increases the amount of computation in the same time. Its total amount of computation is similar to that of a forward simulation process (such as the transfer matrix method) and can often be fulfilled in a few seconds with a personal computer.

Table 2. Refractive indices of each layer.

Layer	Refractive index	Layer	Refractive index
0 (incident media)	1	7	2.3430
1	2.3819	8	1.3785
2	1.3915	9	2.3253
3	2.3764	10	1.3727
4	1.3898	11	2.3082
5	2.3619	12 (exit media)	1.0063
6	1.3848		

This design method is also suited for the 1DPC with defect modes, which reflect the localized state characteristics of the photonic band gap. The target transmittance spectrum is shown as the solid curve in Fig. 4. There is a narrow transmission peak at 1.55 μm in the forbidden band. The reflection transfer function in digital signal domain of the target spectrum can be calculated out as

$$H(z)=\frac{0.3919-0.6005z^{-1}+08892z^{-2}-1.1682z^{-3}+1.3935z^{-4}-1.5227z^{-5}+1.5211z^{-6}-1.3704z^{-7}}{1-1.0312z^{-1}+1.3244z^{-2}-1.5350z^{-3}+1.6213z^{-4}-1.5529z^{-5}+1.3217z^{-6}-0.9413z^{-7}}$$
$$\frac{+1.0711z^{-8}-0.6461z^{-9}-0.0007z^{-10}+0.6466z^{-11}-1.0705z^{-12}+1.3676z^{-13}-1.5166z^{-14}}{+0.4486z^{-8}+0.1026z^{-9}-0.6929z^{-10}+1.1486z^{-11}-1.3740z^{-12}+1.4594z^{-13}-1.4134z^{-14}}$$
$$\frac{+1.5163z^{-15}-1.3866z^{-16}+1.1610z^{-17}-0.8831z^{-18}+0.5956z^{-19}-0.3886z^{-20}}{+1.2608z^{-15}-1.0367z^{-16}+0.7802z^{-17}-0.5282z^{-18}+0.3097z^{-19}-0.1523z^{-20}}. \quad (11)$$

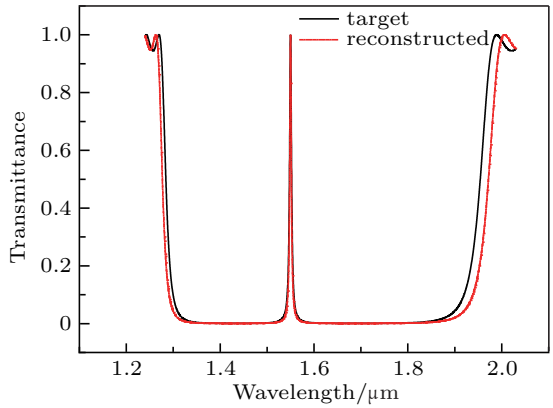


Fig. 4. (color online) Transmittance spectrum of the $(HL)^5(LH)^5$ 1DPC structure.

As shown in Eq. (11) and according to the recursion algorithm, a 1DPC with $1/4\lambda$ $(HL)^5(LH)^5$ configuration is synthesized. The refractive indexes of H and L layers are about 2.3 and 1.4, which are nearly the values of TiO_2 (2.28) and SiO_2 (1.43) respectively. The transmittance spectrum of the synthesized structure almost coincides with the target one and is shown as the dash–dotted curve in Fig. 4.

For the above design examples, the numbers of the sampling points are about 2×10^3 and 2×10^4 in the former and later target spectrum respectively. Computation can be fulfilled within 2–3 seconds with a personal computer (CPU @3.1 GHz and 4 GB RAM). It should be pointed out that a large amount of computation will be needed for designing an arbitrarily given target band gap structure (no transmittance spectrum is obtained in advance). Firstly, a square wave-like transmittance spectrum (at least one period is included) containing the information about the location and the width of the band gap is set as the target. And then the spectrum is sampled into discrete values. A set of reflection transfer function, reflection coefficients, and the refractive index of each layer can

be then searched out if a big deviation (such as a big ripple on both sides of the band gap as shown in Fig. 3) is allowed. Afterwards, whether all the obtained refractive indexes have their own physical meaning should be determined. If any, a refractive index does not have the physical meaning (too small or too big, far from that of a real material), the numerical searching should be redone until a set of physically meaningful refractive indexes is obtained.

4. Conclusions

In this paper, a method of describing 1DPC filters based on the Z-domain DSP technique is presented. The optimized structural parameters of the 1DPC can be numerically calculated from a given target spectrum with a photonic band gap. The analytical expression of the target spectrum in the digital domain is obtained for the first time by the autocorrelation of its impulse response. The feasibility of this method is verified by using two simple 1DPC structures: one with defect mode and the other without defect mode. The spectrum of the synthesized structure with this method coincides well with the target one obtained by the traditional transfer matrix method. This method provides an effective mathematical approach to the function-guided design of nano interference structures. These two examples illustrate how interdisciplinary cooperation between DSP and optics can facilitate the development of nanophotonic technology. The advantage of this design method will be further demonstrated for more complex interference-based band gap structures.

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