Measurement-induced disturbance between two atoms in Tavis–Cummings model with dipole–dipole interaction*

Zhang Guo-Feng(张国锋)[a)†], Wang Xiao(王 骁)[b)], and Lü Guang-Hong(吕广宏)[a)]

[a)] *Department of Physics, School of Physics and Nuclear Energy Engineering, Beihang University, Beijing* 100191, *China*
[b)] *Beijing No. 8 High School, Beijing* 100033, *China*

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Quantum correlation, measured by measurement-induced disturbance (MID), between two two-level atoms is investigated in detail in Tavis–Cummings model with dipole–dipole interaction (DDI). We find that MID can be determined only by the dipole–dipole interaction between the two atoms when the cavity and atoms are at resonance. Moreover, DDI will have different effects on MID for two different kinds of initial states.

Keywords: measurement-induced disturbance (MID), Tavis–Cummings model, dipole–dipole interaction (DDI)

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Recently, a large number of researches have pointed out that quantum correlation is more general and more fundamental than quantum entanglement because there exist some quantum correlations different from entanglement which also have nonzero values even for separable mixed states.[1–4] Quantum entanglement is just a specific kind of quantum correlation, but not only this kind. Since quantum correlations were seldom investigated before, many people took it for granted that quantum entanglement is exactly quantum correlation. However, although this identification is appropriate for pure states, it is problematic when mixed states are involved.[5] In recent years, there has been an ongoing effort to characterize qualitatively and quantitatively the quantum correlations and apply them to quantum information. Five popular measures to describe such quantum correlations are quantum entanglement (QE), measurement-induced disturbance (MID), measurement-induced nonlocality (MIN), quantum discord (QD), and geometric quantum discord (GQD).[6–19] Jiang *et al.* gave a symmetric measure of quantum correlation based on the Hilbert–Schmidt distance.[20] The quantum correlation of coupled qubits in squeezed vacuum reservoir with those in a damping Jaynes–Cummings model was investigated in Refs. [21] and [22]. In this paper, we select MID as the measure to analyze quantum correlation time evolution between two coupled two-level atoms.

According to Luo's theory,[5,23,24] we can apply local measurement $\{\Pi_k\}$ ($\Pi_k\Pi_{k'}=\delta_{kk'}\Pi_k$ and $\sum_k\Pi_k=1$) to any bipartite state ρ and then explore the difference between the state before the measurement and that after the measurement. Here $\Pi_k=\Pi_i^a\otimes\Pi_j^b$ and Π_i^a,Π_j^b are complete projective measurements composed of one-dimensional orthogonal projections for parties a and b. After the measurement, we obtain the state $\Pi(\rho)=\sum_{ij}(\Pi_i^a\otimes\Pi_j^b)\rho(\Pi_i^a\otimes\Pi_j^b)$, which is a classical state. If the measurement Π is induced by the spectral resolutions of the reduced states $\rho^a=\sum_i\rho_i^a\Pi_i^a$ and $\rho^b=\sum_j\rho_j^b\Pi_j^b$, the measurement leaves the marginal information invariant and is in a certain sense the least disturbing. Then we can use quantum mutual information difference between ρ and $\Pi(\rho)$ to measure quantum correlation in ρ. The total correlation in a bipartite state ρ can be well quantified by the quantum mutual information $I(\rho)=S(\rho^a)+S(\rho^b)-S(\rho)$, and $I(\Pi(\rho))$ quantifies the classical correlations in ρ since $\Pi(\rho)$ is a classical state. Here $S(\rho)=-\mathrm{tr}(\rho\log\rho)$ denotes the von Neumann entropy, and the logarithm is always understood as base 2 in this paper. So the quantum correlation can be quantified by the measurement-induced disturbance[5,24]

$$Q(\rho)=I(\rho)-I(\Pi(\rho)). \tag{1}$$

The two atoms are in a perfect one-mode cavity without losses, but there is dipole–dipole interaction between the two atoms. We can obtain a mixed state of atom A and atom B by tracing over the cavity modes at time t. The Hamiltonian of the system within the rotating wave approximation (RWA) can be written as[25]

$$\begin{aligned}H &= \sum_{l=A,B}\omega\sigma_z^l+\omega a^\dagger a+g\sum_{l=A,B}(a^\dagger\sigma_-^l+a\sigma_+^l)\\ &\quad+\Omega(\sigma_-^A\sigma_+^B+\sigma_+^A\sigma_-^B),\end{aligned} \tag{2}$$

where a and $a^\dagger$ denote the annihilation and creation operators for the cavity field, and $\sigma_z^l=|e\rangle_l\langle e|-|g\rangle_l\langle g|$, $\sigma_+^l=|e\rangle_l\langle g|$, $\sigma_-^l=|g\rangle_l\langle e|(l=A,B)$ are the atomic operators with $|e\rangle_l$ and

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†Corresponding author. E-mail: gf1978zhang@buaa.edu.cn

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$|g\rangle_l$ being the excited and ground states of the l atom. We assume that the transition frequencies of atom A and atom B are equal to the cavity frequency ω, i.e., the two atoms and the cavity are coupled resonantly. And both coupling coefficients between the atoms and the field are g. Ω is the dipole–dipole interaction coupling strength between the two atoms. By the way, there exists a conserved quantity $K([H,K]=0)$ for the above Hamiltonian, which is

$$K = a^{\dagger}a + 1 + \frac{\sigma_z^A + \sigma_z^B}{2}. \tag{3}$$

The basis of the subspace $(K = n+2)$ is

$$|n, e_A, e_B\rangle, |n+1, e_A, g_B\rangle, |n+1, g_A, e_B\rangle, |n+2, g_A, g_B\rangle. \tag{4}$$

So, in this basis, the state function $|\psi(t)\rangle$ can be written simply as follows:

$$\begin{aligned} |\psi(t)\rangle \;=\; & C_1(t)|n, e_A, e_B\rangle + C_2(t)|n+1, e_A, g_B\rangle \\ & + C_3(t)|n+1, g_A, e_B\rangle + C_4(t)|n+2, g_A, g_B\rangle. \end{aligned} \tag{5}$$

Based on the Schrödinger equation: $\mathrm{i}d|\psi(t)\rangle/\mathrm{d}t = H|\psi(t)\rangle$, we can obtain the following differential equations:

$$\begin{cases} \mathrm{i}\dot{C}_1(T) = (n+3/2)\omega C_1(T) + \sqrt{n+1}C_2(T) \\ \qquad\qquad +\sqrt{n+1}C_3(T), \\ \mathrm{i}\dot{C}_2(T) = (n+3/2)\omega C_2(T) + \sqrt{n+1}C_1(T) \\ \qquad\qquad +\sqrt{n+2}C_4(T) + \Omega C_3(T), \\ \mathrm{i}\dot{C}_3(T) = (n+3/2)\omega C_3(T) + \sqrt{n+1}C_1(T) \\ \qquad\qquad +\sqrt{n+2}C_4(T) + \Omega C_2(T), \\ \mathrm{i}\dot{C}_4(T) = (n+3/2)\omega C_4(T) + \sqrt{n+2}C_2(T) \\ \qquad\qquad +\sqrt{n+2}C_3(T), \end{cases} \tag{6}$$

where $T = gt$ and all the physics parameters are rescaled by g. Here, we will not give the expressions of $C_i(t)$ $(i = 1,2,3,4)$, for they are redundant, but there are $C_i(t) \sim \mathrm{e}^{\mathrm{i}\omega}$.

The information about the quantum correlation between the atom A and atom B is contained in the reduced density matrix ρ_{AB} which can be obtained from Eq. (5) by tracing out the photonic part of the total pure state. In the standard basis (4), the reduced density matrix ρ_{AB} can be expressed as

$$\rho_{AB}(T) = \begin{pmatrix} |C_1(T)|^2 & 0 & 0 & 0 \\ 0 & |C_2(T)|^2 & C_2(T)C_3^*(T) & 0 \\ 0 & C_3(T)C_2^*(T) & |C_3(T)|^2 & 0 \\ 0 & 0 & 0 & |C_4(T)|^2 \end{pmatrix}, \tag{7}$$

where symbol $*$ refers to the complex conjugate. On the other hand, by taking $\Pi_A^i = |i\rangle\langle i|$ and $\Pi_B^j = |j\rangle\langle j|$ $(i, j = e \text{ or } g)$, we have

$$\Pi(\rho_{AB}(T)) = \begin{pmatrix} |C_1(T)|^2 & 0 & 0 & 0 \\ 0 & |C_2(T)|^2 & 0 & 0 \\ 0 & 0 & |C_3(T)|^2 & 0 \\ 0 & 0 & 0 & |C_4(T)|^2 \end{pmatrix}. \tag{8}$$

After a simple calculation, we have

$$\Pi(\rho_{AB})_A = \Pi(\rho_{AB})_B = \rho_A = \rho_B, \tag{9}$$

where

$$\Pi(\rho_{AB})_A = \mathrm{Tr}_B\Pi(\rho_{AB}), \;\; \rho_A = \mathrm{Tr}_B\rho_{AB}. \tag{10}$$

Thus, we can obtain MID between the two atoms

$$Q(\rho) = S(\Pi(\rho_{AB})) - S(\rho_{AB}). \tag{11}$$

When the initial state is $|\psi(0)\rangle = |n, e_A, e_B\rangle$, after a simple calculation, we have

$$Q(\rho) = \frac{4(1+n)(1-\mathrm{Cos}[T\sqrt{24+16n+\Omega^2}])}{24+16n+\Omega^2}. \tag{12}$$

And for $|\psi(0)\rangle = (|n+1, e_A, g_B\rangle + |n+1, g_A, e_B\rangle)/\sqrt{2}$, we have

$$Q(\rho) = \frac{12+8n+\Omega^2+4(3+2n)\mathrm{Cos}[T\sqrt{24+16n+\Omega^2}]}{24+16n+\Omega^2}. \tag{13}$$

From the above derivation, we can see that the quantum correlation is unrelated to both the frequency of atom and to the cavity. This can be easily understood since for the resonant case the dependency of frequency on $C_i(t)$ $(i = 1,2,3,4)$ can be expressed as $\sim \mathrm{e}^{-\mathrm{i}\omega}$. So the frequency will not appear in Eq. (7) nor in Eq. (8), $Q(\rho)$ is independent of frequency.

The MID evolutions with gt are shown in Figs. 1 and 2. The period will become shorter when DDI and the photon number in the cavity are larger. For initial state $|\psi(0)\rangle = |n, e_A, e_B\rangle$, the more the photon number, the higher value MID can arrive at. On the other hand, the bigger the photon number, the lower value MID can arrive at when $|\psi(0)\rangle = (|n+1, e_A, g_B\rangle + |n+1, g_A, e_B\rangle)/\sqrt{2}$. Moreover, the DDI will have different effects on MID for the different initial states. DDI can enhance the quantum correlation between the two atoms for $|\psi(0)\rangle = (|n+1, e_A, g_B\rangle + |n+1, g_A, e_B\rangle)/\sqrt{2}$, but it will weaken MID for $|\psi(0)\rangle = |n, e_A, e_B\rangle$.

We have investigated the time evolution of the MID between two atoms which are subject to a single-mode cavity. The rigorous solutions are given when the atom and the cavity are at resonance. We find that MID is unrelated to both the cavity and the atom frequency. The dipole–dipole interaction between the atoms will have different effects on MID for the different initial states. We hope that our conclusion would be applied to the quantum information processing and the experiments of quantum correlation.

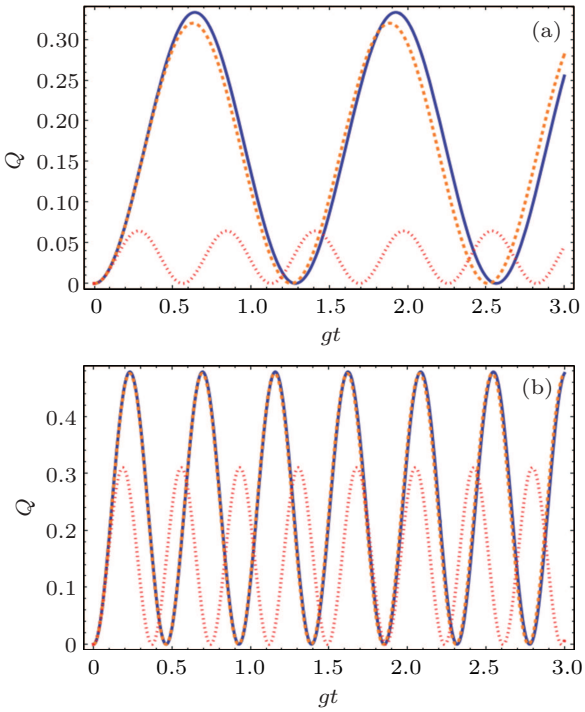


Fig. 1. (color online) Quantum correlation MID as a function of the time (gt) for $|\psi(0)\rangle = |n, e_A, e_B\rangle$. (a) Solid curve for $n = 0$, $\Omega = 0$; dashed curve is for $n = 0$, $\Omega = g$; dotted curve for $n = 0$, $\Omega = 10g$. (b) Solid curve is for $n = 10$, $\Omega = 0$; dashed curve for $n = 10$, $\Omega = g$; dotted curve for $n = 10$, $\Omega = 10g$.

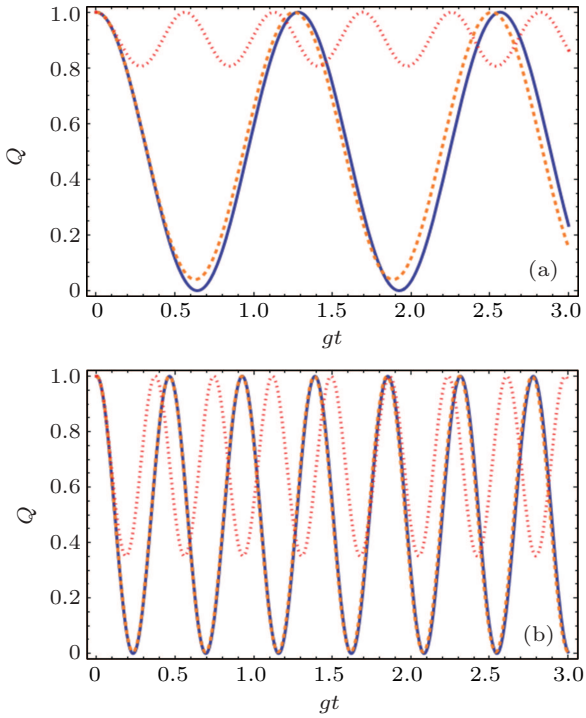


Fig. 2. (color online) Quantum correlation MID as a function of the time (gt) for $|\psi(0)\rangle = (|n+1, e_A, g_B\rangle + |n+1, g_A, e_B\rangle)/\sqrt{2}$. (a) Solid curve is for $n = 0$, $\Omega = 0$; dashed curve for $n = 0$, $\Omega = g$; dotted curve for $n = 0$, $\Omega = 10g$. (b) Solid curve is for $n = 10$, $\Omega = 0$; dashed curve for $n = 10$, $\Omega = g$; dotted curve for $n = 10$, $\Omega = 10g$.

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