Periodic solitons in dispersion decreasing fibers with a cosine profile*

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Periodic solitons are studied in dispersion decreasing fibers with a cosine profile. The variable-coefficient nonlinear Schrödinger equation, which can be used to describe the propagation of solitons, is investigated analytically. Analytic soliton solutions for this equation are derived with the Hirota's bilinear method. Using the soliton solutions, we obtain periodic solitons, and analyze the soliton characteristics. Influences of physical parameters on periodic solitons are discussed. The presented results can be used in optical communication systems and fiber lasers.

Keywords: solitons, dispersion decreasing fibers, analytic soliton solutions, Hirota's bilinear method

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1. Introduction

Dispersion decreasing fibers (DDFs) have been considered as one of the effective media to control solitons in communication systems and fiber lasers.[1–3] For the fabrication of DDFs, their core diameter is varied through tapering the fibers, and then the dispersion of fibers is changed.[2] In DDFs, the dispersion is monotonically and smoothly decreased from an initial value to a smaller value at the end of fibers with some specified profiles.[3] DDFs have generally been used for adiabatic soliton pulse compression and soliton train generation.[4–21]

The soliton propagation in DDFs can be described by the following variable-coefficient nonlinear Schrödinger (vcNLS) equation,[1]

$$\mathrm{i}\frac{\partial u}{\partial z}+\mathrm{i}\beta_1(z)\frac{\partial u}{\partial t}+\beta_2(z)\frac{\partial^2 u}{\partial t^2}+\gamma(z)|u|^2u=0, \tag{1}$$

where $u(z,t)$ is the temporal envelope of DDFs. z is the longitudinal coordinate along DDFs, t is the time in the moving coordinate system. $\beta_1(z)$ is the reciprocal of the group velocity, $\beta_2(z)$ represents the group-velocity dispersion coefficient, and $\gamma(z)$ is the coefficient of Kerr nonlinearity. When $\beta_1(z)=0$, equation (1) can be reduced into the standard vcNLS equation, and great progress has been made in both theoretical and experimental researches for the vcNLS equation.[7,22–27]

Theoretical study of modulation instability (MI) in DDFs has been presented,[22] and the MI induced by cross-phase modulation in DDFs has been solved and analyzed by the perturbation method.[23] Moreover, soliton generation and compression have been numerically studied in DDFs.[7,24] The possibility to generate parabolic pulses via DDFs has been demonstrated experimentally.[25,26] Based on the NLS equation, a novel scheme for the generation of the self-similar parabolic pulse has been proposed, and the corresponding model has been established.[27]

However, to the best of our knowledge, the case of $\beta_1(z)\neq 0$ has not been discussed, and periodic solitons in DDFs with a cosine profile have not been reported in the present literature. In this paper, periodic solitons will be obtained in DDFs with a cosine profile for the first time. Via the Hirota's bilinear method, analytic soliton solutions for Eq. (1) will be derived. Periodic soliton characteristics will be analyzed, and influences of parameters for analytic soliton solutions on solitons will be discussed.

This paper will be structured as follows. In Section 2 analytic soliton solutions for Eq. (1) will be presented. In Section 3 characteristics of periodic solitons will be studied, and the influences of parameters on solitons will be analyzed. Finally in Section 4 we will summarize our results, and present our conclusions.

2. Analytic soliton solutions for Eq. (1)

We introduce the dependent variable transformation[28–31]

$$u(z,t)=\frac{g(z,t)}{f(z,t)}, \tag{2}$$

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where $g(z,t)$ is a complex differentiable function, and $f(z,t)$ is a real one. With symbolic computation, the bilinear forms for Eq. (1) can be derived as

$$\mathrm{i}D_z g\cdot f+\mathrm{i}\beta_1(z)D_t g\cdot f+\beta_2(z)D_t^2 g\cdot f=0, \tag{3}$$

$$\beta_2(z)D_t^2 f\cdot f-\gamma(z)gg^*=0, \tag{4}$$

where the asterisk denotes complex conjugate. D_z and D_t [32] are the Hirota's bilinear operators, and can be defined by

$$D_z^m D_t^l(a\cdot b)=\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)^m\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^l a(z,t)b(z',t')\Big|_{z'=z,t'=t}. \tag{5}$$

With the following power series expansions for $g(z,t)$ and $f(z,t)$

$$g(z,t)=\varepsilon g_1(z,t)+\varepsilon^3 g_3(z,t)+\varepsilon^5 g_5(z,t)+\cdots, \tag{6}$$

$$f(z,t)=1+\varepsilon^2 f_2(z,t)+\varepsilon^4 f_4(z,t)+\varepsilon^6 f_6(z,t)+\cdots, \tag{7}$$

where ε is a formal expansion parameter, bilinear forms of Eqs. (3) and (4) can be solved. Substituting expressions (6) and (7) into bilinear forms (3) and (4) and equating coefficients of the same powers of ε to zero yield the recursion relations for $g_n(z,t)$'s and $f_n(z,t)$'s. Then, the analytic soliton solutions for Eq. (1) can be obtained.

To obtain the analytic soliton solutions for Eq. (1), we assume

$$g(z,t)=g_1(z,t),\quad f(z,t)=1+f_2(z,t), \tag{8}$$

where

$$g_1(z,t)=\mathrm{e}^{\theta_1},$$

$$\theta_1=[a_{11}(z)+\mathrm{i}a_{12}(z)]z+(b_{11}+\mathrm{i}b_{12})t+k_{11}+\mathrm{i}k_{12}, \tag{9}$$

with b_{11}, b_{12}, k_{11}, and k_{12} are real constants. $a_{11}(z)$ and $a_{12}(z)$ are the differentiable functions to be determined. With $g_1(z,t)$, and collecting the coefficient of ε in Eq. (3), we obtain the constraints on $a_{11}(z)$ and $a_{12}(z)$ as

$$a_{11}(z)=\frac{1}{z}\int\left[-b_{11}\beta_1(z)-2b_{11}b_{12}\beta_2(z)\right]\mathrm{d}z,$$

$$a_{12}(z)=\frac{1}{z}\int\left[b_{11}^2\beta_2(z)-b_{12}\beta_1(z)-b_{12}^2\beta_2(z)\right]\mathrm{d}z.$$

Substituting $g_1(z,t)$ into Eq. (4), and collecting the coefficient of ε^2 yield

$$f_2(z,t)=M\mathrm{e}^{\theta_1+\theta_1^*} \tag{10}$$

with

$$M=\frac{\gamma(z)}{8b_{11}^2\beta_2(z)},\quad \beta_2(z)=c\gamma(z),$$

and c is an arbitrary constant.

Without loss of generality, we set $\varepsilon=1$, and can write the analytic soliton solutions as

$$u(z,t)=\frac{g(z,t)}{f(z,t)}=\frac{g_1(z,t)}{1+f_2(z,t)}$$

$$=\frac{1}{\sqrt{M}}\mathrm{e}^{\mathrm{i}[a_{12}(z)z+b_{12}t+k_{12}]}\times\operatorname{sech}\left[a_{11}(z)z+b_{11}t+k_{11}+\frac{1}{2}\ln M\right]. \tag{11}$$

3. Discussion

For the analytic soliton solutions (11), we can choose different values of c, $\beta_1(z)$, $\gamma(z)$, k_{11}, k_{12}, b_{11}, and b_{12} to obtain different properties of solitons. In Fig. 1, the reciprocal of the group velocity $\beta_1(z)$ and Kerr nonlinearity $\gamma(z)$ are chosen to be constants, such as $\beta_1(z)=1$ and $\gamma(z)=2$, we obtain the stable propagation of solitons in optical fibers. The amplitude and shape of solitons do not change during the propagation. When the initial state of solitons is changed, i.e. the values of k_{11}, k_{12}, b_{11}, and b_{12} are different, we can adjust the relevant characteristics of solitons. Compared to Fig. 1(a), changing the values of k_{11} and k_{12} only influences the soliton phases in Fig. 1(b). If we change the values of b_{11} and b_{12}, the velocity of solitons increases as shown in Fig. 1(c). The amplitude and width of solitons are also adjusted. Moreover, when the sign of b_{12} is negative, the propagation direction will change. Thus, we can control the phase, amplitude, width, and velocity of solitons through changing the values of k_{1j} and b_{1j} $(j=1,2)$.

When $\beta_1(z)$ is a function, and $\gamma(z)$ is a constant, such as $\beta_1(z)=\cos(2z)$ and $\gamma(z)=0.01$, solitons propagate periodically in Fig. 2(a). The amplitude and width of solitons are without changes. Increasing the coefficient of $\cos(2z)$ leads to the increase of soliton oscillations in Fig. 2(b). Besides, if we change the coefficient of z in $\cos(2z)$, such as $\beta_1(z)=2\cos(4z)$ in Fig. 2(c), the oscillation period of solitons will change. The bigger their coefficient, the smaller the oscillation period will be. Moreover, changing the Kerr nonlinearity $\gamma(z)$ can change the soliton velocity. In Fig. 2(d), the soliton velocity increases with the increase of the value of $\gamma(z)$. Hence, $\beta_1(z)$ can adjust the soliton period and oscillator strength.

While $\beta_1(z)$ is a constant, and $\gamma(z)$ is a function, solitons also change periodically in Fig. 3(a). Different from Fig. 2(a), the oscillation direction of solitons is opposite, the amplitude of solitons becomes bigger, and the width of solitons becomes narrower. When we increase the coefficient of $\gamma(z)$, soliton oscillations increase in Fig. 3(b). If $\beta_1(z)$ and $\gamma(z)$ are both functions, we can get stable propagation solitons as shown in Fig. 3(c). That is because the dispersion and nonlinearity of solitons can balance each other. When this balance is disturbed somehow, such as $\beta_1(z)=\cos(z)$, and $\gamma(z)=2\cos(2z)$ in Fig. 3(d), solitons are unstable, and change periodically. The period of oscillation is superimposed on $\beta_1(z)$ and $\gamma(z)$. Therefore, in addition to the changes of soliton period and oscillator strength, $\gamma(z)$ can change the soliton width and amplitude.

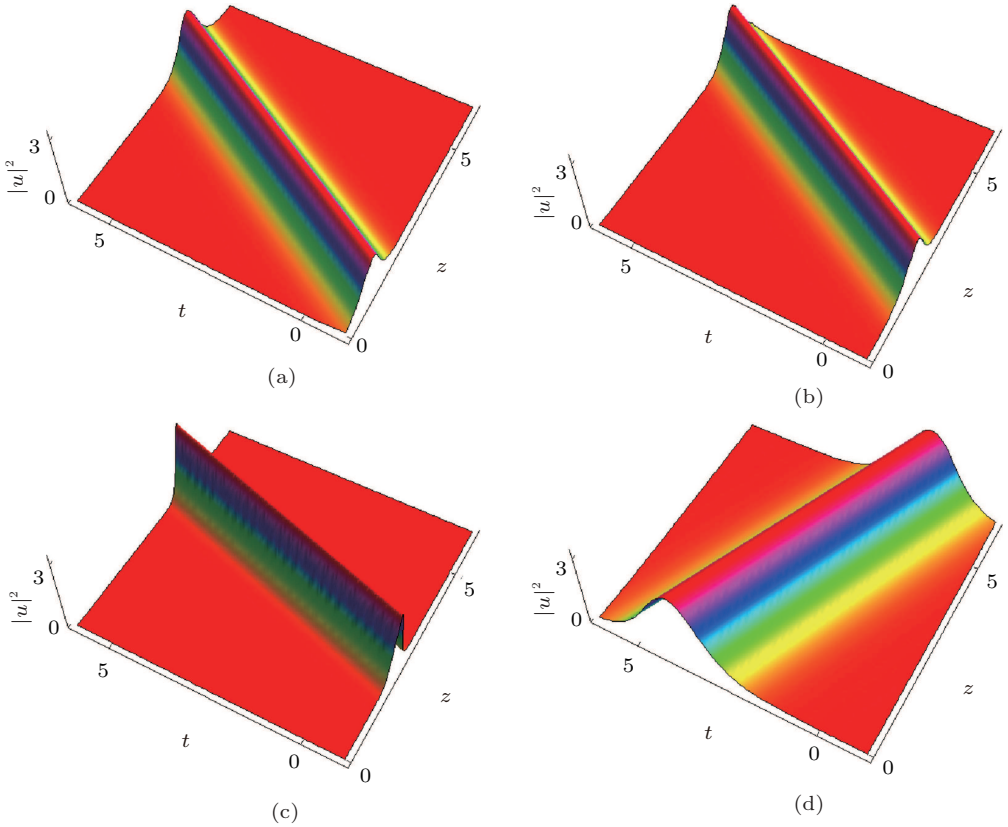


Fig. 1. (color online) Stable propagation of solitons in optical fibers. The parameters are $c = 1$, $\beta_1(z) = 1$, $\gamma(z) = 2$ with (a) $k_{11} = -1$, $k_{12} = 1$, $b_{11} = 1$, $b_{12} = 1$; (b) $k_{11} = 2$, $k_{12} = 1$, $b_{11} = 1$, $b_{12} = 1$; (c) $k_{11} = -1$, $k_{12} = 1$, $b_{11} = 1.2$, $b_{12} = 2$; (d) $k_{11} = -1$, $k_{12} = 1$, $b_{11} = 1.2$, and $b_{12} = -0.5$.

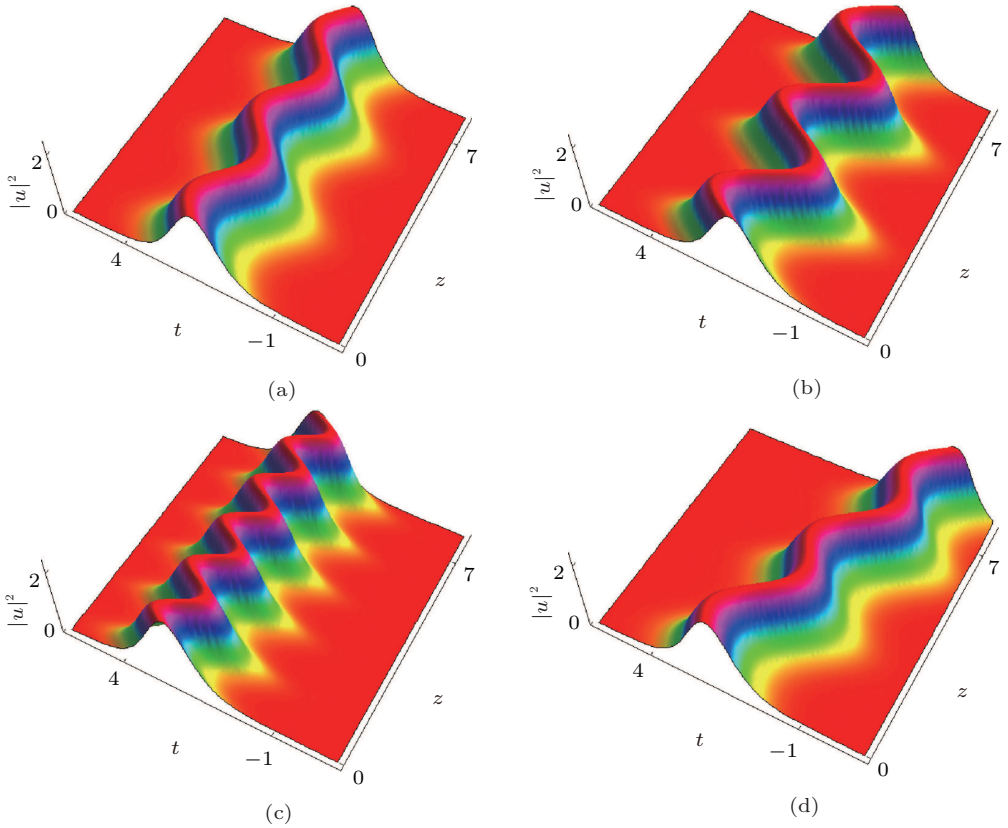


Fig. 2. (color online) Periodic propagation of solitons in optical fibers. The parameters are $k_{11} = -1$, $k_{12} = 1$, $b_{11} = 1$, $b_{12} = -1$, $c = 1$ with (a) $\beta_1(z) = \cos(2z)$, $\gamma(z) = 0.01$; (b) $\beta_1(z) = 2\cos(2z)$, $\gamma(z) = 0.01$; (c) $\beta_1(z) = 2\cos(4z)$, $\gamma(z) = 0.01$; (d) $\beta_1(z) = \cos(2z)$, and $\gamma(z) = 0.2$.

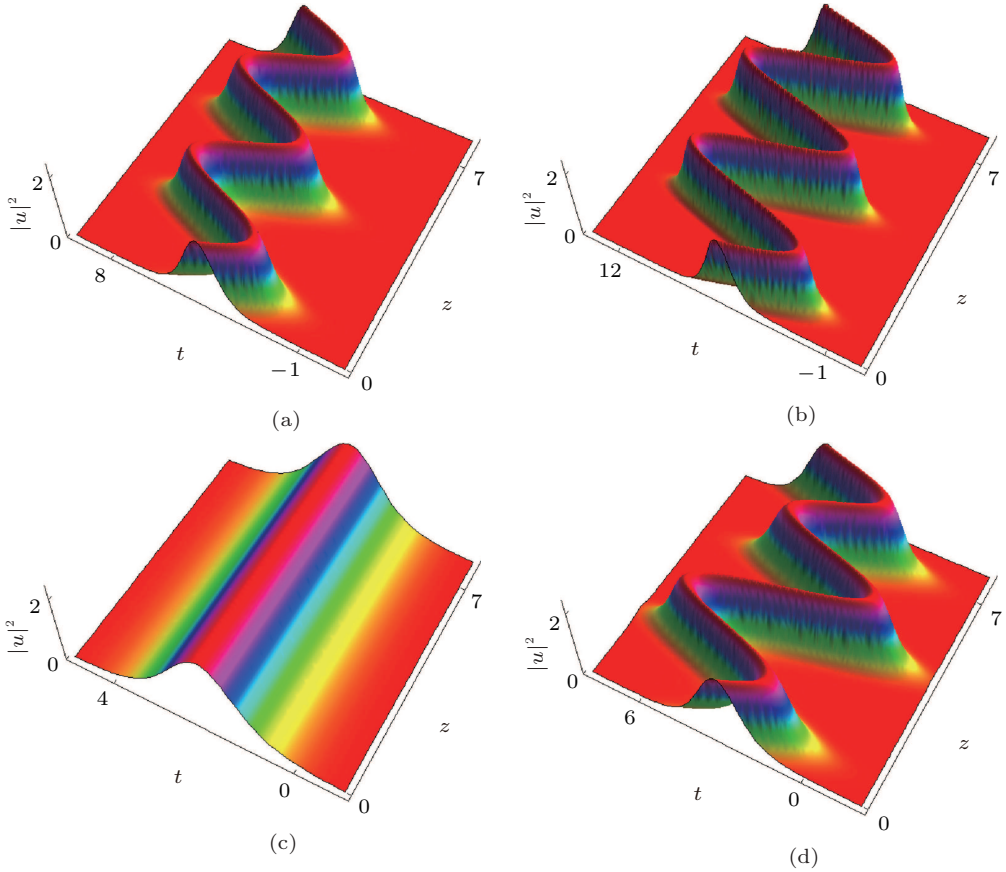


Fig. 3. (color online) Periodic propagation of solitons in optical fibers. The parameters are $k_{11} = -1$, $k_{12} = 1$, $b_{11} = 1$, $b_{12} = -1$, $c = 1$ with (a) $\beta_1(z) = 0.2$, $\gamma(z) = 2\cos(2z)$; (b) $\beta_1(z) = 0.2$, $\gamma(z) = 4\cos(2z)$; (c) $\beta_1(z) = \cos(2z)$, $\gamma(z) = 0.5\cos(2z)$; (d) $\beta_1(z) = \cos(z)$, and $\gamma(z) = 2\cos(2z)$.

4. Conclusions

Analytic study on periodic soliton propagation has been studied in this paper. The dispersion decreasing fibers with the cosine profile have been selected as the transmission medium. Analytic soliton solutions (11) have been obtained for the variable-coefficient nonlinear Schrödinger equation [see Eq. (1)]. Different soliton characteristics have been presented with different choices of the values of c, $\beta_1(z)$, $\gamma(z)$, k_{11}, k_{12}, b_{11}, and b_{12}. We have controlled the phase, amplitude, width and velocity of solitons through changing the values of k_{1j} and b_{1j} ($j = 1, 2$) (see Fig. 1). Besides, the soliton period and oscillator strength have been adjusted with $\beta_1(z)$ and $\gamma(z)$ (see Figs. 2 and 3). Results obtained in this paper have potential applications in the optical communication systems and fiber lasers.

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