Relative ordering of square-norm distance correlations in open quantum systems*

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We investigate the square-norm distance correlation dynamics of the Bell-diagonal states under different local decoherence channels, including phase flip, bit flip, and bit-phase flip channels by employing the geometric discord (GD) and its modified geometric discord (MGD), as the measures of the square-norm distance correlations. Moreover, an explicit comparison between them is made in detail. The results show that there is no distinct dominant relative ordering between them. Furthermore, we obtain that the GD just gradually deceases to zero, while MGD initially has a large freezing interval, and then suddenly changes in evolution. The longer the freezing interval, the less the MGD is. Interestingly, it is shown that the dynamic behaviors of the two geometric discords under the three noisy environments for the Werner-type initial states are the same.

Keywords: decoherence, geometric discord, modified geometric discord, trace distance correlation

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1. Introduction

As is well known, quantum entanglement plays a key role in quantum information theory.[1] However, people have found that entanglement should not be regarded as the unique measure of quantum correlations that are useful for quantum information processing, since other types of nonclassical correlations exist even in some mixed separable (not entangled) states[2–4] and it has been shown both theoretically and experimentally[5–9] that some separable states can improve performance in some computational tasks over their classical counterparts. It opens up new prospects for the study and comprehension of such correlations. Quantum discord, initially introduced by Ollivier and Zurek[4] and by Henderson and Vedral,[10] has been extensively used to capture these nonclassical correlations. Meanwhile, some different methods of computing the discord in the multipartite system have been put forward, such as quantum discord (QD)[11–13] and the geometric discord (GD).[14–17] Recently, the virtues of QD have been widely studied.[18–24] However, the QD demands a minimization procedure for quantum measurements. The GD is defined as the trace distance correlation between the system state and its closest classical state.[14] It is convenient to have an analytical formula of general two-qubit state in contrast to QD and many efforts have been made to study its dynamic behaviors.[25–28]

However, Piani[29] has revealed that the GD is not a good measure for quantum correlation, there is a curious and frustrating problem that it can increase even under trivial local reversible operations of the party whose classicality or nonclassicality is not measured. This makes it a weak method to measure square-norm distance correlation and brings a strong restriction on its application in the quantum computation tasks. Consequently, the above facts inspire the researcher to search for other new methods to capture and quantify nonclasscial correlations of the state. Among them,[30–32] Chang and Luo[30] proposed a modified measure that employs the square root of the density operator to qualify the trace distance correlation of the system. It has been proven that this modified geometric discord (MGD) provides more information and is a greater reliable tool to determine the trace distance correlation due to the square root characteristics.

On the other hand, owing to the unavoidable interaction between a quantum system and its environment, it is significant to have a whole comprehension of the dynamics of quantum correlation in the open quantum system. Some studies have concentrated on the correlations under decoherence,[33–35] and the comparison among various quantum correlation witnesses under a Markovian environment[36,37] and non-Markovian[38] environment. However, to the best of our knowledge, detailed studies of the MGD dynamics under different decoherence channels have been still lacking up to now. Furthermore, since MGD is introduced as a new tool to qualify and quantify quantum correlations in a more intrinsic and informational fashion, it is interesting and necessary to study the relations between it and the original one under different cases. Therefore, the main

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purpose of the present study is to investigate the trace distance correlation dynamics for two-qubit states under various flip channels, including phase flip (PF), bit flip (BF), and bit phase flip (BPF) channels. Also, explicit comparisons between them are discussed in detail.

The rest of this paper is organized as follows. In Section 2, we briefly review the framework of trace distance correlation witnesses for the original method and its modified one. In Section 3, the trace distance correlation dynamic behaviors for the Bell-diagonal states under different local decoherence channels are investigated in detail and a whole comparison between them is also taken into consideration. Finally, we summarize our conclusions in Section 4.

2. Trace distance correlations for original measure and its modified one

The GD, first introduced by Dakić *et al.*[14] in 2010, is a more manageable quantum correlation measure than the QD. For a given two-qubit state, let ρ^{AB} denote the density operator of this two-qubit system AB, and $\rho^{A(B)} = \mathrm{tr}_{B(A)}\rho^{AB}$ be the reduced density operator of the subsystem $B(A)$, then the geometric quantum discord will be defined as the square-norm distance between the system state and its closest state:

$$D_{\mathrm{G}}(\rho) = \min_{\chi_\rho \in \Omega} \|\rho - \chi_\rho\|^2, \tag{1}$$

where $\|\rho - \chi\|^2 = \mathrm{Tr}(\rho - \chi_\rho)^2$ is the square-norm distance in the Hilbert–Schmidt space, Ω represents the set of zero-discord state, and a zero-discord state χ_ρ has the form $\chi_\rho = \sum_{ij} p_{ij}|i\rangle\langle j| \otimes \rho_j^B$, with ρ_j^B being arbitrary states of part B, $\{|i\rangle\}$ being a local basis on part A, and p_{ij} being probabilities. Based on the analysis in Ref. [17], a closed form of expression for geometric discord can be obtained as

$$D_{\mathrm{G}}(\rho) = \frac{1}{4}(\|\boldsymbol{x}\|^2 + \|T\|^2 - \lambda_{\max}), \tag{2}$$

where $x_i = \mathrm{Tr}(\sigma_i^A \rho)$ are components of the local Bloch vector for subsystem A, $T_{ij} = \mathrm{Tr}\rho(\sigma_i \otimes \sigma_j)$ are components of the correlation tensor T, and σ_i $(i = 1,2,3)$ are the three standard Pauli matrices, $\lambda_{\max}$ is the largest eigenvalue of the matrix $K = \boldsymbol{x}\boldsymbol{x}^{\mathrm{T}} + TT^{\mathrm{T}}$ (here the superscript T denotes transpose).

On the other hand, in analogy to Eq. (1), the following MGD introduced by Chang and Luo[30] in 2013, is expressed as

$$D_{\mathrm{H}}(\rho^{AB}) = \min_{\Pi^A} \left\| \sqrt{\rho^{AB}} - \Pi^A\left(\sqrt{\rho^{AB}}\right) \right\|^2, \tag{3}$$

where the min is taken over all local von Neumann measurement $\Pi^A = \{\Pi_k^A\}$ on party A and $\Pi^A(\sqrt{\rho^{AB}}) = \sum_k (\Pi_k^A \otimes I^B)\sqrt{\rho^{AB}}(\Pi_k^A \otimes I^B)$. Consider a bipartite system, for parties A and B, belonging to H^A and H^B with dim $H^A = m$ and dim $H^B = n$. Let $L(H^A)$ be the space consisting of all linear operators on H^A. This is a Hilbert space with the Hilbert–Schmidt inner product $\langle X|Y\rangle = \mathrm{tr}X^+Y$. The Hilbert space $L(H^B)$ and $L(H^A \otimes H^B)$ are defined similarly. Let $\{X_i : i = 0,1,...,m^2-1\}$ and $\{Y_i : j = 0,1,...,n^2-1\}$ be the sets of Hermitian operators which constitute orthonormal bases for $L(H^A)$ and $L(H^B)$, respectively, satisfying $\mathrm{tr}X_iX_{i'} = \delta_{ii'}$ and $\mathrm{tr}Y_jY_{j'} = \delta_{jj'}$. Thus $\{X_i \otimes Y_j\}$ constitutes an orthonormal basis for $L(H^A \otimes H^B)$. Thus, $\sqrt{\rho^{AB}}$, the operator on $H^A \otimes H^B$, can be depicted as

$$\sqrt{\rho^{AB}} = \sum_{ij} \gamma_{ij} X_i \otimes Y_j, \tag{4}$$

where $\gamma_{ij} = \mathrm{tr}\sqrt{\rho^{AB}}(X_i \otimes Y_j)$. The correlation matrix $\sqrt{\rho^{AB}}$ can be defined as $\Gamma = (\gamma_{ij})_{i=0,1,...,m^2-1;j=0,1,...,n^2-1}$. As a result, if $m = 2$ (n can be arbitrary), $X_0 = I^A/\sqrt{2}$, $Y_0 = I^B/\sqrt{n}$, the MGD can be written as

$$D_{\mathrm{H}}(\rho^{AB}) = 1 - \|\boldsymbol{r}\|^2 - z_{\max}, \tag{5}$$

where $\boldsymbol{r} = (\gamma_{00}, \gamma_{01}, ..., \gamma_{0(n^2-1)})$, $\|\boldsymbol{r}\|^2 = \sum_j \gamma_{0j}^2$, and $z_{\max}$ is the largest eigenvalue of the (3×3)-dimensional matrix ZZ^{T} with $Z = (\gamma_{ij})_{i=1,2,3;j=0,1,...,n^2-1}$.

3. Trace distance correlation dynamics under different decoherence channels

In this section, we investigate the relation between the two square-norm distance correlation dynamic behaviors for the two-qubit states under different local decoherence channels. For simplicity, assume that the two-qubit state AB is in a class of state with a maximally mixed marginal, i.e. the Bell diagonal state, then the density operator of this two-qubit system AB in the standard operator base $\{I/\sqrt{2}, \sigma_j/\sqrt{2} : j = 1,2,3\}$ will be described by

$$\rho^{AB} = \frac{1}{4}\left(I^{AB} + \sum_j c_j \sigma_j^A \otimes \sigma_j^B\right), \tag{6}$$

where I^{AB} is the identity operator in a two-qubit Hilbert space and c_i $(0 \le |c_i| \le 1)$ are real numbers satisfying the unit trace and positivity conditions of the density operator. With the purpose of studying the dynamics of square-norm distance correlation, different cases for initial states are considered, such as the general initial state $(|c_1| = 0.7, |c_2| = 0.9, |c_3| = 0.8)$ and the Werner initial state $(|c_1| = 0.6, |c_2| = 0.6, |c_3| = 0.6)$. Subsequently, we focus our attention on the square-norm distance correlation dynamics under their different noisy environments, including PF, BF, and BPF channels.

3.1. Phase flip channel

Here, take the scenario of two qubits under a local identical PF channel,[39] in the operator-sum representation, then

the evolved state of such a system under decoherence will be written as[40]

$$\varepsilon(\rho^{AB}) = \sum_{i,j} \Gamma_i^{(A)} \Gamma_j^{(B)} \rho^{AB} \Gamma_j^{(B)+} \Gamma_i^{(A)+}, \tag{7}$$

where $\Gamma_i^{(k)}$ $(k = A, B)$ are the Kraus operators describing the phase channels A and B. For this channel, the Kraus operator can be represented as $\Gamma_0^{(A)} = \mathrm{diag}(\sqrt{1-p^A}, \sqrt{1-p^A}) \otimes I^B$, $\Gamma_1^{(A)} = \mathrm{diag}(\sqrt{p^A}, -\sqrt{p^A}) \otimes I^B$, $\Gamma_0^{(B)} = I^A \otimes \mathrm{diag}(\sqrt{1-p^B}, \sqrt{1-p^B})$, and $\Gamma_1^{(B)} = I^A \otimes \mathrm{diag}(\sqrt{p^B}, -\sqrt{p^B})$, which are written in the state computational basis $\{|0\rangle_k, |1\rangle_k\}$ $(k = A, B)$ by using $p^A = p^B = p$ and $p^{A(B)}$ $(0 \le p^{A(B)} \le 1)$ as parametrized time in channel $A(B)$. In addition, adopt $p = 1 - \exp(-\gamma t)$, where γ is the decay rate, then the quantum evolution operator $\varepsilon(\rho^{AB})$ will be gained as

$$\varepsilon(\rho^{AB}) = \begin{pmatrix} 1+c'_3 & 0 & 0 & c'_1-c'_2 \\ 0 & 1-c'_3 & c'_1+c'_2 & 0 \\ 0 & c'_1+c'_2 & 1-c'_3 & 0 \\ c'_1-c'_2 & 0 & 0 & 1+c'_3 \end{pmatrix}, \tag{8}$$

where $c'_1 = c_1(1-p)^2$, $c'_2 = c_2(1-p)^2$, and $c'_3 = c_3$.

According to Eqs. (2) and (5), we can easily compute the square-norm distance correlation and its modified version of Eq. (8) as follows:

$$\begin{aligned} D_{\mathrm{G}}[\varepsilon(\rho^{AB})] &= \frac{1}{4}(|c'_1|^2 + |c'_2|^2 + |c'_3|^2 \\ &\quad - \max\{|c'_1|^2, |c'_2|^2, |c'_3|^2\}), \\ D_{\mathrm{H}}[\varepsilon(\rho^{AB})] &= 1 - \frac{1}{4}(h^2 + \max\{|d_1|^2, |d_2|^2, |d_3|^2\}), \end{aligned} \tag{9}$$

where $h = \mathrm{tr}\sqrt{\varepsilon(\rho^{AB})} = \sqrt{\lambda_1} + \sqrt{\lambda_2} + \sqrt{\lambda_3} + \sqrt{\lambda_4}$, $d_1 = \sqrt{\lambda_1} - \sqrt{\lambda_2} + \sqrt{\lambda_3} - \sqrt{\lambda_4}$, $d_2 = -\sqrt{\lambda_1} + \sqrt{\lambda_2} + \sqrt{\lambda_3} - \sqrt{\lambda_4}$, and $d_3 = \sqrt{\lambda_1} + \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}$ with $\lambda_1 = (1 + c'_1 - c'_2 + c'_3)/4$, $\lambda_2 = (1 - c'_1 + c'_2 + c'_3)/4$, $\lambda_3 = (1 + c'_1 + c'_2 - c'_3)/4$, and $\lambda_4 = (1 - c'_1 - c'_2 - c'_3)/4$. To reveal the relative ordering between the two square-norm distance correlation measures, we show the plots of the time evolution of $D_{\mathrm{H}}[\varepsilon(\rho^{AB})]$ and $D_{\mathrm{G}}[\varepsilon(\rho^{AB})]$ versus p for Werner- and general-type initial states under the PF channel in Fig. 1. From the figure, we can easily find that the modified discord $D_{\mathrm{H}}[\varepsilon(\rho^{AB})]$ is always smaller than the GD $D_{\mathrm{G}}[\varepsilon(\rho^{AB})]$ for this given Werner and general types of initial state under the PF channel. The results reveal that for the general state, the GD gradually deceases with the decoherence parameter increasing, and a twist point exists about GD when $p^{\mathrm{SC}} = 1 - 2\sqrt{2}/3$. This is because in this case, $|c'_2|^2 > |c'_1|^2$ and $|c'_3|^2 > |c'_1|^2$ while there is no dominant relation between $|c'_2|^2$ and $|c'_3|^2$. With ρ^{AB} passing through the PF channel, we will initially have

$$D_{\mathrm{G}}[\varepsilon(\rho^{AB})] = \frac{1}{4}(|c'_1|^2 + |c'_3|^2),$$

subsequently, there will be a crossing involving $|c'_2|^2 = |c'_3|^2$ for a critical point $p = p^{\mathrm{SC}}$, at last

$$D_{\mathrm{G}}[\varepsilon(\rho^{AB})] = \frac{1}{4}(|c'_1|^2 + |c'_2|^2).$$

However, for the Werner state, the GD just slowly decays to zero. On the other hand, the MGD first has a large freezing interval, then shows a sudden change in the time evolution. In fact, the smaller the difference between the $|c_1|$, $|c_2|$, $|c_3|$, the longer the freezing interval is. More importantly, we obtain that the larger the c, the longer the freezing interval and the less MGD will be when $|c_1| = |c_2| = |c_3| = c$. This indicates that the decrease of MGD is at the expense of making an effort to gain the big freezing interval. The existence of the freezing interval seems to suggest that the MGD is less sensitive to the external environment than GD. Interestingly, it is worth noticing that the twist point of the MGD is the same as that of the GD. In particular, the results show that the GD can be either smaller than, or equal to, or larger than its modified one, and there is no universal leading relative ordering between them.

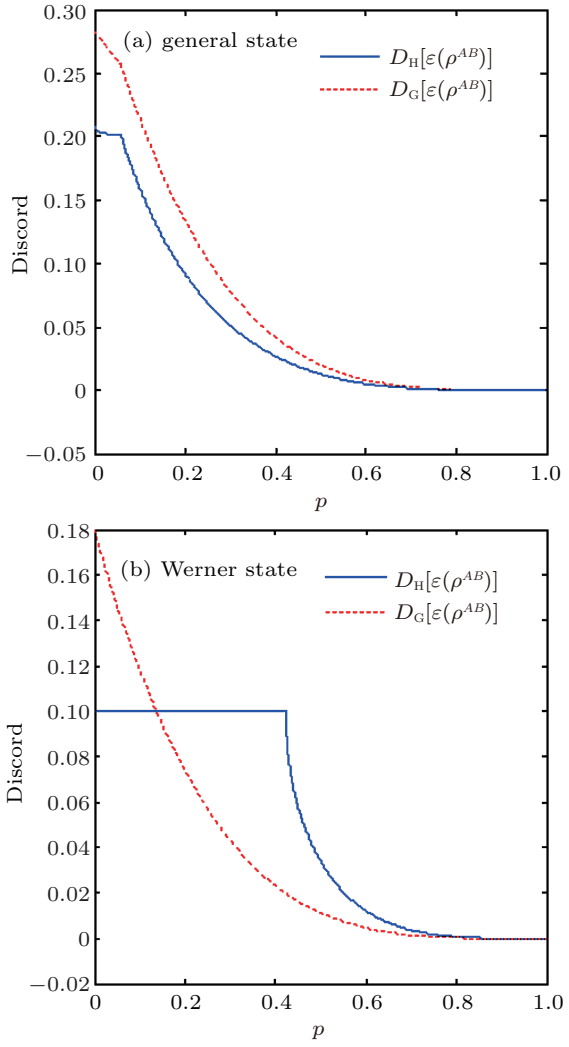


Fig. 1. (color online) Comparison between the two discord measures $D_{\mathrm{H}}[\varepsilon(\rho^{AB})]$ (solid line) and $D_{\mathrm{G}}[\varepsilon(\rho^{AB})]$ (dashed line) for Werner- and general-types of initial states under the PF channel.

3.2. Bit flip and bit-flip channels

Next, we come to the dynamical evolution of the system under the BF and BPF channels. Similarly, we have listed the set of Kraus operators for BF and BPF channels in Table 1.

Table 1. Kraus operators for the BF and BPF channels, where p is decoherence probability.

	Kraus operator
BF	$\Gamma_0 = \sqrt{1-p/2}I,\ \Gamma_1 = \sqrt{p/2}\sigma_1$
BPF	$\Gamma_0 = \sqrt{1-p/2}I,\ \Gamma_1 = \sqrt{p/2}\sigma_2$

In this context, the Bell diagonal form of quantum evolution operator $\varepsilon(\rho^{AB})$ is preserved in Eq. (8), while the values of the correlation vector $\boldsymbol{c}' = (c_1', c_2', c_3')$ are different, which have been given in Table 2.

Table 2. Correlation functions for the BF and BPF channels.

Channels	c_1'	c_2'	c_3'
BF	c_1	$c_2(1-p)^2$	$c_3(1-p)^2$
BPF	$c_1(1-p)^2$	c_2	$c_3(1-p)^2$

Hence, it is easy to compute the two square-norm distance correlations for the general state and Werner state versus the decoherence parameter under BF and BPF channels, and owing to the tedious expressions of the two physical quantities, we will not write them here. The dynamic behaviors of GD and its modified one as a function of the decoherence parameter p for Werner- and general-types of initial states in the cases of BF and BPF channels are shown in Fig. 2. As we can see, the dynamic behaviors of the two GDs under the BF channel are symmetrical to that for the PF channel. The condition for this type of dynamics is obtained by swapping c_1 and c_3 in the PF channel. The MGD can always exhibit freezing behaviors and sudden change in the time evolution. Actually, the so-called "sudden change of GD" is just a signature of the jump of the time derivative of that function in a specific instant. On the contrary, for the BPF channel, both the two GDs of general state decay asymptotically to zero, and there is no sudden change. This can be explained as being due to the fact that the inequalities $|c_2'|^2 > |c_1'|^2$ and $|c_2'|^2 > |c_3'|^2$ remain in the process of time evolution. Moreover, we can find that for the Werner initial state, the dynamic behaviors of the two square-norm distance correlations under the three noisy environments are the same. In particular, from the figure, it is obviously observed that the MGD may effectively exhibit sudden change or freezing behaviors under the action of local noise channels. As is well known, the geometric measure of QD can be explained as a minimal distance from the set of the classical–quantum (CQ) state. When the distance to the CQ state remains constant the distance function is constant, while there will appear a sudden change at the transition point when the distance function is optimized by the time-dependent CQ state in a certain dynamical evolution. This reveals that the location of the CQ state changes in a discontinuous way in the vicinity of the transition point. Actually, the derivation of such features came about as a consequence of the negligibility of the set of zero discord states in comparison with the whole state space, which renders the robustness of GD decoherent. To sum up, the results convince us that the MGD is more robust than the original one against decoherence.

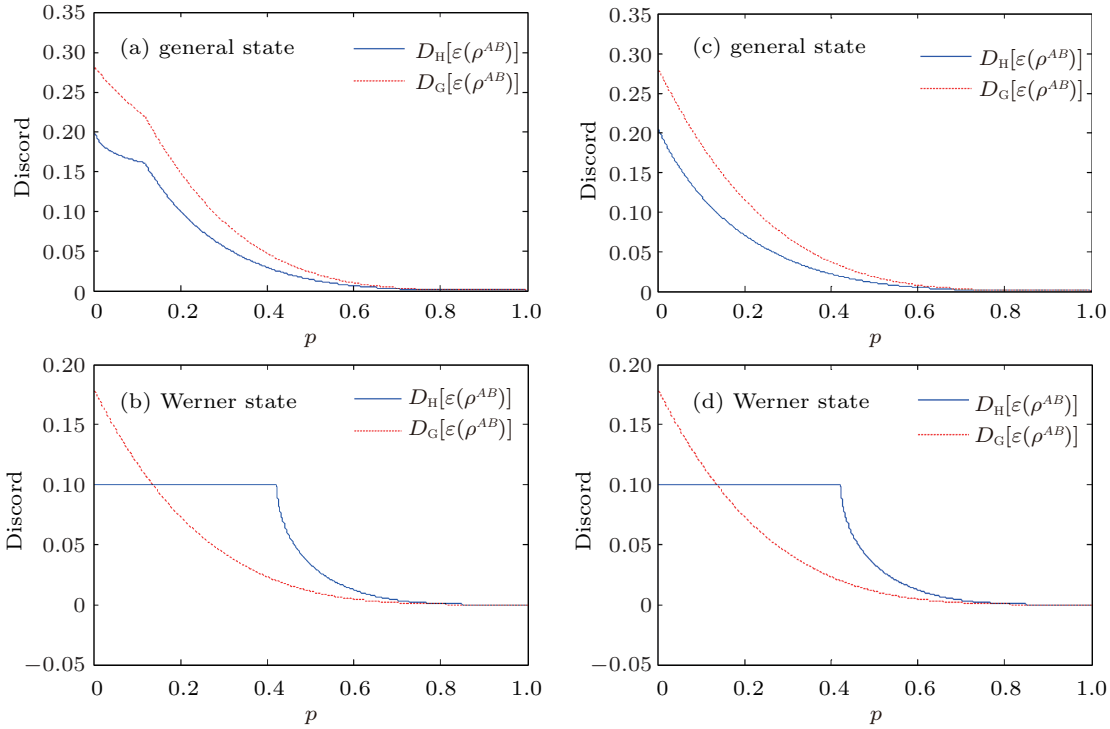


Fig. 2. (color online) Comparison between the two discord measures $D_H[\varepsilon(\rho^{AB})]$ (solid line) and $D_G[\varepsilon(\rho^{AB})]$ (dashed line) for Werner- and general-types of initial states under BF [(a) and (b)] and BPF [(c) and (d)] channels.

4. Conclusions

In this paper, we investigated the square-norm distance correlation dynamics for two-qubit states under different local decoherence channels, including PF, BF, and BPF channels. The Werner- and general-types of initial states are considered. The results show that there is no universal dominant relationship between the two distance correlation measures by using the GD and MGD. Meanwhile, the results show that the GD just slowly decays to zero, while MGD initially exhibits a freezing behavior and subsequently shows a sudden change in the process of time evolution. Furthermore, the results show that the longer the freezing interval, the less the MGD is. Interestingly, for the Werner initial state, there are the same dynamic behaviors between GD and MGD under the PF, BF, and BPF channels. According to the above analysis, we can draw a conclusion that the MGD is more robust than GD against decoherence. Therefore, the MGD can be regarded as a suitable quantification to capture the nonclassical correlations of the physical system. We believe that the results presented here will be helpful to further shed light on the role that the square-norm distance correlation plays in the quantum information processing tasks.

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