# Output regulation for linear multi-agent systems with unmeasurable nodes*

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The output regulation of linear multi-agent systems with partial unmeasurable agents is investigated in this paper. All the agents except the exosystem can be classified into two groups. Agents in the first group can be measured by themselves and their neighbors. State variables are not fully accessible for direct communication and full order Luenberger observers are constructed for the unmeasurable agents. We give a state feedback control law to solve the output regulation problem under the communication topologies based on both measurable and unmeasurable agents. The heterogeneous agents' synchronization problem is a general case of our results. Finally, examples are utilized to show the effectiveness of the obtained results.

**Keywords:** multi-agent systems, output regulation, full order observers

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## 1. Introduction

Complex systems have experienced deep development in recent decades. In such systems, agents interact with other agents to perform tasks beyond the ability of individuals. Obviously, multi-agent systems have advantages over the conventional single control system on reducing cost, improving system efficiency and producing new property.[1–10]

A particularly interesting topic is called the leader–following consensus problem, in which the leader is a particular agent whose motion is independent of the other agents and followed by the others.[11–22] A leader–following consensus problem of a group of autonomous agents with time-varying coupling delays was considered in Ref. [11]. It gave a necessary and sufficient condition in the case when the interconnection topology was fixed and directed; it also gave a sufficient condition in the case when the coupling topology was switched and balanced. The authors in Ref. [18] gave a distributed consensus algorithm for the second-order leader–following problem with unmeasurable velocity of the active leader in real time and designed distributed observers for these unmeasurable velocities. In Ref. [19], time-domain and frequency-domain approaches were used to study the first-order and the second-order leaderless and leader–following consensus algorithms with input delays under a directed network topology. Reference [20] dealt with the leader–follower formations of nonholonomic mobile robots. In Ref. [21], a distributed observer was designed with which the agents could achieve and maintain the desired formation from any initial states and expanded the leader–following problem on formation control. The extension of the case that the input of an agent could only use the information measured at the sampling instants from its neighbors or the virtual leader with a time-varying reference state was provided in Ref. [22].

The output regulation can be seen as a special leader–following problem in which the outputs of the agents track the output of the exosystem. It is an important and interesting problem in control theory which is to control fixed plants so that their output tracks a reference signal (and/or rejects a disturbance) produced by an exogenous system.[23–33] Reference [23] has shown that the solvability of a multi-variable linear regulator problem corresponds to the solvability of a system of two linear matrix equations, which are called regulator equations. Byrnes[24] considered linear distributed parameter systems and provided simple criteria for the solvability of the regulator equations based on the eigenvalues of the exosystem and the system transfer function. In Ref. [25], the output regulation and disturbance rejection were achieved by using an adaptive internal model. For nonlinear multi-agent systems, it has been shown in Ref. [26] that the nonlinear distributed coordinative control can be designed to make the agents asymptotically track the reference and/or rejecting the disturbance. A distributed control approach for an uncertain multi-agent system was given in Ref. [30], in which not all the measurement output could be influenced by the exosystem. The dynamic state feedback control and the dynamic measurement output feedback control were used in Ref. [31] to realize the regulator problem under a switching network. It was shown in Ref. [32] that a distributed measurement output feedback control can be used for the cooperative output regulation of linear multi-agent systems.

The work in Ref. [29] gave a dynamic compensator to handle the cooperative output regulation of linear multi-agent

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systems; it was assumed that all the agents' states are measurable. However, in most practical applications, since sensing devices or transducers are damaged or too expensive, not all the agents' states are completely accessible, or they are no longer available in the current work. The designer only knows the measurement output and input. Hence, observers often appear in multi-agent problems and the estimation of the agents' states are often necessary in the output regulator problem. Motivated by the above discussion, this paper continues the work in Ref. [29] and deals with the problem about partial unmeasurable agents. In comparison with Ref. [29], full order Luenberger observers are designed for unmeasurable agents. In this paper, a dynamic compensator about the exosystem depending on both measurable and unmeasurable states under communication topologies is also considered. This paper is organized as follows. Section 2 introduces the system's dynamic and some necessary preliminaries in the graph theory, and the problem of the output regulation is proposed. The main results are presented in Section 3. Some examples are given in Section 4 and a conclusion is drawn in Section 5.

Some notations used are as follows.[34] Let $\mathbf{R}$ be the set of real numbers, $\mathbf{R}^n$ is the $n$-dimensional vector space and $\mathbf{R}^{n\times n}$ is the matrix space of dimension $n$. For a given vector or matrix $\boldsymbol{A}$, $\boldsymbol{A}^{\mathrm{T}}$ denotes its transpose. The $\mathbf{0}$ denotes a matrix which has appropriate dimensions and all its elements are 0; $\mathbf{1}_N$ represents $(1,1,\ldots,1)^{\mathrm{T}}$ with dimension $N$; $\boldsymbol{A}\otimes\boldsymbol{B}$ denotes the Kronecker product of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. The Kronecker sum of $\boldsymbol{A}\in\mathbf{R}^{n\times n}$ and $\boldsymbol{B}\in\mathbf{R}^{m\times m}$ is defined as $\boldsymbol{A}\oplus\boldsymbol{B}=(\boldsymbol{A}\otimes\boldsymbol{I}_m)+(\boldsymbol{I}_n\otimes\boldsymbol{B})$. The Kronecker product has the following properties:

(1) $(\boldsymbol{A}+\boldsymbol{B})\otimes\boldsymbol{C}=\boldsymbol{A}\otimes\boldsymbol{C}+\boldsymbol{B}\otimes\boldsymbol{C}$,
$\boldsymbol{C}\otimes(\boldsymbol{A}+\boldsymbol{B})=\boldsymbol{C}\otimes\boldsymbol{A}+\boldsymbol{C}\otimes\boldsymbol{B}$;

(2) $(\boldsymbol{A}\otimes\boldsymbol{C})(\boldsymbol{B}\otimes\boldsymbol{D})=\boldsymbol{A}\boldsymbol{B}\otimes\boldsymbol{C}\boldsymbol{D}$;

(3) the eigenvalues of $\boldsymbol{A}\oplus\boldsymbol{B}$ are $mn$ numbers $\lambda_i+\mu_j$, $i=1,2,\ldots,n, j=1,2,\ldots,m$, where $\boldsymbol{A}\in\mathbf{R}^{n\times n}$, $\boldsymbol{B}\in\mathbf{R}^{m\times m}$ and $\boldsymbol{C},\boldsymbol{D}$ have appropriate dimensions, $\lambda_i$ are the eigenvalues of $\boldsymbol{A}$ and $\mu_j$ are the eigenvalues of $\boldsymbol{B}$.

## 2. Problem formulation and preliminaries

### 2.1. Agent's dynamics and algebraic graph theory

Suppose that the multi-agent systems under consideration consist of $N$ agents. Agent $i$ is assumed to have the following dynamics:

$$\begin{aligned}\dot{\boldsymbol{x}}_i(t)&=\boldsymbol{A}_i\boldsymbol{x}_i(t)+\boldsymbol{B}_i\boldsymbol{u}_i(t)+\boldsymbol{D}_i(t),\\ \boldsymbol{y}_i(t)&=\boldsymbol{C}_i\boldsymbol{x}_i(t),\quad i=1,2,\ldots,N,\end{aligned}\tag{1}$$

where $\boldsymbol{x}_i\in\mathbf{R}^n$ is the state of the $i$-th subsystem, $\boldsymbol{u}_i\in\mathbf{R}^m$ is the consensus protocol to be designed under the network topology, $\boldsymbol{D}_i(t)$ represents a disturbance and $\boldsymbol{y}_i\in\mathbf{R}^p$ is the measurement output.

In addition, assume that there exists an external finite dimensional linear system called the exosystem, representing the reference inputs and/or the disturbances, which is assumed to be generated as

$$\begin{aligned}\dot{\boldsymbol{\omega}}(t)&=\boldsymbol{\Gamma}\boldsymbol{\omega}(t),\\ \boldsymbol{D}_i(t)&=\boldsymbol{E}_i\boldsymbol{\omega}(t),\end{aligned}\tag{2}$$

where $\boldsymbol{\omega}\in\mathbf{R}^q$ is the state of the exosystem and $\boldsymbol{E}_i\in\mathbf{R}^{n\times q}$ is associated with the description of the disturbance channel. Then the reference output is expressed as

$$\boldsymbol{y}_{\mathrm{ri}}(t)=\boldsymbol{Q}_i\boldsymbol{\omega}(t),$$

with $y_{ri}(t)\in\mathbf{R}^p$ and $\boldsymbol{Q}_i\in\mathbf{R}^{p\times q}$.

We denote the error between measurement output and reference output as

$$\begin{aligned}\boldsymbol{e}_i(t)&=\boldsymbol{y}_i(t)-\boldsymbol{y}_{\mathrm{ri}}(t)\\ &=\boldsymbol{C}_i\boldsymbol{x}_i(t)-\boldsymbol{Q}_i\boldsymbol{\omega}(t).\end{aligned}\tag{3}$$

The network topology of multi-agent system (1) is usually described as a weighted digraph. The information flow between neighboring agents is represented by a directed edge between the neighboring nodes. Thus, the graph theory is an important tool to analyze the multi-agent systems.

In this section, some preliminary graph theories[35] are presented. Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a weighted directed graph of order $N+1$ with a vertex set $\mathcal{V}=\{v_0,v_1,\ldots,v_N\}$, an edge set $\mathcal{E}=\{e_{ij}=(v_i,v_j)\}\in\mathcal{V}\times\mathcal{V}$ and a weighted adjacency matrix $\mathcal{A}=[a_{ij}]$ with nonnegative adjacency elements $a_{ij}$. If the edge $e_{ij}=(v_i,v_j)\in\mathcal{E}$ , then $a_{ij}>0$, else $a_{ij}=0$. Moreover, we assume $a_{ii}=0$. The set of neighbors of node $v_i$ is denoted as $\mathcal{N}_i=\{v_j\in\mathcal{V}:(v_i,v_j)\in\mathcal{E}\}$. An edge $e_{ij}=(v_i,v_j)\in\mathcal{E}$ indicates that agent $j$ can receive information from agent $i$, $v_i$ is defined as the parent node and $v_j$ is defined as the child node. The degree matrix $\mathcal{D}=[d_{ij}]$ is a diagonal matrix with $d_{ii}=\Sigma_{j=1}^{n}a_{ij}$. The Laplacian matrix with the directed graph is defined as $\mathcal{L}=\mathcal{D}-\mathcal{A}$. Obviously, all the row sums of $\mathcal{L}$ are zero. The node indexed by 0 refers to the exosystem and the other nodes correspond to the agents to be regulated in this paper. The exosystem only sends information to other agents and cannot receive any information from other agents, i.e., $a_{01}=a_{02}=\cdots=a_{0N}=0$ and $a_{i0}>0$ if agent $i$ can receive information from node 0, else $a_{i0}=0$. A digraph is called a directed tree if every node, except the root (node 0), has exactly one parent. A spanning tree is a directed tree that connects all the nodes of the graph. A digraph contains a direct spanning tree if a subset of the edges forms a spanning tree.

### 2.2. Problem statement

A digraph $\bar{\mathcal{G}}=(\bar{\mathcal{V}},\bar{\mathcal{E}},\bar{\mathcal{A}})$ which is used to label the agents except the exosystem is defined as a subgraph of $\mathcal{G}$ with the vertex set $\bar{\mathcal{V}}=\{1,2,\ldots,N\}$ . Agent $i$ can receive signals from itself and its neighbors. Without loss of generality, it

is assumed the first $l$ nodes $\boldsymbol{x}_i$, $i=1,2,\ldots,l$, $1\le l\le N$, in subgraph $\bar{\mathcal{G}}$ are measurable agents and the others are unmeasurable. An observer is a dynamical system with state $\bar{\boldsymbol{x}}_i$, $i=l+1,l+2,\ldots,N$, which asymptotically estimates the state $\boldsymbol{x}_i$, $i=l+1,l+2,\ldots,N$. More precisely, a full order Luenberger observer is given by

$$\begin{aligned}\dot{\bar{\boldsymbol{x}}}_i &= \boldsymbol{A}_i\bar{\boldsymbol{x}}_i+\boldsymbol{B}_i\boldsymbol{u}_i+\boldsymbol{E}_i\boldsymbol{\omega}+\boldsymbol{L}_i(\boldsymbol{C}_i\bar{\boldsymbol{x}}_i-\boldsymbol{C}_i\boldsymbol{x}_i),\\ &\quad i=l+1,l+2,\ldots,N,\end{aligned}\tag{4}$$

where $\boldsymbol{L}_i$ are the gain matrices to be determined later.

**Remark 1** In practical applications, agents' state variables are not fully unmeasurable. Some agents have both measurable and unmeasurable variables, and it may be difficult to distinguish them. Full order observers are given for all the cases. These observers have some advantages. Unmeasurable variables could be observed and be used in the state feedback control. For measurable variables, observers could be expected to have the effect of filters and make the state information more complete.

A dynamic compensator used for both measurable and unmeasurable states is expressed as

$$\begin{aligned}\dot{\zeta}_i(t) &= \boldsymbol{\Gamma}\zeta_i(t)+\alpha\Big(\sum_{j\in\mathcal{N}_i}a_{ij}(\zeta_i(t)-\zeta_j(t))\\ &\quad +a_{i0}(\zeta_i(t)-\boldsymbol{\omega}(t))\Big).\end{aligned}\tag{5}$$

Note that the dynamics of $\zeta_i\in\mathbf{R}^q$ also depends on $\zeta_j, j\in\mathcal{N}_i$ under a communicate topology, so Eq. (5) can always be seen as a distributed observer. Parameter $\alpha$ is an arbitrary constant, which will be used later.

With the full order Luenberger observer, a state feedback control law is given by

$$\begin{cases}\boldsymbol{u}_i=\boldsymbol{K}_{1i}\boldsymbol{x}_i+\boldsymbol{K}_{2i}\zeta_i, & i=1,2,\ldots,l,\\ \boldsymbol{u}_i=\boldsymbol{K}_{1i}\bar{\boldsymbol{x}}_i+\boldsymbol{K}_{2i}\zeta_i, & i=l+1,l+2,\ldots,N,\end{cases}\tag{6}$$

where $\boldsymbol{K}_{1i}\in\mathbf{R}^{m\times n}$ and $\boldsymbol{K}_{2i}\in\mathbf{R}^{m\times q}$ are some constant matrices to be designed.

Let

$$\begin{aligned}&\hat{\boldsymbol{x}}=(\boldsymbol{x}_1^{\mathrm{T}},\boldsymbol{x}_2^{\mathrm{T}},\ldots,\boldsymbol{x}_l^{\mathrm{T}})^{\mathrm{T}},\\ &\zeta=(\zeta_1^{\mathrm{T}},\zeta_2^{\mathrm{T}},\ldots,\zeta_N^{\mathrm{T}})^{\mathrm{T}},\ \ \tilde{\boldsymbol{\omega}}=\mathbf{1}_N\otimes\boldsymbol{\omega},\\ &\check{\boldsymbol{x}}=(\boldsymbol{x}_{l+1}^{\mathrm{T}},\boldsymbol{x}_{l+2}^{\mathrm{T}},\ldots,\boldsymbol{x}_N^{\mathrm{T}},\bar{\boldsymbol{x}}_{l+1}^{\mathrm{T}},\bar{\boldsymbol{x}}_{l+2}^{\mathrm{T}},\ldots,\bar{\boldsymbol{x}}_N^{\mathrm{T}})^{\mathrm{T}},\end{aligned}$$

substituting Eq. (6) into Eqs. (1) and (4) and then combining with Eq. (5) yield the following dynamic closed-loop system:

$$\begin{aligned}\dot{\hat{\boldsymbol{x}}} &= \hat{\boldsymbol{A}}_{\mathrm{c}}\hat{\boldsymbol{x}}+\hat{\boldsymbol{B}}_{\mathrm{c}}\zeta+\hat{\boldsymbol{E}}_{\mathrm{c}}\tilde{\boldsymbol{\omega}},\\ \dot{\check{\boldsymbol{x}}} &= \check{\boldsymbol{A}}_{\mathrm{c}}\check{\boldsymbol{x}}+\check{\boldsymbol{B}}_{\mathrm{c}}\zeta+\check{\boldsymbol{E}}_{\mathrm{c}}\tilde{\boldsymbol{\omega}},\\ \dot{\zeta} &= (\alpha\boldsymbol{H}\oplus\boldsymbol{\Gamma})\zeta-\alpha(\boldsymbol{A}_0\otimes\boldsymbol{I}_q)\tilde{\boldsymbol{\omega}},\end{aligned}\tag{7}$$

where $\boldsymbol{A}_0=\mathrm{diag}\{a_{10},a_{20},\ldots,a_{N0}\}$, $\boldsymbol{H}=\bar{\boldsymbol{L}}+\boldsymbol{A}_0$, $\bar{\boldsymbol{L}}$ is the Laplacian matrix of digraph $\bar{\mathcal{G}}$, and

$$\hat{\boldsymbol{A}}_{\mathrm{c}}=\hat{\boldsymbol{B}}+\hat{\boldsymbol{B}}\hat{\boldsymbol{K}}_1,\quad \hat{\boldsymbol{B}}_{\mathrm{c}}=\begin{pmatrix}\hat{\boldsymbol{B}}\hat{\boldsymbol{K}}_2 & \mathbf{0}\end{pmatrix},\quad \hat{\boldsymbol{E}}_{\mathrm{c}}=\begin{pmatrix}\hat{\boldsymbol{E}} & \mathbf{0}\end{pmatrix},$$

$$\check{\boldsymbol{A}}_{\mathrm{c}}=\begin{pmatrix}\check{\boldsymbol{A}} & \check{\boldsymbol{B}}\check{\boldsymbol{K}}_1\\ -\check{\boldsymbol{L}}\check{\boldsymbol{C}} & \check{\boldsymbol{A}}+\check{\boldsymbol{B}}\check{\boldsymbol{K}}_1+\check{\boldsymbol{L}}\check{\boldsymbol{C}}\end{pmatrix},$$

$$\check{\boldsymbol{B}}_{\mathrm{c}}=\mathbf{1}_2\otimes\begin{pmatrix}\mathbf{0} & \check{\boldsymbol{B}}\check{\boldsymbol{K}}_2\end{pmatrix},\quad \check{\boldsymbol{E}}_{\mathrm{c}}=\mathbf{1}_2\otimes\begin{pmatrix}\mathbf{0} & \check{\boldsymbol{E}}\end{pmatrix},$$

with

$$\begin{aligned}&\hat{\boldsymbol{A}}=\mathrm{diag}\{\boldsymbol{A}_1,\boldsymbol{A}_2\ldots,\boldsymbol{A}_l\};\\ &\hat{\boldsymbol{B}}=\mathrm{diag}\{\boldsymbol{B}_1,\boldsymbol{B}_2\ldots,\boldsymbol{B}_l\};\\ &\hat{\boldsymbol{E}}=\mathrm{diag}\{\boldsymbol{E}_1,\boldsymbol{E}_2\ldots,\boldsymbol{E}_l\};\\ &\check{\boldsymbol{A}}=\mathrm{diag}\{\boldsymbol{A}_{l+1},\boldsymbol{A}_{l+2}\ldots,\boldsymbol{A}_N\};\\ &\check{\boldsymbol{B}}=\mathrm{diag}\{\boldsymbol{B}_{l+1},\boldsymbol{B}_{l+2}\ldots,\boldsymbol{B}_N\};\\ &\check{\boldsymbol{C}}=\mathrm{diag}\{\boldsymbol{C}_{l+1},\boldsymbol{C}_{l+2}\ldots,\boldsymbol{C}_N\};\\ &\check{\boldsymbol{E}}=\mathrm{diag}\{\boldsymbol{E}_{l+1},\boldsymbol{E}_{l+2}\ldots,\boldsymbol{E}_N\};\\ &\check{\boldsymbol{L}}=\mathrm{diag}\{\boldsymbol{L}_{l+1},\boldsymbol{L}_{l+2}\ldots,\boldsymbol{L}_N\};\\ &\hat{\boldsymbol{K}}_1=\mathrm{diag}\{\boldsymbol{K}_{11},\boldsymbol{K}_{12},\ldots,\boldsymbol{K}_{1l}\};\\ &\hat{\boldsymbol{K}}_2=\mathrm{diag}\{\boldsymbol{K}_{21},\boldsymbol{K}_{22},\ldots,\boldsymbol{K}_{2l}\};\\ &\check{\boldsymbol{K}}_1=\mathrm{diag}\{\boldsymbol{K}_{1(l+1)},\boldsymbol{K}_{1(l+2)},\ldots,\boldsymbol{K}_{1N}\};\\ &\check{\boldsymbol{K}}_2=\mathrm{diag}\{\boldsymbol{K}_{2(l+1)},\boldsymbol{K}_{2(l+2)},\ldots,\boldsymbol{K}_{2N}\},\end{aligned}$$

and $\mathbf{1}_2=\begin{pmatrix}1 & 1\end{pmatrix}^{\mathrm{T}}$. Let $\boldsymbol{\chi}=(\hat{\boldsymbol{x}}^{\mathrm{T}},\check{\boldsymbol{x}}^{\mathrm{T}},\zeta^{\mathrm{T}})^{\mathrm{T}}$ and

$$\begin{aligned}\mathcal{A}_{\mathrm{c}}&=\begin{pmatrix}\hat{\boldsymbol{A}}_{\mathrm{c}} & \mathbf{0} & \hat{\boldsymbol{B}}_{\mathrm{c}}\\ \mathbf{0} & \check{\boldsymbol{A}}_{\mathrm{c}} & \check{\boldsymbol{B}}_{\mathrm{c}}\\ \mathbf{0} & \mathbf{0} & \alpha\boldsymbol{H}\oplus\boldsymbol{\Gamma}\end{pmatrix},\\ \mathcal{E}_{\mathrm{c}}&=\begin{pmatrix}\hat{\boldsymbol{E}}_{\mathrm{c}}\\ \check{\boldsymbol{E}}_{\mathrm{c}}\\ -\alpha(\boldsymbol{A}_0\otimes\boldsymbol{I}_q)\end{pmatrix}.\end{aligned}\tag{8}$$

Closed-loop system (7) is rewritten as

$$\dot{\boldsymbol{\chi}}=\mathcal{A}_{\mathrm{c}}\boldsymbol{\chi}+\mathcal{E}_{\mathrm{c}}\tilde{\boldsymbol{\omega}}.\tag{9}$$

**Definition 1** The cooperative output regulation of multi-agent systems can be solved by state feedback control law (6) if the following two conditions are satisfied:

1) The closed-loop system (9) is asymptotically stable without disturbance, i.e., all the eigenvalues of matrix $\mathcal{A}_{\mathrm{c}}$ are assigned in the open left half plane.

2) For all initial conditions $(\boldsymbol{x}_i(0),\boldsymbol{\omega}(0))$, the solution $(\boldsymbol{x}_i(t),\boldsymbol{\omega}(t))$ of

$$\begin{aligned}\dot{\boldsymbol{\chi}} &= \mathcal{A}_{\mathrm{c}}\boldsymbol{\chi}+\mathcal{E}_{\mathrm{c}}\tilde{\boldsymbol{\omega}},\\ \dot{\boldsymbol{\omega}} &= \boldsymbol{\Gamma}\boldsymbol{\omega}\end{aligned}$$

also satisfies

$$\lim_{t\to\infty}\boldsymbol{e}_i(t)=\lim_{t\to\infty}(\boldsymbol{C}_i\boldsymbol{x}_i(t)-\boldsymbol{Q}_i\boldsymbol{\omega}(t))=0.\tag{10}$$

## 3. Main results

The following standard assumptions are necessary to solve the output regulation problem.

**Assumption 1** The pairs $(\boldsymbol{A}_i, \boldsymbol{B}_i)$, $i = 1,2,\ldots,N$ are stabilizable, i.e., there exist matrices $\boldsymbol{K}_i$ such that $\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_i$ are Hurwitz.

**Assumption 2** The pairs $(\boldsymbol{C}_i, \boldsymbol{A}_i)$, $i = l+1, l+2,\ldots,N$ are detectable, i.e., $\boldsymbol{A}_i + \boldsymbol{L}_i\boldsymbol{C}_i$ are stable for some $\boldsymbol{L}_i$.

**Assumption 3** All the eigenvalues of $\boldsymbol{\Gamma}$ do not have negative real parts.

The following lemma appeared in Refs. [11] and [29] is also necessary.

**Lemma 1** Matrix $\boldsymbol{H} = \bar{\boldsymbol{L}} + \boldsymbol{A}_0$ is positive stable if and only if digraph $\mathcal{G}$ contains a spanning tree and node 0 as its root.

**Theorem 1** If Assumptions 1 and 2 hold, then $\hat{\boldsymbol{A}}_{\rm c}$ and $\check{\boldsymbol{A}}_{\rm c}$ defined in Eq. (7) are Hurwitz.

**Proof** If the pairs $(\boldsymbol{A}_i, \boldsymbol{B}_i)$, $i = 1,2,\ldots,N$ are stabilizable and the pairs $(\boldsymbol{C}_i, \boldsymbol{A}_i)$, $i = l+1, l+2,\ldots,N$ are detectable, then there exist matrices $\boldsymbol{K}_{1i}$, $i = 1,2,\ldots,N$, $\boldsymbol{L}_i$, $i = l+1, l+2,\ldots,N$ such that $\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_{1i}$ and $\boldsymbol{A}_i + \boldsymbol{L}_i\boldsymbol{C}_i$ are Hurwitz.

Obviously, $\hat{\boldsymbol{A}}_{\rm c} = \hat{\boldsymbol{B}} + \hat{\boldsymbol{B}}\hat{\boldsymbol{K}}_1$ is Hurwitz if and only if $\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_{1i}$ are Hurwitz.

Let

$$\boldsymbol{S} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{I} \end{pmatrix}, \quad \check{\boldsymbol{A}}_{\rm cs} = \begin{pmatrix} \check{\boldsymbol{A}} + \check{\boldsymbol{B}}\check{\boldsymbol{K}}_1 & \check{\boldsymbol{B}}\check{\boldsymbol{K}}_1 \\ \boldsymbol{0} & \check{\boldsymbol{A}} + \check{\boldsymbol{L}}\check{\boldsymbol{C}} \end{pmatrix},$$

then we have $\check{\boldsymbol{A}}_{\rm c} = \boldsymbol{S}\check{\boldsymbol{A}}_{\rm cs}\boldsymbol{S}^{-1}$, i.e., $\check{\boldsymbol{A}}_{\rm c}$ is similar to $\check{\boldsymbol{A}}_{\rm cs}$. Then $\check{\boldsymbol{A}}_{\rm c}$ is Hurwitz if and only if $\check{\boldsymbol{A}}_{\rm cs}$ is Hurwitz.

**Theorem 2** Let Assumptions 1–3 hold, if there exist $\boldsymbol{\Pi}_i$ and $\boldsymbol{U}_i$ for all $i = 1,2,\ldots,N$ satisfying the following regulator equations:

$$\begin{aligned} \boldsymbol{\Pi}_i\boldsymbol{\Gamma} &= \boldsymbol{A}_i\boldsymbol{\Pi}_i + \boldsymbol{B}_i\boldsymbol{U}_i + \boldsymbol{E}_i, \\ 0 &= \boldsymbol{C}_i\boldsymbol{\Pi}_i - \boldsymbol{Q}_i, \end{aligned} \tag{11}$$

and digraph $\mathcal{G}$ contains a spanning tree with root 0, then the state feedback control law (6) solves the output regulation problem of partial unmeasurable linear multi-agent systems.

**Proof** A coordinate transformation is represented by $\boldsymbol{z}_i = \boldsymbol{x}_i - \boldsymbol{\Pi}_i\boldsymbol{\omega}$, $i = 1,2,\ldots,N$, $\bar{\boldsymbol{z}}_i = \bar{\boldsymbol{x}}_i - \boldsymbol{\Pi}_i\boldsymbol{\omega}$, $i = l+1, l+2,\ldots,N$, with $\boldsymbol{\Pi}_i$ as the solution of Eq. (11). Let $\hat{\boldsymbol{z}} = (\boldsymbol{z}_1^{\rm T}, \boldsymbol{z}_2^{\rm T}, \ldots, \boldsymbol{z}_l^{\rm T})^{\rm T}$ and $\check{\boldsymbol{z}} = (\boldsymbol{z}_{l+1}^{\rm T}, \boldsymbol{z}_{l+2}^{\rm T}, \ldots, \boldsymbol{z}_N^{\rm T}, \bar{\boldsymbol{z}}_{l+1}^{\rm T}, \bar{\boldsymbol{z}}_{l+2}^{\rm T}, \ldots, \bar{\boldsymbol{z}}_N^{\rm T})^{\rm T}$, then

$$\hat{\boldsymbol{z}} = \hat{\boldsymbol{x}} - \begin{pmatrix} \hat{\boldsymbol{\Pi}} & \boldsymbol{0} \end{pmatrix}\tilde{\boldsymbol{\omega}}, \quad \check{\boldsymbol{z}} = \check{\boldsymbol{x}} - \boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & \check{\boldsymbol{\Pi}} \end{pmatrix}\tilde{\boldsymbol{\omega}}, \tag{12}$$

where

$$\begin{aligned} \hat{\boldsymbol{\Pi}} &= \mathrm{diag}\{\boldsymbol{\Pi}_1, \boldsymbol{\Pi}_2, \ldots, \boldsymbol{\Pi}_l\}, \\ \check{\boldsymbol{\Pi}} &= \mathrm{diag}\{\boldsymbol{\Pi}_{l+1}, \boldsymbol{\Pi}_{l+2}, \ldots, \boldsymbol{\Pi}_N\}. \end{aligned}$$

Taking the derivative of Eq. (12) yields

$$\dot{\hat{\boldsymbol{z}}} = \dot{\hat{\boldsymbol{x}}} - \begin{pmatrix} \hat{\boldsymbol{\Pi}} & \boldsymbol{0} \end{pmatrix}\dot{\tilde{\boldsymbol{\omega}}}, \quad \dot{\check{\boldsymbol{z}}} = \dot{\check{\boldsymbol{x}}} - \boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & \check{\boldsymbol{\Pi}} \end{pmatrix}\dot{\tilde{\boldsymbol{\omega}}}, \tag{13}$$

by using $\dot{\tilde{\boldsymbol{\omega}}} = \boldsymbol{1}_N \otimes \boldsymbol{\Gamma}\boldsymbol{\omega} = (\boldsymbol{I}_N \otimes \boldsymbol{\Gamma})\tilde{\boldsymbol{\omega}}$ and substituting Eqs. (2), (7) and (12) into Eq. (13), we have

$$\begin{aligned} \dot{\hat{\boldsymbol{z}}} &= \hat{\boldsymbol{A}}_{\rm c}\hat{\boldsymbol{z}} + \hat{\boldsymbol{B}}_{\rm c}\zeta + \hat{\boldsymbol{E}}_{zc}\tilde{\boldsymbol{\omega}}, \\ \dot{\check{\boldsymbol{z}}} &= \check{\boldsymbol{A}}_{\rm c}\check{\boldsymbol{z}} + \check{\boldsymbol{B}}_{\rm c}\zeta + \check{\boldsymbol{E}}_{zc}\tilde{\boldsymbol{\omega}}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \hat{\boldsymbol{E}}_{zc} &= \hat{\boldsymbol{A}}_{\rm c}\begin{pmatrix} \hat{\boldsymbol{\Pi}} & \boldsymbol{0} \end{pmatrix} + \hat{\boldsymbol{E}}_{\rm c} - \begin{pmatrix} \hat{\boldsymbol{\Pi}} & \boldsymbol{0} \end{pmatrix}(\boldsymbol{I}_N \otimes \boldsymbol{\Gamma}), \\ \check{\boldsymbol{E}}_{zc} &= \check{\boldsymbol{A}}_{\rm c}(\boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & \check{\boldsymbol{\Pi}} \end{pmatrix}) \\ &\quad + \check{\boldsymbol{E}}_{\rm c} - (\boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & \check{\boldsymbol{\Pi}} \end{pmatrix})(\boldsymbol{I}_N \otimes \boldsymbol{\Gamma}). \end{aligned} \tag{15}$$

Let the matrix gain be $\boldsymbol{K}_{2i} = \boldsymbol{U}_i - \boldsymbol{K}_{1i}\boldsymbol{\Pi}_i$ and the first equation of regulation equations (11) is rewritten as

$$\boldsymbol{\Pi}_i\boldsymbol{\Gamma} = (\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_{1i})\boldsymbol{\Pi}_i + \boldsymbol{B}_i\boldsymbol{K}_{2i} + \boldsymbol{E}_i, \tag{16}$$

then Eq. (16) implies

$$\begin{aligned} &\begin{pmatrix} \hat{\boldsymbol{\Pi}} & \boldsymbol{0} \end{pmatrix}(\boldsymbol{I}_N \otimes \boldsymbol{\Gamma}) \\ &= \begin{pmatrix} \hat{\boldsymbol{\Pi}}(\boldsymbol{I}_l \otimes \boldsymbol{\Gamma}) & \boldsymbol{0} \end{pmatrix} \\ &= \begin{pmatrix} (\hat{\boldsymbol{B}} + \hat{\boldsymbol{B}}\hat{\boldsymbol{K}}_1)\hat{\boldsymbol{\Pi}} + \hat{\boldsymbol{B}}\hat{\boldsymbol{K}}_2 + \hat{\boldsymbol{E}} & \boldsymbol{0} \end{pmatrix} \\ &= \hat{\boldsymbol{A}}_{\rm c}\begin{pmatrix} \hat{\boldsymbol{\Pi}} & \boldsymbol{0} \end{pmatrix} + \hat{\boldsymbol{B}}_{\rm c} + \hat{\boldsymbol{E}}_{\rm c}. \end{aligned} \tag{17}$$

Since

$$\begin{aligned} &\boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & (\check{\boldsymbol{A}} + \check{\boldsymbol{B}}\check{\boldsymbol{K}}_1)\check{\boldsymbol{\Pi}} \end{pmatrix} \\ &= \begin{pmatrix} \boldsymbol{0} & (\check{\boldsymbol{A}} + \check{\boldsymbol{B}}\check{\boldsymbol{K}}_1)\check{\boldsymbol{\Pi}} \\ \boldsymbol{0} & (\check{\boldsymbol{A}} + \check{\boldsymbol{B}}\check{\boldsymbol{K}}_1)\check{\boldsymbol{\Pi}} \end{pmatrix} \\ &= \begin{pmatrix} \check{\boldsymbol{A}} & \check{\boldsymbol{B}}\check{\boldsymbol{K}}_1 \\ -\check{\boldsymbol{L}}\check{\boldsymbol{C}} & \check{\boldsymbol{A}} + \check{\boldsymbol{B}}\check{\boldsymbol{K}}_1 + \check{\boldsymbol{L}}\check{\boldsymbol{C}} \end{pmatrix}\begin{pmatrix} \boldsymbol{0} & \check{\boldsymbol{\Pi}} \\ \boldsymbol{0} & \check{\boldsymbol{\Pi}} \end{pmatrix} \\ &= \check{\boldsymbol{A}}_{\rm c}(\boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & \check{\boldsymbol{\Pi}} \end{pmatrix}), \end{aligned}$$

then equation (16) also implies

$$\begin{aligned} &(\boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & \check{\boldsymbol{\Pi}} \end{pmatrix})(\boldsymbol{I}_N \otimes \boldsymbol{\Gamma}) \\ &= \boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & \check{\boldsymbol{\Pi}}(\boldsymbol{I}_{N-l} \otimes \boldsymbol{\Gamma}) \end{pmatrix} \\ &= \boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & (\check{\boldsymbol{A}} + \check{\boldsymbol{B}}\check{\boldsymbol{K}}_1)\check{\boldsymbol{\Pi}} + \check{\boldsymbol{B}}\check{\boldsymbol{K}}_2 + \check{\boldsymbol{E}} \end{pmatrix} \\ &= \check{\boldsymbol{A}}_{\rm c}(\boldsymbol{1}_2 \otimes \begin{pmatrix} \boldsymbol{0} & \check{\boldsymbol{\Pi}} \end{pmatrix}) + \check{\boldsymbol{B}}_{\rm c} + \check{\boldsymbol{E}}_{\rm c}. \end{aligned} \tag{18}$$

Substituting Eqs. (17) and (18) into Eq. (15) yields $\hat{\boldsymbol{E}}_{zc} = -\hat{\boldsymbol{B}}_{\rm c}$ and $\check{\boldsymbol{E}}_{zc} = -\check{\boldsymbol{B}}_{\rm c}$; Eq. (14) can be presented as

$$\begin{aligned} \dot{\hat{\boldsymbol{z}}} &= \hat{\boldsymbol{A}}_{\rm c}\hat{\boldsymbol{z}} + \hat{\boldsymbol{B}}_{\rm c}(\zeta - \tilde{\boldsymbol{\omega}}), \\ \dot{\check{\boldsymbol{z}}} &= \check{\boldsymbol{A}}_{\rm c}\check{\boldsymbol{z}} + \check{\boldsymbol{B}}_{\rm c}(\zeta - \tilde{\boldsymbol{\omega}}). \end{aligned} \tag{19}$$

For $\zeta - \tilde{\boldsymbol{\omega}}$, we have

$$\begin{aligned} \dot{\zeta} - \dot{\tilde{\boldsymbol{\omega}}} &= (\alpha\boldsymbol{H} \oplus \boldsymbol{\Gamma})\zeta - \alpha(\boldsymbol{A}_0 \otimes \boldsymbol{I}_q)\tilde{\boldsymbol{\omega}} - (\boldsymbol{I}_N \otimes \boldsymbol{\Gamma})\tilde{\boldsymbol{\omega}} \\ &= (\alpha\boldsymbol{H} \oplus \boldsymbol{\Gamma})\zeta - (\alpha(\boldsymbol{H} \otimes \boldsymbol{I}_q) + (\boldsymbol{I}_N \otimes \boldsymbol{\Gamma}))\tilde{\boldsymbol{\omega}} \\ &= (\alpha\boldsymbol{H} \oplus \boldsymbol{\Gamma})(\zeta - \tilde{\boldsymbol{\omega}}). \end{aligned} \tag{20}$$

Let $\boldsymbol{Z} = (\hat{\boldsymbol{z}}^{\mathrm{T}}, \check{\boldsymbol{z}}^{\mathrm{T}}, (\zeta - \tilde{\boldsymbol{\omega}})^{\mathrm{T}})^{\mathrm{T}}$, Eqs. (19) and (20) can be combined as

$$\dot{\boldsymbol{Z}} = \mathcal{A}_{\mathrm{c}} \boldsymbol{Z}, \tag{21}$$

where $\mathcal{A}_{\mathrm{c}}$ is defined in Eq. (8). The eigenvalues of matrix $\alpha \boldsymbol{H} \oplus \boldsymbol{\Gamma}$ can be expressed as

$$\lambda(\alpha \boldsymbol{H} \oplus \boldsymbol{\Gamma}) = \{\alpha \lambda_i(\boldsymbol{H}) + \lambda_j(\boldsymbol{\Gamma}) \mid i = 1, \ldots, N, j = 1, \ldots, q\}.$$

Clearly, there exists $\alpha < 0$ such that all the eigenvalues of $\alpha \boldsymbol{H} \oplus \boldsymbol{\Gamma}$ have negative real parts, i.e., $\alpha \boldsymbol{H} \oplus \boldsymbol{\Gamma}$ is Hurwitz. According to Theorem 1, $\mathcal{A}_{\mathrm{c}}$ is Hurwitz, and one has $Z \to 0$, $(t \to \infty)$, i.e., $\boldsymbol{z}_i \to 0$, $(t \to \infty)$.

Substituting $\boldsymbol{z}_i = \boldsymbol{x}_i - \boldsymbol{\Pi}_i \boldsymbol{\omega}$ into Eq. (3) yields

$$\begin{aligned} \boldsymbol{e}_i &= \boldsymbol{C}_i(\boldsymbol{z}_i + \boldsymbol{\Pi}_i \boldsymbol{\omega}) - \boldsymbol{Q}_i \boldsymbol{\omega} \\ &= \boldsymbol{C}_i \boldsymbol{z}_i + (\boldsymbol{C}_i \boldsymbol{\Pi}_i - \boldsymbol{Q}_i)\boldsymbol{\omega} \\ &= \boldsymbol{C}_i \boldsymbol{z}_i, \end{aligned} \tag{22}$$

then $\lim\limits_{t \to \infty} \boldsymbol{e}_i = \lim\limits_{t \to \infty} \boldsymbol{C}_i \boldsymbol{z}_i = 0$. This completes the prove.

**Remark 2** Define error $\boldsymbol{x}_i^* = \boldsymbol{x}_i - \bar{\boldsymbol{x}}_i$, then

$$\begin{aligned} \dot{\boldsymbol{x}}_i^* &= \dot{\boldsymbol{x}}_i - \dot{\bar{\boldsymbol{x}}}_i \\ &= \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{B}_i \boldsymbol{u}_i + \boldsymbol{E}_i \boldsymbol{\omega}_i \\ &\quad - (\boldsymbol{A}_i \bar{\boldsymbol{x}}_i + \boldsymbol{B}_i \boldsymbol{u}_i + \boldsymbol{E}_i \boldsymbol{\omega} + \boldsymbol{L}_i(\boldsymbol{C}_i \bar{\boldsymbol{x}}_i - \boldsymbol{C}_i \boldsymbol{x}_i)) \\ &= (\boldsymbol{A}_i + \boldsymbol{L}_i \boldsymbol{C}_i) \boldsymbol{x}_i^*. \end{aligned} \tag{23}$$

By the detectability of $(\boldsymbol{C}_i, \boldsymbol{A}_i)$, $\boldsymbol{x}_i^* \to 0$, $(t \to \infty)$. Observer (4) can be seen as a dynamical system, which asymptotically estimates the unmeasurable states $\boldsymbol{x}_i$, $i = l+1, l+2, \ldots, N$. The observers have dimension $n$, which has the same dimension with states $\boldsymbol{x}_i$. So they are also full order observers.

**Remark 3** Assume $l = N$, i.e., all the agents' states are measurable. The full order Luenberger observer is invalid and then the output regulation problem of multi-agent systems in Ref. [29] is a special case of the one considered in this paper. If $l = 0$, the output regulation of unmeasurable multi-agent systems is also solved.

**Remark 4** Our research includes some recent results of heterogeneous agents' synchronization problems. Let $\boldsymbol{\omega}(t) = \boldsymbol{x}_{\mathrm{s}}(t)$ and $\boldsymbol{y}_{\mathrm{ir}}(t) = \boldsymbol{y}_{\mathrm{s}}(t)$, Eq. (2) is rewritten as

$$\begin{aligned} \dot{\boldsymbol{x}}_{\mathrm{s}}(t) &= \boldsymbol{A}_{\mathrm{s}} \boldsymbol{x}_{\mathrm{s}}(t), \\ \boldsymbol{y}_{\mathrm{s}}(t) &= \boldsymbol{C}_{\mathrm{s}} \boldsymbol{x}_{\mathrm{s}}(t), \end{aligned}$$

with $\boldsymbol{\Gamma} = \boldsymbol{A}_{\mathrm{s}}$, $\boldsymbol{Q}_i = \boldsymbol{C}_{\mathrm{s}}$, and $\boldsymbol{E}_i = 0$. It can be seen as the virtual reference system in Ref. [36] with $\boldsymbol{y}_{\mathrm{s}}(t)$ as the synchronous trajectory, and Eq. (1) can be written as

$$\begin{cases} \dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}_i \boldsymbol{x}_i(t) + \boldsymbol{B}_i \boldsymbol{u}_i(t), \\ \boldsymbol{y}_i(t) = \boldsymbol{C}_i \boldsymbol{x}_i(t), \end{cases}$$

which has the same formation with agents $P_i$ in Ref. [36]. Regulator error (3) is rewritten as $\boldsymbol{e}_i(t) = \boldsymbol{y}_i(t) - \boldsymbol{y}_{\mathrm{s}}(t)$. It has been shown in Ref. [36] that the agents' outputs $\boldsymbol{y}_i(t)$ reach a common trajectory $\boldsymbol{y}_{\mathrm{s}}(t)$, $\boldsymbol{y}_1(t) = \boldsymbol{y}_2(t) = \cdots = \boldsymbol{y}_{\mathrm{s}}(t)$, $t \geq 0$, by output feedback controllers with internal state $\boldsymbol{x}_{\mathrm{ri}}(t)$. Our results use the state feedback controllers. It solves the asymptotic synchronization problem as the agents asymptotically approach the same trajectory $\boldsymbol{y}_{\mathrm{s}}(t)$ quicker than that in Ref. [36] for all initial states.

## 4. Examples

Examples are used to illustrate the proposed design technique. We give examples from three cases: 1) partial agents' states are unmeasurable; 2) all agents can access the state information of others under the communication networks; and 3) the agents' states cannot be measured by themselves and neighbors. For the sake of brevity and clarity, we consider the multi-agent systems consisting of four agents with

$$\begin{aligned} &\boldsymbol{A}_i = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}, \ \boldsymbol{B}_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ &\boldsymbol{C}_i = \begin{pmatrix} 1 & 0 \end{pmatrix}, \ \boldsymbol{E}_i = \begin{pmatrix} 0 & 0 \\ 0 & -i \end{pmatrix}, \quad i = 1, 2, 3, 4, \end{aligned}$$

and node 0 (exosystem) with

$$\boldsymbol{\Gamma} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \ \boldsymbol{Q}_i = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

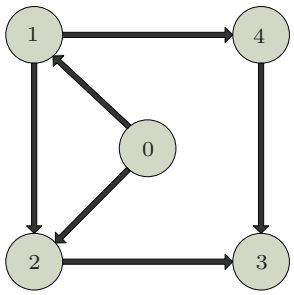



**Fig. 1.** (color online) Communication networks for the agents; 0 is the exosystem.

Digraph $\mathcal{G}$ contains a spanning tree and node 0 as its root, which is shown in Fig. 1. The $(\boldsymbol{U}_i = \begin{pmatrix} 1 & i \end{pmatrix}, \boldsymbol{\Pi}_i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, i = 1, 2, 3, 4)$ are the solutions of regulator equations (11). The Laplacian matrix of digraph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ and matrix $\boldsymbol{H}$ can be written as

$$\bar{\boldsymbol{L}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & -1 \\ -2 & 0 & 0 & 2 \end{pmatrix},$$

$$\boldsymbol{H} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & -1 & 2 & -1 \\ -2 & 0 & 0 & 2 \end{pmatrix}.$$

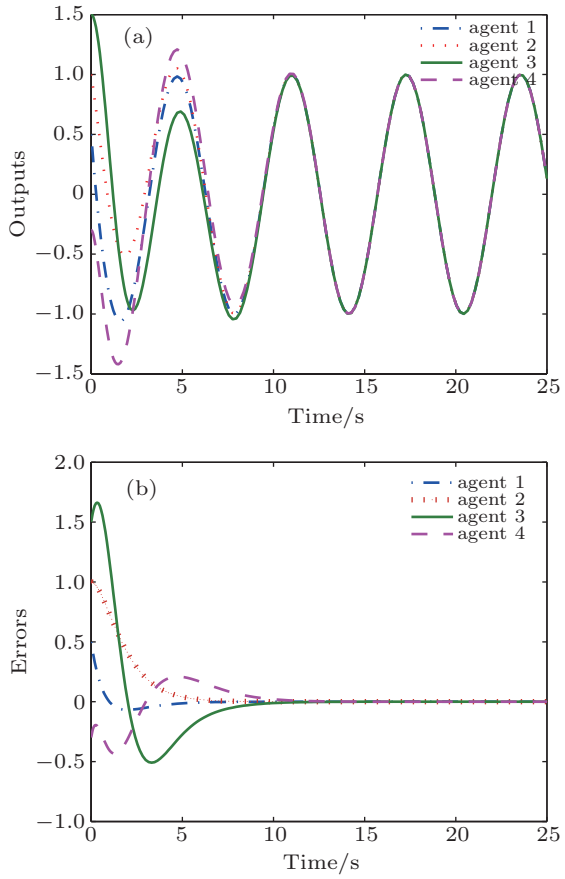



**Fig. 2.** (color online) Partial agents' states are unmeasurable: (a) measurement outputs, (b) tracking errors of the four agents.

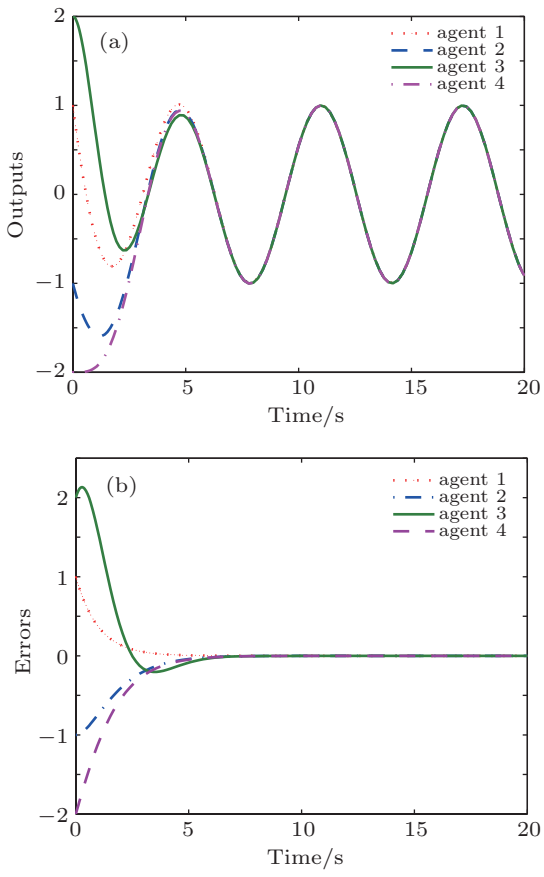



**Fig. 3.** (color online) All the agents' states are measurable: (a) measurement outputs, (b) tracking errors of the four agents.

Since the pairs $(\boldsymbol{A}_i, \boldsymbol{B}_i)$, $i = 1,2,3,4$ are stabilizable, there exist matrices $\boldsymbol{K}_{1i} = \begin{pmatrix} 1 & -2 \end{pmatrix}$ such that $\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_i$ are Hurwitz. Then $\boldsymbol{K}_{2i} = \begin{pmatrix} 0 & i+2 \end{pmatrix}$, $i = 1,2,3,4$. For case 1, assume $l = 2$, we have $\boldsymbol{L}_i = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $i = 3,4$. The numerical results of the measurement outputs and the regulation error are shown in Fig. 2. As expressed in Remark 2, if $l = 4, l = 0$, the numerical results for cases 2 and 3 are obtained and shown in Figs. 3 and 4. No matter whether the agents' states are measurable or unmeasurable, it is seen that the regulator errors under the feedback control law (6) converge to zero asymptotically.

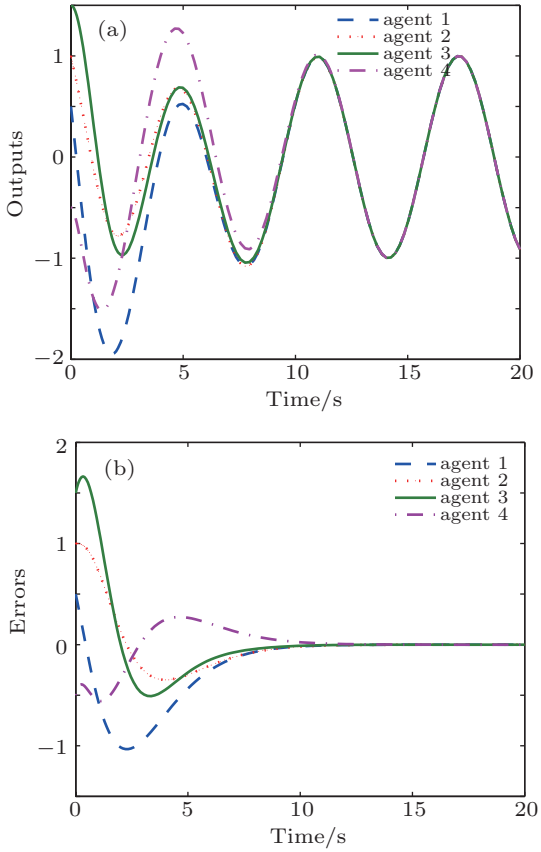



**Fig. 4.** (color online) The agents' states cannot be measured by themselves and neighbors: (a) measurement outputs, (b) tracking errors of the four agents.

## 5. Conclusion

In this paper, we investigate the output regulation problem of multi-agent systems which consist of two groups. The first group of subsystems' states can be measured by themselves and others, and the second groups are not measurable. We establish a dynamic compensator for the exosystem based on all the agents. At last, a state feedback control law is utilized for the output regulation problems. Future work including extending the results of this paper to second-order and high-order multi-agent systems will be presented.

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