Tracking of a third-order maneuvering target under an arbitrary topology*

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We explore the tracking problem of a maneuvering target. Tracking agents with third-order kinematics can communicate with each other via wireless network. The communication network topology is arbitrary rather than switches among several fixed topologies. The information sharing and interaction among agents are position, velocity, and acceleration. Some sufficient conditions of tracking strategy have been proposed. Finally, a numerical example is employed to demonstrate the effectiveness of proposed tracking strategy.

Keywords: third-order multi-agent systems, tracking strategy, arbitrary topology

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1. Introduction

Multi-agent systems and their collective behavior have a variety of applications such as flocking, cooperative control of unmanned aerial vehicles, and formation control of mobile robots. Therefore, multi-agent systems have attracted increasing attention of researchers from various disciplines of engineering, biology, and science over the past decade. Consensus of multi-agent systems is a fundamental problem in this research area. We investigate the tracking problem of a maneuvering target by a group of identical third-order agents, which can be categorized as consensus problem of leader-following multi-agent systems. The consensus problem for multi-agent systems with first-order and second-order kinematics has been intensively studied.[1–11] The authors in Ref. [1] explored the consensus problem of first-order multi-agent systems with multiple time delays. Some second-order consensus or formation algorithms for multi-agent systems under fixed topologies has been studied in Refs. [2]–[6].

Although there are many results on the tracking problem of multi-agent systems with first-order or second-order kinematics, there are only a few results on high-order multi-agent systems. Considering the behavior of a group of agents with different positions and speeds, the acceleration sometimes have an effect on the final state. As mentioned in Ref. [12], when one of a flock of birds perceives danger, the bird flock may change direction abruptly. It is obvious that the other birds in this flock need to track the leading bird not only on their position and velocity, but also on acceleration. Therefore, it is essential to investigate high-order tracking problems of multi-agent systems. Then, some researchers became interested in the high-order differential equations.[13–17] Specifically, in Ref. [13], Sun investigated the third-order leader-following consensus problem of nonlinear multi-agent systems in undirected network topologies. Yang[16] studied the consensus problems of high-order continuous-time multi-agent systems with time delays and switching topologies.

Most of work in this area has only considered multi-agent systems under fixed topologies or switching topologies regardless of second-order or high-order kinematics. In Refs. [7]–[11] and [16], the consensus problems for multi-agent systems under switching topologies have been discussed. In these papers, the communication network topology switches among several fixed topologies. While, in practice, the velocity of each maneuvering agent is time-varying and the communication radius of each agent is finite. Consequently, the communication topology between the target agent and the tracking agents may change from time to time rather than switches among several fixed topologies. The tracking problem of a maneuvering target under arbitrary topologies is of vital necessity.

Few researchers have paid attention on the consensus problem of multi-agent systems under arbitrary topologies. In Ref. [18], Kim addressed the consensus problem of multi-agent systems, consisting of a set of identical MIMO LTI systems, under a time-varying network which has a well-defined average. By employing some knowledge of complex dynamical network, the author has discussed the consensus problem of first-order leader-following multi-agent system under an arbitrary topology in Ref. [19].

We will discuss the third-order multi-agent consensus problem under an arbitrary topology in which the communication topology changes continuously rather than switches

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among several fixed topologies. Based on Ref. [19], some sufficient conditions and controller design principles are given to ensure that the third-order nonlinear tracking agents capture the maneuvering target eventually under an arbitrary topology.

Throughout this paper, the following notations will be used: $\boldsymbol{R}^{p\times q}$ is the set of $p\times q$ real matrix; $\boldsymbol{I}_n \in \boldsymbol{R}^{n\times n}$ denotes an $n\times n$ identity matrix; the Kronecker product is denoted by $\otimes$; $\overline{\lambda}(\boldsymbol{A}) := \sup_{i\in\mathfrak{v}}\{\lambda_i(\boldsymbol{A}(t))\}$ denotes the supremum eigenvalue of matrix $\boldsymbol{A}(t)$; $\underline{\lambda}(\boldsymbol{A}) := \inf_{i\in\mathfrak{v}}\{\lambda_i(\boldsymbol{A}(t))\}$ denotes the infimum eigenvalue of matrix $\boldsymbol{A}(t)$; and the Euclidean norm of $\boldsymbol{X}$ is denoted by $\|\boldsymbol{X}\|$.

2. Preliminary

A brief summary of the relevant results in graph theory and control stability, by merging which we are able to analyze the tracking problem, are recalled.

A weighted graph $\boldsymbol{G} := \{\boldsymbol{N}, \boldsymbol{E}, \boldsymbol{A}\}$ is denoted by a node set $\boldsymbol{N} = \{1, 2, \ldots, n\}$, an edge set $\boldsymbol{E} \subseteq \boldsymbol{N}\times\boldsymbol{N}$, and a weighted adjacency matrix $\boldsymbol{A} = [a_{ij}] \in \boldsymbol{R}^{n\times n}$ with nonnegative elements. The set of neighbors of node i is denoted by $\boldsymbol{N}_i = \{j | j \in \boldsymbol{N}, (j,i) \in \boldsymbol{E}\}$. When $j \notin N_i$, i.e., node j cannot access node i, $a_{ij} = 0$; otherwise, $a_{ij} > 0$.

The Laplacian matrix $\boldsymbol{L} = [l_{ij}] \in \boldsymbol{R}^{n\times n}$ of graph $\boldsymbol{G}$ is defined as $\boldsymbol{L} := \boldsymbol{D} - \boldsymbol{A}$, where

$$\boldsymbol{D} := \mathrm{diag}\Big\{\sum_{j\in N_1} a_{1j}, \sum_{j\in N_2} a_{2j}, \ldots, \sum_{j\in N_n} a_{nj}\Big\}.$$

In the view of graph theory, every agent can be treated as a node. Then, the topology of tracking agents and the target can be treated as a dynamic graph when it is analyzed in theory.

To analyze the tracking problem, Definition 1 and Lemma 1 are essential to be introduced.

Definition 1 A nonlinear system is Lyapunov globally uniformly stable if there is a function $\boldsymbol{V}$ such that $\boldsymbol{V}$ is positive definite and $\dot{\boldsymbol{V}}(x) \le 0$. Then, every trajectory of $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \boldsymbol{f}(0) = 0$ converges to zero as $t \to \infty$.

Lemma 1[20] Let $\boldsymbol{A}$ be a real symmetric $(n\times n)$ matrix and $\overline{\lambda}(\boldsymbol{A}) \ge \lambda_i(\boldsymbol{A}) \ge \underline{\lambda}(\boldsymbol{A})$ $(i \in \boldsymbol{N})$, then the following inequality equation on inner product is established:

$$\overline{\lambda}(\boldsymbol{A})\langle \boldsymbol{X}, \boldsymbol{X}\rangle \ge \langle \boldsymbol{A}X, \boldsymbol{X}\rangle \ge \underline{\lambda}(\boldsymbol{A})\langle \boldsymbol{X}, \boldsymbol{X}\rangle.$$

Assumption 1 There exists a finite constant $l \ge 0$ such that the vector F satisfies

$$\|\boldsymbol{F}(t, x_1(t), x_2(t), x_3(t))\| \le l(\|x_1(t)\| + \|x_2(t)\| + \|x_3(t)\|)$$

uniformly in t, for arbitrary $x_1(t), x_2(t), x_3(t)$ in $\boldsymbol{R}^3$.

3. Problem description

Considering a team of n identical tracking agents which are distributed at n distinct position in the three-dimensional (3D) space and denoted by $\boldsymbol{P} := \{x_{\boldsymbol{p}i} \in \boldsymbol{R}^3, i \in N\}$, where $x_{\boldsymbol{p}i} = \{x_{\boldsymbol{p}i}, y_{\boldsymbol{p}i}, z_{\boldsymbol{p}i}\}$. The kinematic equation of the i-th tracking agent is described by

$$\begin{aligned}
\dot{x}_{\boldsymbol{p}i}^{(0)}(t) &= x_{\boldsymbol{p}i}^{(1)}(t),\\
\dot{x}_{\boldsymbol{p}i}^{(1)}(t) &= x_{\boldsymbol{p}i}^{(2)}(t),\\
\dot{x}_{\boldsymbol{p}i}^{(2)}(t) &= f\Big(t, x_{\boldsymbol{p}i}^{(0)}(t), x_{\boldsymbol{p}i}^{(1)}(t), x_{\boldsymbol{p}i}^{(2)}(t)\Big) + u_{\boldsymbol{p}i}(t),
\end{aligned} \tag{1}$$

where $x_{\boldsymbol{p}i}^{(d)} \in \boldsymbol{R}^3$ $(d = 0, 1, 2)$ are position, velocity, and acceleration states of the i-th tracking agent, respectively. The $u_{\boldsymbol{p}i} \in \boldsymbol{R}^3$ is the control input of the i-th tracking agent.

In this paper, a_{ij}, the weighted value of agent j to agent i, is continuously time-varying, and denoted as $a_{ij}(t)$, which means that the topology of multi-agent system is arbitrary. The communication topology for the tracking agents is assumed to be modeled by a time-varying graph $\boldsymbol{G}(t) := \{N, E(t), \boldsymbol{A}(t)\}$ with node set N, time-varying edge $E(t) \subseteq N\times N$, and continuous adjacency matrix $\boldsymbol{A}(t) = [a_{ij}(t)] \in \boldsymbol{R}^{n\times n}$. The topology is symmetric, i.e., the corresponding Laplacian matrix $\boldsymbol{L}(t)$ satisfies $\boldsymbol{L}(t) = \boldsymbol{L}^{\mathrm{T}}(t)$.

The goal of each tracking agent, which is regarded as a point of the dynamic graph, is to capture a maneuvering target. The maneuvering target is assumed to possess the following third-order kinematics:

$$\begin{aligned}
\dot{x}_t^{(0)}(t) &= x_t^{(1)}(t),\\
\dot{x}_t^{(1)}(t) &= x_t^{(2)}(t),\\
\dot{x}_t^{(2)}(t) &= f(t, x_t^{(0)}(t), x_t^{(1)}(t), x_t^{(2)}(t)),
\end{aligned} \tag{2}$$

where $x_t^{(d)} \in \boldsymbol{R}^3$ $(d = 0, 1, 2)$ are position, velocity, and acceleration states of the target agent, respectively.

The case $b_i(t) > 0$ indicates that the i-th tracking agent can obtain the position, velocity, and acceleration states of the target at time t; the case $b_i(t) = 0$ indicates that information of the maneuvering target is not accessible by the i-th tracking agent. Denote $\boldsymbol{B}(t) := \mathrm{diag}\{b_1(t), b_2(t), \ldots, b_n(t)\}$.

The objective of this paper is to design a tracking strategy which ensures that the tracking agents effectively capture the target agent in 3D space under an arbitrary topology.

Definition 2 The third-order tracking agents (1) are said to capture the target (2) successfully when

$$\lim_{t\to\infty} \sum_{d=0}^{2} \Big\| x_{\boldsymbol{p}i}^{(d)}(t) - x_t^{(d)}(t) \Big\| = 0,\ i \in N. \tag{3}$$

4. Main results

This section presents a tracking strategy by which tracking agents can effectively capture a third-order maneuvering target under an arbitrary communication topology.

The tracking control protocol, consisting of the disagreement vectors between the i-th tracking agent and its neighbor agents and the disagreement vectors between the i-th tracking agent and the maneuvering target, is adopted in the following form for the i-th tracking agent:

$$u_i(t) = -\gamma \sum_{d=0}^{2} \Big\{ \sum_{j\in \boldsymbol{N}_i(t)} a_{ij}(t)\delta_{ij}^{(d)}(t) + b_i(t)\delta_{\boldsymbol{p}i}^{(d)}(t) \Big\}, \quad (4)$$

where $\gamma > 0$ is the control parameter to be designed, $\delta_{ij}^{(d)} = x_{\boldsymbol{p}i}^{(d)} - x_{\boldsymbol{p}j}^{(d)}(t)$ $(d = 0,1,2)$ are the position, velocity, and acceleration disagreement vectors between the i-th tracking agent and the j-th tracking agent, respectively, and $\delta_{\boldsymbol{p}i}^{(d)} = x_{\boldsymbol{p}i}^{(d)} - x_{\boldsymbol{t}}^{(d)}(t)$ $(d = 0,1,2)$ are the position, velocity, and acceleration disagreement vectors between the i-th tracking agent and the target, respectively. Then, from Definitions 1 and 2, the tracking problem can be thought of as uniform stability problem of the overall disagreement system (5) on the original point.

$$\begin{aligned}
\dot{\hat{\delta}}^{(0)}(t) &= \hat{\delta}^{(1)}(t),\\
\dot{\hat{\delta}}^{(1)}(t) &= \hat{\delta}^{(2)}(t),\\
\dot{\hat{\delta}}^{(2)}(t) &= F\Big(t,\hat{\delta}^{(0)}(t),\hat{\delta}^{(1)}(t),\hat{\delta}^{(2)}(t)\Big)\\
&\quad -\gamma\tilde{\boldsymbol{L}}(t)\big\{\hat{\delta}^{(0)}(t)+\hat{\delta}^{(1)}(t)+\hat{\delta}^{(2)}(t)\big\}, \quad (5)
\end{aligned}$$

where $\hat{\delta}^{(d)}(t) = \mathrm{col}\{\delta_1^{(d)}(t),\delta_2^{(d)}(t),\ldots,\delta_n^{(d)}(t)\} \in \boldsymbol{R}^{n\times 3}$ are the collective position, velocity, and acceleration disagreement vectors between tracking agents and the target agent, at time t, where $d = 1,2,3$. $\tilde{\boldsymbol{L}}(t) = (\boldsymbol{L}(t) + \boldsymbol{B}(t)) \otimes \boldsymbol{I}_3$ is the associated Laplacian matrix of the dynamic graph which includes the target. $F(t,\hat{\delta}^{(0)}(t),\hat{\delta}^{(1)}(t),\hat{\delta}^{(2)}(t)) = \mathrm{col}\{f_1(t),f_2(t),\ldots,f_n(t)\}$ satisfies Assumption 1, where

$$\begin{aligned}
f_i(t) &= f\Big(t,\hat{\delta}_i^{(0)},\hat{\delta}_i^{(1)}(t),\hat{\delta}_i^{(2)}(t)\Big)\\
&= f\Big(t,x_{\boldsymbol{p}i}^{(0)}(t),x_{\boldsymbol{p}i}^{(1)}(t),x_{\boldsymbol{p}i}^{(2)}(t)\Big)\\
&\quad -f\Big(t,x_t^{(0)}(t),x_t^{(1)}(t),x_t^{(2)}(t)\Big),\ i\in N,
\end{aligned}$$

is the collective nonlinear self-dynamic disagreement to the target at time t.

Assumption 2 The eigenvalues of Laplacian matrix $\tilde{\boldsymbol{L}}(t)$ are non-negative and satisfy $\underline{\lambda} \le \lambda(\tilde{L}(t)) \le \overline{\lambda}$.

Assumption 3 Laplacian matrix $\tilde{\boldsymbol{L}}(t)$ is differentiable and the eigenvalues of $\dot{\tilde{\boldsymbol{L}}}(t)$ are non-positive and satisfy $\dot{\underline{\lambda}} \le \lambda(\dot{\tilde{\boldsymbol{L}}}(t)) \le \dot{\overline{\lambda}}$.

The following theorem provides a sufficient condition for successful tracking in the case of an arbitrary topology in which the communication topology changes continuously under Assumption 2 and 3.

Theorem 1 The tracking agents can capture the target with third-order nonlinear kinematics under an arbitrary topology if there exists a positive parameter γ satisfying $0 < \gamma \le \gamma_{\mathrm{m}}$, where

$$\gamma_{\mathrm{m}} = \max\left\{ \frac{\frac{5}{4}l\overline{\lambda} + \sqrt{\Big(\frac{5}{4}l\overline{\lambda}\Big)^2 + 3l\Big(\frac{1}{2}\underline{\lambda}^2 - \frac{1}{2}\dot{\overline{\lambda}}\overline{\lambda}\Big)}}{\underline{\lambda}^2 - \dot{\overline{\lambda}}\overline{\lambda}}, \right.$$
$$\left. \frac{\frac{5}{4}l\overline{\lambda} + \sqrt{\Big(\frac{5}{4}l\overline{\lambda}\Big)^2 + 3l\Big(\frac{1}{4}\underline{\lambda}^2 - \frac{1}{2}\dot{\overline{\lambda}}\overline{\lambda}\Big)}}{\frac{1}{2}\underline{\lambda}^2 - \dot{\overline{\lambda}}\overline{\lambda}}, \frac{3l}{\underline{\lambda} - \frac{1}{2}\overline{\lambda}l} \right\},$$

and $0 < l < 2\underline{\lambda}/\overline{\lambda}$.

Proof A continuously differentiable Lyapunov function $V(t)$ is employed as a basic tool.

$$\begin{aligned}
V(t) &= \frac{1}{8}\big\langle\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t)\big\rangle\\
&\quad +\frac{1}{8}\big\langle\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)\big\rangle + \frac{1}{4}\big\langle\hat{\delta}^{(2)}(t),\hat{\delta}^{(2)}(t)\big\rangle\\
&\quad +\frac{1}{2}\Big\langle\hat{\delta}^{(2)}(t)+\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)+\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t),\hat{\delta}^{(2)}(t)\\
&\quad +\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)+\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t)\Big\rangle\\
&= \frac{1}{8}\big\|\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t)\big\|^2 + \frac{1}{8}\big\|\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)\big\|^2 + \frac{1}{4}\big\|\hat{\delta}^{(2)}(t)\big\|^2\\
&\quad +\frac{1}{2}\big\|\hat{\delta}^{(2)}(t)+\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)+\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t)\big\|^2. \quad (6)
\end{aligned}$$

Obviously, $V(t) > 0$. When $\|[\hat{\delta}^{(0)}(t),\hat{\delta}^{(1)}(t),\hat{\delta}^{(2)}(t)]\| \to \infty$, $V(t) \to \infty$.

The differential of Lyapunov function (6) along trajectory (5) is

$$\dot{V}|_{(5)} = -V_1 - V_2 - V_3 + V_4, \quad (7)$$

where

$$\begin{aligned}
V_1 &= -\frac{1}{2}\langle\gamma\dot{\tilde{\boldsymbol{L}}}^{(1)}(t)\hat{\delta}^{(0)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t)\rangle\\
&\quad -\frac{1}{2}\langle\gamma\dot{\tilde{\boldsymbol{L}}}^{(1)}(t)\hat{\delta}^{(1)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)\rangle\\
&\quad +\langle\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(2)}(t),\hat{\delta}^{(2)}(t)\rangle + \frac{1}{2}\langle\gamma\dot{\tilde{\boldsymbol{L}}}(t)\hat{\delta}^{(0)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t)\rangle\\
&\quad +\frac{1}{4}\langle\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)\rangle,\\
V_2 &= \langle\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t),\hat{\delta}^{(2)}(t)\rangle + \frac{3}{2}\langle\gamma\dot{\tilde{\boldsymbol{L}}}(t)\hat{\delta}^{(0)}(t),\hat{\delta}^{(2)}(t)\rangle\\
&\quad +\frac{1}{2}\langle\gamma\dot{\tilde{\boldsymbol{L}}}(t)\hat{\delta}^{(0)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)\rangle\\
&\quad +\frac{1}{4}\langle\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(2)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t)\rangle,\\
V_3 &= -\frac{1}{2}\langle\gamma\dot{\tilde{\boldsymbol{L}}}^{(1)}(t)\hat{\delta}^{(1)}(t),\hat{\delta}^{(2)}(t)\rangle\\
&\quad -\frac{1}{4}\langle\gamma\dot{\tilde{\boldsymbol{L}}}^{(1)}(t)\hat{\delta}^{(1)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t)\rangle\\
&\quad -\frac{1}{2}\langle\gamma\dot{\tilde{\boldsymbol{L}}}^{(1)}(t)\hat{\delta}^{(0)}(t),\hat{\delta}^{(2)}(t)\rangle\\
&\quad -\frac{1}{4}\langle\gamma\dot{\tilde{\boldsymbol{L}}}^{(1)}(t)\hat{\delta}^{(0)}(t),\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)\rangle,\\
V_4 &= \Big\langle\frac{3}{2}\hat{\delta}^{(2)}(t)+\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)+\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t),\\
&\quad F(t,\hat{\delta}^{(0)}(t),\hat{\delta}^{(1)}(t),\hat{\delta}^{(2)}(t))\Big\rangle.
\end{aligned}$$

According to Lemma 1, the following derivation can be obtained:

$$\begin{aligned}V_1 \le & -\frac{1}{2}\gamma^2\dot{\bar{\lambda}}\bar{\lambda}\|\hat{\delta}^{(0)}(t)\|^2-\frac{1}{2}\gamma^2\dot{\bar{\lambda}}\bar{\lambda}\|\hat{\delta}^{(1)}(t)\|^2+\gamma\underline{\lambda}\|\hat{\delta}^{(2)}(t)\|^2\\ &+\frac{1}{2}\gamma^2\underline{\lambda}^2\|\hat{\delta}^{(0)}(t)\|^2+\frac{1}{4}\gamma^2\underline{\lambda}^2\|\hat{\delta}^{(1)}(t)\|^2\\ =&\left(\frac{1}{2}\gamma^2\underline{\lambda}^2-\frac{1}{2}\gamma^2\dot{\bar{\lambda}}\bar{\lambda}\right)\|\hat{\delta}^{(0)}\|^2\\ &+\left(\frac{1}{4}\gamma^2\underline{\lambda}^2-\frac{1}{2}\gamma^2\dot{\bar{\lambda}}\bar{\lambda}\right)\|\hat{\delta}^{(1)}\|^2+\gamma\underline{\lambda}\|\hat{\delta}^{(2)}(t)\|^2,\\ V_4 \le & \left\|\frac{3}{2}\hat{\delta}^{(2)}(t)+\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(1)}(t)+\frac{1}{2}\gamma\tilde{\boldsymbol{L}}(t)\hat{\delta}^{(0)}(t)\right\|\\ &\times\|F(t,\hat{\delta}^{(0)}(t),\hat{\delta}^{(1)}(t),\hat{\delta}^{(2)}(t))\|.\end{aligned}$$

According to Assumption 1 and $\|\boldsymbol{X}+\boldsymbol{Y}\|\le\|\boldsymbol{X}\|+\|\boldsymbol{Y}\|$, the following equation can be derived:

$$\begin{aligned}V_4 \le & \left\{\frac{3}{2}\|\hat{\delta}^{(2)}(t)\|+\frac{1}{2}\gamma\bar{\lambda}\|\hat{\delta}^{(1)}(t)\|+\frac{1}{2}\gamma\bar{\lambda}\|\hat{\delta}^{(0)}(t)\|\right\}\\ &\times l\{\|\hat{\delta}^{(0)}(t)\|+\|\hat{\delta}^{(1)}(t)\|+\|\hat{\delta}^{(2)}(t)\|\}.\end{aligned}$$

For $\|\boldsymbol{X}\|\|\boldsymbol{Y}\|\le\frac{1}{2}(\|\boldsymbol{X}\|^2+\|\boldsymbol{Y}\|^2)$, it can be derived that

$$\begin{aligned}V_4 \le & \left(\frac{3}{4}l+\frac{5}{4}\gamma l\bar{\lambda}\right)\|\hat{\delta}^{(0)}(t)\|^2+\left(\frac{3}{4}l+\frac{5}{4}\gamma l\bar{\lambda}\right)\|\hat{\delta}^{(1)}(t)\|^2\\ &+\left(3l+\frac{1}{2}\gamma l\bar{\lambda}\right)\|\hat{\delta}^{(2)}(t)\|^2.\end{aligned}$$

For $\gamma>0$, $\lambda(\tilde{\boldsymbol{L}}(t))>0$, and $\lambda(\dot{\tilde{\boldsymbol{L}}}(t))<0$, it is clear that $V_2\ge0$ and $V_3\ge0$ defined by Eq. (7) are positive definite. Then,

$$\begin{aligned}\dot{V}|_{(5)} \le & V_4-V_1\\ =&\left(\frac{3}{4}l+\frac{5}{4}\gamma l\bar{\lambda}-\frac{1}{2}\gamma^2\underline{\lambda}^2+\frac{1}{2}\gamma^2\dot{\bar{\lambda}}\bar{\lambda}\right)\|\hat{\delta}^{(0)}(t)\|^2\\ &+\left(\frac{3}{4}l+\frac{5}{4}\gamma l\bar{\lambda}-\frac{1}{4}\gamma^2\underline{\lambda}^2+\frac{1}{2}\gamma^2\dot{\bar{\lambda}}\bar{\lambda}\right)\|\hat{\delta}^{(1)}(t)\|^2\\ &+\left(3l+\frac{1}{2}\gamma\bar{\lambda}l-\gamma\underline{\lambda}\right)\|\hat{\delta}^{(2)}(t)\|^2.\end{aligned}$$

From Definition 1, we can obtain that Theorem 1 ensures the uniform stability of the disagreement system (5) on original point. The disagreement between the tracking agents and the target will remain at 0 when $t\to\infty$. Namely, tracking agents finally capture the target. We complete the proof.

Corollary 1 The tracking agents can capture the target with third-order integral kinematics under an arbitrary topology if Assumptions 2 and 3 satisfy without any special condition on the control parameter γ.

Proof Since the kinematics of tracking agents and the target are integral and the nonlinear function F is replaced by 0, the differential of Lyapunov function becomes

$$\begin{aligned}\dot{V}|_{(5)} \le & -V_1\\ =&\left(-\frac{1}{2}\gamma^2\underline{\lambda}^2+\frac{1}{2}\gamma^2\dot{\bar{\lambda}}\bar{\lambda}\right)\|\hat{\delta}^{(0)}(t)\|^2\\ &+\left(-\frac{1}{4}\gamma^2\underline{\lambda}^2+\frac{1}{2}\gamma^2\dot{\bar{\lambda}}\bar{\lambda}\right)\|\hat{\delta}^{(1)}(t)\|^2-\gamma\underline{\lambda}\|\hat{\delta}^{(2)}(t)\|^2.\end{aligned}$$

For $\gamma>0$, $\dot{\bar{\lambda}}<0$, $\bar{\lambda}>0$, and $\underline{\lambda}>0$, it is obvious that $\dot{V}|_{(5)}\le0$.

Note that Theorem 1 and Corollary 1 can be used to design the control parameter of the third-order tracking problem under an arbitrary topology.

5. A numerical example

In this section, we present a numerical example to illustrate the previous development. Consider a scenario where a group of three tracking agents (1) tracking a maneuvering target (2) in 3D space. The nonlinear function is specified as

$$f=0.25\sin(0.3x^{(0)}(t))-0.15x^{(1)}(t)-0.5\sin(x^{(2)}(t)). \quad (8)$$

The arbitrary communication topology of the tracking system is specified as a time-varying matrix. $\boldsymbol{L}(t)+\boldsymbol{B}(t)$ is specified as

$$\begin{pmatrix}\tilde{l}_{11} & \tilde{l}_{12} & \tilde{l}_{13}\\ \tilde{l}_{21} & \tilde{l}_{22} & \tilde{l}_{23}\\ \tilde{l}_{31} & \tilde{l}_{32} & \tilde{l}_{33}\end{pmatrix}, \quad (9)$$

where

$$\begin{aligned}&\tilde{l}_{11}=(e^2-1)\tan(t)+e\arctan(t),\\ &\tilde{l}_{12}=(1-e)\tan(t)-2e\arctan(t),\quad \tilde{l}_{13}=e\arctan(t),\\ &\tilde{l}_{21}=\tilde{l}_{12}^{\mathrm{T}},\quad \tilde{l}_{22}=\tan(t)-2e^2\arctan(t),\\ &\tilde{l}_{23}=2\tan(t)+e^2\arctan(t),\quad \tilde{l}_{31}=\tilde{l}_{13}^{\mathrm{T}},\quad \tilde{l}_{32}=\tilde{l}_{23}^{\mathrm{T}},\\ &\tilde{l}_{33}=3(e-e^2)\tan(t)+\arctan(t).\end{aligned}$$

The initial position, velocity, and acceleration of the target in 3D space are

$$\begin{aligned}&x_t^{(0)}(0)=[2,5,5],\quad x_t^{(1)}(0)=[0.2,0.9,0.6],\\ &x_t^{(2)}(0)=[0.6,0.8,0.3],\end{aligned}$$

respectively. The initial positions of three tracking agents in 3D space are

$$\begin{aligned}&x_{\boldsymbol{p}1}^{(0)}(0)=[3,6,3],\quad x_{\boldsymbol{p}2}^{(0)}(0)=[8,4,0],\\ &x_{\boldsymbol{p}3}^{(0)}(0)=[0,6,4],\end{aligned}$$

respectively. The initial velocities of three tracking agents in 3D space are

$$\begin{aligned}&x_{\boldsymbol{p}1}^{(1)}(0)=[0.8,0.3,1],\quad x_{\boldsymbol{p}2}^{(1)}(0)=[0.2,0.7,0.2],\\ &x_{\boldsymbol{p}3}^{(1)}(0)=[0.5,0.2,1],\end{aligned}$$

respectively. The initial accelerations of three tracking agents are 0. The tracking strategy is (4) with control parameter $\gamma=0.8$.

Figure 1 shows the group behavior of tracking agents and the target. From the figure, it is clearly seen that the tracking agents eventually capture the maneuvering target under the arbitrary topology (9) with the proposed tracking strategy (4).

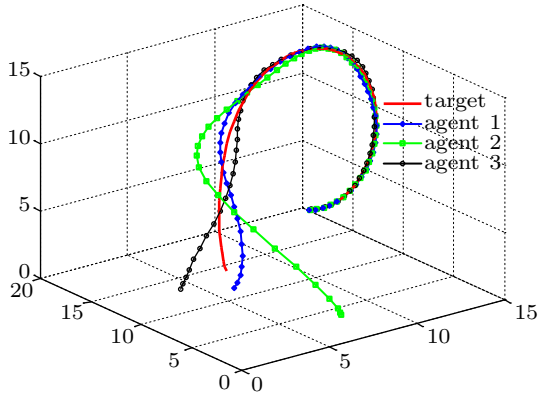


Fig. 1. Trajectories of the tracking agents and the maneuvering target.

6. Conclusions

We have proposed a tracking strategy for the capture of a maneuvering target by a group of third-order tracking agents distributed in 3-dimensional space. It is assumed that the tracking agents and the target can communicate with each other under an arbitrary topology. The simulation results indicate that the proposed tracking strategy enables the tracking agents capture the target eventually.

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