Robust networked H_∞ synchronization of nonidentical chaotic Lur'e systems*

Yang De-Dong(杨德东)†

School of Control Science and Engineering, Hebei University of Technology, Tianjin 300130, *China*

(Received 22 April 2013; revised manuscript received 21 June 2013; published 3 December 2013)

We mainly investigate the robust networked H_∞ synchronization problem of nonidentical chaotic Lur'e systems. In the design of the synchronization scheme, some network characteristics, such as nonuniform sampling, transmission-induced delays, and data packet dropouts, are considered. The parameters of master–slave chaotic Lur'e systems often allow differences. The sufficient condition in terms of linear matrix inequality (LMI) is obtained to guarantee the dissipative synchronization of nonidentical chaotic Lur'e systems in network environments. A numerical example is given to illustrate the validity of the proposed method.

Keywords: synchronization, Lur'e systems, networked control, dissipativity

PACS: 05.45.Gg, 05.45.Xt **DOI:** 10.1088/1674-1056/23/1/010504

1. Introduction

Since Carroll and Pecora's work,[1] chaotic synchronization has been widely discussed and developed due to its potential applications in the design of chaos generators, image encryption, secure communication, and information science, including the integral-order and fractional-order chaotic systems.[2–6] Moreover, a broad class of nonlinear systems can be modeled as Lur'e systems with the sector restricted nonlinearity, such as the classical Chua's circuit systems. Thus, various master–slave synchronization schemes for chaotic Lur'e systems have been proposed in recent years.[7–19]

Generally, a master–slave synchronization controller can be modeled as a continuous-time case. Some synchronization schemes have been proposed to achieve the performance requirements, such as the impulsive feedback control[9,12,13] and the delay feedback control.[10,14] There also exist some results about Lur'e systems with special properties.[7,15,16,18] However, with the development of the microprocessor and its interfacing technology, the remote digital controller is more popularly for communication engineering. Some special characteristics, such as nonuniform sampling, transmission-induced delays, and data packet dropouts, should be considered in the controller design. Recently, hybrid system models have been generally built via a zero-order hold (ZOH) for the sampled-data control.[11,17] However, those schemes did not consider the transmission-induced delays and data packet dropouts. Moreover, a robust synchronization is out of their results. In this paper, the robust networked H_∞ controller design for the dissipative synchronization of nonidentical chaotic Lur'e systems is discussed. This paper follows the idea in Ref. [7] and extends the original result to the networked case.

The following notations are used. Throughout this paper, superscripts T and -1 represent the transpose and the inverse of a matrix, respectively, $\mathbf{N}$ denotes the natural number set, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, $\mathbf{R}^{n\times m}$ is the set of all $n\times m$ real matrices, the identity matrices and zero matrices are denoted by $\boldsymbol{I}$ and $\mathbf{0}$, respectively, notation $*$ always denotes the symmetric block in one symmetric matrix, the standard notation $>$ $(<)$ is used to denote the positive (negative) definite ordering of matrices, inequality $\boldsymbol{X}>\boldsymbol{Y}$ shows that matrix $\boldsymbol{X}-\boldsymbol{Y}$ is positive definite.

2. Problem formulation

Consider the following nonidentical master–slave chaotic Lur'e systems:[7]

$$\begin{aligned}
\mathcal{R}\ &: \begin{cases} \dot{\boldsymbol{\mu}}(t)=\boldsymbol{R}\boldsymbol{\mu}(t)+\boldsymbol{S}r(t), \\ d(t)=\boldsymbol{T}\boldsymbol{\mu}(t)+\boldsymbol{U}r(t), \end{cases} \\
\mathcal{M}\ &: \begin{cases} \dot{\boldsymbol{x}}(t)=\boldsymbol{A}_1\boldsymbol{x}(t)+\boldsymbol{B}_1\boldsymbol{\sigma}(\boldsymbol{C}_1\boldsymbol{x}(t))+\boldsymbol{D}d(t), \\ \boldsymbol{p}(t)=\boldsymbol{H}\boldsymbol{x}(t), \end{cases} \\
\mathcal{S}\ &: \begin{cases} \dot{\boldsymbol{z}}(t)=\boldsymbol{A}_2\boldsymbol{z}(t)+\boldsymbol{B}_2\boldsymbol{\sigma}(\boldsymbol{C}_2\boldsymbol{z}(t))+\boldsymbol{u}(t), \\ \boldsymbol{q}(t)=\boldsymbol{H}\boldsymbol{z}(t), \end{cases}
\end{aligned} \tag{1}$$

with master system $\mathcal{M}$, slave system $\mathcal{S}$, and linear filter $\mathcal{R}$. Here, $\boldsymbol{x}(t),\boldsymbol{z}(t)\in\mathbf{R}^n$, $\boldsymbol{\mu}(t)\in\mathbf{R}^{n_r}$ are the state vectors, and $\boldsymbol{p}(t),\boldsymbol{q}(t)\in\mathbf{R}^l$, $d(t)\in\mathbf{R}$ are the output vectors. The message signal $r(t)\in\mathbf{R}$, which is assumed to be binary valued, is low pass filtered by $\mathcal{R}$. The $\boldsymbol{u}(t)\in\mathbf{R}^n$ is the control input. The $\boldsymbol{A}_1,\boldsymbol{A}_2\in\mathbf{R}^{n\times n}$, $\boldsymbol{B}_1,\boldsymbol{B}_2\in\mathbf{R}^{n\times n_h}$, $\boldsymbol{C}_1,\boldsymbol{C}_2\in\mathbf{R}^{n_h\times n}$, $\boldsymbol{D}\in\mathbf{R}^{n\times 1}$, $\boldsymbol{H}\in\mathbf{R}^{l\times n}$, $\boldsymbol{R}\in\mathbf{R}^{n_r\times n_r}$, $\boldsymbol{S}\in\mathbf{R}^{n_r\times 1}$, $\boldsymbol{T}\in\mathbf{R}^{1\times n_r}$, and $U\in\mathbf{R}$ are known system parameters. It is assumed that $\boldsymbol{\sigma}_i(\cdot):\mathbf{R}^{n_h}\to\mathbf{R}^{n_h}$

*Project supported by the Natural Science Foundation of China (Grant No. 61203076), the Natural Science Foundation of Tianjin City, China (Grant No. 13JCQNJC03500), the Natural Science Foundation of Hebei Province, China (Grant No. F2012202100), and the Excellent Young Technological Innovation Foundation in Hebei University of Technology, China (Grant No. 2011005).

†Corresponding author. E-mail: Dedongyang@gmail.com

© 2014 Chinese Physical Society and IOP Publishing Ltd

satisfies a sector condition with $\sigma_i(\cdot)$, $i = 1,2,\ldots,n_h$, belonging to sectors $[0,\omega_i]$, i.e.,

$$\sigma_i(\xi)(\sigma_i(\xi)-\omega_i\xi)\le 0,\quad \forall\xi;\ i=1,2,\ldots,n_h.$$

Assumption 1[7] For the master system $\mathcal{M}$, there exist initial states $\boldsymbol{x}_0$ and a positive real constant δ_x such that

$$\|\boldsymbol{x}(t)\|_2\le\delta_x,\quad \forall t\in[t_0,\infty), \tag{2}$$

for all continuous time reference inputs r that satisfy $\|r\|_\infty = \sup_{t\ge t_0}|r(t)|\le 1$.

Remark 1 Due to the boundedness of the chaotic Lur'e systems, the above assumption is reasonable. As described in Ref. [7], for a zero external input d, the upper bound δ_x can be a measure for the "size" of the attractor of the chaotic master system.

The robust networked H_∞ synchronization controller related with system (1) is described by

$$\begin{aligned}&\mathcal{C}:\ \boldsymbol{u}(t)=\boldsymbol{K}(\boldsymbol{p}(t_{i_k})-\boldsymbol{q}(t_{i_k})+\boldsymbol{\epsilon}),\\ &t_{i_k}+\tau_{i_k}\le t<t_{i_{k+1}}+\tau_{i_{k+1}},\end{aligned} \tag{3}$$

where $\boldsymbol{\epsilon}\in\mathbf{R}^l$ is the noise signal in network transmission channels, $\boldsymbol{K}\in\mathbf{R}^{n\times l}$ denotes the controller gain matrix, $\boldsymbol{p}(t_{i_k})$ and $\boldsymbol{q}(t_{i_k})$ are the available measurements of $\boldsymbol{p}(t)$ and $\boldsymbol{q}(t)$ at sampling instant t_{i_k}, $k=0,1,\ldots,\infty$, $i_k\in\mathbf{N}$ denotes the serial number of the available data packet such that $\{i_0,i_1,i_2,\ldots\}\subseteq\{0,1,2,3,\ldots\}$, and τ_{i_k} denotes the network-induced delay calculated from instant t_{i_k}.

Assumption 2 The sensor is clock-driven; and the controller and actuator are event-driven. The clocks in all the devices are synchronized. The $\boldsymbol{p}(t)$ and $\boldsymbol{q}(t)$ are online measurable and the measurements are transmitted with a single packet.

Assumption 3[20] There exist three constants $h>0$, $\tau_{\min}\ge 0$, and $\tau_{\max}\ge 0$ such that

$$\begin{aligned}&t_{k+1}-t_k\le h,\\ &\tau_{\min}\le\tau_{i_k}\le\tau_{\max},\ k=0,1,\ldots,\infty,\end{aligned} \tag{4}$$

where h denotes the upper bound of the interval between two consecutive sampling instants, and $\tau_{\min}$ and $\tau_{\max}$ denote the minimum and maximum of the network-induced delays, respectively. It is assumed that $\delta_{\max}$ denotes the admitted maximum of successive data packet dropouts in the network transmissions. Considering condition (4) and successive data packet dropouts, we can obtain the following result:

$$\begin{aligned}t_{i_{k+1}}+\tau_{i_{k+1}}-t_{i_k}&\le(i_{k+1}-i_k)h+\tau_{\max}\\ &\le(1+\delta_{\max})h+\tau_{\max},\ k=0,1,\ldots,\infty.\end{aligned} \tag{5}$$

The error signal concept is provided by defining a tracking error[7]

$$\boldsymbol{\nu}(t)=d(t)-\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{e}(t),$$

where $\boldsymbol{\beta}=[\,1\ 0\ \cdots\ 0\,]^{\mathrm{T}}$. Let the error be $\boldsymbol{e}(t)=\boldsymbol{x}(t)-\boldsymbol{z}(t)$. Then, by considering the controller (3), the synchronization scheme is represented as

$$\begin{cases}\dot{\boldsymbol{e}}(t)=\boldsymbol{\rho}(\boldsymbol{e}(t),\boldsymbol{e}(t_{i_k}),\boldsymbol{z}(t),\boldsymbol{\mu}(t),r(t))+\boldsymbol{\varphi}(\boldsymbol{x}(t)),\\ \dot{\boldsymbol{\mu}}(t)=\boldsymbol{R}\boldsymbol{\mu}(t)+\boldsymbol{S}r(t),\\ \boldsymbol{\nu}(t)=-\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{e}(t)+\boldsymbol{T}\boldsymbol{\mu}(t)+\boldsymbol{U}r(t),\\ t_{i_k}+\tau_{i_k}\le t<t_{i_{k+1}}+\tau_{i_{k+1}},\end{cases} \tag{6}$$

where

$$\begin{aligned}&\boldsymbol{\rho}(\boldsymbol{e}(t),\boldsymbol{e}(t_{i_k}),\boldsymbol{z}(t),\boldsymbol{\mu}(t),r(t))\\ &=\boldsymbol{A}_2\boldsymbol{e}(t)+\boldsymbol{B}_2\boldsymbol{\eta}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))+\boldsymbol{D}\boldsymbol{T}\boldsymbol{\mu}(t)+\boldsymbol{D}\boldsymbol{U}r(t)\\ &\quad-\boldsymbol{K}\boldsymbol{H}\boldsymbol{e}(t_{i_k})-\boldsymbol{K}\boldsymbol{\epsilon},\\ &\boldsymbol{\eta}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))\\ &=\boldsymbol{\sigma}(\boldsymbol{C}_2\boldsymbol{e}(t)+\boldsymbol{C}_2\boldsymbol{z}(t))-\boldsymbol{\sigma}(\boldsymbol{C}_2\boldsymbol{z}(t)),\\ &\boldsymbol{\varphi}(\boldsymbol{x}(t))=\Delta\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}_1\boldsymbol{\sigma}(\boldsymbol{C}_1\boldsymbol{x}(t))-\boldsymbol{B}_2\boldsymbol{\sigma}(\boldsymbol{C}_2\boldsymbol{x}(t)),\end{aligned}$$

and $\Delta\boldsymbol{A}=\boldsymbol{A}_1-\boldsymbol{A}_2$. Assume that nonlinearity $\boldsymbol{\eta}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))$ belongs to the sector bound $[0,\omega_i]$, i.e.,

$$0\le\frac{\eta_i(\boldsymbol{c}_{2i}\boldsymbol{e},\boldsymbol{z})}{\boldsymbol{c}_{2i}\boldsymbol{e}}=\frac{\sigma_i(\boldsymbol{c}_{2i}\boldsymbol{e}+\boldsymbol{c}_{2i}\boldsymbol{z})-\sigma_i(\boldsymbol{c}_{2i}\boldsymbol{z})}{\boldsymbol{c}_{2i}\boldsymbol{e}}\le\omega_i,$$

where $\boldsymbol{c}_{2i}$ denotes the i-th row vector of $\boldsymbol{C}_2$. This implies

$$\begin{aligned}&\eta_i(\boldsymbol{c}_{2i}\boldsymbol{e},\boldsymbol{z})(\eta_i(\boldsymbol{c}_{2i}\boldsymbol{e},\boldsymbol{z})-\omega_i\boldsymbol{c}_{2i}\boldsymbol{e})\le 0,\\ &\forall\boldsymbol{e},\boldsymbol{z};\quad i=1,2,\ldots,n_h.\end{aligned} \tag{7}$$

Set $\phi_0:=(1+\delta_{\max})h+\tau_{\max}$, then the initial condition of $\boldsymbol{e}(t)$ on $(t_0-\phi_0,\ t_0]$ is supplemented as $\boldsymbol{e}(t)=\boldsymbol{\phi}(t)$, $t\in(t_0-\phi_0,\ t_0]$, where $\boldsymbol{\phi}(t)$ is a continuous function on $(t_0-\phi_0,\ t_0]$. The synchronization error system (6) can be expressed as

$$\begin{cases}\dot{\boldsymbol{e}}(t)=\boldsymbol{\rho}(\boldsymbol{e}(t),\boldsymbol{e}(t-\tau(t)),\boldsymbol{z}(t),\boldsymbol{\mu}(t),r(t))\\ \qquad\quad+\boldsymbol{\varphi}(\boldsymbol{x}(t)),\\ \dot{\boldsymbol{\mu}}(t)=\boldsymbol{R}\boldsymbol{\mu}(t)+\boldsymbol{S}r(t),\\ \boldsymbol{\nu}(t)=-\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{e}(t)+\boldsymbol{T}\boldsymbol{\mu}(t)+\boldsymbol{U}r(t),\\ \boldsymbol{e}(t)=\boldsymbol{\phi}(t),\ t\in(t_0-\phi_0,\ t_0],\end{cases} \tag{8}$$

where $\tau(t)=t-t_{i_k}$. It is explicit that $\tau_{\min}\le\tau(t)<(1+\delta_{\max})h+\tau_{\max}$ because of inequalities (4) and (5) and $\tau(t)<t_{i_{k+1}}+\tau_{i_{k+1}}-t_{i_k}$. The time-varying delay $\tau(t)$ is piecewise-continuous with derivative $\dot{\tau}(t)=1$ for $t\ne t_{i_k}+\tau_{i_k}$ and the derivative does not exist at the instant $t_{i_k}+\tau_{i_k}$.

3. Main result

In this section, the dissipativity with finite L_2-gain γ of system (8) will be derived for the synchronization schemes. Before beginning the proof procedure, two useful lemmas are introduced.

Lemma 1 Let $\boldsymbol{X}$ be any $m\times n$ matrix, for any constant $\varepsilon>0$ and any positive-definite symmetric matrix $\boldsymbol{T}$, such that

$$2\boldsymbol{\zeta}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{\varphi}\le\varepsilon\boldsymbol{\zeta}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{T}^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{\zeta}+\frac{1}{\varepsilon}\boldsymbol{\varphi}^{\mathrm{T}}\boldsymbol{T}\boldsymbol{\varphi}$$

for all $\zeta \in \mathbf{R}^m$, $\varphi \in \mathbf{R}^n$, and $\boldsymbol{T} \in \mathbf{R}^{n\times n}$.

Lemma 2 For any constant symmetric matrix $\boldsymbol{G} \in \mathbf{R}^{n\times n}$, $\boldsymbol{G} = \boldsymbol{G}^{\mathrm{T}} > 0$, scalar $\upsilon > 0$, vector function $\psi : [t-\upsilon, t] \to \mathbf{R}^n$ such that the integrations in the following are well defined, then

$$\upsilon \int_{t-\upsilon}^{t} \psi^{\mathrm{T}}(\beta)\boldsymbol{G}\psi(\beta)\mathrm{d}\beta \geq \left(\int_{t-\upsilon}^{t} \psi(\beta)\mathrm{d}\beta\right)^{\mathrm{T}} \boldsymbol{G}\left(\int_{t-\upsilon}^{t} \psi(\beta)\mathrm{d}\beta\right).$$

Theorem 1 Given a scalar $\phi_0 > 0$ composed of h, $\delta_{\max}$, $\tau_{\max}$ and a diagonal matrix $\boldsymbol{\Omega} = \mathrm{diag}\{\omega_1, \omega_2, \ldots, \omega_{n_h}\}$, the synchronization error system (8) is dissipative with finite L_2-gain in network environments if there are matrices $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0$, $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > 0$, $\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{n_h}\} \geq 0$, any matrices $\boldsymbol{N}_1$, $\boldsymbol{N}_2$, $\boldsymbol{M}$, $\boldsymbol{V}$ with appropriate dimensions such that the following linear matrix inequality (LMI) condition holds:

$$\begin{bmatrix} \boldsymbol{\Theta} & \phi_0 \boldsymbol{N} \\ * & -\phi_0 \boldsymbol{Q} \end{bmatrix} < 0, \tag{9}$$

where

$$\boldsymbol{\Theta} = \begin{bmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} & \boldsymbol{\Phi}_{13} & \boldsymbol{\Phi}_{14} & \boldsymbol{\Phi}_{15} & \boldsymbol{\Phi}_{16} & \boldsymbol{\Phi}_{17} & \boldsymbol{\Phi}_{18} \\ * & \boldsymbol{\Phi}_{22} & \boldsymbol{\Phi}_{23} & \boldsymbol{\Phi}_{24} & \boldsymbol{\Phi}_{25} & \boldsymbol{\Phi}_{26} & \boldsymbol{\Phi}_{27} & \boldsymbol{\Phi}_{28} \\ * & * & \boldsymbol{\Phi}_{33} & \boldsymbol{\Phi}_{34} & \boldsymbol{\Phi}_{35} & \boldsymbol{\Phi}_{36} & \boldsymbol{\Phi}_{37} & \boldsymbol{\Phi}_{38} \\ * & * & * & \boldsymbol{\Phi}_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \boldsymbol{\Phi}_{55} & \boldsymbol{\Phi}_{56} & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Phi}_{66} & 0 & 0 \\ * & * & * & * & * & * & \boldsymbol{\Phi}_{77} & 0 \\ * & * & * & * & * & * & * & \boldsymbol{\Phi}_{88} \end{bmatrix}, \quad \boldsymbol{N} = \begin{bmatrix} \boldsymbol{N}_1 \\ 0 \\ \boldsymbol{N}_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} \end{bmatrix},$$

$$\begin{aligned}
\boldsymbol{\Phi}_{11} &= \boldsymbol{N}_1 + \boldsymbol{N}_1^{\mathrm{T}} + \beta\beta^{\mathrm{T}} + \boldsymbol{M}\boldsymbol{A}_2 + \boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{M}^{\mathrm{T}}, \\
\boldsymbol{\Phi}_{12} &= \boldsymbol{P}_{11} + \boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{M}^{\mathrm{T}} - \boldsymbol{M}, \\
\boldsymbol{\Phi}_{13} &= -\boldsymbol{N}_1 + \boldsymbol{N}_2^{\mathrm{T}} - \boldsymbol{V}\boldsymbol{H} + \boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{M}^{\mathrm{T}}, \\
\boldsymbol{\Phi}_{14} &= \boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{\Omega}\boldsymbol{\Lambda} + \boldsymbol{M}\boldsymbol{B}_2, \\
\boldsymbol{\Phi}_{15} &= -2\beta\boldsymbol{T} + \boldsymbol{M}\boldsymbol{D}\boldsymbol{T}, \\
\boldsymbol{\Phi}_{16} &= -2\beta\boldsymbol{U} + \boldsymbol{M}\boldsymbol{D}\boldsymbol{U}, \\
\boldsymbol{\Phi}_{17} &= -\boldsymbol{V}, \\
\boldsymbol{\Phi}_{18} &= \boldsymbol{M}, \\
\boldsymbol{\Phi}_{22} &= \phi_0\boldsymbol{Q} - \boldsymbol{M} - \boldsymbol{M}^{\mathrm{T}}, \\
\boldsymbol{\Phi}_{23} &= -\boldsymbol{V}\boldsymbol{H} - \boldsymbol{M}^{\mathrm{T}}, \\
\boldsymbol{\Phi}_{24} &= \boldsymbol{M}\boldsymbol{B}_2, \\
\boldsymbol{\Phi}_{25} &= 2\boldsymbol{P}_{12} + \boldsymbol{M}\boldsymbol{D}\boldsymbol{T}, \\
\boldsymbol{\Phi}_{26} &= \boldsymbol{M}\boldsymbol{D}\boldsymbol{U}, \\
\boldsymbol{\Phi}_{27} &= -\boldsymbol{V}, \\
\boldsymbol{\Phi}_{28} &= \boldsymbol{M}, \\
\boldsymbol{\Phi}_{33} &= -\boldsymbol{N}_2 - \boldsymbol{N}_2^{\mathrm{T}} - \boldsymbol{V}\boldsymbol{H} - \boldsymbol{H}^{\mathrm{T}}\boldsymbol{V}^{\mathrm{T}}, \\
\boldsymbol{\Phi}_{34} &= \boldsymbol{M}\boldsymbol{B}_2, \\
\boldsymbol{\Phi}_{35} &= \boldsymbol{M}\boldsymbol{D}\boldsymbol{T}, \\
\boldsymbol{\Phi}_{36} &= \boldsymbol{M}\boldsymbol{D}\boldsymbol{U}, \\
\boldsymbol{\Phi}_{37} &= -\boldsymbol{V}, \\
\boldsymbol{\Phi}_{38} &= \boldsymbol{M}, \\
\boldsymbol{\Phi}_{44} &= -2\boldsymbol{\Lambda}, \\
\boldsymbol{\Phi}_{55} &= \boldsymbol{R}^{\mathrm{T}}\boldsymbol{P}_{22} + \boldsymbol{P}_{22}\boldsymbol{R} + \boldsymbol{T}^{\mathrm{T}}\boldsymbol{T}, \\
\boldsymbol{\Phi}_{56} &= \boldsymbol{P}_{22}\boldsymbol{S} + 2\boldsymbol{T}^{\mathrm{T}}\boldsymbol{U}, \\
\boldsymbol{\Phi}_{66} &= -\gamma^2\boldsymbol{I} + \boldsymbol{U}^{\mathrm{T}}\boldsymbol{U}, \\
\boldsymbol{\Phi}_{77} &= -\gamma^2\boldsymbol{I}, \\
\boldsymbol{\Phi}_{88} &= -\gamma^2\boldsymbol{I}.
\end{aligned}$$

Moreover, the controller gain matrix is $\boldsymbol{K} = \boldsymbol{M}^{-1}\boldsymbol{V}$.

Proof The following storage function is considered:

$$\boldsymbol{V}(t) = \begin{bmatrix} \boldsymbol{e}^{\mathrm{T}}(t) & \boldsymbol{\mu}^{\mathrm{T}}(t) \end{bmatrix} \boldsymbol{P} \begin{bmatrix} \boldsymbol{e}(t) \\ \boldsymbol{u}(t) \end{bmatrix} + \int_{-\phi_0}^{0} \int_{t+\theta}^{t} \dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Q}\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta,$$

where $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} \end{bmatrix} > 0$, $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > 0$, and $\phi_0 = (1+\delta_{\max})h + \tau_{\max}$. The supply rate with finite L_2-gain γ is described as

$$s(\boldsymbol{w}(t), \boldsymbol{u}(t)) = \gamma^2 \boldsymbol{w}(t)^{\mathrm{T}}\boldsymbol{w}(t) - \boldsymbol{\nu}^{\mathrm{T}}(t)\boldsymbol{\nu}(t),$$

where the exogenous input $\boldsymbol{w}(t) = [r(t); \epsilon; \beta_{\boldsymbol{m}}\delta_x]$ consists of the reference input, the disturbance signal, and a constant signal. System (1) is said to be dissipative with respect to the storage function and the supply rate if $\dot{V} \leq s(\boldsymbol{w}, \boldsymbol{\nu}), \forall \boldsymbol{w}, \boldsymbol{\nu}$. According to condition (11), it is satisfied with $\|\varphi(\boldsymbol{x}(t))\| \leq \beta_{\mathrm{m}}\delta_x$ and $\beta_{\mathrm{m}} = \|\Delta\boldsymbol{A}\|_2 + \max_i\{\omega_i\}\|\boldsymbol{B}_1\|_2\|\boldsymbol{C}_1\|_2 + \max_i\{\omega_i\}\|\boldsymbol{B}_2\|_2\|\boldsymbol{C}_2\|_2$.

The following equations hold for any matrices $\boldsymbol{N}_1, \boldsymbol{N}_2, \boldsymbol{M}$ with appropriate dimensions:

$$\begin{aligned}
&[\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{N}_1 + \boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{N}_2] \\
&\times [\boldsymbol{e}(t) - \boldsymbol{e}(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{e}}(s)\mathrm{d}s] = 0, \qquad (10) \\
&[\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{M} + \boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{M} + \dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{M}] \\
&\times [-\dot{\boldsymbol{e}}(t) + \boldsymbol{A}_2\boldsymbol{e}(t) + \boldsymbol{B}_2\eta(\boldsymbol{C}_2\boldsymbol{e}(t), \boldsymbol{z}(t)) \\
&+ \boldsymbol{D}\boldsymbol{T}\boldsymbol{\mu}(t) + \boldsymbol{D}\boldsymbol{U}r(t) - \boldsymbol{K}\boldsymbol{H}\boldsymbol{e}(t-\tau(t))
\end{aligned}$$

$$-\boldsymbol{K}\boldsymbol{\epsilon}+\boldsymbol{\varphi}(\boldsymbol{x}(t))]=0. \tag{11}$$

On the other hand, for any $\boldsymbol{\Lambda}=\mathrm{diag}\{\lambda_1,\lambda_2,\ldots,\lambda_{n_h}\}\geq 0$, it follows from inequality (7) that

$$\begin{aligned}0 \leq & -2\sum_{i=1}^{n_h}\lambda_i\eta_i(\boldsymbol{c}_{2i}\boldsymbol{e},\boldsymbol{z})(\eta_i(\boldsymbol{c}_{2i}\boldsymbol{e},\boldsymbol{z})-\omega_i\boldsymbol{c}_{2i}\boldsymbol{e})\\ = & -2\boldsymbol{\eta}^{\mathrm{T}}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))\boldsymbol{\Lambda}\boldsymbol{\eta}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))\\ & +2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{\Omega}\boldsymbol{\Lambda}\boldsymbol{\eta}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t)).\end{aligned} \tag{12}$$

For any positive real constant π, the following condition is satisfied:

$$\pi(\beta_{\mathrm{m}}^2\delta_x^2-\boldsymbol{\varphi}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{\varphi}(\boldsymbol{x}(t)))\geq 0. \tag{13}$$

Combining Eqs. (10)–(13), the condition $\dot{V}(t)-s\leq 0$ is checked

$$\begin{aligned}&\dot{V}(t)-s\\ \leq\ & 2\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}_{11}\boldsymbol{e}(t)+4\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}_{12}\boldsymbol{\mu}(t)\\ & +2\dot{\boldsymbol{\mu}}^{\mathrm{T}}(t)\boldsymbol{P}_{22}\boldsymbol{\mu}(t)+\phi_0\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{Q}\dot{\boldsymbol{e}}(t)\\ & -\int_{t-\phi_0}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Q}\dot{\boldsymbol{e}}(s)\,\mathrm{d}s\\ & +2[\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{N}_1+\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{N}_2]\\ & \times[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{\boldsymbol{e}}(s)\,\mathrm{d}s]\\ & +2[\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{M}+\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{M}+\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{M}]\\ & \times[-\dot{\boldsymbol{e}}(t)+\boldsymbol{A}_2\boldsymbol{e}(t)+\boldsymbol{B}_2\boldsymbol{\eta}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))\\ & +\boldsymbol{D}\boldsymbol{T}\boldsymbol{\mu}(t)+\boldsymbol{D}\boldsymbol{U}r(t)-\boldsymbol{K}\boldsymbol{H}\boldsymbol{e}(t-\tau(t))\\ & -\boldsymbol{K}\boldsymbol{\epsilon}+\boldsymbol{\varphi}(\boldsymbol{x}(t))]-2\boldsymbol{\eta}^{\mathrm{T}}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))\boldsymbol{\Lambda}\boldsymbol{\eta}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))\\ & +2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{\Omega}\boldsymbol{\Lambda}\boldsymbol{\eta}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))\\ & +\pi(\beta_{\mathrm{m}}^2\delta_x^2-\boldsymbol{\varphi}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{\varphi}(\boldsymbol{x}(t)))\\ & -\gamma^2(\boldsymbol{r}^{\mathrm{T}}(t)\boldsymbol{r}(t)+\boldsymbol{\epsilon}^{\mathrm{T}}\boldsymbol{\epsilon}+\beta_{\mathrm{m}}^2\delta_x^2)+\boldsymbol{\nu}^{\mathrm{T}}(t)\boldsymbol{\nu}(t).\end{aligned}$$

By using Lemma 1 and Lemma 2 and setting $\pi=\gamma^2$, the following result can be obtained:

$$\dot{V}(t)-s\leq\boldsymbol{\zeta}^{\mathrm{T}}(t)(\boldsymbol{\Theta}+\phi_0\boldsymbol{N}\boldsymbol{Q}^{-1}\boldsymbol{N}^{\mathrm{T}})\boldsymbol{\zeta}(t),$$

where $\boldsymbol{\zeta}(t)=[\boldsymbol{e}^{\mathrm{T}}(t)\ \ \dot{\boldsymbol{e}}^{\mathrm{T}}(t)\ \ \boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\ \ \boldsymbol{\eta}^{\mathrm{T}}(\boldsymbol{C}_2\boldsymbol{e}(t),\boldsymbol{z}(t))\ \ \boldsymbol{\mu}^{\mathrm{T}}(t)\ \ \boldsymbol{r}^{\mathrm{T}}(t)\ \ \boldsymbol{\epsilon}^{\mathrm{T}}\ \ \boldsymbol{\varphi}^{\mathrm{T}}(\boldsymbol{x}(t))]^{\mathrm{T}}$, matrices $\boldsymbol{\Theta}$ and $\boldsymbol{N}$ are defined in inequality (9). It is explicit that if $\boldsymbol{\Theta}+\phi_0\boldsymbol{N}\boldsymbol{Q}^{-1}\boldsymbol{N}^{\mathrm{T}}<0$ holds, then $\dot{V}(t)-s<0$ for any nonzero $\boldsymbol{\zeta}(t)$. By utilizing the Schur complement,[21] the condition in Theorem 1 can be obtained, and the proof is completed.

4. Illustrative example

In this section, a numerical example is given to demonstrate the effectiveness of the proposed networked H_∞ synchronization scheme.

Consider the following representation of Chua's circuit systems as in Refs. [7], [11], and [17]:

$$\begin{cases}\dot{x}_1(t)=a[x_2(t)-h(x_1(t))],\\ \dot{x}_2(t)=x_1(t)-x_2(t)+x_3(t),\\ \dot{x}_3(t)=-bx_2(t),\end{cases}$$

where $a=9$, $b=14.286$, and $h(x_1(t))=m_1x_1(t)+0.5(m_0-m_1)(|\,x_1(t)+c\,|-|\,x_1(t)-c\,|)$ with $m_0=-1/7$, $m_1=2/7$, and $c=1$. The generated chaotic attractor is shown in Fig. 1. It is clear that the system can be represented in Lur'e form with $\boldsymbol{A}=\begin{bmatrix}-am_1 & a & 0\\ 1 & -1 & 1\\ 0 & -b & 0\end{bmatrix}$, $\boldsymbol{B}=\begin{bmatrix}-a(m_0-m_1)\\ 0\\ 0\end{bmatrix}$, and $\boldsymbol{C}=[1\ 0\ 0]$, and $\sigma(x_1(t))=0.5(|\,x_1(t)+c\,|-|\,x_1(t)-c\,|)$ belongs to sector $[0,\omega]$ with $\omega=1$. The parameters of the designed synchronization scheme are offered as $\boldsymbol{A}_2=\boldsymbol{A}$, $\boldsymbol{B}_2=\boldsymbol{B}$, $\boldsymbol{C}_2=\boldsymbol{C}$, $\boldsymbol{D}=[1;1;1]$, $\boldsymbol{H}=\boldsymbol{I}$, and $\boldsymbol{\beta}=[1;0;0]$. For the reference model, $\mathcal{R}$ is chosen with cut-off frequency 10 Hz, the same as that in Ref. [7]. A perturbation of element a_{11} of the $\boldsymbol{A}$ matrix $\delta a_{11}=0.001$ is taken in the master system with respect to the nominal slave system.

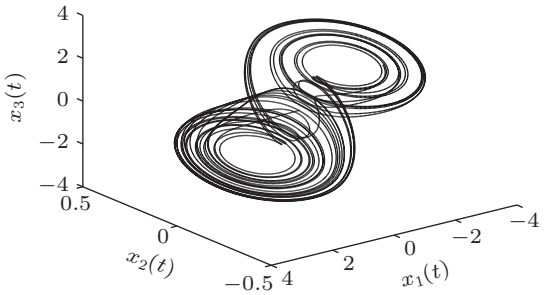


Fig. 1. Chaotic attractor of Chua's circuit systems.

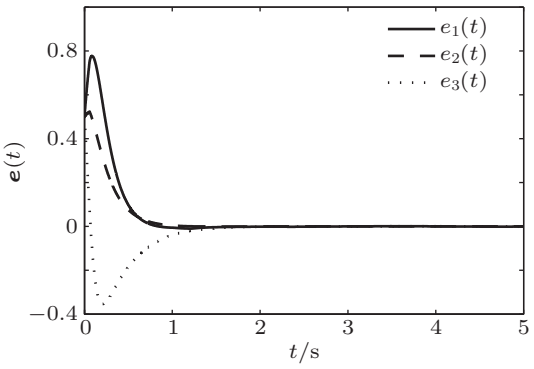


Fig. 2. Response curves of the error system.

Set $h=0.01$, $\delta_{\max}=5$, and $\tau_{\max}=0.01$, LMI (9) in Theorem 1 is feasible, with the minimal value of γ and the controller gain matrix being

$$\gamma_{\min}=3.42,$$

$$\boldsymbol{K} = \begin{bmatrix} 4.3646 & 0.6786 & -1.7671 \\ 1.4985 & 0.8053 & 0.3809 \\ -3.4511 & -1.9073 & 4.0412 \end{bmatrix}.$$

The initial conditions of the master and slave systems are chosen as $\boldsymbol{x}(0) = [\,1\ \ 1\ \ 1\,]^{\mathrm{T}}$ and $\boldsymbol{z}(0) = [\,0.5\ \ 0.5\ \ 0.5\,]^{\mathrm{T}}$, respectively. The response curves of the error system are shown in Fig. 2. It shows that the synchronization error converges to zero asymptotically.

5. Conclusion

In the present work, the robust networked H_∞ synchronization scheme for chaotic nonidentical chaotic Lur'e systems is proposed. The obtained result depends on the network characteristics. In future works, more performance requirements for the synchronization of chaotic Lur'e systems will be considered in a uniform network topological structure.

References

[1] Carroll T L and Pecora L M 1991 *IEEE Trans. Circ. Syst.* **38** 453
[2] Wang X Y, Zhang N, Ren X L and Zhang Y L 2011 *Chin. Phys. B* **20** 020507
[3] Wang X Y and Liu L T 2013 *Chin. Phys. B* **22** 050503
[4] Zhou P and Zhu W 2011 *Nonlinear Anal.: Real World Appl.* **12** 811
[5] Zhou P, Ding R and Cao Y X 2012 *Nonlinear Dynam.* **70** 1263
[6] Wang Z, Huang X, Li N and Song X N 2012 *Chin. Phys. B* **21** 050506
[7] Suykens J A K, Curran P F, Vandewalle J and Chua L O 1997 *IEEE Trans. Circ. Syst. I: Fund. Theory Appl.* **44** 891
[8] Liu F, Ren Y, Shan X M and Qiu Z L 2001 *Acta Phys. Sin.* **50** 2318 (in Chinese)
[9] Lu J G and Hill D J 2007 *IEEE Trans. Circ. Syst. II: Exp. Briefs* **54** 710
[10] Han Q L 2007 *IEEE Trans. Circ. Syst. I. Regul. Pap.* **54** 1573
[11] Lu J G and Hill D J 2008 *IEEE Trans. Circ. Syst. II: Exp. Briefs* **55** 586
[12] Ma T D, Zhang H G and Fu J 2008 *Chin. Phys. B* **17** 4407
[13] Chen W H, Lu X and Chen F 2008 *Phys. Lett. A* **372** 4210
[14] Li T, Yu J and Wang Z 2009 *Commun. Nonlinear Sci. Numer. Simul.* **14** 1796
[15] Guo H, Zhong S and Gao F 2009 *Appl. Math. Comput.* **212** 86
[16] Ji D H, Park J H and Won S C 2009 *Phys. Lett. A* **373** 1044
[17] Zhang C K, He Y and Wu M 2009 *IEEE Trans. Circ. Syst. II: Exp. Briefs* **56** 320
[18] Yin C, Zhong S M and Chen W F 2011 *Commun. Nonlinear Sci. Numer. Simul.* **16** 1632
[19] Lee S M, Kwon O M and Park J H 2011 *Chin. Phys. B* **20** 010506
[20] Yang D and Cai K Y 2010 *Proceedings of the 8th World Congress on Intelligent Control and Automation (WCICA)*, p. 4823
[21] Boyd S, Ghauoi L E, Feron E and Balakrishan V 1994 *Linear Matrix Inequalities in System and Control Theory* (Philadelphia: SIAM)