Surface structures of equilibrium restricted curvature model on two fractal substrates*

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With the aim to probe the effects of the microscopic details of fractal substrates on the scaling of discrete growth models, the surface structures of the equilibrium restricted curvature (ERC) model on Sierpinski arrowhead and crab substrates are analyzed by means of Monte Carlo simulations. These two fractal substrates have the same fractal dimension $d_{\rm f}$, but possess different dynamic exponents of random walk $z_{\rm rw}$. The results show that the surface structure of the ERC model on fractal substrates are related to not only the fractal dimension $d_{\rm f}$, but also to the microscopic structures of the substrates expressed by the dynamic exponent of random walk $z_{\rm rw}$. The ERC model growing on the two substrates follows the well-known Family–Vicsek scaling law and satisfies the scaling relations $2\alpha + d_{\rm f} \approx z \approx 2z_{\rm rw}$. In addition, the values of the scaling exponents are in good agreement with the analytical prediction of the fractional Mullins–Herring equation.

Keywords: equilibrium restricted curvature model, Sierpinski arrowhead, crab fractal substrate, dynamic scaling

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1. Introduction

The study of kinetic roughening of growing surfaces has very important theoretical significance and application value. In the statistical physics and condensed matter physics community, considerable efforts have been devoted to this field over the last several decades and many valuable breakthroughs have been achieved so far.[1–4] The theoretical works mainly focused on the numerical and analytical analyzes of various discrete atomistic growth models and continuum dynamic equations.[1–7]

It is widely known that the surface morphology usually has self-affine properties with simple space and time scaling properties, which can be described by the coarse-grained height variable $h(i,t)$ or the growth height over the i-th lattice site at time t. The roughness of the surface is usually characterized by the surface width W, which is defined as the standard deviation or the root mean square fluctuation of the surface height

$$W(L,t) = \sqrt{\frac{1}{L^d}\sum_{i=1}^{L^d}[h(i,t) - \langle h(t)\rangle]^2}, \tag{1}$$

where L and d are the linear size and the dimension of the substrate, respectively; $\langle h(t)\rangle$ denotes the average height over all lattice sites. In a finite system, it is generally hypothesized that the surface width satisfies the Family–Vicsek scaling relation,[8] which is written as

$$W(L,t) = L^{\alpha} f(\frac{t}{L^z}), \tag{2}$$

where the scaling function $f(u)$ behaves as $f(u) \sim u^{\beta}$ for $u \ll 1$ and becomes constant in the $u \gg 1$ limit. The scaling exponents α and z are called the roughness and dynamic exponent, respectively. The ratio $\beta = \alpha/z$ is called the growth exponent that describes the short time behavior of the surface fluctuation. These dynamic scaling exponents describe the asymptotic behavior of the growing surface on the large length and sufficiently late time scale, which can be used to determine the universality class of the model being researched.

The most prominent continuum dynamic equation is the nonlinear Kardar–Parisi–Zhang (KPZ) equation,[9] which is believed to describe a class of local growth models, such as the Eden model,[10] the ballistic deposition model[11] and the restricted solid-on-solid (RSOS) model[12] on regular lattices. Different from the KPZ equation, the fourth-order continuum linear Mullins–Herring (MH) equation describes the dynamics of the growth model under curvature constraint [13]

$$\frac{\partial h(\boldsymbol{x},t)}{\partial t} = -\nu\nabla^4 h(\boldsymbol{x},t) + \eta(\boldsymbol{x},t), \tag{3}$$

where $\eta(\boldsymbol{x},t)$ is an uncorrelated Gaussian noise

$$\langle \eta(\boldsymbol{x},t)\eta(\boldsymbol{x}',t')\rangle = 2D\delta^d(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t'), \tag{4}$$

with D describing the strength of the noise. Employing the scaling approach, the MH equation can be solved exactly by $\alpha = (4-d)/2$, $\beta = (4-d)/8$, $z = 4$.[13]

The ERC model has a restrction on the local surface curvature $\nabla^2 h$ rather than on the height difference $|\nabla h| = |h(\boldsymbol{x}) - h(\boldsymbol{x}')|$.[13] The scaling exponents of numerical simulations on the regular lattice are, respectively, $\alpha = 1.45 \pm 0.01$,

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$\beta = 0.365 \pm 0.001$, $z = \alpha/\beta = 3.97 (d = 1)$, and $\alpha = 0.97 \pm 0.01$, $\beta = 0.24 \pm 0.01$, $z = \alpha/\beta = 4.04 (d = 2)$,[13] where d is the substrare dimension. Within numerical accuracy, the numerical values are obviously consistent with the analytical predictions from Eq. (3); the scaling relation $2\alpha + d \approx z$ is also well satisfied. This equilibrium growth model was accordingly believed to belong to the universality class of the MH equation.[13]

Among the previous numerical simulations and analysis of discrete atomistic models, most were performed on regular or Euclidean substrates with an integer dimension, however, few were devoted to fractal substrates. As a result, there is no simple and clear understanding about the interplay between the dynamical growth rules of discrete models and the self-similarity of fractal structures until these recent works.[14–19]

The RSOS growing on the Sierpinski gasket and the checkerboard fractal embedded in three dimensions were studied in 2008.[17] These were two kinds of fractal substrates that had the same fractal dimension $d_{\rm f} = 2$. On a regular lattice, the RSOS model was generally believed to be described by the KPZ equation. The results obtained there, however, showed that the RSOS model on fractal substrates did not appear to follow the original KPZ equation.[17] The growing surfaces on fractal substrates were rougher than that of regular substrates. In Ref. [14], the equilibrium RSOS growth on fractal substrates including Sierpinski gasket, checkerboard fractal and Sierpinski carpet, were discussed. The numerical simulations of the discrete model agreed well with the analytical predictions of the fractional langevin equation introduced to describe the growth process; the scaling relations $2\alpha + d_{\rm f} \approx z \approx z_{\rm rw}$ hold, where $z_{\rm rw}$ is the dynamic exponent of random walk of the fractal substrates. After that, the validity of the fractional langevin equation and the scaling relation were confirmed by a model of random deposition with relaxation on both Sierpinski gasket and checkerboard fractal substrates.[16] By means of numerical simulations, Horowitz *et al.*[15] investigated the growth of ballistic aggregates on deterministic fractal substrates and observed the discrete scale invariance. They argued that the interplay between the spatial discrete scale invariance of the fractal substrate and the dynamics of the physical process occurring in those media were general phenomena. The systematic deviations of the data calculated there[15] was subsequently conjectured to be the finite-size effects of the Ballistic Deposition model.[19] Some of us analyzed numerically the growth of the modified family model and the etching model on Sierpinski carpet and found that both models on the fractal substrate still satisfied the Family–Vicsek scaling law and the modified family model was well described by the fractional Langevin equation introduced in Ref. [14]. The growth process of the etching model on the fractal substrate, however, can no longer be described by the original KPZ equation.[15]

In one of their recent works, Kim *et al.*[20] performed numerical simulations on the surface structures of the ERC model on Sierpinski gasket substrate and obtained the results $\alpha \approx 1.54$, $\beta \approx 0.323$, and $z = \alpha/\beta \approx 4.78$. The scaling exponents derived satisfied the scaling relations $2\alpha + d_{\rm f} \approx z$ and $z \approx 2z_{\rm rw}$ very well.[20] To describe this discrete model on fractal substrates, they introduced the fractional Langevin equation,[20] written as

$$\frac{\partial h(\boldsymbol{x},t)}{\partial t} = -\nu \nabla^{2z_{\rm rw}} h(\boldsymbol{x},t) + \eta(\boldsymbol{x},t), \tag{5}$$

where $\eta(\boldsymbol{x},t)$ is the uncorrelated noise

$$\langle \eta(\boldsymbol{x},t)\eta(\boldsymbol{x}',t')\rangle = 2D\delta^{d_{\rm f}}(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t'), \tag{6}$$

where $d_{\rm f}$ is the fractal dimension of the substrate.

Employing the scaling analysis used previously, equation (5) can be solved exactly by

$$\alpha = \frac{(2z_{\rm rw} - d_{\rm f})}{2}, \quad \beta = \frac{1}{2}\frac{(1-d_{\rm f})}{2z_{\rm rw}}, \quad z = 2z_{\rm rw}, \tag{7}$$

where $z_{\rm rw} = 2d_{\rm f}/d_{\rm s}$, $d_{\rm s}$ is the spectral dimension of the fractal substrate. Equation (7) indicated that the scaling exponents depended on the independent parameters, $z_{\rm rw}$ and $d_{\rm f}$ (or $d_{\rm s}$ and $d_{\rm f}$), which are related to the microscopic details of fractal substrates. It can be seen that there exists a scaling relation $2\alpha + d_{\rm f} = z$.

To probe the effects of the microscopic details of fractal substrates on the scaling behavior of the growth model, in this work, we analyze numerically the surface structures of the ERC model on Sierpinski arrowhead and crab substrates. These two substrates have the same fractal dimension $d_{\rm f}$, but different spectral dimensions $d_{\rm s}$.[21] Our results show that the values of the scaling exponents are related to both the fractal dimension $d_{\rm f}$ and the spectral dimensions $d_{\rm s}$ (or $z_{\rm rw}$), which are determined by the microscopic details of the fractal substrates. The dynamic scaling of the ERC model on both Sierpinski arrowhead and crab substrates can be well described by the fractional Mullins–Herring (FMH) equation introduced in Ref. [20], with the satisfaction of the scaling relation $2\alpha + d_{\rm f} \approx z \approx 2z_{\rm rw}$.

The rest of the paper is organized as follows: in Section 2, we briefly describe the structures of Sierpinski arrowhead and crab substrates, as well as the ERC model on fractal media. The numerical simulations and corresponding discussions are given in Section 3. In the last section, we present our conclusions.

2. Equilibrium restricted curvature model on two fractal substrates

2.1. Sierpinski arrowhead and crab lattice

As shown in Figs. 1 and 2, the fractal substrates discussed in this work are called, respectively, Sierpinski arrowhead and crab. The former is named the "Sierpinski arrowhead" because

it is very similar to the well-known Sierpinski gasket, whereas the latter is termed "crab" because the overall shape of the generated lattice looks like a crab.[21] These two lattices are generated from the arc-fractal system. The lattice sites are the invariant set of points of the arc-fractal. The morphologies of these two lattices are obviously different from each other due to the iteration rules. For the Sierpinski arrowhead, the rule for orientating the arc at each iteration is "in–out–in", while the rule is "out–in–out" for the crab. The hollow circles added between the real sites in Figs. 1 and 2 are not part of the lattice but reference sites in order to determine the linear size of substrates. Since the space between these sites is uniform, the linear size is then equal to the total number of hollow and solid circles along L. So at the n-th iteration, the linear size is given by $L_n = 2^n + 1$ for the Sierpinski arrowhead lattice and $L_n = 3 \times 2^{n-1}$ for the crab lattice. Both lattices have the same fractal dimension $d_f = \ln 3/\ln 2 \approx 1.585$[21] and the total number of sites on these two lattices at the n-th iteration is $N_n = 3^n + 1$. For the ERC model on these two fractal structures, their lattice sites are defined on the vertices of the regular hexagon. The growth proceeds on the lattice sites of the fractal substrate and, accordingly, the height variable is defined on those lattice sites.

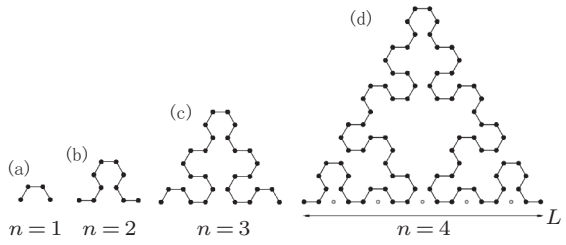


Fig. 1. Sierpinski arrowhead lattice at the first, the second, the third and the fourth iterations with $L = 17$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$ and (d) $n = 4$.

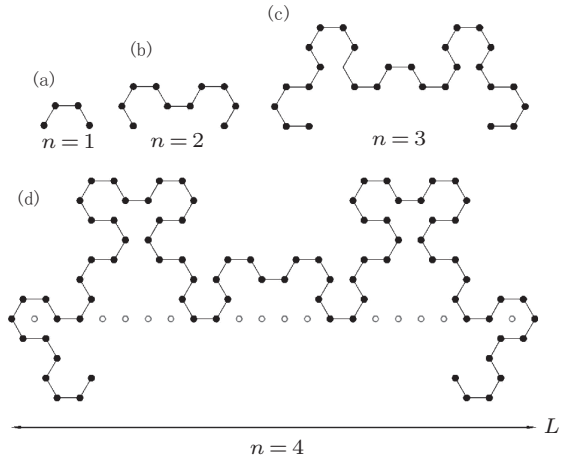


Fig. 2. Crab lattice at the first, the second, the third and the fourth iterations with $L = 24$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$ and (d) $n = 4$.

In the fractal substrates, the connection paths between two points apart from each other are quite restricted due to the nearby vacancies generated by iteration rules as compared to the case for the ordinary lattice. Therefore, the routes of the Sierpinski arrowhead and crab lattices like a one-dimensional linear chain. The number of the nearest-neighbor sites for these two lattices depends on the sites, that is, the coordination number of sites on these lattices varies between two and three except for the two end points. Asymptotically, one third of the sites have three nearest neighbors, while the remaining two thirds have two nearest neighbors. Our fractal lattices only have two end points at which two copies can join to form a bigger lattice.

In our simulations, we consider the neighboring height interactions among sites possess the same shortest Euclidean distance to a given site. Here, we introduce the chemical distance,[22] which is defined as the shortest distance along the connected path on a fractal lattice and is larger than the geometrical distance because the path detour holes created by iteration rules. Therefore, for a site whose coordinate number is three, the chemical distances between this point and its neighboring sites are different; one of them is quite a lot larger than the other two, even though their geometrical distances are the same.

2.2. Equilibrium restricted curvature model

The dynamic growth rule of the ERC model on Sierpinski arrowhead and crab substrates embedded in $d = 2$ dimensions is to randomly select a site x on the substrate lattice and then to deposit or evaporate a particle on the site with equal probability, provided that the restriction of the local curvature $\left|\nabla^2 h\right| = |h(x+1) + h(x-1) - 2h(x)| \leq N$ is obeyed between the selected site and the nearest-neighbor sites after the deposition or evaporation, where N is a preassigned positive integer. If this restriction of the local curvature is not satisfied on any of the sites, the deposition or evaporation process is forbidden and the height on the corresponding sites remains the same. It is found that the values of the exponents are independent of N as long as N satisfies the relation $N \geq 2(d-1)$,[13] where d is the dimension of substrate. Considering that the dimension of the fractal substrates discussed in this paper is $d_f \approx 1.585$, we assumed $N = 4$. The discrete curvature is defined as $\nabla^2 h_x = \sum_\delta (h_\delta - h_x)$, where the sum is taken for the nearest neighbors of site x. Thus, the ERC model is analogous to the equilibrium RSOS model,[12,23] except that the restriction is on the local curvature $\nabla^2 h$ rather than on the height difference $|\nabla h| = |h(x) - h(x')|$. Since there is a restriction only on the local curvature, the height difference between the nearest neighbors can be arbitrarily large, in contrast to the case for the RSOS model. This is why a high value of α is possible in the RC model. When a particle is deposited or evaporated at the corner sites on the edge, the nearest-neighbor sites to be examined for the ERC condition are limited due to the system boundary. In order to reduce the finite size effect, it is necessary to set up a certain boundary condition. The periodic

boundary and the free boundary are usually employed in critical phenomena; the former is known to be more successful in reducing the size effects.[14,19,24] Accordingly, the periodic boundary condition is employed in our simulations.

3. Numerical simulations and discussion

For the discrete models on fractal substrate, particles are deposited randomly on occupied sites of the substrate only by following the rules described. The ERC growths on different fractal substrates are performed by starting from a flat initial substrate at time $t = 0$, i.e., $h(x,t = 0) = 0$. The simulation time is measured in units of the number of trial Monte Carlo steps. The growth process is monitored by calculating the width $W(t)$ of the surface for the system generated up to the ninth generation, i.e., the linear sizes are $L = 9$, 17, 33, 65, 513 for the Sierpinski arrowhead and $L = 6$, 12, 24, 48, 96, 768 for the crab substrate. Since the dynamic exponent z is larger than four, the growth process leading to the saturation region will spend a great amount of computer time, that would considerably limit the system size discussed. The largest size simulated to the saturation width here is generated up to $n = 6$.

3.1. Simulation on Sierpinski arrowhead

The mean-square surface width is calculated on a Sierpinski arrowhead substrate for the system sizes of $L = 9, 17, 33, 65, 513$ and the results are illustrated in Fig. 3. The surface widths are averaged over 500–50000 independent runs varying with the system size simulated.

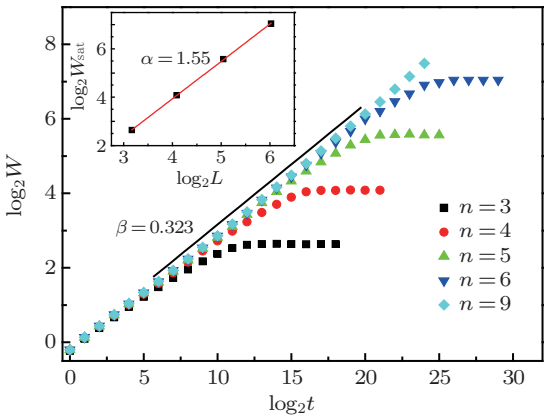


Fig. 3. (color online) The log–log plot of the interface width $W(t)$ as a function of time t in the ERC model on a Sierpinski arrowhead substrate for the system of sizes $L = 9$, 17, 33, 65, 513. The inset is the log–log plot of the saturated interface width versus the system sizes $L = 9$, 17, 33, 65.

Starting from a flat surface, the fluctuations of the surface width increase with time and eventually reach the saturated states when the parallel correlation length reaches the size of the system. The standard procedure is followed to calculate the growth, roughness and dynamic exponents. In the early time $t \ll L^z$, the lateral correlations develop and lead to the typical growth regime with $W(t) \sim t^{\beta}$. As time increases, the correlation length of the surface height increases as $\xi(t) \sim t^{1/z}$. Based on the regression fit of the data from the biggest size in the early time, we obtain $\beta = 0.323 \pm 0.002$.

In the saturated state regime $t \gg L^z$, the saturated width follows a power-law $W_{\rm sat}(L) \sim L^{\alpha}$. To determinate the roughness exponent describing the saturation of the interface fluctuation, the relation $W_{\rm sat}(L) \sim L^{\alpha}$ is assumed with respect to the linear sizes of $L = 9$, 17, 33, 65. In the inset of Fig. 3, the results for $W_{\rm sat}(L)$ are plotted on a double-logarithmic scale, with a power-law fit which yields $\alpha = 1.55 \pm 0.02$.

With the calculated values of β and α, the corresponding exponent z is obtained as $z = \alpha/\beta \approx 4.80$, which is obviously larger than $z = 4$ for on the regular substrate.[13] Due to the limitation of the computer power, the largest system that reaches the saturated area in the calculation is $L = 65$, where a simulation up to $t = 10^8$ is required to obtain the saturation width.

It was known that the fractal dimension and the dynamic exponent of random walk of the Sierpinski arrowhead are $d_{\rm f} = 1.585$ and $z_{\rm rw} = 2.322$, respectively.[21] The analytical results predicted from Eq. (7) are $\beta = 0.329$, $\alpha = 1.53$ and $z = 4.64$. Within the margin of the numerical errors, it could be clearly concluded that our numerical estimates here can match the analytical predictions derived from Eq. (7), as well as satisfy the identities $2\alpha + d_{\rm f} \approx z$ and $z \approx 2z_{\rm rw}$.

Although the Sierpinski arrowhead and the Sierpinski gasket have the same fractal dimension $d_{\rm f}$ and dynamic exponent of random walk $z_{\rm rw}$, they actually have different structures. One of the main differences of these two fractals is the lack of the uniformity of the Sierpinski arrowhead in the coordination number. A Sierpinski gasket has a uniform coordination number four, while the number of nearest-neighbor sites for the Sierpinski arrowhead depends on the sites. The coordination number of sites on the Sierpinski arrowhead varies between two and three besides the end points. Generally, the growth on sites with a larger coordination number, compared to the growth on other sites, may be suppressed due to the extra restrictions. However, our calculations here are very similar to the results of the ERC model on the Sierpinski gasket.[20] It seems that the fractal substrates will share the same dynamic scaling exponents if they have the same fractal dimension and dynamic exponent of random walk.

The scaled data for the surface width collapse excellently to a single curve with the measured values of the exponents on Sierpinski arrowhead, as shown in Fig. 4, which implies that the ERC on the Sierpinski arrowhead also obeys well the widely-established Family–Vicsek dynamic scaling expressed by Eq. (2).

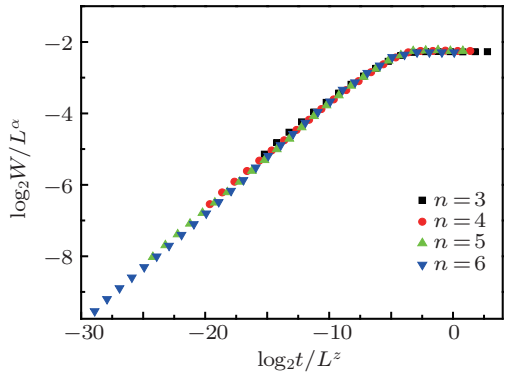


Fig. 4. (color online) Scaling plot of $\log_2(W/L^\alpha)$ against $\log_2(t/L^z)$ with $\alpha = 1.55$ and $z = 4.80$ for system sizes $L = 9$, 17, 33, 65 on Sierpinski arrowhead substrates.

For the sake of clearness, the scaling exponents of the ERC model on a Sierpinski arrowhead derived respectively by numerical and analytical methods are shown in Table 1.

Table 1. The dynamic scaling exponents of the ERC model on a Sierpinski arrowhead.

Scaling exponents	α	β	z	$2\alpha + d_f$
Numerical results	1.55±0.02	0.323±0.002	4.80	4.68
Analytical results	1.53	0.329	4.64	4.64

3.2. Simulation on crab fractal

The numerical simulations of the ERC model on a crab fractal substrate are carried out with the system sizes $L = 6$, 12, 24, 48, 96, 768. The surface widths calculated are shown in Fig. 5. The inset displays the saturated widths versus the system sizes L. Based on the scaling relation $W(t) \sim t^\beta$, at an early time, we obtain $\beta = 0.350 \pm 0.001$. In the long-time region, the saturated width follows a power-law $W_{\rm sat}(L) \sim L^\alpha$ and the exponent α is estimated from the data in the inset as $\alpha = 1.80 \pm 0.01$. Based on the values of α and β calculated, we have $z = \alpha/\beta \approx 5.14$ for the ERC model on crab substrates, which are obviously different from the values calculated for the Sierpinski arrowhead and the Sierpinski gasket.[20]

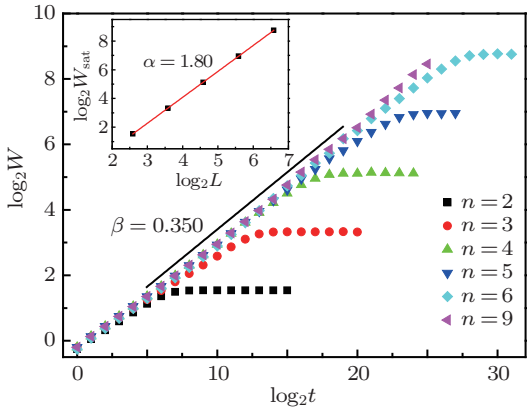


Fig. 5. (color online) Log–log plot of the interface width $W(t)$ as a function of time t in the restricted curvature model on crab substrate for the system of sizes $L = 6$, 12, 24, 48, 96, 768. The inset is the log–log plot of the saturated interface width versus the system size with $L = 12$, 24, 48, 96.

The fractal dimension and the dynamic exponent of random walk of the crab substrate are $d_f = 1.585$ and $z_{\rm rw} = 2.578$. Accordingly, the corresponding scaling exponents predicted from Eq. (7) are respectively $\alpha = 1.79$, $\beta = 0.346$ and $z = 5.16$. Within the numerical accuracy, the numerical values calculated here are also in agreement with the analytical estimations of Eq. (7). As a result, we can conclude that the ERC model on the crab substrate should belong to the universality described by the fractal MH equation given as Eq. (5).

The scaling plot of W/L^α against t/L^z with $\alpha = 1.80$ and $z = 5.14$ for various system sizes $L = 12$, 24, 48, 96 are shown in Fig. 6, where the excellent data collapse is obtained, with the exception of a slight deviation of the early-time data. This result implies that the ERC model on the crab substrate also satisfy the Family–Vicsek scaling expressed in Eq. (2). Moreover, the scaling relations $2\alpha + d_f \approx z$ and $z \approx 2z_{\rm rw}$ are also obeyed.

In order to be compared clearly, the scaling exponents of both the numerical and analytical analyzes are shown in Table 2.

Table 2. The dynamic scaling exponents of the ERC model on crab substrate.

Scaling exponents	α	β	z	$2\alpha + d_f$
Numerical results	1.80±0.01	0.350±0.001	5.14	5.18
Analytical results	1.79	0.346	5.16	5.16

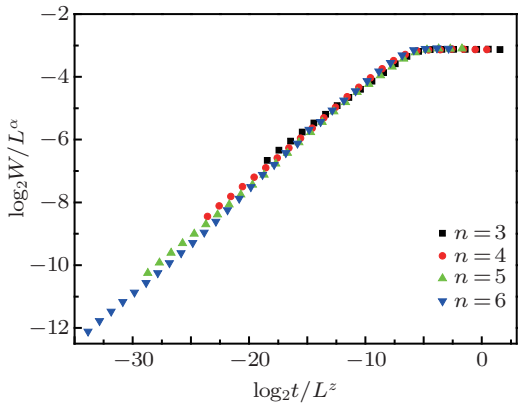


Fig. 6. (color online) The scaling plot of $\log_2(W/L^\alpha)$ against $\log_2(t/L^z)$ with $\alpha = 1.80$ and $z = 5.14$ for system sizes $L = 12$, 24, 48, 96 on crab substrates.

4. Conclusions

In this work, we have made extensively numerical simulations on the surface structures of the ERC model growing on both Sierpinski arrowhead and crab substrates embedded in two-dimensional Euclidian lattices. These two fractal substrates are generated from the arc-fractal system and have the same fractal dimension d_f, but different microscopic structures indicated by the dynamic exponent of random walk $z_{\rm rw}$, which actually could result in the differences of the corresponding dynamic exponents. It is consequently concluded that the surface structure of the ERC model developing on the Sierpinski

arrowhead and crab substrates are affected observably by the microscopic details of the substrates characterized by the dynamic exponent of random walk $z_{\rm rw}$.

Our results show that the dynamic scaling of the ERC model on these two fractal substrates follow the Family–Vicsek scaling function with the satisfaction of the relations $2\alpha + d_{\rm f} \approx z \approx 2z_{\rm rw}$. Moreover, within the margin of numerical errors, the exponents calculated here are consistent with the analytical predictions of Eq. (7) derived from Eq. (5), which implies that the fractal MH equation can well describe the dynamic scaling behaviors of the ERC model on the substrates discussed in this work. In other words, our results have proved the validity of the fractal MH equation on describing the scaling behavior of the discrete models on fractal substrates.

In conjunction with the analyzes of the growth of the ERC model on the regular lattice[13] and on the Sierpinski gasket,[20] we seems to get a general picture with respect to the surface structures of the ERC model: Whether on the regular or fractal lattices, the surface growth of the ERC model always follows the Family–Vicsek scaling and satisfies the scaling relations $2\alpha + d \approx z$ or $2\alpha + d_{\rm f} \approx z \approx 2z_{\rm rw}$, respectively. The fractal structures of substrates would have significant influences on the surface morphology of the ERC model. On the regular substrates, this model belongs to the universality class described by the usual MH equation, while on the fractal substrates discussed in this paper and Ref. [20], this model should belong to the universality class described by the fractal MH equation. It is reasonable to believe that the fractal MH equation should have a wider application in the field of dynamic processes involving fractal structures. Further studies may be necessary to probe in-depth the effects of the fractal substrates on the surface structures.

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