Stochastic resonance for a metapopulation system driven by multiplicative and additive colored noises*

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We investigate the stochastic resonance (SR) phenomenon induced by the periodic signal in a metapopulation system with colored noises. The analytical expression of signal-to-noise is derived in the adiabatic limit. By numerical calculation, the effects of the addictive noise intensity, the multiplicative noise intensity and two noise self-correlation times on SNR are respectively discussed. It shows that: (i) in the case that the addictive noise intensity M takes a small value, a SR phenomenon for the curve of SNR appears; however, when M takes a large value, SNR turns into a monotonic function on the multiplicative noise intensity Q. (ii) The resonance peaks in the plots of the multiplicative noise intensity Q versus its self-correlation time τ_1 and the addictive noise intensity M versus its self-correlation time τ_2 translate in parallel. Meanwhile, a parallel translation also appears in the plots of τ_1 versus Q and τ_2 versus M. (iii) The interactive effects between self-correlation times τ_1 and τ_2 are opposite.

Keywords: metapopulation, Levins model, colored noises, signal-to-noise ratio

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1. Introduction

In the last three decades, the phenomenon of stochastic resonance (SR) has been investigated extensively in bistable systems,[1–4] asymmetric bistable systems,[5,6] excited linear systems,[7–9] coupled oscillators,[10–12] even in linear systems.[13,14] The output signal of the system may be amplified in the presence of an optimal level of noise and a weak periodic signal. The concept of noise-induced SR has been widely applied to many fields, ranging from electronic circuits and lasers to chemical systems and the connection with some situations of biological interests.[15–21] Yang and Liu[22] investigated the stochastic resonance in a cancer development system driven by colored noises. Guo *et al.*[23–25] studied the stochastic resonance in a monostable system driven by a square-wave signal and a time-delayed bistable system subject to a square-wave signal and multiplicative and additive noise. Bai *et al.*[26] discussed the stochastic resonance in a gene system driven by a multiplicative signal and correlated noises. Jia[27] studied the stochastic resonance in a cancer growth system driven by time delay and correlated noises. Wang[28] investigated the stochastic resonance in a gene transcriptional system subject to noises.

The concept of a metapopulation system was introduced by Levins when he studied the extinction phenomenon of the species because of the influence of the environment. Based on the Levins model, ecologists did a large amount of research on the metapopulation system.[29–35] Nowadays, the Levins model is honored as the mother of the metapopulation system. However, the previous work was confined to the deterministic Levins model; the external environment and the undulation effect in the metapopulation system were ignored. Under the practical condition, an immense amount of concrete research shows that noises play a very important role in the nonlinear system.

In fact, there has been a lot of research on stochastic resonance in nonlinear systems. However, most of them only discussed a part of SR phenomena, which were induced by the noises and their self-correlation times on the signal-to-noise ratio (SNR). In this paper, we study completely and systematically all of the SR phenomena that are evoked by the multiplicative, additive colored noises and their self-correlation times on SNR in the Levins model, which is also called the metapopulation system. We discuss all possible stochastic resonance phenomena between any two variational parameters in all noises and self-correlation times. On the other hand, the metapopulation system we investigated is a fresh nonlinear biological system; the stochastic resonance phenomenon based on this biological nonlinear system has not yet been studied in the SR field. The research on the signal-to-noise in metapopulation systems is useful to understand how biotic population make out and amplify the signals of food and natural enemies by means of appropriate noises and self-correlation times. In

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other words, we hope to master the optimal intensities and self-correlation times of noises for the wildlife population capturing the advantageous signals from the outside world.

2. Model

Levins' deterministic model is defined as

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = cx(1-x) - ex, \quad x \in [0,1].$$

The Levins model is assumed to satisfy some conditions, as follows: (i) the metapopulation exists in the shape of discrete habitat patches. (ii) All of the local populations of the metapopulation are confronted with the same extinction risk. (iii) All of the habitat patches are not isolated and can be invaded or occupied once again. (iv) The trends of all local populations are not synchronous. In the Levins model, c denotes the diffusion rate of the metapopulation invading the empty habitat patches, e denotes the extinction rate of the local populations. We denote by a multiplicative Gaussian colored noise $\xi(t)$ the effects of surroundings, such as the temperature, climate, and frost, on the extinction rate of a metapopulation system. Meanwhile, we denote by another additive Gaussian colored noise $\eta(t)$ the interaction effects of all individuals in the metapopulation system and consider a weak low-frequency signal added to the stochastic system. With the varying of the intensities of two noises $\xi(t)$ and $\eta(t)$, the SNR of the metapopulation system will also fluctuate accordingly. Therefore, we can rewrite Levins' deterministic model as follows:

$$\begin{aligned}\frac{\mathrm{d}x(t)}{\mathrm{d}t} &= cx(1-x) - [e+\xi(t)]x + \eta(t) + A\cos(\omega t),\\ & x \in [0,1],\end{aligned} \tag{1}$$

where $\xi(t)$ and $\eta(t)$ are the multiplicative noise and the additive noise, respectively. The two noise terms are characterized by their mean and variance

$$\begin{aligned}&\langle \xi(t)\rangle = \langle \eta(t)\rangle = 0,\\ &\langle \xi(t)\xi(t')\rangle = \frac{Q}{\tau_1}\exp\left[-\frac{|t-t'|}{\tau_1}\right],\\ &\langle \eta(t)\eta(t')\rangle = \frac{M}{\tau_2}\exp\left[-\frac{|t-t'|}{\tau_2}\right],\\ &\langle \xi(t)\eta(t')\rangle = \langle \eta(t)\xi(t')\rangle = 0,\end{aligned} \tag{2}$$

where Q and M describe the intensities of the multiplicative noise and the additive noise, respectively, τ_1 and τ_2 are the self-correlation times of the noises and $A\cos(\omega t)$ is a weak low-frequency signal, i.e., $A \ll 1$ and $\omega \ll 1$. Equation (1) can be considered as describing an overdamped motion of the state variable moving in a quasi-"free energy potential"

$$V(x) = \frac{1}{3}cx^3 + \frac{1}{2}(e-c)x^2. \tag{3}$$

In the absence of an external periodic force and noise terms, the fixed points of Eq. (1) strongly depend on c and e. We can obtain an unstable point $x_0 = 0$ and a stable point $x_s = 1 - e/c$.

3. Output SNR

By applying the Novikov theorem[29] and Fox's approach,[30] the approximate Fokker–Planck equation (FPE) corresponding to Eq. (1) can be written as

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\mu(x)P(x,t) + \frac{\partial^2}{\partial x^2}\sigma^2(x)P(x,t). \tag{4}$$

The drift coefficient $\mu(x)$ and the diffusion coefficient $\sigma^2(x)$ are

$$\begin{aligned}\mu(x) &= f(x) + \frac{Qx}{1-\tau_1 f'(x_s)} + A\cos(\omega t),\\ \sigma^2(x) &= \frac{Qx^2}{1-\tau_1 f'(x_s)} + \frac{M}{1-\tau_2 f'(x_s)},\end{aligned} \tag{5}$$

with

$$f(x) = cx(1-x) - ex. \tag{6}$$

The FPE is valid for $1 - \tau_i f'(x_s) > 0$ $(i = 1,2)$. The steady-state probability distribution function corresponding to the Fokker–Planck equation, equation (4) can be written as

$$\begin{aligned}P_{\mathrm{st}}(x,t) &= \frac{N}{\sigma(x)}\exp\left[\int_x \frac{\mu(x')}{\sigma^2(x')}\mathrm{d}x'\right]\\ &= \frac{N}{\sigma(x)}\exp[U(x)],\end{aligned} \tag{7}$$

where N is the normalization constant and $U(x)$ is the modified potential function, which is described as

$$\begin{aligned}U(x) &= -\frac{cd_1x}{Q} + \frac{\ln(Qx^2d_2 + Md_1)}{2Q}(Q + cd_1 - d_1e)\\ &\quad + \frac{\arctan\left(Qxd_2/\sqrt{QMd_1d_2}\right)}{Q\sqrt{QMd_1d_2}}\\ &\quad \times [A\cos(\omega t)Qd_1d_2 + d_1^2cM],\end{aligned} \tag{8}$$

where

$$d_1 = 1 - \tau_1 f'(x_s), \quad d_2 = 1 - \tau_2 f'(x_s).$$

The mean first-passage time (MFPT) of the metapopulation system can be given by the Kramers time

$$T(x_s \to x_0) = \frac{2\pi}{\sqrt{|V''(x_0)V''(x_s)|}}\exp[U(x_s) - U(x_0)], \tag{9}$$

where $V(x)$ and $U(x)$ are defined by Eqs. (3) and (8), respectively. Between the two states x_0 and x_s, the transition rate W (the traverse direction is $x_s \to x_0$) can be obtained as follows:

$$W = \frac{1}{T} = \frac{\sqrt{|V''(x_0)V''(x_s)|}}{2\pi}\exp[U(x_0) - U(x_s)]. \tag{10}$$

By virtue of the theory SR proposed by McNamara and Wiesenfeld,[1] we can obtain the expression of SNR of the system in the adiabatic limit as follows:

$$\mathrm{SNR} = \frac{\pi W_1^2 A^2}{4W_0}\left[1 - \frac{W_1^2 A^2}{2(W_0^2+\omega^2)}\right]^{-1}, \tag{11}$$

where

$$\begin{aligned} W_0 &= 2W|_{A\cos(\omega t)=0} \\ &= \frac{e-c}{\pi}\exp\left\{\frac{\ln(d_1 M)}{2Q}(Q-ed_1+cd_1)+\frac{cd_1(1-e/c)}{Q}\right. \\ &\quad +\frac{\ln[Q(1-e/c)^2 d_2+Md_1]}{2}\left(\frac{d_1 e}{Q}-1-\frac{d_1 c}{Q}\right) \\ &\quad \left.-\frac{\arctan\left[(1-e/c)Qd_2/\sqrt{QMd_1d_2}\right]Md_1^2 c}{Q\sqrt{QMd_1d_2}}\right\}, \\ W_1 &= \left.\frac{2\mathrm{d}W}{\mathrm{d}A\cos(\omega t)}\right|_{A\cos(\omega t)=0} \\ &= \frac{e-c}{\pi\sqrt{QMd_1d_2}}\arctan\left[\frac{(1-e/c)Qd_2}{\sqrt{QMd_1d_2}}\right]d_1 d_2 \\ &\quad \times\exp\left\{\frac{\ln(d_1 M)}{2Q}(Q-ed_1+cd_1)+\frac{cd_1(1-e/c)}{Q}\right. \\ &\quad +\frac{\ln[Q(1-e/c)^2 d_2+Md_1]}{2}\left(\frac{d_1 e}{Q}-1-\frac{d_1 c}{Q}\right) \\ &\quad \left.-\frac{\arctan\left[(1-e/c)Qd_2/\sqrt{QMd_1d_2}\right]Md_1^2 c}{Q\sqrt{QMd_1d_2}}\right\}. \end{aligned} \tag{12}$$

4. Discussion

By virtue of Eqs. (11)–(12), we discuss the effect of the external force and the noises as well as the system parameters on the SNR.

The SNR as a function of the multiplicative noise intensity Q is plotted in Fig. 1 for different values of the multiplicative noise self-correlation time τ_1. If the additive noise intensity $M = 0.01$, just like that in Fig. 1(a), with the increase of Q, the function of SNR is non-monotonic. The curves show the conventional SR phenomena. The peak value of SNR stays invariant with the increase of τ_1, but its position moves to the right. When $M = 0.3$, figure 1(b) shows that the function of SNR changes into a monotonic one. The SNR decreases with the increasing of Q and increases with the increasing of τ_1.

Figure 2 is a plot of SNR as a function of the additive noise intensity M for different values of τ_1. The SNR is a non-monotonic function on the additive noise intensity M. As seen from Fig. 2, SNR increases monotonically as τ_1 increases. From Fig. 2, we can also see that the peaks of SNR become higher and its position shifts to the right slowly with the increase of τ_1.

Figure 3(a) shows that there is one peak in the SNR curves for all small values of τ_2. With the increase of τ_2, the peak value becomes smaller and the position moves to the right gradually. Figure 3(b) shows that in the case of $M = 0.3$, the SNR is a monotonic decreasing function on the multiplicative noise intensity Q. Simultaneously, the additive noise self-correlation time τ_2 can enhance SNR.

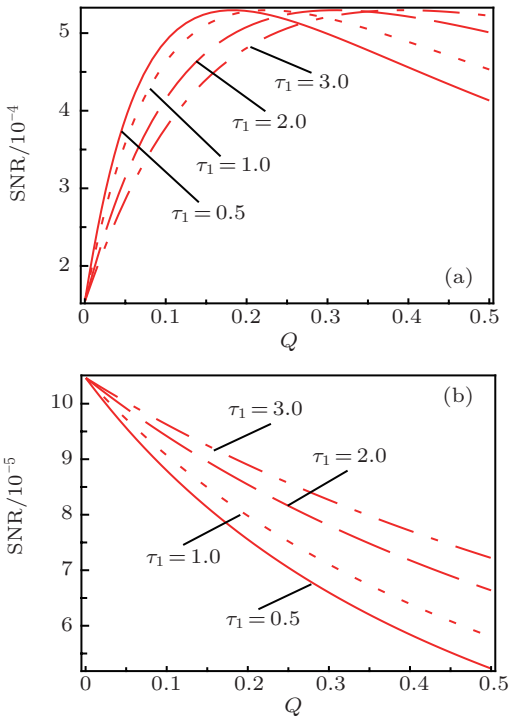


Fig. 1. (color online) The SNR versus multiplicative noise intensity Q for different values of τ_1 at $e = 0.2$, $c = 0.8$, $\tau_2 = 0.3$, $A = 0.01$, $\omega = 0.01$, and (a) $M = 0.01$, (b) $M = 0.3$.

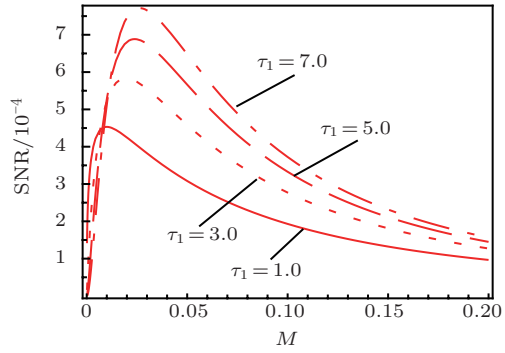


Fig. 2. (color online) The SNR versus additive noise intensity M for different values of τ_1 at $Q = 0.5$, $c = 0.8$, $e = 0.2$, $\tau_2 = 0.3$, $A = 0.01$, $\omega = 0.01$.

Figure 4 is a plot of SNR as a function of additive noise intensity M for different values of τ_2. Comparing this figure with Fig. 1(a), we can easily see that the behaviors of these plots are similar, the SR is observed in both cases. The resonance peak value stays invariant with the increase of τ_2, however, its position shifts to the right quickly. The resonance peak broadens gradually with the increase of τ_2.

Figure 5 is a plot of SNR as a function of additive noise self-correlation time τ_2 for different values of τ_1. From Fig. 5, we can see that the SNR is a non-monotonic function of additive noise self-correlation time τ_2. With the increase of τ_1, the peak of SNR is enhanced but the peak's position on τ_2 almost stays invariant. The multiplicative noise self-correlation time τ_1 plays an important role in enhancing the value of the SNR.

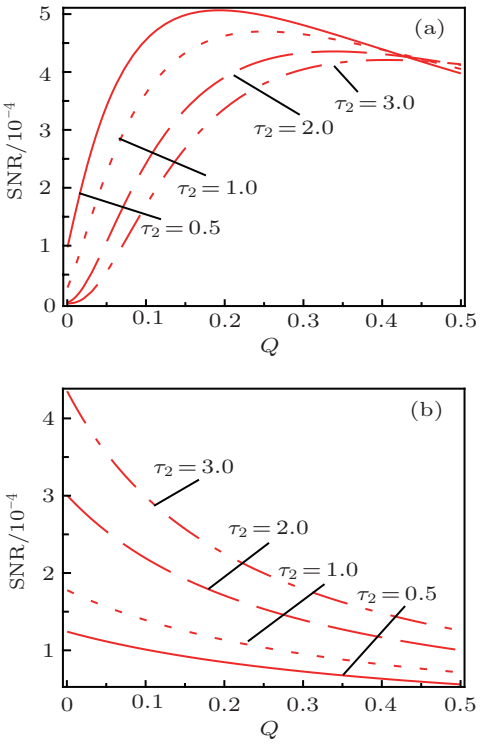


Fig. 3. (color online) The SNR versus multiplicative noise intensity Q for different values of τ_2 at $e = 0.2$, $c = 0.8$, $\tau_1 = 0.3$, $A = 0.01$, $\omega = 0.01$, and (a) $M = 0.01$, (b) $M = 0.3$.

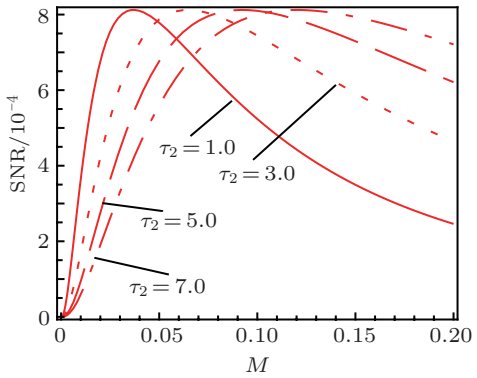


Fig. 4. (color online) The SNR versus additive noise intensity M for different values of τ_2 at $Q = 0.1$, $c = 0.8$, $e = 0.2$, $\tau_1 = 0.3$, $A = 0.01$, $\omega = 0.01$.

The curves for the SNR versus multiplicative noise self-correlation time τ_1 with different additive noise self-correlation time τ_2 are shown in Fig. 6. Different from that in Fig. 5, we can observe in Fig. 6 that the peak of the SNR descends gradually and its position moves to the left quickly with the increase of τ_2. Figure 6 shows that the additive noise self-correlation time τ_2 can restrain the appearance of the peak of the SNR.

In Fig. 7, we analyze the influence of the multiplicative noise intensity Q on SNR when the SNR is shown to be a function of the additive noise intensity. It is easy to see that the curve of SNR is originally a non-monotonic function on M. The peak of SNR declines rapidly and its position shifts to the left with the increase of Q. Finally, the phenomenon of SR disappears and the SNR becomes a monotonic decreasing function on the additive noise intensity M. Figure 7 illustrates that the multiplicative noise intensity Q plays an important role in restraining the phenomenon of the stochastic resonance.

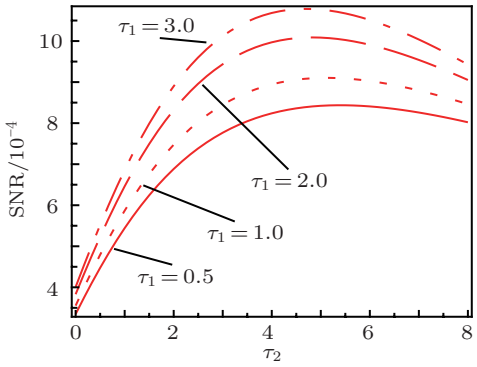


Fig. 5. (color online) The SNR versus additive noise self-correlation time τ_2 for different values of τ_1 at $Q = 0.1$, $M = 0.01$, $e = 0.2$, $c = 0.8$, $A = 0.01$, $\omega = 0.01$.

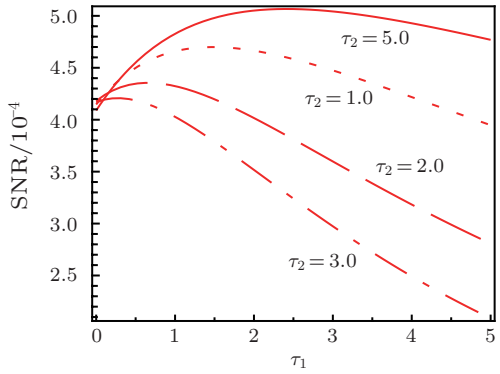


Fig. 6. (color online) The SNR versus multiplicative noise self-correlation time τ_1 for different values of τ_2 at $Q = 0.1$, $M = 0.01$, $c = 0.8$, $e = 0.2$, $A = 0.01$, $\omega = 0.01$.

Figure 8 presents the curves for SNR versus the multiplicative noise intensity Q with different values of M. As seen in Fig. 8, when the additive noise intensity M takes a smaller value, a peak of SNR appears. With the increase of additive noise intensity M, the peak rises to the left in the beginning. Finally, the SNR degenerates into a monotonic decreasing function on the multiplicative noise intensity Q. For the case of $Q > 0.5$, the additive noise intensity M plays a role in weakening the SNR.

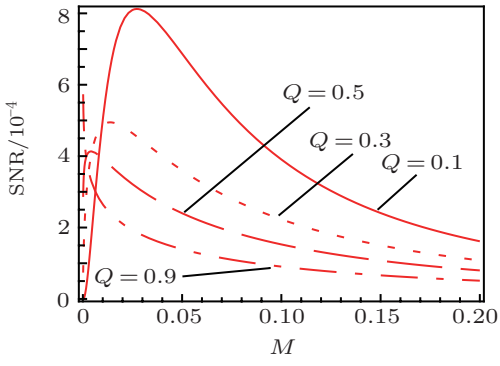


Fig. 7. (color online) The SNR versus additive noise intensity M for different values of Q at $\tau_1 = 0.3$, $\tau_2 = 0.3$, $c = 0.8$, $e = 0.2$, $A = 0.01$, $\omega = 0.01$.

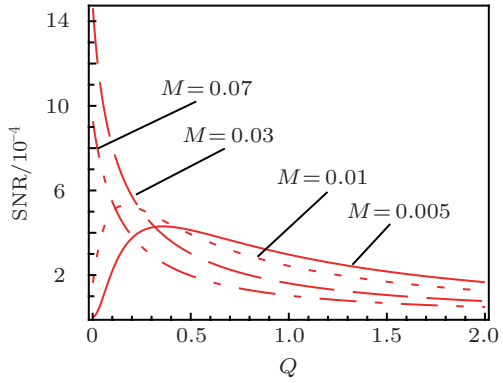


Fig. 8. (color online) The SNR versus multiplicative noise intensity Q for different values of M at $\tau_1 = 0.3$, $\tau_2 = 0.3$, $c = 0.8$, $e = 0.2$, $A = 0.01$, $\omega = 0.01$.

Figure 9 is a plot of SNR as a function of multiplicative noise self-correlation time τ_1 for different values of Q. From Fig. 5, we can observe that the peak value of the stochastic resonance remains invariant with the increase of Q, but its position shifts to the right quickly. The resonance peak widens step by step with the increase of Q.

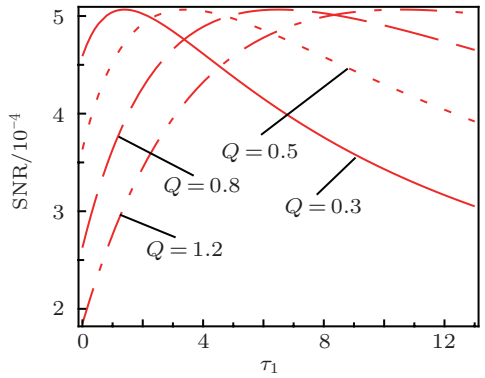


Fig. 9. (color online) The SNR versus multiplicative noise self-correlation time τ_1 for different values of Q at $\tau_2 = 0.3$, $M = 0.01$, $c = 0.8$, $e = 0.2$, $A = 0.01$, $\omega = 0.01$.

Figure 10 reflects the effect of the additive noise self-correlation time τ_2 on SNR with the variation of Q. We can see that the SNR is a non-monotonic function on additive noise self-correlation time τ_2. With the increase of small values of Q, the peak of SNR is reduced promptly. The multiplicative noise intensity Q plays a key role in suppressing the phenomenon of the SNR.

Figure 11 shows the changes of the SNR with multiplicative noise self-correlation time τ_1 for different values of M. As shown in Fig. 11, the SNR is a monotonic increasing function on τ_1. On the other hand, with the increase of additive noise intensity M, the SNR is weakened promptly. Therefore, the additive noise intensity M and the multiplicative noise self-correlation time τ_1 play the opposite roles in motivating the phenomenon of the SNR.

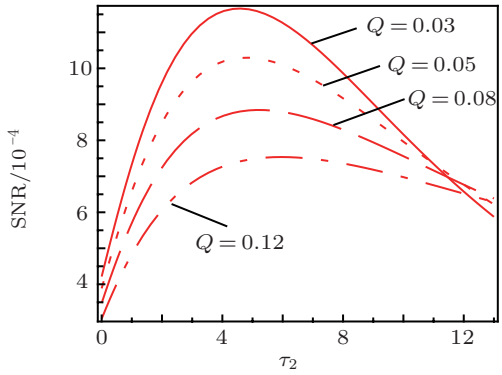


Fig. 10. (color online) The SNR versus additive noise self-correlation time τ_2 for different values of Q at $\tau_1 = 0.3$, $M = 0.01$, $c = 0.8$, $e = 0.2$, $A = 0.01$, $\omega = 0.01$.

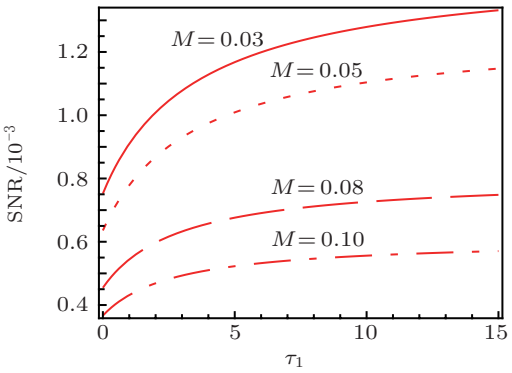


Fig. 11. (color online) The SNR versus additive noise self-correlation time τ_1 for different values of M at $\tau_2 = 0.3$, $Q = 0.1$, $c = 0.8$, $e = 0.2$, $A = 0.01$, $\omega = 0.01$.

Figure 12 displays the influence of the additive noise self-correlation time τ_2 on the SNR for different values of M. Combining Fig. 12 with Fig. 9, we find that the two figures show similar properties: a single resonance peak arises with the increase of τ_2, the peak value of the stochastic resonance stays invariant with the increase of M, but its position moves to the right gradually and the resonance peak widens stage by stage with the increase of M.

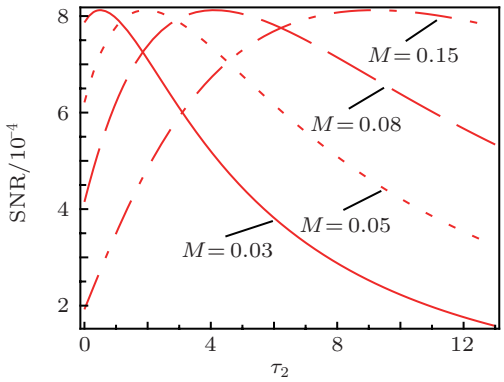


Fig. 12. (color online) The SNR versus additive noise self-correlation time τ_2 for different values of M at $\tau_1 = 0.3$, $Q = 0.1$, $c = 0.8$, $e = 0.2$, $A = 0.01$, $\omega = 0.01$.

5. Conclusion

Using the SNR theory proposed by McnNamara and Wiesenfeld,[1,2] we have investigated the SR phenomenon in

the metapopulation system subjected to both colored noises and a periodic signal. Our results indicate that the SNR is a non-monotonic function of the multiplicative noise intensity Q for the case of smaller values of additive noise intensity M from Figs. 1(a) and 3(a); further, it is a monotonic decreasing function on the multiplicative noise intensity Q for the case of larger values of additive noise intensity M from Figs. 1(b) and 3(b). There is always a resonance peak in the SNR–M plot. Meanwhile, in the SNR–M plot, τ_1 can strengthen the phenomenon of SR. However, τ_2 can only change the position of the resonance peak, but cannot change the value of the peak. Figures 7, 8 and 10 show that with the increase of the multiplicative noise intensity Q and the additive noise intensity M, the phenomenon of stochastic resonance tends to disappear.

The study indicates that the appropriate noise intensity and self-association time can make a contribution to the perception and spread to the weak signals of the outside world for the metapopulation system. According to the proper internal noise and environment noise, the metapopulation system can better detect and percept the weak signal of food, bait and natural enemies submerged by external noises. The metapopulation system is useful to the study of different spatial ecological and evolutionary characteristics of the population maintained by diffusion, migration and gene flow. Metapopulation theory plays an important role in many reality problems, such as species differentiation, increasingly fragmented landscape differentiation, preservation of genetic diversity, altruism in the group selection and even some abstract research mechanisms.

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