Monogamy of quantum correlations in the one-dimensional anisotropic *XY* model*

Xu Shuai(徐 帅), Song Xue-Ke(宋学科), and Ye Liu(叶 柳)†

School of Physics & Material Science, Anhui University, Hefei 230039, *China*

(Received 9 April 2013; revised manuscript received 4 June 2013; published 3 December 2013)

In this paper, the monogamy properties of some quantum correlations, including the geometric quantum discord, concurrence, entanglement of formation and entropy quantum discord, in the anisotropic spin-1/2 *XY* model with staggered Dzyaloshinskii–Moriya (DM) interaction have been investigated using the quantum renormalization group (QRG) method. We summarize the monogamy relation for different quantum correlation measures and make an explicit comparison. Through mathematical calculations and analysis, we obtain that no matter whether the QRG steps are carried out, the monogamy of the given states are always unaltered. Moreover, we conclude that the geometric quantum discord and concurrence obey the monogamy property while other quantum correlation measures, such as entanglement of formation and quantum discord, violate it for this given model.

Keywords: monogamy relation, geometric quantum discord, entanglement of formation, entropy quantum discord

PACS: 03.65.Ud, 03.67.–a, 03.67.Mn **DOI:** 10.1088/1674-1056/23/1/010302

1. Introduction

In the last few decades, there have been several discoveries in the field of quantum information science, which used quantum mechanical principles to enhance the abilities of computing and communication.[1] The most significant concept in quantum information processing is quantum entanglement due to its important features in developing the idea of quantum computers and other quantum information devices.[1,2] Therefore, a variety of computable measures describing quantum entanglement in the composite states were proposed: concurrently, entanglement of formation (EOF), negativity and so on.[3–9] However, in recent years, some studies have displayed that entanglement is not a unique measure of quantum correlations because other types of nonclassical correlations exist that are not captured by entanglement, subsequently, quantum discord (QD) appeared; some theoretical studies indicated that the quantifier of quantum discord[10–16] is present in almost all quantum states and plays an important role in many quantum states. In addition, a deformation of quantum discord, named geometric quantum discord (GQD)[17], has attracted a great deal of attention, mainly attributed to its simple calculation. Moreover, it has been illustrated that GQD can give us several interesting operational interpretations, such as its optimality, as a resource for remote state preparation[18] and quantum random access code protocols.

On the other hand, the concept of monogamy has received considerate investigation since it was introduced as an important binding theme for quantum correlations of states shared between several quantum systems.[19] The existence of monogamy due to the fact that tripartite quantum correlations of quantum systems have a principle, that is, if two of the parties are highly quantum correlated, these parties have little or no quantum correlations with any other parties.[20–26] Nevertheless, as mentioned above, GQD, concurrence, EOF and QD are introduced to describe the nonclassical correlations of the corresponding systems, so this motivates us to apply these correlations to study their dynamic behaviors of the monogamy relation. In this paper, we study the monogamy relation of some quantum correlation measures in the anisotropic spin-1/2 *XY* model with staggered Dzyaloshinskii–Moriya (DM) interaction using the quantum renormalization group method (QRG).

The rest of the paper is organized as follows. In Section 2, we employ the RG method to investigate the anisotropic spin-1/2 *XY* model with staggered DM interaction. In Section 3, the analytical expressions of different measurements of quantum correlations are obtained and the monogamy relations are discussed in detail. Finally, we give a summary in Section 4.

2. Quantum renormalization of *XY* model

The QRG method, which is explicitly designed to investigate statistical continuum limit problems, is a very effective method for processing the system that is at zero temperature or ultimate cryogenic circumstances (especially the multi-body system). The key point to the QRG scheme is that it reduces

*Project supported by the National Natural Science Foundation of China (Grants Nos. 11074002 and 61275119), the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20103401110003), and the Personal Development Foundation of Anhui Province, China (Grant No. 2008Z018).

†Corresponding author. E-mail: yeliu@ahu.edu.cn

© 2014 Chinese Physical Society and IOP Publishing Ltd

the effective degrees of freedom of the system through a recursive procedure until a controllable state is obtained in which the low energy part of the system is unchanged. In this paper, Kadanoff's block approach is implemented where we consider three sites as a block (marked as 1, 2, 3) and the Hamiltonian of each block can be exactly diagonalized and solved. By selecting a number of low-lying eigenstates of the blocks, the full Hamiltonian is projected onto these eigenstates giving the effective renormalized Hamiltonian.[27,28] Here, the RG method is summarized as the following steps.

(i) Decomposing the Hamiltonian into two parts: $H = H^{\mathrm{B}} + H^{\mathrm{BB}}$, where H^{B} and H^{BB} are the block Hamiltonian and the interblock Hamiltonian, respectively.

(ii) According to the fact that the low-energy part of H and H^{eff} are the same, we construct a projection operator T to connect the original Hamiltonian and the effective Hamiltonian.

(iii) Renormalization of H^{B} and H^{BB}: H^{eff} up to the first-order correction is $H^{\mathrm{eff}} = T^{+}H^{\mathrm{B}}T + T^{+}H^{\mathrm{BB}}T$.

(iv) Iteration: iterate these until a controllable state is obtained. More detailed process is in Refs. [27]–[31].

The Hamiltonian of the one-dimensional anisotropic XY model with staggered DM interaction in the z direction on a periodic chain of N sites is

$$H_0 = \frac{J}{4}\sum_{i=1}^{N}\Big[(1+\gamma)\,\boldsymbol{\sigma}_i^x\boldsymbol{\sigma}_{i+1}^x + (1-\gamma)\,\boldsymbol{\sigma}_i^y\boldsymbol{\sigma}_{i+1}^y + (-1)^{\mathrm{i}}D\left(\boldsymbol{\sigma}_i^x\boldsymbol{\sigma}_{i+1}^y - \boldsymbol{\sigma}_i^y\boldsymbol{\sigma}_{i+1}^x\right)\Big], \tag{1}$$

where γ is the anisotropy parameter, J is the exchange constant, D is the strength of the DM interaction in the direction of z and $\boldsymbol{\sigma}_i^{\alpha}$ $(\alpha = x, y)$ are standard Pauli matrices at site i. For this model, it becomes an isotropic XX model when $D = 0$.

The original Hamiltonian H_0 effects on the effective Hilbert space, then the effective Hamiltonian H^{eff}, which is the necessary condition for RG, can be realized. In order to produce the H^{eff}, which has a similar structure to H_0, we achieve a π rotating around the x axis for all sites and leave all odd sites unchanged.[29] Hence, we can obtain the transformed Hamiltonian as follows:

$$H = \frac{J}{4}\sum_{i=1}^{N}\Big[(1+\gamma)\,\boldsymbol{\sigma}_i^x\boldsymbol{\sigma}_{i+1}^x - (1-\gamma)\,\boldsymbol{\sigma}_i^y\boldsymbol{\sigma}_{i+1}^y + D\left(\boldsymbol{\sigma}_i^x\boldsymbol{\sigma}_{i+1}^y + \boldsymbol{\sigma}_i^y\boldsymbol{\sigma}_{i+1}^x\right)\Big]. \tag{2}$$

According to the above method, Ref. [32] gives the degenerate ground states of the block Hamiltonian

$$|\varphi_0\rangle = \frac{\sqrt{2(1+D^2)}}{2q}\left[\frac{-\sqrt{2}(1-D\mathrm{i})q}{2(1+D^2)}|\uparrow\uparrow\downarrow\rangle + \frac{(1-D\mathrm{i})\gamma}{1+D^2}|\uparrow\downarrow\uparrow\rangle + \frac{-\sqrt{2}(1-D\mathrm{i})q}{2(1+D^2)}|\downarrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle\right], \tag{3}$$

$$|\varphi_0\rangle' = \frac{1}{2}\left[\frac{-\sqrt{2}(1-D\mathrm{i})}{q}|\uparrow\uparrow\uparrow\rangle + |\uparrow\downarrow\downarrow\rangle + \frac{-\sqrt{2}\gamma}{q}|\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle\right], \tag{4}$$

where $|\uparrow\rangle, |\downarrow\rangle$ are the eigenstates of $\boldsymbol{\sigma}_z$ and $q = \sqrt{1+D^2+\gamma^2}$. The effective Hamiltonian of the renormalized chain can be written as

$$H^{\mathrm{eff}} = \frac{J'}{4}\sum_{k=1}^{N/3}\Big[(1+\gamma')\boldsymbol{\sigma}_k^x\boldsymbol{\sigma}_{k+1}^x - (1-\gamma')\boldsymbol{\sigma}_k^y\boldsymbol{\sigma}_{k+1}^y + D'(\boldsymbol{\sigma}_k^x\boldsymbol{\sigma}_{k+1}^y + \boldsymbol{\sigma}_k^y\boldsymbol{\sigma}_{k+1}^x)\Big], \tag{5}$$

where the iterative relationship is

$$J' = \frac{1+D^2+3\gamma^2}{2q^2}J,\quad \gamma' = \frac{3\gamma+3D^2\gamma+\gamma^3}{1+D^2+3\gamma^2},\quad D' = -D. \tag{6}$$

The most significative information given in the RG method is the fixed points. We can obtain the stable and unstable fixed points by solving $\gamma' \equiv \gamma$. The stable fixed point is located in $\gamma = \sqrt{1+D^2}$ and $\gamma \to \infty$, which corresponds to the phase of system, the unstable fixed point is $\gamma = 0$ that corresponds to the phase transition point of system; the system can be separated into two phases: the spin-fluid phase ($\gamma = 0$ and $\gamma \to \infty$) and the Néel phase ($0 < |\gamma| < \sqrt{1+D^2}$).

3. Monogamy analysis

In this section, we will calculate the geometric quantum discord, concurrence (C), entanglement of formation and entropy quantum discord to investigate the monogamy of our model using the renormalization group method. The density matrix of a ground state is composed, i.e.

$$\boldsymbol{\rho}_{123} = |\varphi_0\rangle\langle\varphi_0|, \tag{7}$$

where $|\varphi_0\rangle$ has been introduced in Eq. (3). The results will be the same if we consider $|\varphi_0'\rangle$ constructing the density matrix. It is easy to obtain the reduced matrix of $\boldsymbol{\rho}$ by tracing out any one of the three particles

$$\boldsymbol{\rho}_{13} = \langle\downarrow_2|\varphi_0\rangle\langle\varphi_0|\downarrow_2\rangle + \langle\uparrow_2|\varphi_0\rangle\langle\varphi_0|\uparrow_2\rangle = \begin{pmatrix} \frac{\gamma^2}{2q^2} & 0 & 0 & \frac{\sqrt{\gamma^2+\gamma^2D^2}}{2q^2}\mathrm{e}^{\mathrm{i}\varphi} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{\sqrt{\gamma^2+\gamma^2D^2}}{2q^2}\mathrm{e}^{-\mathrm{i}\varphi} & 0 & 0 & \frac{1+D^2}{2q^2} \end{pmatrix}, \tag{8}$$

$$\boldsymbol{\rho}_{12} = \langle\downarrow_3|\varphi_0\rangle\langle\varphi_0|\downarrow_3\rangle + \langle\uparrow_3|\varphi_0\rangle\langle\varphi_0|\uparrow_3\rangle$$

$$= \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{-\sqrt{1+D^2}}{2\sqrt{2}q}\mathrm{e}^{-\mathrm{i}\varphi} \\ 0 & \frac{\gamma^2}{2q^2} & \frac{-\gamma}{2\sqrt{2}q} & 0 \\ 0 & \frac{-\gamma}{2\sqrt{2}q} & \frac{1}{4} & 0 \\ \frac{-\sqrt{1+D^2}}{2\sqrt{2}q}\mathrm{e}^{\mathrm{i}\varphi} & 0 & 0 & \frac{1+D^2}{2q^2} \end{pmatrix}, \tag{9}$$

where $\sin\varphi = D/\sqrt{1+D^2}$, $\cos\varphi = 1/\sqrt{1+D^2}$, $\rho_{13} = \mathrm{Tr}_2(|\varphi_0\rangle\langle\varphi_0|)$ and analogously for ρ_{12}.

In a general way, the concept of monogamy constrains the sharing of quantum correlations among subsystems of a multiparticle quantum state. For a tripartite quantum state, the bipartite quantum correlation measure Q is monogamous if

$$Q(\rho_{13}) + Q(\rho_{12}) \leq Q(\rho_{1:23}). \tag{10}$$

Here, $Q(\rho_{1:23})$ is the quantum correlation between subsystem 1 and subsystems 2 and 3 taken together, $Q(\rho_{13})$ is the quantum correlation between subsystems 1 and 2 and analogously for $Q(\rho_{12})$.

3.1. GQD

In this subsection, we will research the monogamy of the given states by the geometric measure of discord D^{g}, which was first introduced in Ref. [17] as the minimal square Hilbert–Schmidt distance to the set of classical-quantum states (CQ)

$$D^{\mathrm{g}}(\rho_{\mathrm{AB}}) = \min_{\chi\in CQ} \|\rho_{\mathrm{AB}} - \chi\|^2. \tag{11}$$

Here, $\|A\|^2 = \mathrm{Tr}(A^+A)$ is the square of the Hilbert–Schmidt norm of Hermitian operators; the minimum is taken over all classical-quantum states χ. Reference [33] proposed an easily computable exact expression for the GQD about a two-qubit system.

The GQD can be written as

$$D^{\mathrm{g}}(\rho_{\mathrm{AB}}) = \frac{1}{4}(\|\boldsymbol{x}\|^2 + \|\boldsymbol{R}\|^2 - \lambda_{\max}), \tag{12}$$

where $\boldsymbol{x} = (\boldsymbol{x}_1 + \boldsymbol{x}_2 + \boldsymbol{x}_3)^{\mathrm{T}}$, $\boldsymbol{R}$ is the matrix with elements $R_{ij} = \mathrm{Tr}(\rho_{\mathrm{AB}}(\sigma_i \otimes \sigma_j))$ and $\lambda_{\max}$ is the largest eigenvalue of the matrix $\boldsymbol{K} = \boldsymbol{x}\boldsymbol{x}^{\mathrm{T}} + \boldsymbol{R}\boldsymbol{R}^{\mathrm{T}}$ ($\boldsymbol{R}^{\mathrm{T}}$ is the transpose of matrix $\boldsymbol{R}$).[14] In addition, $x_i = \mathrm{Tr}(\rho_{\mathrm{AB}}(\sigma_i \otimes \boldsymbol{I}))$ and $y_i = \mathrm{Tr}(\rho_{\mathrm{AB}}(\boldsymbol{I} \otimes \sigma_i))$ are the parts of the local Bloch vector. According to this, the general result for the geometric quantum discord of two-qubit states is

$$D^{\mathrm{g}}(\rho_{\mathrm{AB}}) = \frac{1}{4}[8(|\rho_{14}|^2 + |\rho_{23}|^2) + \lambda_3 - \max\{\lambda_1, \lambda_2, \lambda_3\}], \tag{13}$$

where $\lambda_1 = 4(\rho_{14} + \rho_{23})^2$, $\lambda_2 = 4(\rho_{14} - \rho_{23})^2$, and $\lambda_3 = 2(\rho_{11} - \rho_{33})^2 + 2(\rho_{22} - \rho_{44})^2$ are the eigenvalues of the matrix $\boldsymbol{K}$.

Through some calculations, we have the mathematical formulas of the GQD as follows:

$$D^{\mathrm{g}}(\rho_{13}) = \frac{1}{4}\left(\frac{1}{2} - \frac{|\gamma|\sqrt{1+D^2}}{q^2}\right),$$
$$D^{\mathrm{g}}(\rho_{12}) = \frac{1}{4}\left(1 - \frac{|\gamma|\sqrt{1+D^2}}{q^2} - \frac{2\gamma^2(1+D^2)}{q^4}\right). \tag{14}$$

Using the consequences in Ref. [34], if the pure states $|\varphi\rangle_{123}$ have the monogamous property, it will satisfy the following condition

$$D^{\mathrm{g}}(\rho_{13}) + D^{\mathrm{g}}(\rho_{12}) \leq D^{\mathrm{g}}(\rho_{1:23}), \tag{15}$$

where $D^{\mathrm{g}}(\rho_{1:23}) = 1 - \sum_i k_i^2$ and k_i are the eigenvalues of the matrix $\rho_{23} = \mathrm{Tr}_1(\rho_{123})$.

According to this, it is easy to obtain

$$D^{\mathrm{g}}(\rho_{1:23}) = 1 - \left(\frac{3q^2 - 2\gamma^2}{4q^2}\right)^2 - \left(\frac{q^2 + 2\gamma^2}{4q^2}\right)^2. \tag{16}$$

In order to facilitate our conclusion, we define a parameter F, where

$$F(\gamma, D) = D^{\mathrm{g}}(\rho_{13}) + D^{\mathrm{g}}(\rho_{12}) - D^{\mathrm{g}}(\rho_{1:23}). \tag{17}$$

In Fig. 1, we illustrate the evolution of the F versus γ and D for different RG steps. Notice that the iterative relationship we adopt is described in Eq. (6). It reveals that no matter how much the anisotropy parameter (γ) or the strength of DM interaction (D) is, F is always negative, i.e., the monogamous property shown in the Eq. (16) is accurate in the anisotropic spin-1/2 XY model with staggered Dzyaloshinskii–Moriya interaction using the QRG method.

Moreover, we can obtain a more visual similar result from Fig. 2, which plots the evolution of the F versus γ in terms of RG iteration steps in the XY model without DM interaction. Therefore, comparing Fig. 1 with Fig. 2, we can conclude that the geometric quantum discord of the XY model obeys the monogamy properties; as the time of iteration or the strength of the DM interaction D increase, the phenomenon becomes more and more evident.

3.2. Concurrence, entanglement of formation and entropy quantum discord

In this subsection, we calculate other measures to study the monogamy for a reduced density matrix ρ_{ij} of two qubits that arises after integrating out other degrees of freedom. The EOF between two qubits is evaluated as

$$E^{\mathrm{f}}(\varphi) = H\left(\frac{1+\sqrt{1-C^2}}{2}\right). \tag{18}$$

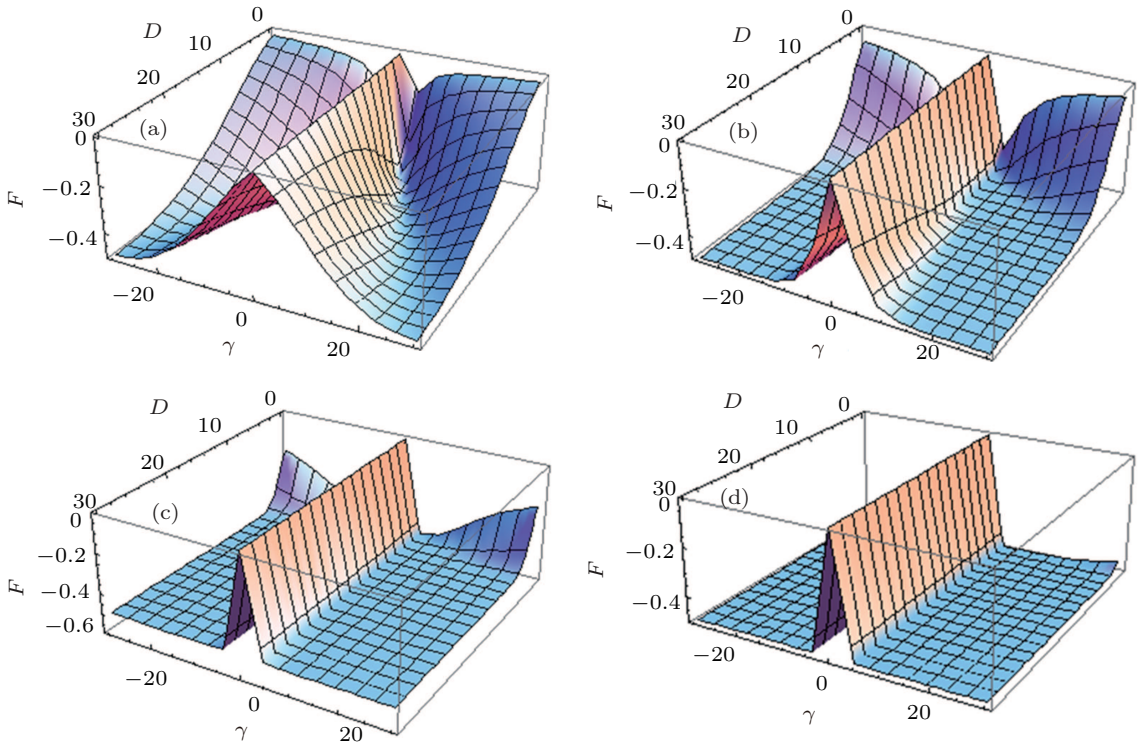


Fig. 1. (color online) Plots of F versus γ and D in terms of RG iteration steps in the XY model: (a) the initial state; (b) after the first step; (c) after the second step; (d) after the third step. The maximum of the coordinate axes of F are zero in the images.

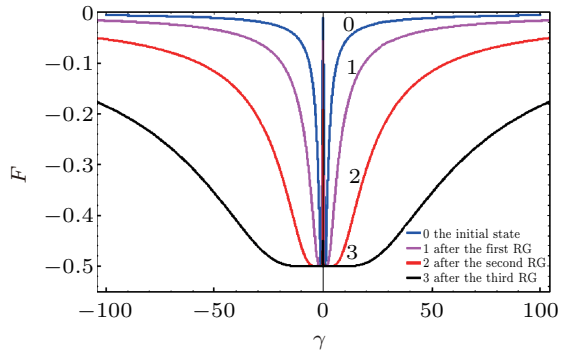


Fig. 2. (color online) The evolution of the F versus γ in terms of RG iteration steps in the XY model at a fixed value of $D = 0$.

Here, H is a binary entropy function $H(x) = -x\log_2 x - (1-x)\log_2(1-x)$ and C denotes the concurrence defined as $C = \mathrm{Max}\{\xi_1 - \xi_2 - \xi_3 - \xi_4, 0\}$, where ξ_i $(i = 1, 2, 3, 4)$ are the square roots of the eigenvalues in descending order of the operator R_{ij}, $R_{ij} = \rho_{ij}(\sigma_1^y \otimes \sigma_2^y)\rho_{ij}^*(\sigma_1^y \otimes \sigma_2^y)$ and then, by some calculations, we have the mathematical formulas of the concurrence and EOF as follows:

$$
\begin{aligned}
C(\rho_{13}) &= \frac{1}{2} - \frac{|\gamma|\sqrt{1+D^2}}{q^2}, \\
C(\rho_{12}) &= \frac{\left|\sqrt{1+D^2} - |\gamma|\right|}{\sqrt{2}q}, \\
E^{\mathrm{f}}(\rho_{13}) &= -\left(\frac{1+\sqrt{1-C(\rho_{13})^2}}{2}\right) \\
&\quad \times \log_2\left(\frac{1+\sqrt{1-C(\rho_{13})^2}}{2}\right) \\
&\quad - \left(\frac{1-\sqrt{1-C(\rho_{13})^2}}{2}\right) \\
&\quad \times \log_2\left(\frac{1-\sqrt{1-C(\rho_{13})^2}}{2}\right), \\
E^{\mathrm{f}}(\rho_{12}) &= -\left(\frac{1+\sqrt{1-C(\rho_{12})^2}}{2}\right) \\
&\quad \times \log_2\left(\frac{1+\sqrt{1-C(\rho_{12})^2}}{2}\right) \\
&\quad - \left(\frac{1-\sqrt{1-C(\rho_{12})^2}}{2}\right) \\
&\quad \times \log_2\left(\frac{1-\sqrt{1-C(\rho_{12})^2}}{2}\right). \qquad (19)
\end{aligned}
$$

In the previous works, the concurrence,[22] EOF and entropy quantum discord of any tripartite states have monogamous inequalities as follow:

$$
\begin{aligned}
&C^2(\rho_{13}) + C^2(\rho_{12}) \le C^2(\rho_{1:23}), \\
&E^{\mathrm{f}}(\rho_{13}) + E^{\mathrm{f}}(\rho_{12}) \le E^{\mathrm{f}}(\rho_{1:23}), \\
&D(\rho_{13}) + D(\rho_{12}) \le D(\rho_{1:23}), \qquad (20)
\end{aligned}
$$

where

$$
C^2(\rho_{1:23}) = 4\det\rho_1,
$$

$$
\begin{aligned}
E^{\mathrm{f}}(\rho_{1:23}) &= D(\rho_{1:23}) \\
&= S(\rho_1) \\
&= -\left(\frac{1}{4}+\frac{\gamma^2}{2q^2}\right)\log_2\left(\frac{1}{4}+\frac{\gamma^2}{2q^2}\right) \\
&\quad -\left(\frac{1}{4}+\frac{1+D^2}{2q^2}\right)\log_2\left(\frac{1}{4}+\frac{1+D^2}{2q^2}\right)
\end{aligned}
$$

and

$$
\rho_1 = \mathrm{Tr}_{23}(\rho_{123}) = \begin{pmatrix} \frac{1}{4}+\frac{\gamma^2}{2q^2} & 0 \\ 0 & \frac{1}{4}+\frac{1+D^2}{2q^2} \end{pmatrix}.
$$

Whether quantum correlations of our model violate the above inequality will prove that it is monogamous or not.

In order to achieve our goals, we will define some new parameters F^{C}, F^{E} and F^{D} as

$$
\begin{aligned}
F^{\mathrm{C}} &= C^2(\rho_{13}) + C^2(\rho_{12}) - C^2(\rho_{1:23}), \\
F^{\mathrm{E}} &= E^{\mathrm{f}}(\rho_{13}) + E^{\mathrm{f}}(\rho_{12}) - E^{\mathrm{f}}(\rho_{1:23}), \\
F^{\mathrm{D}} &= D(\rho_{13}) + D(\rho_{12}) - D(\rho_{1:23}). \qquad (21)
\end{aligned}
$$

We illustrate the evolutions of the F^{C}, F^{E} and F^{D} as functions of γ and D in Fig. 3, from which it is not difficult to find that the concurrence of the given XY model obeys the monogamy property, while entanglement of formation violates the monogamy inequality. Moreover, Ref. [35] proposed that EOF can be written as a function of the quantum discord and a quantum conservation law in an arbitrary tripartite pure system, $E^{\mathrm{f}}(\rho_{13}) + E^{\mathrm{f}}(\rho_{12}) = D(\rho_{13}) + D(\rho_{12})$, which means that the monogamy of EOF and QD are identical in our model.

The evolutions of the F^{C}, F^{E} and F^{D} as functions of γ at a fixed value of $D = 0$ are shown in Fig. 4. It is more evident than in Fig. 3 that the monogamy relation of concurrence is satisfied while those of EOF and QD are not established in our model without DM interaction. In addition, no matter how many times of the iteration of the renormalization group happens, entanglement of formation and entropy quantum discord can comply with the monogamy relation approximatively when the anisotropy parameter is close to one of the critical points ($|\gamma| = \sqrt{1+D^2}$), while the two quantum correlation measures obviously violate the monogamy property when the anisotropy parameter is large or close to zero. Therefore, we can obtain the result that the anisotropy parameter not only influences quantum correlations, but also has an important effect on the monogamy property of quantum correlations for this model.

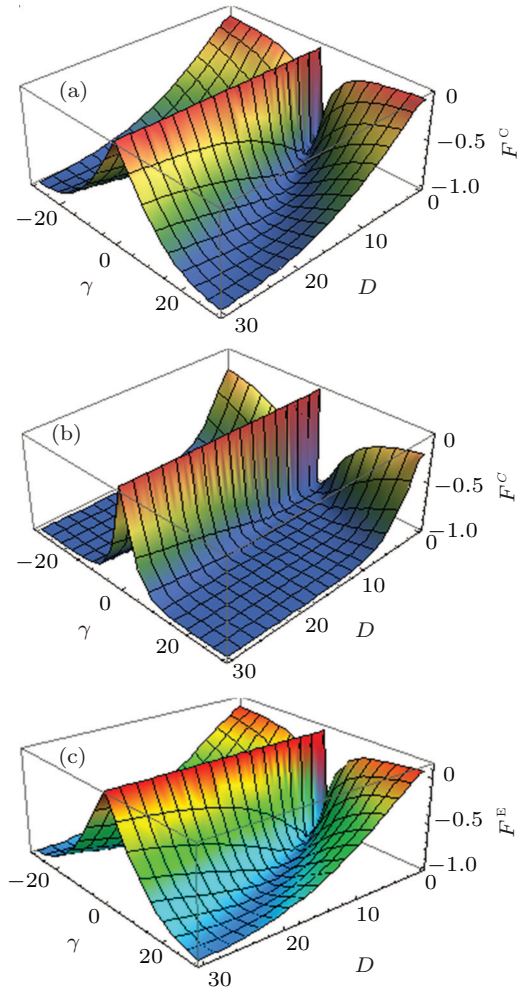


Fig. 3. (color online) Three-dimensional graphs of the F^{C} as a function of γ and D ((a) the initial state and (b) after the first step RG), the F^{E} versus γ and D ((c) the initial state) in terms of RG iterations steps in the XY model. The maximum of the coordinate axes of F^{C} are zero in the images.

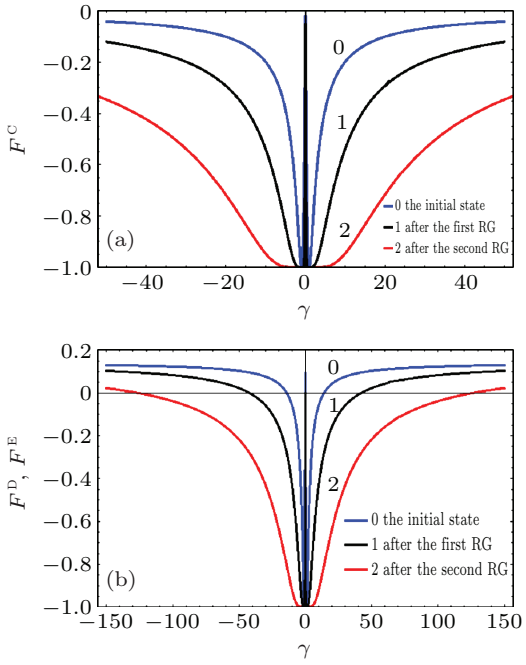


Fig. 4. (color online) Evolutions of the F^{C}, F^{E} and F^{D} as functions of γ in terms of RG iterations at a fixed value of $D = 0$.

4. Conclusions

In this paper, we investigated the monogamy property of quantum correlations using some different quantum correlation measures in the anisotropic spin-1/2 XY model using the QRG method. As the degenerate ground states of the block Hamiltonian are concerned, we used the reduced density matrix of the system to obtain the analytical expression of different measurements of quantum correlations and then the relating monogamy of GQD, concurrence, EOF and QD were discussed. In addition, through mathematical calculations, we found that, no matter how much the anisotropy parameter (γ) or the strength of DM interaction (D) is, the monogamy inequality of the geometric quantum discord and concurrence are always established, but the other measurements are not. Therefore, We conclude that the geometric quantum discord and the concurrence obey the monogamy property while other measures, such as entanglement of formation and quantum discord, violate it in the given XY model. Moreover, we found that the strength of the DM interaction D or anisotropy parameter not only influences the quantum correlations, but also has an important effect on the monogamy property of quantum correlations for this model; no matter whether the QRG steps are carried out, the monogamy of the given states is always unaltered.

As we know, the first example of a measurement of quantum correlations beyond entanglement is the geometric quantum discord that obeys the monogamy inequality; however, the question remains as to why the entropy quantum discord has a different result. Perhaps, another perfect monogamy relation that all arbitrary tripartite pure states obey will be found in the near future.

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[2] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
[3] Hill S and Wootters W K 1997 *Phys. Rev. Lett.* **78** 5022
[4] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[5] Ge G Q, Qin C, Yin M and Huang Y H 2011 *Chin. Phys. B* **20** 080304
[6] Vedral V, Plenio M B, Rippin M A and Knight P L 1997 *Phys. Rev. Lett.* **78** 2275
[7] Horodecki M, Horodecki P and Horodecki R 1998 *Phys. Rev. Lett.* **80** 5239
[8] Li Y L and Li X M 2008 *Chin. Phys. B* **17** 0812
[9] Vidal G and Werner R F 2002 *Phys. Rev. A* **65** 032314
[10] Ollivier H and Zurek W H 2001 *Phys. Rev. Lett.* **88** 017901
[11] Ferraro A, Aolito L, Cavalcanti D, Cucchietti F M and Acín A 2010 *Phys. Rev. A* **81** 052318
[12] Ding B F, Wang X Y and Zhao H P 2011 *Chin. Phys. B* **20** 100302
[13] Sarandy M S 2009 *Phys. Rev. A* **80** 022108
[14] Luo S and Zhang Q 2009 *J. Stat. Phys.* **136** 165
[15] Wang B, Xu Z Y, Chen Z Q and Feng M 2010 *Phys. Rev. A* **81** 014101
[16] Han W, Zhang Y J and Xia Y J 2013 *Chin. Phys. B* **22** 010306
[17] Dakić B, Vedral V and Brukner C 2010 *Phys. Rev. Lett.* **105** 190502
[18] Tufarelli T, Girolami D, Vasile R, Bose S and Adesso G 2012 arXiv:1205.0251
[19] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865
[20] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
[21] Bennett C H, Bernstein H J, Popescu S and Schumacher B 1996 *Phys. Rev. A* **53** 2046
[22] Coffman V, Kundu J and Wootters W K 2000 *Phys. Rev. A* **61** 052306
[23] Koashi M and Winter A 2004 *Phys. Rev. A* **69** 022309
[24] Song X K, Wu T and Ye L 2013 *Mod. Phys. Lett. B* **27** 1350049
[25] Man Z X, Xia Y J and An N B 2010 *New. J. Phys.* **12** 033020
[26] Osborne T J and Verstraete F 2006 *Phys. Rev. Lett.* **96** 220503
[27] Martín-Delgado M A and Sierra G 1996 *Int. J. Mod. Phys. A* **11** 3145
[28] Langari A 1998 *Phys. Rev. B* **58** 14467
[29] Langari A 2004 *Phys. Rev. B* **69** 100402
[30] Jafari R and Langari A 2007 *Phys. Rev. B* **76** 014412
[31] Jafari R and Langari A 2006 *Physica A* **364** 213
[32] Ma F W, Liu S X and Kong X M 2011 *Phys. Rev. A* **84** 042302
[33] Rau A R P 2009 *J. Phys. A* **42** 412002
[34] Streltsov A, Adesso G, Piani M and Bruß D 2001 *Phys. Rev. Lett.* **109** 050503
[35] Fanchini F F, Cornelio M F, de Oliveira M C and Caldeira A O 2011 *Phys. Rev. A* **84** 012313