Implementation of a one-dimensional quantum walk in both position and phase spaces*

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Quantum walks have been investigated as they have remarkably different features in contrast to classical random walks. We present a quantum walk in a one-dimensional architecture, consisting of two coins and a walker whose evolution is in both position and phase spaces alternately controlled by the two coins respectively. By analyzing the dynamics evolution of the walker in both the position and phase spaces, we observe an influence on the quantum walk in one space from that in the other space, which behaves like decoherence. We propose an implementation of the two-coin quantum walk in both position and phase spaces via cavity quantum electrodynamics (QED).

Keywords: quantum walks, cavity quantum electrodynamics

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1. Introduction

Quantum walks (QWs) have attracted interest because they can be used to develop new quantum algorithms,[1–4] are one of the most promising resources for the simulation of quantum systems[5–10] and are emerging as an alternative to the standard circuit model for quantum computing.[11,12] Different experimental implementations of QWs have been performed with trapped atoms,[13] ions,[14–16] energy levels in nuclear magnetic resonance schemes,[17] photons in waveguide structures and in linear optical elements.[18–23] With decoherence, the spreading of a QW is transmitted from a ballistic to diffusive scaling and with quantumness losing, the QW is transmitted to random walk (RW).[24–29]

In this paper, we present a discrete-time QW in one-dimensional (1D) architecture. The QW consists of a walker walking in position and phase spaces alternately and two coins driving the walker in position and phase spaces respectively. As we know that position and phase are not independent degrees of freedom, the dynamics of the QW in either position and phase spaces influences that in the other space. We propose a scheme to implement the QW via cavity quantum electrodynamics (QED) with a single-mode cavity field interacting with two-level atomic states. By analyzing the standard deviation of the probability distribution of the QW in both position and phase spaces, we find that the influence on the dynamics of the QW in either the position and phase spaces from that in the other space is one kind of decoherence, transmitting QW to RW.

The structure of this paper is organized as follows. In Section 2, we briefly review QWs in position space and in phase space respectively. In Section 3, we present the implementation of the 1D two-coin QW via cavity QED consisting of a single-mode cavity field, which interacts with two-level atomic states. In Section 4, we analyze the results of numerical simulations on the dynamics of the QW. Finally, in Section 5, we summarize and drive a brief conclusion.

2. Quantum walks in both position and phase spaces

2.1. Quantum walks in position space

First, we briefly recall the ideal standard QW in position space which consists of one walker, usually a quantum particle confined in 1D position space. The basis of the walker in position space is $\{|x\rangle\}$, where x can be any integer and $|x\rangle$ is the eigenstate of the position operator $\hat{x} = \hat{a} + \hat{a}^{\dagger}$. A coin controls the position state to increase or decrease by 1 for each step, which presents the walker moving towards left or right for one step. Usually, one can use a two-level particle with internal states $|0\rangle$ and $|1\rangle$ as a quantum coin. Besides, we apply the Hadamard operator

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{1}$$

to realize the coin flipping before each step the walk takes. The unitary operation for one step of the QW is given by

$$\hat{E} \equiv \sum_{x} (|x+1\rangle \langle x| \otimes |0\rangle \langle 0|$$

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$$+|x-1\rangle\langle x|\otimes|1\rangle\langle 1|)\left(\hat{I}\otimes\hat{H}\right), \tag{2}$$

where $\hat{I}$ is the identity operator applied on the walker.

We assume the walker is prepared initially in a vacuum state $|0\rangle$ and the coin is initially prepared in a superposition state

$$|\varphi_0\rangle=\frac{1}{\sqrt{2}}(|0\rangle+\mathrm{i}|1\rangle). \tag{3}$$

Then, the full initial state of the system is

$$|\phi_0\rangle=|0\rangle\otimes|\varphi_0\rangle. \tag{4}$$

Therefore, the state of the walker+coin system after n steps evolves to

$$|\phi_n\rangle=\left(\hat{E}\right)^n|\phi_0\rangle. \tag{5}$$

2.2. Quantum walks in phase space

Before discussing about QWs in both position and phase spaces, we briefly review QWs in phase space by introducing the finite-dimensional orthogonal phase state representation[30,31]

$$\left|\theta_k=\frac{2\pi k}{d}\right\rangle=\frac{1}{\sqrt{d}}\sum_{n=0}^{d-1}\mathrm{e}^{\mathrm{i}n\theta_k}|n\rangle,\quad k\in\mathbb{Z}, \tag{6}$$

where $\{|n\rangle, n<d\}$ is the Fock state. We choose a walker, also a quantum particle, which walks in phase space with a basis of phase states $\{|\theta_k\rangle\}$. For the rotation operator $\hat{R}_l=\mathrm{e}^{\mathrm{i}\theta_l\hat{n}}, l\in\mathbb{Z}$, we have $\hat{R}_l|\theta_k\rangle=|\theta_{k+l}\rangle$. We also consider a two-level particle with internal states $|0\rangle$ and $|1\rangle$ as a coin. We choose the Hadamard coin flipping. The unitary operation of one step of the QW in phase space is given by

$$\hat{F}\equiv\mathrm{e}^{\mathrm{i}\frac{2\pi}{d}\hat{n}\otimes\hat{\sigma}_z}\left(\hat{I}\otimes\hat{H}\right), \tag{7}$$

where $2\pi/d$ is the step size and $\hat{\sigma}_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ is one of the Pauli operators applied on the coin. We assume the walker+coin system is prepared in the state initially

$$\left|\phi_0'\right\rangle=|\theta_0\rangle\otimes|\varphi_0\rangle. \tag{8}$$

Therefore, the state of the walker+coin system after m steps is

$$|\phi_m\rangle=\left(\hat{F}\right)^m\left|\phi_0'\right\rangle. \tag{9}$$

3. 1D two-coin quantum walks and physical implementation

We present the 1D QW in both position and phase spaces, consisting of one walker that walks in position and phase spaces alternately and two coins which respectively control the walker to take steps in position and phase spaces. We propose a scheme to implement the QW via cavity QED of single-mode cavity field interacting with two-level atomic states. Cavity QED is considered to be one of the most promising physical systems to implement quantum computation, quantum information processing and quantum communication.[32,33] Although cavity QED provides good conditions for realizations of different schemes of generating entangled states, quantum control and quantum logic gates,[34–40] the decoherence from spontaneous emission of atoms and decay of the cavity adds serious restrictions to further development of cavity QED experimental techniques.[41] However, in our scheme, we neglect the intrinsic decoherence and noise of the system in order to purely focus on the influence on QWs in one space from the other space, which behaves like decoherence. We choose the single-mode cavity field to present the walker and two two-level atomic states to present the coins. One of the two atoms resonantly interacts with the single-mode cavity field and there is large detuning between the cavity field and the transition frequency of the other atom. We mark the former atom as atom 1 and the latter one as atom 2. The two atoms alternately interact with the same cavity field individually. According to the Jaynes–Cummings model, we get the effective interaction Hamiltonian of the cavity and atom 1 ($\hbar=1$)

$$\hat{\mathbb{H}}_1=g_1\hat{x}\hat{\sigma}_{z1},\quad \omega_{\mathrm{a}1}=\omega_{\mathrm{c}} \tag{10}$$

and the effective interaction Hamiltonian of the cavity and atom 2, derived from[42]

$$\hat{\mathbb{H}}_2=\frac{g_2^2}{\Delta}\hat{a}^\dagger\hat{a}\hat{\sigma}_{z2},\quad \omega_{\mathrm{a}2}-\omega_{\mathrm{c}}=\Delta\gg g_2, \tag{11}$$

where g_1 and g_2 are the coupling coefficients of atom 1, atom 2 and the cavity field, $\omega_{\mathrm{a}1}$, $\omega_{\mathrm{a}2}$ and ω_{c} are transition frequencies of atom 1, atom 2 and the frequency of the cavity field, respectively. We use atom 1 to control the walker to walk in position space and atom 2 to control the walker to walk in phase space. The conditional position shift operator takes the form

$$\hat{u}_1=\mathrm{e}^{-\mathrm{i}\hat{\mathbb{H}}_1t_1}=\mathrm{e}^{-\mathrm{i}g_1\left(\hat{a}^\dagger+\hat{a}\right)\hat{\sigma}_{z1}t_1}. \tag{12}$$

Defining $\Delta\alpha=-g_1t_1$ for simplification, the above equation turns out to be

$$\hat{u}_1=\mathrm{e}^{\mathrm{i}\Delta\alpha\left(\hat{a}^\dagger+\hat{a}\right)\hat{\sigma}_{z1}}. \tag{13}$$

The conditional phase shift operator takes the form

$$\hat{u}_2=\mathrm{e}^{-\mathrm{i}\hat{\mathbb{H}}_2t_2}=\mathrm{e}^{-\mathrm{i}\frac{g_2^2}{\Delta}\hat{a}^\dagger\hat{a}\hat{\sigma}_{z2}t_2}. \tag{14}$$

Defining $\Delta\theta=-g_2^2t_2/\Delta$ for simplification, equation (14) turns out to be

$$\hat{u}_2=\mathrm{e}^{\mathrm{i}\Delta\theta\hat{a}^\dagger\hat{a}\hat{\sigma}_{z2}}. \tag{15}$$

The Hadamard operator is applied on the atomic states of Eq. (1), which is realized by a periodic sequence of $\pi/2$ pulses. The QW starts with the cavity field in the coherent state

$$|\alpha\rangle = \mathrm{e}^{-\frac{|\alpha|^2}{2}} \sum_{n=0} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \tag{16}$$

and atom 1 (2) in the state $|\varphi_0\rangle_{\mathrm{a1(a2)}}$ in Eq. (3). Then, the initial state of the walker+coins system is

$$|\phi_0\rangle = |\alpha\rangle_{\mathrm{c}} \otimes |\varphi_0\rangle_{\mathrm{a1}} \otimes |\varphi_0\rangle_{\mathrm{a2}}. \tag{17}$$

We have introduced the following protocol to implement the QW in position and phase spaces alternatively. Firstly, the coin flipping operator

$$\hat{H}_1 = \hat{I}_{\mathrm{c}} \otimes \hat{H}_{\mathrm{a1}} \otimes \hat{I}_{\mathrm{a2}} \tag{18}$$

is applied on atom 1, leaving the state of the cavity field and the state of atom 2 unchanged. Secondly, the conditional position shift operator

$$\begin{aligned} \hat{E}_1 \;=\; & D(\mathrm{i}\Delta\alpha) \otimes |0\rangle_{\mathrm{a1}} \langle 0| \otimes \hat{I}_{\mathrm{a2}} \\ & + D(-\mathrm{i}\Delta\alpha) \otimes |1\rangle_{\mathrm{a1}} \langle 1| \otimes \hat{I}_{\mathrm{a2}}, \end{aligned} \tag{19}$$

with the displacement operator $D(\alpha) = \mathrm{e}^{\alpha\hat{a}^\dagger - \alpha^* \hat{a}}$ applied on the walker+coins system to realize one step in position space. Thirdly, we apply the coin flipping operator

$$\hat{H}_2 = \hat{I}_{\mathrm{c}} \otimes \hat{I}_{\mathrm{a1}} \otimes \hat{H}_{\mathrm{a2}} \tag{20}$$

to atom 2, leaving the state of the cavity field and the state of atom 1 unchanged. Fourthly, we apply the conditional phase shift operator

$$\begin{aligned} \hat{E}_2 \;=\; & \mathrm{e}^{\mathrm{i}\Delta\theta \hat{a}^\dagger \hat{a}} \otimes \hat{I}_{\mathrm{a1}} \otimes |0\rangle_{\mathrm{a2}} \langle 0| \\ & + \mathrm{e}^{-\mathrm{i}\Delta\theta \hat{a}^\dagger \hat{a}} \otimes \hat{I}_{\mathrm{a1}} \otimes |1\rangle_{\mathrm{a2}} \langle 1| \end{aligned} \tag{21}$$

on the state of the system to realize one step in phase space. Then, we repeat the process: at each odd step, the walker walks in position space and at each even step, the walker takes steps in the phase space. Therefore, the state of the system after N steps comes out

$$|\phi_N\rangle = \left(\hat{E}_1 \hat{H}_1\right)^{(N \bmod 2)} \left(\hat{E}_2 \hat{H}_2 \hat{E}_1 \hat{H}_1\right)^{\lfloor \frac{N}{2} \rfloor} |\phi_0\rangle. \tag{22}$$

By tracing out the coin, we get the reduced density matrix of the the walker only

$$\rho_{\mathrm{w}} = \mathrm{Tr}_{\mathrm{c}} |\phi_{\mathrm{N}}\rangle \langle \phi_{\mathrm{N}}|. \tag{23}$$

To obtain the probability distribution of the walk $P(x)$ in position space after N steps, the final state of the system is projected into the position state $|x\rangle$

$$P(x) = \langle x| \rho_{\mathrm{w}} |x\rangle, \tag{24}$$

where $|x\rangle$ is the eigenstate of $\hat{x}$. Similarly, the probability distribution of the walker in phase space after N steps is defined by

$$P(\theta) = \langle \theta| \rho_{\mathrm{w}} |\theta\rangle. \tag{25}$$

In order to investigate the dispersion of the distribution, which is a significant signature of QWs, we calculate the standard deviation of the distribution. For the distribution in position space, the standard deviation is defined by

$$\sigma_{\mathrm{po}} = \sqrt{\langle x^2\rangle - \langle x\rangle^2}. \tag{26}$$

Similarly, for the distribution in phase space, the standard deviation is defined by

$$\sigma_{\mathrm{ph}} = \sqrt{\langle \theta^2\rangle - \langle \theta\rangle^2}. \tag{27}$$

In general, the standard deviation of the probability distribution in either the position and phase space is linear in N for QW, in contrast to $\sqrt{N}$ for RW.

4. Numerical simulations and analysis

In this paper, the QW is taking in position and phase spaces alternatively. Therefore, we investigate the influence on the QW in one space from that in the other space.

For analyzing the influence on the QW in position space from that in the phase space, we choose a coherent state $|\alpha = 10^{-6}\rangle$ as the initial state of the cavity field (the walker state), a superposition state $|\varphi_0\rangle$ in Eq. (3) and a Hadamard operation as coin flipping. The walker takes odd steps with size $\Delta\alpha = 0.8$ in position space and takes even steps with size $\Delta\theta = 0.1, 0.3$ in phase space. The ln–ln plots of the standard deviation of the probability distribution of the walker in position space with step sizes in position space $\Delta\alpha = 0.8$ and different step sizes in phase space $\Delta\theta = 0.1, 0.3$ are shown in Figs. 1 and 2 respectively. Figures 1(a) and 2(a) show the position standard deviation as a function of the number of the steps N with different step sizes in phase space $\Delta\theta$ and the standard deviation in position space at only odd steps are shown in Figs. 1(b) and 2(b). Though the walker walks in position space only at the odd steps, the QW in phase space at even steps influences that in position space; the position distribution and standard deviation for each odd step are different compared with those of the next even steps. The difference depends on the step size $\Delta\theta$ of the walker in the phase space. In Fig. 1(a), with small step size $\Delta\theta = 0.1$ in the phase space, the difference between the standard deviation of the position distribution at the odd and the adjacent even steps is not so obvious. Whereas, in Fig. 2(a), with a large step size $\Delta\theta = 0.3$, the standard deviation of the position distribution for each even step is obviously

smaller than that of the adjacent odd step. The standard deviation of the position distribution for the odd steps is linear on the number of the steps for the first 16 steps with the small step size in the phase space $\Delta\theta = 0.1$ shown in Fig. 1(b) and for the first 6 steps with the larger step size $\Delta\theta = 0.3$ shown in Fig. 2(b), respectively. After the first few steps, the standard deviation of the position distribution is not linear on the number of the steps because the impact of the QW in phase space is more significant with the number of steps increasing, then the walk in position space loses quantumness. Figures 1(c) and 2(c) show the position distributions at the 9th and 10th steps with the same step size in position space and different step sizes in phase space. With small step size in phase space $\Delta\theta = 0.1$, the position distributions at the adjacent two steps are almost the same in Fig. 1(c). Whereas with the larger step size in phase space $\Delta\theta = 0.3$ compared with the position distribution at the 9th step, that of the 10th step is still symmetric but squeezed a little due to the impact of the QW in phase space.

Now, we investigate the impact of the QW in position space on that in phase space. We choose $|\alpha = 3\rangle$ as the initial state of the cavity field and the superposition state in Eq. (3) as the initial state of the atoms, step size in phase space $\Delta\theta = 0.39$ and different step sizes $\Delta\alpha = 0.1, 0.3$ in position space. The ln–ln plots of the probability distribution of the walker in phase space are shown in Figs. 3 and 4. We can see for both $\Delta\alpha = 0.1$ and 0.3, the influence of the QW in position space is not as obvious as that of the QW in phase space. As a signature of QWs, the standard deviation of the phase distribution is linear on the number of steps for the first 20 steps and drops down after that in Figs. 3 and 4. Though the impact of the QW in position space on the standard deviation of the phase distribution is not so obvious, the probability distribution is still affected. In Fig. 3(c), the phase distributions of the walker at the 8th and 9th steps with step size in the phase space $\Delta\theta = 0.39$ and step size in position space $\Delta\alpha = 0.1$ are almost the same. Whereas, with the larger step size in position space $\Delta\alpha = 0.3$ compared with the phase distribution at the 8th step, that of the 9th step is not symmetric any more due to the impact of the QW in phase space.

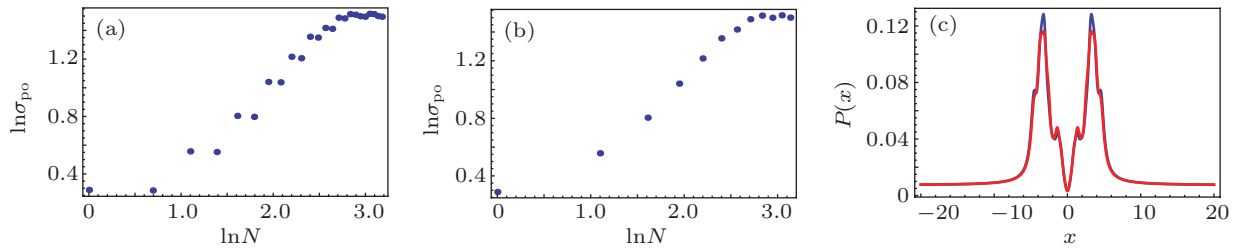


Fig. 1. (color online) (a) The ln–ln plot of the standard deviation of the position distribution σ_{po} with the initial state of the walker $|\alpha = 10^{-6}\rangle$, the step size in position space $\Delta\alpha = 0.8$ and the step size in phase space $\Delta\theta = 0.1$ as a function of the number of the steps N for the first 22 steps. The ln–ln plot only for the odd steps is shown in panel (b). (c) The probability distribution of the walker in position space at the 9th step (in blue solid line) and the 10th step (in red solid line).

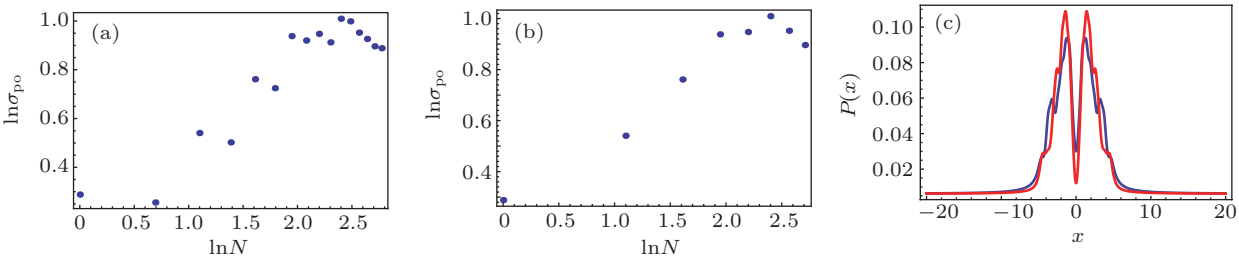


Fig. 2. (color online) (a) The ln–ln plot of the standard deviation of the position distribution σ_{po} with the initial walker state $|\alpha = 10^{-6}\rangle$, the step size in position space $\Delta\alpha = 0.8$ and the step size in phase space $\Delta\theta = 0.3$ as a function of the number of the steps N for the first 15 steps. The ln–ln plot only for the odd steps is shown in panel (b). (c) The probability distribution of the walker in position space at the 9th step (in blue solid line) and the 10th step (in red solid line).

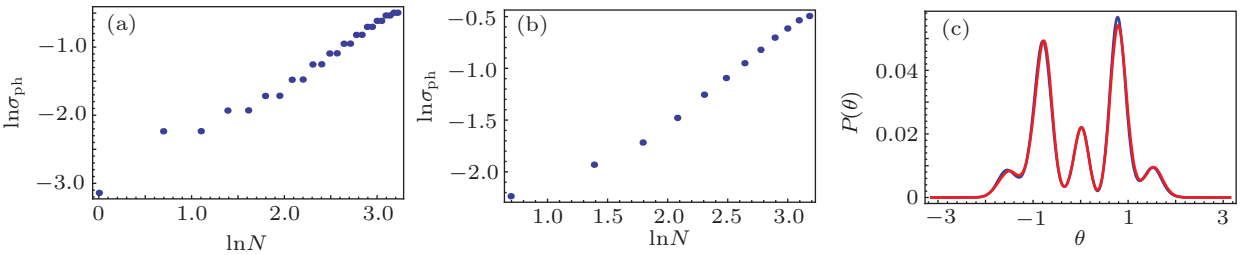


Fig. 3. (color online) (a) The ln–ln plot of the standard deviation of the phase distribution σ_{ph} with the initial walker state $|\alpha = 3\rangle$, the step size in phase space $\Delta\theta = 0.39$ and the step size in position space $\Delta\alpha = 0.1$ as a function of the number of the steps N for the first 22 steps. The ln–ln plot only for the even steps is shown in panel (b). (c) The probability distribution of the walker in phase space at the 8th step (in blue solid line) and the 9th step (in red solid line).

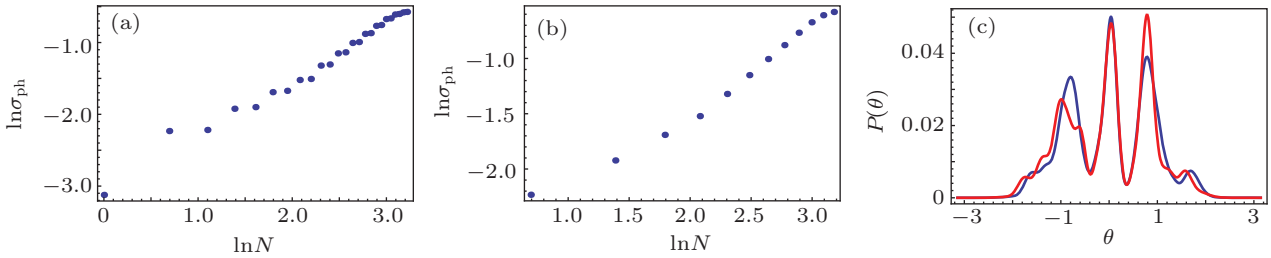


Fig. 4. (color online) (a) The ln–ln plot of the standard deviation of the phase distribution $\sigma_{\rm ph}$ with the initial walker state $|\alpha = 3\rangle$, the step size in phase space $\Delta\theta = 0.39$ and the step size in position space $\Delta\alpha = 0.3$ as a function of the number of the steps N for the first 22 steps. The ln–ln plot only for the even steps is shown in panel (b). (c) The probability distribution of the walker in phase space at the 8th step (in blue solid line) and the 9th step (in red solid line).

5. Conclusion

In summary, we have introduced the two-coin QW, which proceeds in position and phase spaces alternately, and have proposed a scheme of implementation via cavity QED with a single-mode cavity field interacting with two-level atomic states. By numerical simulations, we prove two-coin QW keeps quantumness for the first several steps via the linearity of the standard deviation on the number of the steps. With the number of the steps increasing, the impact on the QW in one space from that in the other space becomes more significant and the QW is transmitted to RW by losing quantumness. We also analyze the impact on the QW in one space from that on the other space. The QW in phase space causes relatively greater influence on the standard deviation of the position distribution of the walker and less influence on the probability distribution of the walk in position space. Whereas the QW in position space leads to relatively smaller influence in the standard deviation of the phase distribution and causes the asymmetric probability distribution of the walk in phase space.

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