Simulation of electron cyclotron current drive and electron cyclotron resonant heating for the HL-2A tokamak*

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A quasi-linear formalism is developed for relativistic particles. It is self-consistent including spatial diffusion. An attempt is made to simulate the process of electron cyclotron resonant heating (ECRH) and electron cyclotron current drive (ECCD) for the HL-2A tokamak. Temperature oscillating regimes in Tore Supra diagnosed by MHD activity seem to be reproduced in the simulation. The special feature in this paper is to see the resonance in the long time scale for relativistic plasma.

Keywords: relativistic particles, quasi-linear formalism, three-dimensional diffusion

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1. Introduction

The interaction of radio-frequency waves with plasma in a magnetic confinement device has been a very important discipline of plasma physics. To approach a more realistic description of wave-plasma interaction on a time scale longer than kinetic time scales, the bounce-average is needed. The long time evolution of the kinetic distribution can be treated by the Fokker–Planck equation. The behavior of the plasma and the most interesting macroscopic effects are obtained by balancing the diffusion term with the collision term.

For relativistic particles, the action and angle variables initiated by Kaufman[1] are introduced. "There has been a gradual evolution over the years away from the averaging approach and towards the transformation approach" said Littlejohn.[2] The technique of the area-conserved transformation proposed by Lichtenberg and Lieberman[3] is employed. A new invariant which is actually an implicit Hamiltonian is formed by using the bounce average and from which the bounce frequency and processional frequency can be calculated. Using new action and angle variables, the relativistic quasi-linear equation is derived including spatial diffusion. For circulating particles, under the conditions of small Larmor radius and first harmonic resonance, the diffusion coefficient is compatible with the numerical code developed in Frascati.[4] An attempt is made to simulate the process of electron cyclotron resonant heating (ECRH) and electron cyclotron current drive (ECCD) for the LH-2A tokamak. The distribution function is obtained after the wave power is put in. The temperature and the driven current are calculated for HL-2A. Temperature oscillating regimes in Tore Supra diagnosed by MHD activity[5,6] seem to be reproduced in the simulation. The special feature in this paper is seeing the resonance on a long time scale for relativistic plasma.

The rest of this paper is organized as follows. In Section 2, exact guiding center variables are derived for the relativistic particles. In Section 3, the bounce-averaged quasi-linear equation is obtained. Numerical results of electron cyclotron current drive and resonant heating for HL-2A are given in Section 4. The summary is presented in the last section.

2. Exact guiding center variables

In tokamak configuration, the relativistic Hamiltonian of a charged particle can be expressed as

$$H=\sqrt{\left[\left(P_R-\frac{e}{c}A_R\right)^2+\left(P_Z-\frac{e}{c}A_Z\right)^2+\frac{(P_\phi-eRA_\phi/c)^2}{R^2}\right]c^2+m_0^2c^4}+e\Phi, \tag{1}$$

where A_R, A_Z, and A_ϕ are the vector potential components of the magnetic field; Φ is the electrical potential which is assumed to be function of Ψ; m_0 is the rest mass; e is the charge; P_R, P_ϕ and P_Z are the canonical momenta conjugate to R, ϕ, and Z respectively, and expressed as

$$P_R=m_0u_R+\frac{e}{c}A_R, \tag{2}$$

$$P_\phi=Rm_0u_\phi+\frac{e}{c}RA_\phi, \tag{3}$$

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$$P_Z = m_0 u_Z + \frac{e}{c} A_Z, \tag{4}$$

with $u = \gamma v$ and $\gamma = (1 + u^2/c^2)^{1/2}$ being the relativistic factor. The magnetic field can be expressed as

$$B = \nabla\phi \times \nabla\Psi + I\nabla\phi, \tag{5}$$

where Ψ is related to the poloidal flux of the magnetic field, I is related to the poloidal current, and R is the major radius. Then, in tokamaks, we have

$$A_R = 0, \ A_Z = -I \ln \frac{R}{R_0}, \ A_\phi = -\frac{\Psi}{R}. \tag{6}$$

We introduce a generating function[6,7] for changing to the guiding center variables

$$F_1 = -\frac{m_0 \Omega_0 R_0^2}{2} \exp\Big(\frac{X}{m_0 \Omega_0 R_0}\Big) \times \Big(\ln \frac{R}{R_0} - \frac{X}{m_0 \Omega_0 R_0}\Big)^2 \tan\alpha - ZX, \tag{7}$$

where

$$X = m_0 \Omega_0 R_0 \ln \frac{R_C}{R_0}, \tag{8}$$

and Ω_0 is the toroidal gyrofrequency taken to be the absolute value for electron, ρ is the Larmor radius, α is the gyrophase, subscripts 0 and C refer to the values at the magnetic axis and in the guiding center respectively, X and α are the new coordinates conjugate to the momenta

$$P_X = Z + \rho \sin\alpha + \frac{\rho^2}{4R_C} \sin 2\alpha, \tag{9}$$

$$P_\alpha = \frac{1}{2} m_0 \Omega_C \rho^2, \tag{10}$$

where P_X is actually the guiding center of the Z coordinate, Z_C. That the moment turns to be coordinate often occurs during area-conserved canonical transformation.[3] The other two canonical variables P_ϕ and ϕ do not change in the new coordinates. The old coordinates are related to new ones through the following four identical equations:

$$P_R = m_0 \Omega_C \rho \, e^{\rho \cos\alpha / R_C} \sin\alpha, \tag{11}$$

$$P_Z = -X, \tag{12}$$

$$R = R_C \exp\Big(-\frac{\rho \cos\alpha}{R_C}\Big), \tag{13}$$

$$Z = P_X - \rho \sin\alpha - \frac{\rho^2}{4R_C} \sin 2\alpha. \tag{14}$$

The Jacobian in the area-conserved transformation is unity,[3] that is,

$$d\tau = J \, dP_\alpha \, dP_x \, dP_\phi \, d\alpha \, dX \, d\phi, \tag{15}$$

$$J = 1. \tag{16}$$

The exact Hamiltonian for the relativistic particles is

$$H = \sqrt{\Big\{2m_0 \Omega_C P_\alpha \Big[\Big(\frac{R_C}{R}\Big)^2 \sin^2\alpha + \cos^2\alpha\Big] + \frac{1}{R^2}(P_\phi + e\Psi)^2\Big\} c^2 + m_0^2 c^4} + e\Phi. \tag{17}$$

It is suitable for particle simulation from which we can obtain equations of motion and Vlasov's equation.

3. Quasi-linear equation

For the gyro-kinetics the Hamiltonian in Eq. (17) could be averaged as

$$H = \sqrt{(2m_0 \Omega_C P_\alpha + m_0^2 u_\phi^2) c^2 + m_0^2 c^4} + e\Phi. \tag{18}$$

To derive the quasi-linear equation we form a new invariant[3]

$$\Pi = \frac{1}{2\pi} \oint P_x \, dX. \tag{19}$$

For the trapped particles in the large aspect ratio configuration, that is, $\varepsilon << 1$, we obtain

$$\Pi_t = \frac{8qR_0 m_0 (\varepsilon \Omega_0 P_\alpha / m_0)^{0.5}}{\pi} [\mathrm{E}(k_1) - (1 - k_1^2)\mathrm{K}(k_1)], \tag{20}$$

which is the toroidal magnetic flux enclosed by drift surface, $\mathrm{E}(k_1)$ and $\mathrm{K}(k_1)$ are complete elliptic functions. The bounce frequency and the procession frequency are obtained as[7,8]

$$\omega_{bt} = \frac{\pi (\varepsilon \Omega_0 P_\alpha / m_0)^{0.5}}{2\gamma q R_0 \mathrm{K}(k_1)}, \tag{21}$$

$$\omega_{\zeta t} = \frac{2\Omega_0 P_\alpha}{\gamma \Omega_p m_0 R_0^2} \Big[\frac{\mathrm{E}(k_1)}{\mathrm{K}(k_1)} - \frac{1}{2}\Big] + \frac{4\Omega_0 P_\alpha \hat{s}}{\gamma \Omega_p m_0 R_0^2} \Big[\frac{\mathrm{E}(k_1)}{\mathrm{K}(k_1)} - (1 - k_1^2)\Big], \tag{22}$$

where Ω_p is the poloidal gyro-frequency, $\varepsilon = r/R_0$, $k_1^2 = u_{\phi 0}^2 / (4\varepsilon \Omega_0 P_\alpha)$, $\hat{s}$ is the magnetic shear.

For the circulating particles

$$\Pi_c = \frac{\Omega_0 m_0 r^2}{2} + \frac{2qR_0 m_0 u_{\phi 0} \sigma}{\pi} \mathrm{E}(k), \tag{23}$$

$$\omega_{bc} = \frac{\pi u_{\phi 0} \sigma}{2\gamma q R_0 \mathrm{K}(k)}, \tag{24}$$

$$\omega_{\zeta c} = q\omega_{bc}\sigma + \frac{u_{\phi 0}^2}{2\gamma \Omega_p r R_0} \Big[\frac{E(k)}{K(k)} - \Big(1 - \frac{k^2}{2}\Big)\Big] + \frac{u_{\phi 0}^2 \hat{s}}{\gamma \Omega_p R_0^2} \frac{\mathrm{E}(k)}{\mathrm{K}(k)}, \tag{25}$$

where σ represents the direction of the circulating particle and $k^2 = k_1^{-2}$. The bounce-averaged gyro-frequency for the trapped particles is

$$\Omega = \frac{\Omega_0}{\gamma} \Big[1 - 2\varepsilon \Big(\frac{\mathrm{E}(k_1)}{\mathrm{K}(k_1)} - \frac{1}{2}\Big)\Big], \tag{26}$$

while for the circulating particles, it is

$$\Omega = \frac{\Omega_0}{\gamma} \Big\{1 - \frac{2\varepsilon}{k^2(1+\varepsilon)} \Big[\frac{\mathrm{E}(k)}{\mathrm{K}(k)} - \Big(1 - \frac{k^2}{2}\Big)\Big]\Big\}. \tag{27}$$

New momenta Π, P_α, P_ζ are conjugate to η, α, ζ in which $P_\zeta = -e\Psi_0/c = \Psi^\bullet$ which is actually the position variable.[6] In the extended phase space the Hamiltonian is

written as follows:[3]

$$\bar{H}(\bar{p},\bar{q}) = H(p,q,t) - H, \tag{28}$$

where $\bar{p}_{n+1} = -H$, $\bar{q}_{n+1} = t$.

According to Liouville's theorem, the distribution function, f, satisfies Vlasov's equation

$$\frac{\partial f}{\partial t} + \dot{\eta}\frac{\partial f}{\partial \eta} + \dot{\xi}\frac{\partial f}{\partial \xi} + \dot{\alpha}\frac{\partial f}{\partial \alpha} + \dot{\Phi}\frac{\partial f}{\partial \Phi} + \dot{P}_\xi\frac{\partial f}{\partial P_\xi} + \dot{P}_\alpha\frac{\partial f}{\partial P_\alpha} + \dot{H}\frac{\partial f}{\partial H} = 0, \tag{29}$$

where f can be divided into two parts, the averaged part and the oscillatory part

$$f = \bar{f} + \tilde{f}. \tag{30}$$

The linear solution of Eq. (29)

$$\tilde{f} = -\frac{\omega H_1\frac{\partial \bar{f}}{\partial H} + nH_1\frac{\partial \bar{f}}{\partial P_\xi} + mH_1\frac{\partial \bar{f}}{\partial \Phi} + lH_1\frac{\partial \bar{f}}{\partial P_\alpha}}{\omega - n\omega_\xi - m\omega_\eta - l\Omega}, \tag{31}$$

$\bar{f}$ in Eq. (30) is a function of H, P_α, $\Psi^\bullet$ only, independent of Φ, and satisfies the quasi-linear equation

$$\frac{\partial \bar{f}}{\partial \tau} + \hat{L}D\hat{L}\bar{f} = C(\bar{f}), \tag{32}$$

where

$$\tau = t\nu,\ \ \hat{L} = \frac{\partial}{\partial H} + \frac{n}{\omega}\frac{\partial}{\partial \Psi^\bullet} + \frac{l}{\omega}\frac{\partial}{\partial P_\alpha}$$

is the differential operator, C is the collision operator, ν is the collision frequency, and D is the diffusion coefficient expressed as

$$D = \pi H_1^2\delta(\omega - n\omega_\xi - m\omega_\eta - l\Omega). \tag{33}$$

For one harmonic

$$H_{1l} = \left[\left(e\Phi_k - \frac{e}{c}v_\xi A_{\varphi k}\right)J_l(k_\perp\rho) - \frac{e}{c}v_\perp A_{Zk}\frac{J_{l-1}+J_{l+1}}{2} - \frac{e}{c}v_\perp A_{Rk}\frac{J_{l-1}-J_{l+1}}{2\mathrm{i}}\right]\mathrm{i}^l\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_\mathrm{c} - \mathrm{i}l\alpha_0) \times \exp(\mathrm{i}n\xi + \mathrm{i}m\eta + l\Omega - \mathrm{i}\omega t), \tag{34}$$

where $\boldsymbol{A}$ is the vector potential of incident electromagnetic wave, $\boldsymbol{r}_\mathrm{c}$ is the guiding center position, α_0 is the angle between $\boldsymbol{k}$ and $-R$ direction in the cylindrical coordinates.

For the relativistic circulating particles the resonant term in Eq. (33) is written as

$$\delta\left[\frac{\omega}{\gamma}\left(\gamma - \frac{l\gamma\Omega}{\omega} - N_{//}\frac{qR_0\gamma\omega_b}{c}\right)\right] = \frac{c}{qR_0\omega\omega_b}\delta(N_\mathrm{res} - N_{//}), \tag{35}$$

where γ is the relativistic factor

$$N_\mathrm{res} = \frac{2\mathrm{K}(k)}{\pi}\frac{\gamma - \gamma\Omega/\omega}{\beta_\mathrm{th}P_{//}},$$

which is consistent with the code developed in Frascati[4] if k approaches zero. However, for barely circulating particles, the expression of the resonant term in Eq. (33) is different from Eq. (35). From Eq. (33), we obtain

$$D = \frac{2}{\pi}K(k)\frac{P_\perp^2}{\gamma P_{//}}D_\perp\delta(N_\mathrm{res} - N_{//}). \tag{36}$$

Here, $P_\perp$ and $P_{//}$ are normalized with respect to the thermal momentum $P_\mathrm{t} = (m_0T)^{0.5}$. The steady state is described by a relativistic Maxwellian with temperature T_t

$$\delta(N_\mathrm{res} - N_{//}) = \frac{\pi^{-0.5}}{\Delta N_{//}}\exp\left[-\frac{(N_\mathrm{res} - N_{//})^2}{\Delta N_{//}^2}\right], \tag{37}$$

$$D_\perp = \frac{\pi\omega e^2A_\perp^2\beta_\mathrm{th}}{\nu T^2},$$

and

$$\hat{L} = \frac{\gamma\partial}{P\partial P} + \frac{nT_t}{\omega}\frac{\partial}{\partial\Psi^\bullet} + \frac{l\Omega_0}{\omega(1+\varepsilon)}\frac{\partial}{P^2\mu\partial\mu}. \tag{38}$$

The quasi-linear formalism has been approved in scholarly research exchange (SRX).[9]

4. Numerical results for HL-2A

There are two gyrotrons in our institute. The frequency is 68 GHz. The power is about 1 MW together. The waves are injected normally into the plasma from low field side. The averaged density is about $3\times10^{13}\mathrm{m}^{-3}$. The experiment condition is as follows: $\Omega_0/\omega \cong 1.07$, $R_0 = 1.65$ m, $r = 0.06$ m., $N_{//} = 0.2$ and $\Delta N_{//} = 0.1$. For $D_\perp = 0.2$, numerical results are given in Figs. 1, 2, 3, and 4.

Both the driven current and temperature oscillation are similar to those in Tore Supra, and called global oscillations. The oscillating temperature is due to the nonlinearly coupled predator-prey system mentioned in Ref. [5].

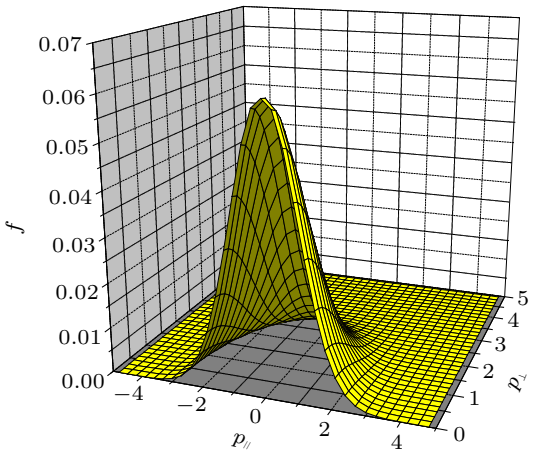


Fig. 1. (color online) Distribution function f versus $P_\perp$ and $P_{//}$.

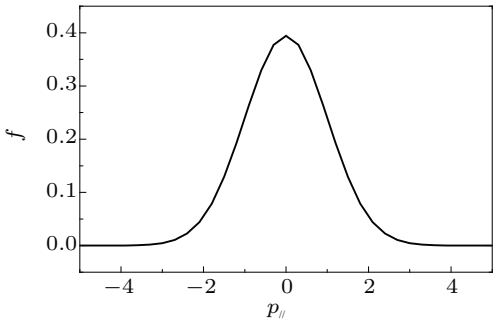


Fig. 2. Distribution function f versus $P_{//}$.

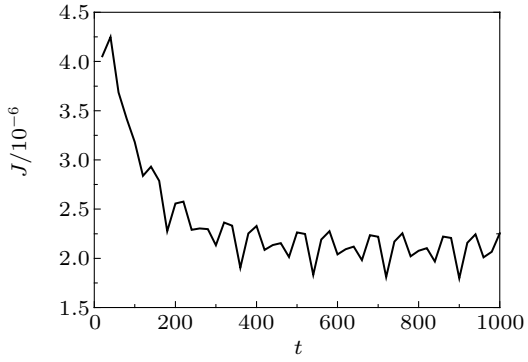


Fig. 3. Driven current (in A) versus time.

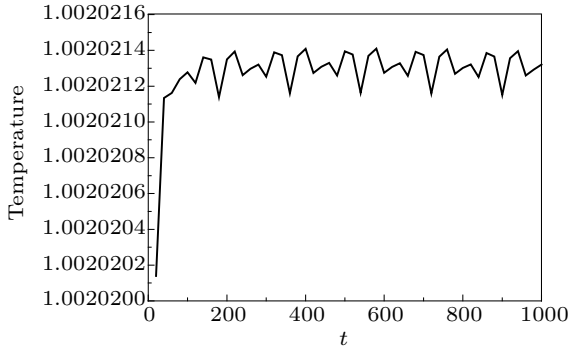


Fig. 4. Temperature versus time normalized by m_0c^2.

5. Summary

The bounce-averaged quasi-linear Fokker–Planck equation for relativistic particles is rigorously obtained in canonical variables including spatial diffusion. The spatial diffusion coefficient obtained is similar to the one obtained by Hazeltine *et al.*[10] For circulating particles, under the conditions of small Larmor radius and first harmonic resonance, the diffusion coefficient is compatible with the numerical code developed by Frascati.[4] An attempt is made to simulate the process of ECRH and ECCD for the HL-2A tokamak. For the HL-2A parameters the distribution function, the temperature and the driven current are calculated. Both the driven current and temperature oscillation are similar to those in Tore Supra, and called global oscillations. Compared with the experimental results in HL-2A, the calculated results are correct qualitatively. Temperature oscillating regimes in Tore Supra diagnosed by MHD activity seem to be reproduced in the simulation. The special feature in this paper is seeing the resonance on a long time scale for relativistic plasma.

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